

1.13: Logarithms

Learning Objectives

- Compute logs using different bases
- Perform basic arithmetic operations using logs
- State the relationship between logs and proportional change

The log transformation reduces positive skew. This can be valuable both for making the data more interpretable and for helping to meet the assumptions of inferential statistics.

Basics of Logarithms (Logs)

Logs are, in a sense, the opposite of exponents. Consider the following simple expression:

$$10^2 = 100 \quad (1.13.1)$$

Here we can say the base of 10 is raised to the second power. Here is an example of a log:

$$\log_{10}(100) = 2 \quad (1.13.2)$$

This can be read as: The log base ten of 100 equals 2. The result is the power that the base of 10 has to be raised to in order to equal the value (100). Similarly,

$$\log_{10}(1000) = 3 \quad (1.13.3)$$

since 10 has to be raised to the third power in order to equal 1,000.

These examples all used base 10, but any base could have been used. There is a base which results in "natural logarithms" and that is called e and equals approximately 2.718. It is beyond the scope here to explain what is "natural" about it. Natural logarithms can be indicated either as: $\ln(x)$ or $\log_e(x)$.

Changing the base of the log changes the result by a multiplicative constant. To convert from \log_{10} to natural logs, you multiply by 2.303. Analogously, to convert in the other direction, you divide by 2.303.

$$\ln X = 2.303 \log_{10} X \quad (1.13.4)$$

Taking the antilog of a number undoes the operation of taking the log. Therefore, since $\log_{10}(1000) = 3$, the *antilog*₁₀ of 3 is $10^3 = 1,000$. Taking the antilog of a number simply raises the base of the logarithm in question to that number.

Logs and Proportional Change

A series of numbers that increase proportionally will increase in equal amounts when converted to logs. For example, the numbers in the first column of Table 1.13.1

increase by a factor of 1.5 so that each row is 1.5 times as high as the preceding row. The \log_{10} transformed numbers increase in equal steps of 0.176.

Table 1.13.1: Proportional raw changes are equal in log units

Raw	Log
4.0	0.602
6.0	0.778
9.0	0.954
13.5	1.130

As another example, if one student increased their score from 100 to 200 while a second student increased theirs from 150 to 300, the percentage change (100%) is the same for both students. The log difference is also the same, as shown below.

$$\text{Log}_{10}(100) = 2.000 \quad (1.13.5)$$

$$\log_{10}(200) = 2.301$$

$$\text{Difference} : 0.301$$

$$\log_{10}(150) = 2.176$$

$$\log_{10}(300) = 2.477$$

$$\text{Difference} : 0.301$$

Arithmetic Operations

Rules for logs of products and quotients are shown below.

$$\log(AB) = \log(A) + \log(B) \quad (1.13.6)$$

$$\log\left(\frac{A}{B}\right) = \log(A) - \log(B) \quad (1.13.7)$$

For example,

$$\log_{10}(10 \times 100) = \log_{10}(10) + \log_{10}(100) = 1 + 2 = 3. \quad (1.13.8)$$

Similarly,

$$\log_{10}\left(\frac{100}{10}\right) = \log_{10}(100) - \log_{10}(10) = 2 - 1 = 1. \quad (1.13.9)$$

Contributors and Attributions

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