

10.11: Correlation

Learning Objectives

- State the standard error of z'
- Compute a confidence interval on ρ

The computation of a confidence interval on the population value of Pearson's correlation (ρ) is complicated by the fact that the sampling distribution of r is not normally distributed. The solution lies with Fisher's z' transformation described in the section on the sampling distribution of Pearson's r . The steps in computing a confidence interval for ρ are:

1. Convert r to z' .
2. Compute a confidence interval in terms of z' .
3. Convert the confidence interval back to r .

Let's take the data from the case study [Animal Research](#) as an example. In this study, students were asked to rate the degree to which they thought animal research is wrong and the degree to which they thought it is necessary. As you might have expected, there was a negative relationship between these two variables: the more that students thought animal research is wrong, the less they thought it is necessary. The correlation based on 34 observations is -0.654 . The problem is to compute a 95% confidence interval on ρ based on this r of -0.654 .

The conversion of r to z' can be done using a calculator. This calculator shows that the z' associated with an r of -0.654 is -0.78 .

The sampling distribution of z' is approximately normally distributed and has a standard error of

$$\frac{1}{\sqrt{N-3}} \quad (10.11.1)$$

For this example, $N = 34$ and therefore the standard error is 0.180. The Z for a 95% confidence interval ($Z_{0.95}$) is 1.96, as can be found using the normal distribution calculator (setting the shaded area to 0.95 and clicking on the "Between" button). The confidence interval is therefore computed as:

$$\text{Lower limit} = -0.775 - (1.96)(0.18) = -1.13 \quad (10.11.2)$$

$$\text{Upper limit} = -0.775 + (1.96)(0.18) = -0.43 \quad (10.11.3)$$

The final step is to convert the endpoints of the interval back to r using a calculator. The r associated with a z' of -1.13 is -0.81 and the r associated with a z' of -0.43 is -0.40 . Therefore, the population correlation (ρ) is likely to be between -0.81 and -0.40 . The 95% confidence interval is:

$$-0.81 \leq \rho \leq -0.40 \quad (10.11.4)$$

To calculate the 99% confidence interval, you use the Z for a 99% confidence interval of 2.58 as follows:

$$\text{Lower limit} = -0.775 - (2.58)(0.18) = -1.24 \quad (10.11.5)$$

$$\text{Upper limit} = -0.775 + (2.58)(0.18) = -0.32 \quad (10.11.6)$$

Converting back to r , the confidence interval is:

$$-0.84 \leq \rho \leq -0.31 \quad (10.11.7)$$

Naturally, the 99% confidence interval is wider than the 95% confidence interval.

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