7.3: Sample Size Calculation for a Proportion

A confidence interval for a population proportion p and q = 1 – p, with specific margin of error E is given by:

$n=p^{*} \cdot q^{*}\left(\frac{z_{\alpha / 2}}{E}\right)^{2}$ Always round up to the next whole number.

Note: If the sample size is determined before the sample is selected, the p* and q* in the above equation are our best guesses. Often times statisticians will use p* = q* = 0.5; this takes the guesswork out of determining p* and provides the "worst case scenario" for n. In other words, if p* = 0.5 is used, then you are guaranteed that the margin of error will not exceed E but you also will have to sample the largest possible sample size. Some texts will use p or π instead of p*.

A study found that 73% of prekindergarten children ages 3 to 5 whose mothers had a bachelor's degree or higher were enrolled in early childhood care and education programs.

a. How large a sample is needed to estimate the true proportion within 3% with 95% confidence?
b. How large a sample is needed if you had no prior knowledge of the proportion?

Solution

a) Use $n=p^{*} \cdot q^{*}\left(\frac{z_{\alpha / 2}}{E}\right)^{2}=0.73 \cdot 0.27\left(\frac{1.96}{0.03}\right)^{2}=841.3104$. Since we cannot have 0.3104 of a person, we need to round up to the next whole person and use n = 842. Don't round down since we may not get within our margin of error for a smaller sample size.

b) Since no proportion is given, use the planning value of p* = 0.5.

$$n=0.5 \cdot 0.5\left(\frac{1.96}{0.03}\right)^{2}=1067.1111$$

Round up and use $n=1,068$.

Note the sample sizes of 842 and 1,068. If you have a prior knowledge about the sample proportion then you may not have to sample as many people to get the same margin of error. The larger the sample size, the smaller the confidence interval.