

## 7.3: Areas Under Normal Distributions

### Learning Objectives

- State the proportion of a normal distribution within 1 and within 2 standard deviations of the mean
- Use the calculator "Calculate Area for a given  $X$ "
- Use the calculator "Calculate  $X$  for a given Area"

Areas under portions of a normal distribution can be computed by using calculus. Since this is a non-mathematical treatment of statistics, we will rely on computer programs and tables to determine these areas. Figure 7.3.1 shows a normal distribution with a mean of 50 and a standard deviation of 10. The shaded area between 40 and 60 contains 68% of the distribution.

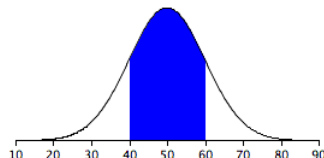


Figure 7.3.1: Normal distribution with a mean of 50 and standard deviation of 10.  
68% of the area is within one standard deviation (10) of the mean (50)

Figure 7.3.2 shows a normal distribution with a mean of 100 and a standard deviation of 20. As in Figure 7.3.1, 68% of the distribution is within one standard deviation of the mean.

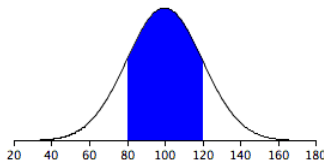


Figure 7.3.2: Normal distribution with a mean of 100 and standard deviation of 20.  
68% of the area is within one standard deviation (20) of the mean (100).

The normal distributions shown in Figures 7.3.1 and 7.3.2 are specific examples of the general rule that 68% of the area of any normal distribution is within one standard deviation of the mean.

Figure 7.3.3 shows a normal distribution with a mean of 75 and a standard deviation of 10. The shaded area contains 95% of the area and extends from 55.4 to 94.6. For all normal distributions, 95% of the area is within 1.96 standard deviations of the mean. For quick approximations, it is sometimes useful to round off and use 2 rather than 1.96 as the number of standard deviations you need to extend from the mean so as to include 95% of the area.

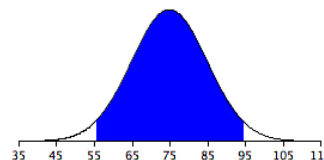


Figure 7.3.3: A normal distribution with a mean of 75 and a standard deviation of 10. 95% of the area is within 1.96 standard deviations of the mean.

The normal calculator can be used to calculate areas under the normal distribution. For example, you can use it to find the proportion of a normal distribution with a mean of 90 and a standard deviation of 12 that is above 110. Set the mean to 90 and the standard deviation to 12. Then enter "110" in the box to the right of the radio button "Above." At the bottom of the display you will see that the shaded area is 0.0478. See if you can use the calculator to find that the area between 115 and 120 is 0.0124.

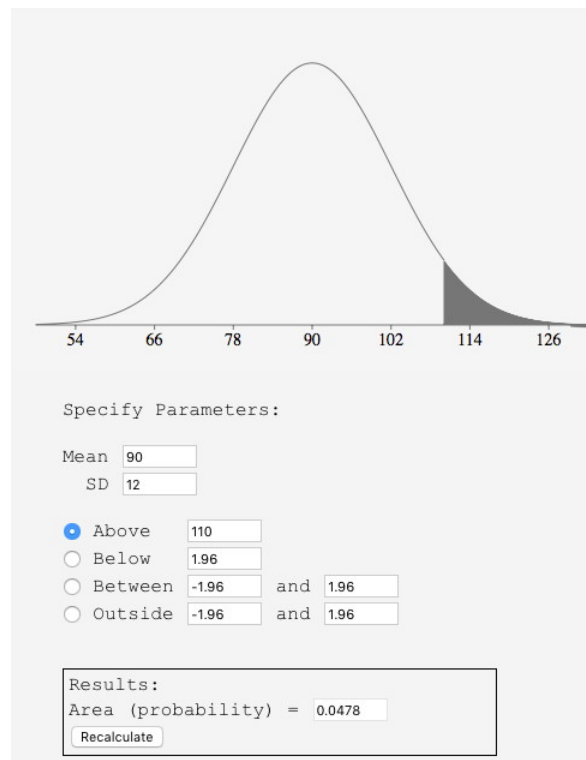


Figure 7.3.4: Display from calculator showing the area above 110.

Say you wanted to find the score corresponding to the 75<sup>th</sup> percentile of a normal distribution with a mean of 90 and a standard deviation of 12. Using the inverse normal calculator, you enter the parameters as shown in Figure 7.3.5 and find that the area below 98.09 is 0.75.

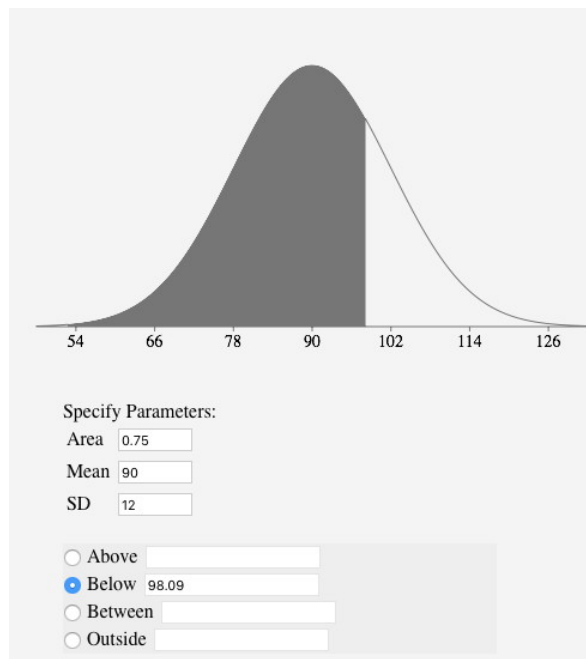


Figure 7.3.5: Display from normal calculator showing that the 75<sup>th</sup> percentile is 98.09.

### Normal and Inverse Normal Calculator

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