13.1.1: Factorial Notations and Square Tables

2x2 Designs

We've just started talking about a 2x2 Factorial design, which means that we have two IVs (the number of numbers indicates how many IVs we have) and each IV has two levels (the numbers represent the number of level for each IV). We said this means the IVs are crossed. To illustrate this, take a look at the following tables. Table 13.1.1.1 is a conceptual version. Although not exactly accurate, many call these types of tables a Punnett Square because it shows the combination of different levels of two categories.

Table 13.1.1.1- Conceptual Example of a 2x2 Factorial Design

IV Levels	**IV1 Level I**	**IV1 Level 2**
IV2 Level 1	DV	DV
IV2 Level 2	DV	DV

Our study on distraction is a 2x2 design, so what would that look like in this type of table?

Example 13.1.1.1

Create a "Punnett's Square" for the IVs and DV of the Distraction scenario.

Solution

Table 13.1.1.2- Factorial Design of Distraction Scenario

IV Levels	**IV1 (Distraction): Yes**	**IV1 (Distraction): No**
IV2 (Reward): Yes	DV = Number of differences spotted	DV = Number of differences spotted
IV2 (Reward): No	DV = Number of differences spotted	DV = Number of differences spotts

You could have just as easily made IV1 the reward and IV2 the Distraction, and the table would still be correct.

Let's talk about this crossing business. Here's what it means for the design. For the first level of Distraction (Yes), we measure the number of differences spotted performance for the people who were rewarded, as well as for the people who were not rewarded. So, for the people who were distracted we also manipulated whether or not they earned a reward. In the second level of the Distraction IV (No), we also manipulate reward, with some people earning a reward and some people not. We collect how many differences were spotted in all conditions.

We could say the same thing, but talk from the point of view of the second IV. For example, for participants who were rewarded, some are distracted and some are not. Similarly, for participants who were not rewarded, we distract some of the participants and don't distract some of them.

Each of the four squares representing a DV, is called a condition. So, we have 2 IVs, each with 2 levels, for a total of 4 conditions. This is why we call it a 2x2 design. 2x2 = 4. The notation tells us how to calculate the total number of conditions.

Factorial Notation

Anytime all of the levels of each IV in a design are fully crossed, so that they all occur for each level of every other IV, we can say the design is a fully factorial design. We use a notation system to refer to these designs. The rules for notation are as follows. Each IV get's it's own number. The number of levels in the IV is the number we use for the IV. Let's look at some examples:

2x2 = There are two IVS, the first IV has two levels, the second IV has 2 levels. There are a total of 4 conditions, 2x2 = 4.

2x3 = There are two IVs, the first IV has two levels, the second IV has three levels. There are a total of 6 conditions, 2x3 = 6

3x2 = There are two IVs, the first IV has three levels, the second IV has two levels. There are a total of 6 conditions, 3x2=6.

4x4 = There are two IVs, the first IV has 4 levels, the second IV has 4 levels. There are a total of 16 condition, 4x4=16

2x3x2 = There are a total of three IVs. The first IV has 2 levels. The second IV has 3 levels. The third IV has 2 levels. There are a total of 12 condition. 2x3x2 = 12.

Let's practice a little with this notation.

Exercise 13.1.1.1

What is the factorial design notation for a study with two IVs, one has 2 levels and the other has 3 levels?

Answer

2x3

There are two IVs, so there are two numbers. Each number represents the number of levels for each IV.

Exercise 13.1.1.2

What is the factorial design notation with a study with the following IVs:

2 (task presentation: computer or paper) by

2 (task difficulty: easy or hard) by

2 (student: high school or college)

Answer

2x2x2

There are three IVs, so there are three numbers. Each IV only has two levels, so there are three two's! Notice that there are no threes. The number of IVs is represented in the *number* of numbers.

Okay, let's try something a little more challenging.

Let's do a couple more to make sure that we have this notation business down.

Exercise 13.1.1.5

For one of Dr. MO's dissertation studies, participants read about a character, then rated that character on several personality traits (DV). The race and gender of the character were varied systematically. Here are the IVs:

- Race of participant: White or Black
- Gender of participant: Woman or man
- Race of character: White or Black or none/neutral (the character's race was not mentioned)
- Gender of character: Woman or man or none/neutral (the character's gender was not mention)

What kind of factorial design was this study? 2x2? 2x3? Something else (what?)?

Answer

This was a 2x2x3x3 study because:

- Race of participant: White or Black = 2
- Gender of participant: Woman or man = 2
- Race of character: White or Black or none/neutral = 3
- Gender of character: Woman or man or none/neutral -= 3

Dr. MO wanted 30 participants in each cell, so she had to have 1,080 participants!

