LibreTexts

4.2: Introduction to Probability

When we speak of the probability of something happening, we are talking how likely it is that "thing" will happen based on the conditions present. For instance, what is the probability that it will rain? That is, how likely do we think it is that it will rain today under the circumstances or conditions today? To define or understand the conditions that might affect how likely it is to rain, we might look out the window and say, "it's sunny outside, so it's not very likely that it will rain today." Stated using probability language: given that it is sunny outside, the probability of rain is low. "Given" is the word we use to state what the conditions are. As the conditions change, so does the probability. Thus, if it were cloudy and windy outside, we might say, "given the current weather conditions, there is a high probability that it is going to rain."

In these examples, we spoke about whether or not it is going to rain. Raining is an example of an event, which is the catch-all term we use to talk about any specific thing happening; it is a generic term that we specified to mean "rain" in exactly the same way that "conditions" is a generic term that we specified to mean "sunny" or "cloudy and windy."

It should also be noted that the terms "low" and "high" are relative and vague, and they will likely be interpreted different by different people (in other words: given how vague the terminology was, the probability of different interpretations is high). Most of the time we try to use more precise language or, even better, numbers to represent the probability of our event. Regardless, the basic structure and logic of our statements are consistent with how we speak about probability using numbers and formulas.

Let's look at a slightly deeper example. Say we have a regular, six-sided die (note that "die" is singular and "dice" is plural, a distinction that Dr. Foster has yet to get correct on his first try) and want to know how likely it is that we will roll a 1. That is, what is the probability of rolling a 1, given that the die is not weighted (which would introduce what we call a bias, though that is beyond the scope of this chapter). We could roll the die and see if it is a 1 or not, but that won't tell us about the probability, it will only tell us a single result. We could also roll the die hundreds or thousands of times, recording each outcome and seeing what the final list looks like, but this is time consuming, and rolling a die that many times may lead down a dark path to gambling or, worse, playing Dungeons & Dragons. What we need is a simple equation that represents what we are looking for and what is possible.

To calculate the probability of an event, which here is defined as rolling a 1 on an unbiased die, we need to know two things: how many outcomes satisfy the criteria of our event (stated different, how many outcomes would count as what we are looking for) and the total number of outcomes possible. In our example, only a single outcome, rolling a 1, will satisfy our criteria, and there are a total of six possible outcomes (rolling a 1, rolling a 2, rolling a 3, rolling a 4, rolling a 5, and rolling a 6). Thus, the probability of rolling a 1 on an unbiased die is 1 in 6 or 1/6. Put into an equation using generic terms, we get:

$$\text{Probability of an event } = \frac{\text{number of outcomes that satisfy our criteria}}{\text{total number of possible outcomes}} \tag{4.2.1}$$

We can also using P() as shorthand for probability and A as shorthand for an event:

$$P(A) = \frac{\text{number of outcomes that count a } A}{\text{total number of possible outcomes}} \tag{4.2.2}$$

Using this equation, let's now calculate the probability of rolling an even number on this die:

$$P(\text{Even Number}) = \frac{2, 4, \text{or } 6}{1, 2, 3, 4, 5, \text{or } 6} = \frac{3}{6} = \frac{1}{2}$$

So we have a 50% chance of rolling an even number of this die. The principles laid out here operate under a certain set of conditions and can be elaborated into ideas that are complex yet powerful and elegant. However, such extensions are not necessary for a basic understanding of statistics, so we will end our discussion on the math of probability here. Now, let's turn back to more familiar topics.