6.7: Geometric Distribution

Consider these two random variables, which both start with repeated Bernoulli trials:

1. Flip a fair coin 10 times. Let X = the number of heads.
2. Flip a fair coin repeatedly until you get a head. Let X = the number of total flips.

The first random variable is a binomial random variable where $n = 10$ and $p = 0.5$. The possible values of X are {0,1,2,3,4,5,6,7,8,9,10}

The second random variable is unusual in that there are an infinite number of possibilities for X. The possible number of flips until you get a head are {1, 2, 3, ...}. This is called the geometric distribution and its features are shown in the box.

Geometric Probability Distribution (parameter= p)

Two possible outcomes (Success/Failure) or (Yes/No)

$p = P$(yes/success) on one trial

$q = 1\text{-}p = P$ (no/failure) on one trial

X = Number of independent trials until the first success. (1, 2, 3, ...)

$$\mu = \frac{1}{p}$$

$$\sigma^2 = \frac{1-p}{p^2}$$

$$\sigma = \sqrt{\frac{1-p}{p^2}}$$

$$P(x) = p(1-p)^{x-1}$$

Example: Free throw shooting

Let's again return to the example of Draymond Green, a 70% free throw shooter. Now let X = the number of free throws Draymond takes until he makes a shot. X follows a geometric distribution.

Solution

The expected number of shots: $\mu = \frac{1}{p} = 1.43$ shots

The variance: $\sigma^2 = \frac{1-0.7}{0.7^2} = 0.612$

The probability that Draymond Green takes exactly 3 shots to make a free throw:

$P(X = 3) = 0.7(0.3)^2 = 0.063$

The probability that Draymond Green takes 3 or more shots to make a free throw:

Since $P(X \geq 3) = P(3) + P(4) + \dots$ is an infinite sum, is better to use Rule of Complement.

$P(X \geq 3) = 1 - P(1) - P(2) = (0.7)(0.3)^0 + (0.7)(0.3)^1 = 1 - 0.91 = 0.09$

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