3.9: Additional Measures

Learning Objectives

- Compute the trimean
- Compute the geometric mean directly
- Compute the geometric mean using logs
- Use the geometric to compute annual portfolio returns
- Compute a trimmed mean

Although the mean, median, and mode are by far the most commonly used measures of central tendency, they are by no means the only measures. This section defines three additional measures of central tendency: the trimean, the geometric mean, and the trimmed mean. These measures will be discussed again in the section "Comparing Measures of Central Tendency."

Trimean

The trimean is a weighted average of the 25^{th} percentile, the 50^{th} percentile, and the 75^{th} percentile. Letting $P25$ be the 25^{th} percentile, $P50$ be the 50^{th} and $P75$ be the 75^{th} percentile, the formula for the trimean is:

$$Trimean = \frac{P25 + 2P50 + P75}{4} \tag{3.9.1}$$

As you can see from the formula, the median is weighted twice as much as the 25^{th} and 75^{th} percentiles. Table $3.9.1$ shows the number of touchdown (TD) passes thrown by each of the 31 teams in the National Football League in the 2000 season. The relevant percentiles are shown in Table $3.9.2$.

Table $3.9.1$: *Number of touchdown passes*

$$\begin{array}{cccccccc} 37 & 33 & 33 & 32 & 29 & 28 & 28 & 23 \\ 22 & 22 & 22 & 21 & 21 & 21 & 20 & 20 \\ 19 & 19 & 18 & 18 & 18 & 18 & 16 & 15 \\ 14 & 14 & 14 & 12 & 12 & 9 & 6 & \end{array} \tag{3.9.2}$$

Table $3.9.2$: *Percentiles*

Percentile	Value
25	15
50	20
75	23

The trimean is therefore

$$\frac{15 + 2 \times 20 + 23}{4} = \frac{78}{4} = 19.5. \tag{3.9.3}$$

Geometric Mean

The geometric mean is computed by multiplying all the numbers together and then taking the n^{th} root of the product. For example, for the numbers $1, 10$ and 100, the product of all the numbers is:

$$1 \times 10 \times 100 = 1,000. \tag{3.9.4}$$

Since there are three numbers, we take the cubed root of the product $(1,000)$ which is equal to 10. The formula for the geometric mean is therefore

$$(\Pi X)^{1/N} \tag{3.9.5}$$

where the symbol Π means to multiply. Therefore, the equation says to multiply all the values of X and then raise the result to the $1/N$th power. Raising a value to the $\frac{1}{N}^{th}$ power is, of course, the same as taking the N^{th} root of the value. In this case, $1000^{1/3}$ is the cube root of $1,000$.

The geometric mean has a close relationship with logarithms. Table $3.9.3$ shows the logs (base 10) of these three numbers. The arithmetic mean of the three logs is 1. The anti-log of this **arithmetic mean** of 1 is the **geometric mean**. The anti-log of 1 is $10^1 = 10$. Note that the geometric mean only makes sense if all the numbers are positive.

Table $3.9.3$: *Logarithms*

X	$\log_{10}(X)$
1	0
10	1
100	2

The geometric mean is an appropriate measure to use for averaging rates. For example, consider a stock portfolio that began with a value of $\$1,000$ and had annual returns of $13\%, 22\%, 12\%, -5\%, and -13\%$ Table $3.9.4$ shows the value after each of the five years.

Table $3.9.4$: *Portfolio Returns*

Year	Return	Value
1	13%	1,130
2	22%	1,379
3	12%	1,544
4	-5%	1,467
5	-13%	1,276

The question is how to compute average annual rate of return. The answer is to compute the geometric mean of the returns. Instead of using the percents, each return is represented as a multiplier indicating how much higher the value is after the year. This multiplier is 1.13 for a 13% return and 0.95 for a 5% loss. The multipliers for this example are $1.13, 1.22, 1.12, 0.95, \text{ } and \text{ } 0.87$ The geometric mean of these multipliers is 1.05. Therefore, the average annual rate of return is 5%. Table $3.9.5$ shows how a portfolio gaining 5% a year would end up with the same value ($\$1,276$) as shown in Table $3.9.4$.

Table $3.9.5$: *Portfolio Returns*

Year	Return	Value
1	5%	1,050
2	5%	1,103
3	5%	1,158
4	5%	1,216
5	5%	1,276

Trimmed Mean

To compute a trimmed mean, you remove some of the higher and lower scores and compute the mean of the remaining scores. A mean trimmed 10% is a mean computed with 10% of the scores trimmed off: 5% from the bottom and 5% from the top. A mean trimmed 50% is computed by trimming the upper 25% of the scores and the lower 25% of the scores and computing the mean of the remaining scores. The trimmed mean is similar to the median which, in essence, trims the upper 49% and the lower 49% of the scores. Therefore the trimmed mean is a hybrid of the mean and the median. To compute the mean trimmed 20% for the touchdown pass data shown in Table $3.9.1$, you remove the lower 10% of the scores ($6, 9, \ and\ 12$) as well as the upper 10% of the scores ($33, 33, \ and\ 37$) and compute the mean of the remaining 25 scores. This mean is 20.16.