7.5: Standard Normal Distribution

Learning Objectives

- State the mean and standard deviation of the standard normal distribution
- Use a Z table
- Use the normal calculator
- Transform raw data to Z scores

As discussed in the introductory section, normal distributions do not necessarily have the same means and standard deviations. A normal distribution with a mean of 0 and a standard deviation of 1 is called a standard normal distribution.

Areas of the normal distribution are often represented by tables of the standard normal distribution. A portion of a table of the standard normal distribution is shown in Table 7.5.1.

Table 7.5.1: A portion of a table of the standard normal distribution

Z	Area below
-2.5	0.0062
-2.49	0.0064
-2.48	0.0066
-2.47	0.0068
-2.46	0.0069
-2.45	0.0071
-2.44	0.0073
-2.43	0.0075
-2.42	0.0078
-2.41	0.008
-2.4	0.0082
-2.39	0.0084
-2.38	0.0087
-2.37	0.0089
-2.36	0.0091
-2.35	0.0094
-2.34	0.0096
-2.33	0.0099
-2.32	0.0102

The first column titled "Z" contains values of the standard normal distribution; the second column contains the area below Z. Since the distribution has a mean of 0 and a standard deviation of 1, the Z column is equal to the number of standard deviations below (or above) the mean. For example, a Z of -2.5 represents a value 2.5 standard deviations below the mean. The area below Z is 0.0062

The same information can be obtained using the following Java applet. Figure 7.5.1 shows how it can be used to compute the area below a value of -2.5 on the standard normal distribution. Note that the mean is set to 0 and the standard deviation is set to 1.

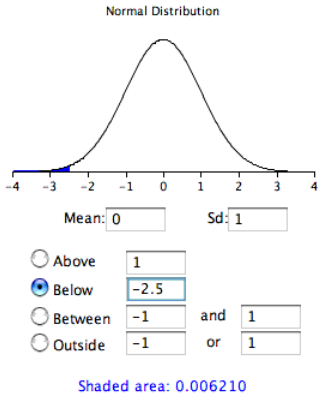

Figure 7.5.1: An example from the applet

Calculate Areas

A value from any normal distribution can be transformed into its corresponding value on a standard normal distribution using the following formula:

$$Z = \frac{X - \mu}{\sigma} \tag{7.5.1}$$

where Z is the value on the standard normal distribution, X is the value on the original distribution, μ is the mean of the original distribution, and σ is the standard deviation of the original distribution.

Example 7.5.1

As a simple application, what portion of a normal distribution with a mean of 50 and a standard deviation of 10 is below 26?

Solution

Applying the formula, we obtain

$$Z = \frac{26 - 50}{10} = -2.4 \tag{7.5.2}$$

From Table 7.5.1, we can see that 0.0082 of the distribution is below -2.4. There is no need to transform to Z if you use the applet as shown in Figure 7.5.2.

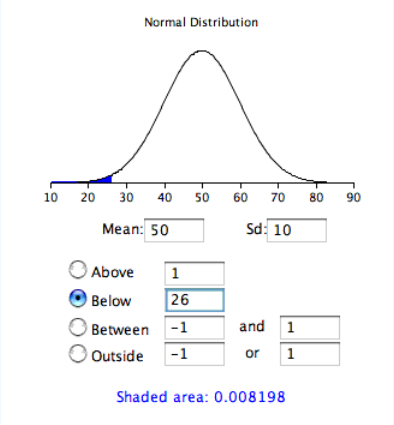

Figure 7.5.2: Area below 26 in a normal distribution with a mean of 50 and a standard deviation of 10

If all the values in a distribution are transformed to Z scores, then the distribution will have a mean of 0 and a standard deviation of 1. This process of transforming a distribution to one with a mean of 0 and a standard deviation of 1 is called standardizing the distribution.
