8.4: Hypothesis Test for One Proportion

When you read a question, it is essential that you correctly identify the parameter of interest. The parameter determines which model to use. Make sure that you can recognize and distinguish between a question regarding a population mean and a question regarding a population proportion.

The z-test is a statistical test for a population proportion. It can be used when $np \geq 10$ and $nq \geq 10$.

Definition: z-Test

The formula for the *test statistic* is:

$$Z = \frac{\hat{p} - p_0}{\sqrt{\left(\frac{p_0 q_0}{n}\right)}}. \tag{8.4.1}$$

where

n is the sample size

$\hat{p} = \frac{x}{n}$ is the sample proportion (sometimes already given as a %) and

p_0 is the hypothesized population proportion,

$$q_0 = 1 - p_0.$$

Use the phrases in Figure 8-24 to help with setting up the hypotheses.

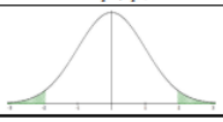
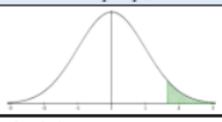
Two-tailed Test	Right-tailed Test	Left-tailed Test
$H_0: p = p_0$ $H_1: p \neq p_0$	$H_0: p = p_0$ $H_1: p > p_0$	$H_0: p = p_0$ $H_1: p < p_0$
		
Claim is in the Null Hypothesis		
=	≤	≥
Is equal to	Is less than or equal to	Is greater than or equal to
Is exactly the same as	Is at most	Is at least
Has not changed from	Is not more than	Is not less than
Is the same as	Within	Is more than or equal to
Claim is in the Alternative Hypothesis		
≠	>	<
Is not	More than	Less than
Is not equal to	Greater than	Below
Is different from	Above	Lower than
Has changed from	Higher than	Shorter than
Is not the same as	Longer than	Smaller than
	Bigger than	Decreased
	Increased	Reduced

Figure 8-24

Note we will not be using the t-distribution with proportions. We will use a standard normal z distribution for testing a proportion since this test uses the normal approximation to the binomial distribution (never use the t-distribution).

If you are doing a left-tailed z-test the critical value will be negative. If you are performing a right-tailed z-test the critical value will be positive. If you were performing a two-tailed z-test then your critical values would be ±critical value. The p-value will always be a positive number between 0 and 1. The most important step in any method you use is setting up your null and alternative hypotheses. The critical values and p-value can be found using a standard normal distribution the same way that we did for the one sample z-test.

It has been found that 85.6% of all enrolled college and university students in the United States are undergraduates. A random sample of 500 enrolled college students in a particular state revealed that 420 of them were undergraduates. Is there sufficient evidence to conclude that the proportion differs from the national percentage? Use α = 0.05. Show that all three methods of hypothesis testing yield the same results.

Solution

At this point you should be more comfortable with the steps of a hypothesis test and not have to number each step, but know what each step means.

Critical Value Method

Step 1: State the hypotheses: The key words in this example, "proportion" and "differs," give the hypotheses:

H_0: p = 0.856

H_1: p ≠ 0.856 (claim)

Step 2: Compute the test statistic. Before finding the test statistic, find the sample proportion $\hat{p} = \frac{420}{500} = 0.84$ and q_0 = 1 – 0.856 = 0.144.

Next, compute the test statistic:

$$z = \frac{\hat{p} - p_0}{\sqrt{\left(\frac{p_0 q_0}{n}\right)}} = \frac{0.84 - 0.856}{\sqrt{\left(\frac{0.856 \cdot 0.144}{500}\right)}} = -1.019. \quad (8.4.2)$$

Step 3: Draw and label the curve with the critical values. See Figure 8-25.

Use α = 0.05 and technology to compute the critical values $z_{\alpha/2}$ and $z_{1-\alpha/2}$.

