4.7: Variance Sum Law II - Correlated Variables

Learning Objectives

- State the variance sum law when X and Y are not assumed to be independent
- Compute the variance of the sum of two variables if the variance of each and their correlation is known
- Compute the variance of the difference between two variables if the variance of each and their correlation is known

Recall that when the variables X and Y are independent, the variance of the sum or difference between X and Y can be written as follows:

$$\sigma^2_{X\pm Y} = \sigma^2_X + \sigma^2_Y \tag{4.7.1}$$

which is read: "The variance of X plus or minus Y is equal to the variance of X plus the variance of Y."

When X and Y are correlated, the following formula should be used:

$$\sigma^2_{X\pm Y} = \sigma^2_X + \sigma^2_Y \pm 2\rho\sigma_X\sigma_Y \tag{4.7.2}$$

where ρ is the correlation between X and Y in the **population**.

Example 4.7.1

If the variance of verbal SAT were $10,000$, the variance of quantitative SAT were $11,000$ and the correlation between these two tests were 0.50, what is the variance of total SAT (verbal + quantitative) and the difference (verbal - quantitative)?

Solution

Since the two variables are correlated, we use Equation 4.7.2 instead of Equation 4.7.1 for uncorrelated (independent) variables. Hence, the variance of the sum is

$$\sigma^2_{verbal+quant} = 10,000 + 11,000 + 2 \times 0.5 \times \sqrt{10,000} \times \sqrt{11,000} \tag{4.7.3}$$

which is equal to $31,488$. The variance of the difference is also determined by Equation 4.7.2:

$$\sigma^2_{verbal-quant} = 10,000 + 11,000 - 2 \times 0.5 \times \sqrt{10,000} \times \sqrt{11,000} \tag{4.7.4}$$

which is equal to $10,512$

If the variances and the correlation are computed in a sample, then the following notation is used to express the variance sum law:

$$s^2_{X\pm Y} = s^2_X + s^2_Y \pm 2r\, s_X\, s_Y \tag{4.7.5}$$