Last one! This one has a few more questions to better understand the scenario.

Exercise 13.1.1.6

Dr. MO has more Star Wars collectibles than can fit in her office, and she'd like to sell some. Her research question is whether she'd get a better price through CraigsList or eBay? Also, should she take the picture or use a stock photo?

1. What is the DV? (It is not explicitly labeled.)
2. For each IV, what are the levels?
3. Is this a 2x2 factorial design? If not, what kind of design is it?
4. List out each of the *combinations* of the levels of the IVs.

Answer

1. The DV is the price, or how much Dr. MO could earn for selling each collectible.
2. IV1's levels are CraigsList or eBay, so the IV name could be something like "website" or "platform". IVs' levels are personal photo or stock photo, so the IV name could be something like "Photo Type."
3. This is a 2x2 factorial design: 2 (Platform: CraigsList or eBay) by 2 (Photo Type: Personal or Stock)
4. List out each of the *combinations* of the levels of the IVs:
    - Posted on CraigsList with a personal photo.
    - Posted on Craigslists with a stock photo.
    - Posted on eBay with a personal photo.
    - Posted on eBay with a stock photo.

Just for fun, let's illustrate a 2x3 design using the same kinds of tables we looked at before for the 2x2 design.

Table 13.1.1.3- Conceptual Example of a 2x3 Factorial Design

IV Levels	**IV1 Level I**	**IV1 Level 2**
IV2 Level 1	DV	DV
IV2 Level 2	DV	DV
IV2 Level 3	DV	DV

Our very first example of tiem spent studying is a 2x3 design, so what would that look like in this type of table?

Example 13.1.1.2

Create a "Punnett's Square" for the IV of Mindset (Growth or Fixed) and the IV of Job (Full-Time, Part-Time, or None) for time spend studying.

Solution

Table 13.1.1.4- Factorial Design of Studying Scenario

IV Levels	**IV1 (Mindset): Growth**	**IV1 (Mindset): Fixed**
IV2 (Job): Full-Time	DV = Minutes spent studying	DV = Minutes spent studying
IV2 (Job): Part-Tiime	DV = Minutes spent studying	DV = Minutes spent studying
IV2 (Job): None	DV = Minutes spent studying	DV = Minutes spent studying

You could have just as easily made IV1 the Job and IV2 the Mindset, and the table would still be correct.

All we did was add another row for the second IV. It's a 2x3 design, so it should have 6 conditions. As you can see there are now 6 cells to measure the DV.

You might have noticed in the list of notation for different factorial designs that you can have three IVs (that's the 2x3x2 design). In fact, you can have as many IVs with as many levels as you'd like, but the Central Limit Theorem shows (through complicated math that we aren't going to go into) that each condtion (or cell) should have at least 30-50 participants, that can get expensive quickly!

If a 2x2 has 4 conditions, and you want at least 30 participants in each condition, then you'd need 120 participants. If you have a 2x3, then you'd need at least 180 participants (2 * 3 * 30 = 180). So, for a 2x3x2, how many participants would you want?

$$\text{Participants } = 2 * 3 * 2 = 12$$
$$\text{Participants } = 12 * 30 = 360$$

Students always want to know how we would represent more than two IVs in a Punnett's Square, and the answer is that we don't. We create two Punnett Squares.

Let's say that we were looking at time spend studying for those with different mindsets and who have different jobs for different kinds of schools (community colleges or universities). That could look like:

Table 13.1.1.5- Factorial Design of Studying Scenario FOR COMMUNITY COLLEGE STUDENTS

IV Levels	**IV1 (Mindset): Growth**	**IV1 (Mindset): Fixed**
IV2 (Job): Full-Time	DV = Minutes spent studying MATH	DV = Minutes spent studying MATH
IV2 (Job): Part-Tiime	DV = Minutes spent studying	DV = Minutes spent studying
IV2 (Job): None	DV = Minutes spent studying	DV = Minutes spent studying

AND:

Table 13.1.1.5- Factorial Design of Studying Scenario FOR UNIVERSITY STUDENTS

IV Levels	**IV1 (Mindset): Growth**	**IV1 (Mindset): Fixed**
IV2 (Job): Full-Time	DV = Minutes spent studying MATH	DV = Minutes spent studying MATH
IV2 (Job): Part-Tiime	DV = Minutes spent studying	DV = Minutes spent studying
IV2 (Job): None	DV = Minutes spent studying	DV = Minutes spent studying

You could have just as easily made IV1 the Job and IV2 the Mindset, or even made a table for only students with Growth Mindset (and had IV1 be the type of school) and another table for only students with Fixed Mindset and the table would still be correct. It doesn't matter statistically which IV is placed where, it's more about interpreting and understanding what is besting tested.

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