6.2: Uniform Distribution

The continuous uniform distribution models the probability that is the same on an interval from a to b. We use the following probability density function (PDF) to graph a straight line.

$$f(x)= \begin{cases} \frac{1}{b-a}, & \text{for } a \le x \le b \\ 0, & \text{elsewhere} \end{cases}$$

The probability is found by taking the area between two points within the rectangle formed from the x-axis, between the endpoints a and b, the length, and f(x) = 1/(b-a), the height. When working with continuous distributions it is helpful to draw a picture of the distribution, then shade in the area of the probability that you are trying to find. See Figure 6-5.

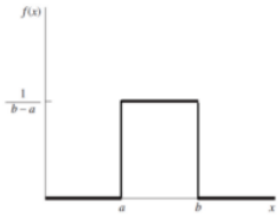

Figure 6-5

If a continuous random variable X has a uniform distribution with starting point a and ending point b then the distribution is denoted as X~U(a,b).

Area of a Rectangle = length*height

To find the probability (area) under the uniform distribution, use the following formulas.

- $P(X \ge x) = P(X > x) = \left(\frac{1}{b-a}\right) \cdot (b-x)$
- $P(X \le x) = P(X < x) = \left(\frac{1}{b-a}\right) \cdot (x-a)$
- $P(x_1 \le X \le x_2) = P(x_1 < X < x_2) = \left(\frac{1}{b-a}\right) \cdot (x_2 - x_1)$

The arrival time between trains at a train stop is uniformly distributed between 0 and 15 minutes. A student does not check the schedule and has arrived at the train stop.

a. Compute the probability they wait more than 10 minutes.
b. Compute the probability of waiting between 2 and 8 minutes.

Solution

a) First plug in the endpoints a = 0 and b = 15 into the PDF to get the height of the rectangle. The height is f(x)= $\frac{1}{15-0} = \frac{1}{15}$. Draw and label the distribution with the a, b and the height as in Figure 6- 6. The probability is the area of the shaded rectangle P(X > 10). Draw a vertical line at x = 10. We want x values that are greater than 10, so shade the area to the right of 10, stopping at b = 15. To find the area of the shaded rectangle in Figure 6-6, we can take the length times the height. The length would be b – a = 15 – 10 = 5 and the height is f(x) = 1/15.

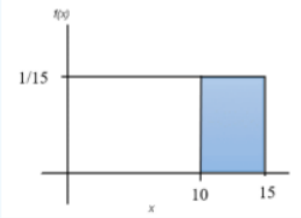

Figure 6-6

The area of the shaded rectangle is 5 $(\frac{1}{15}) = \frac{1}{3}$ = 0.3333 or P(X > 10) = 0.3333, which is the probability of waiting more than 10 minutes. Note that this would be the same if we asked P(X ≥ 10) = 0.3333 since there is no area at the line X = 10.

b) The area will be length times height. Draw the picture and shade the rectangle between 2 and 8, see Figure 6-7. The length is b – a = 8 – 2 = 6 and the height is still f(x) = 1/15. P(2 ≤ X ≤ 8) = 6$(\frac{1}{15})$ = 0.4

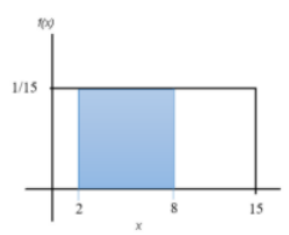

Figure 6-7