3.3.4: Measures of Central Tendency- Mean

The mean is also called the average. And, we're guessing you might already now what the average of a bunch of numbers is? It's the sum of the numbers, divided by the number of number right? How do we express that idea in a formula? Just like this:

$$\bar{X} = \frac{\sum X}{N}$$

In which $\bar{X}$ (read as "x-bar") is the *mean of the sample*. Sometimes Dr. MO adds parentheses to show that the scores (all of the X's) are summed before anything is divided because PEMDAS says that you calculate anything in parentheses first.

$$\bar{X} = \frac{(\sum X)}{N}$$

"That looks like Greek to me". Yup. The $\sum$ symbol is Greek, called sigma, and it stands for the operation of summing. We sum up *all* of the numbers, then divide the sum by N, which is the total number of numbers. Sometimes you will see $\bar{X}$ to refer to the mean of all of the numbers.

In plain English, the formula looks like:

$$\text{Mean} = \frac{\text{Sum of my numbers}}{\text{Count of my numbers}}$$

"Well, why didn't you just say that?". We just did!

Let's compute the mean for these five numbers:

Table 3.3.4.1-Five Scores

Scores
2
3
6
7
9

Add 'em up: $\sum = 27 = (2 + 3 + 6 + 7 + 9)$

Divide 'em by the number of scores: $\frac{27}{5} = 5.4$

Or, to put the numbers in the formula, it looks like this:

$$\bar{X} = \frac{\sum X}{N} = \frac{3 + 7 + 9 + 2 + 6}{5} = \frac{27}{5} = 5.4$$

Is the mean a good measure of central tendency? By now, you should know: it depends.

What does the mean mean?

It is not enough to know the formula for the mean, or to be able to use the formula to compute a mean for a set of numbers. We believe in your ability to add and divide numbers. What you really need to know is what the mean really "means". This requires that you know what the mean does, and not just how to do it. Puzzled? Let's explain.

Can you answer this question: What happens when you divide a sum of numbers by the number of numbers? What are the consequences of doing this? What is the formula doing? What kind of properties does the result give us? FYI, the answer is not that we compute the mean.

OK, so what happens when you divide any number by another number? Of course, the key word here is divide. We literally carve the number up top in the numerator into pieces. How many times do we split the top number? That depends on the bottom number in the denominator. Watch:

$$\frac{12}{3} = 4$$

So, we know the answer is 4. But, what is really going on here is that we are slicing and dicing up 12 aren't we. Yes, and we slicing 12 into three parts. It turns out the size of those three parts is 4. So, now we are thinking of 12 as three different pieces $12 = 4 + 4 + 4$. I know this will be obvious, but what kind of properties do our pieces have? You mean the fours? Yup. Well, obviously they are all fours. Yes. The pieces are all the same size. They are all equal. So, division equalizes the numerator by the denominator…

"Umm, I think I learned this in elementary school, what does this have to do with the mean?". The number on top of the formula for the mean is just another numerator being divided by a denominator isn't it. In this case, the numerator is a sum of all the values in your data. What if it was the sum of all of the 500 happiness ratings? The sum of all of them would just be a single number adding up all the different ratings. If we split the sum up into equal parts representing one part for each person's happiness what would we get? We would get 500 identical and equal numbers for each person. It would be like taking all of the happiness in the world, then dividing it up equally, then to be fair, giving back the same equal amount of happiness to everyone in the world. This would make some people more happy than they were before, and some people less happy right. Of course, that's because it would be equalizing the distribution of happiness for everybody. This process of equalization by dividing something into equal parts is what the mean does. See, it's more than just a formula. It's an idea. This is just the beginning of thinking about these kinds of ideas. We will come back to this idea about the mean, and other ideas, in later chapters.

> *Pro tip: The mean is the one and only number that can take the place of every number in the data, such that when you add up all the equal parts, you get back the original sum of the data.*
