

5.10: Multinomial Distribution

Learning Objectives

- Define multinomial outcomes
- Compute probabilities using the multinomial distribution

The binomial distribution allows one to compute the probability of obtaining a given number of binary outcomes. For example, it can be used to compute the probability of getting 6 heads out of 10 coin flips. The flip of a coin is a binary outcome because it has only two possible outcomes: heads and tails. The multinomial distribution can be used to compute the probabilities in situations in which there are more than two possible outcomes.

For example, suppose that two chess players had played numerous games and it was determined that the probability that Player *A* would win is 0.40, the probability that Player *B* would win is 0.35, and the probability that the game would end in a draw is 0.25. The multinomial distribution can be used to answer questions such as: "If these two chess players played 12 games, what is the probability that Player *A* would win 7 games, Player *B* would win 2 games, and the remaining 3 games would be drawn?" The following formula gives the probability of obtaining a specific set of outcomes when there are three possible outcomes for each event:

$$p = \frac{n!}{n_1!n_2!n_3!} p_1^{n_1} p_2^{n_2} p_3^{n_3} \quad (5.10.1)$$

where

- p is the probability,
- n is the total number of events
- n_1 is the number of times Outcome 1 occurs,
- n_2 is the number of times Outcome 2 occurs,
- n_3 is the number of times Outcome 3 occurs,
- p_1 is the probability of Outcome 1
- p_2 is the probability of Outcome 2, and
- p_3 is the probability of Outcome 3.

Example 5.10.1: Chess

For the chess example discussed above:

- $n = 12$ (12 games are played),
- $n_1 = 7$ (number won by Player *A*),
- $n_2 = 2$ (number won by Player *B*),
- $n_3 = 3$ (the number drawn),
- $p_1 = 0.40$ (probability Player *A* wins)
- $p_2 = 0.35$ (probability Player *B* wins)
- $p_3 = 0.25$ (probability of a draw)

$$P = \frac{12!}{7!2!3!} \times 40^7 \times 35^2 \times 25^3 = 0.0248 \quad (5.10.2)$$

The formula for k outcomes is

$$p = \frac{n!}{n_1!n_2!n_3!} p_1^{n_1} p_2^{n_2} \dots p_k^{n_k} \quad (5.10.3)$$

Note that the binomial distribution is a special case of the multinomial distribution when $k = 2$.

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