Excel: $z_{\alpha/2}$ =NORM.S.INV(0.025) = –1.96 and $z_{1-\alpha/2}$ =NORM.S.INV(0.975) = 1.96.

TI-Calculator: $z_{\alpha/2}$ = invNorm(0.025,0,1) = –1.96 and $z_{1-\alpha/2}$ = invNorm(0.975,0,1) = 1.96.

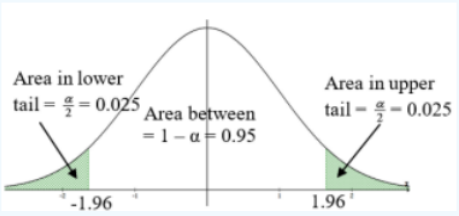


Figure 8-25

Step 4: State the decision. Since the test statistic is not in the shaded rejection area, do not reject H_0.

Step 5: State the summary. At the 5% level of significance, there is not enough evidence to conclude that the proportion of undergraduates in college for this state differs from the national average of 85.6%.

P-value Method

The hypotheses and test statistic stay the same.

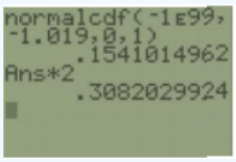
H_0: p = 0.856

H_1: p ≠ 0.856 (claim)

$$Z = \frac{\hat{p} - p_0}{\sqrt{\left(\frac{p_0 q_0}{n}\right)}} = \frac{0.84 - 0.856}{\sqrt{\left(\frac{0.856 \cdot 0.144}{500}\right)}} = -1.019$$

To find the p-value we need to find the P(Z > |1.019|) the area to the left of z = –1.019 and to the right of z = 1.019. First, find the area below (since the test statistic is negative) z = –1.019 using the normalcdf we get 0.1541. Then, double this area to get

the p-value = 0.3082.



Since the p-value > α the decision is to not reject H_0.

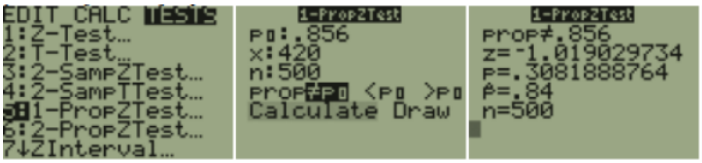
Summary: There is not enough evidence to conclude that the proportion of undergraduates in college for this state differs from the national average of 85.6%.

There is a shortcut for this test on the TI Calculators, which will quickly find the test statistic and p-value.

The rejection rule for the two methods are:

- P-value method: reject H_0 when the p-value $\leq \alpha$.
- Critical value method: reject H_0 when the test statistic is in the critical region.

TI-84: Press the [STAT] key, arrow over to the [TESTS] menu, arrow down to the option [5:1-PropZTest] and press the [ENTER] key. Type in the hypothesized proportion (p_0), x, sample size, arrow over to the $\neq$, <, > sign that is the same in the problem's alternative hypothesis statement then press the [ENTER] key, arrow down to [Calculate] and press the [ENTER] key.



The calculator returns the z-test statistic and the p-value. Note: sometimes you are not given the x value but a percentage instead. To find the x to use in the calculator, multiply $\hat{p}$ by the sample size and round off to the nearest integer. The calculator will give you an error message if you put in a decimal for x or n. For example, if $\hat{p}$ = 0.22 and n = 124 then 0.22*124 = 27.28, so use x = 27.

TI-89: Go to the [Apps] **Stat/List Editor**, then press [2nd] then F6 [Tests], then select **5: 1-PropZ-Test**. Type in the hypothesized proportion (p0), x, sample size, arrow over to the $\neq$, <, > sign that is the same in the problem's alternative hypothesis statement then press the [ENTER] key to calculate. The calculator returns the z-test statistic and the pvalue. Note: sometimes you are not given the x value but a percentage instead. To find the x value to use in the calculator, multiply $\hat{p}$ by the sample size and round off to the nearest integer. The calculator will give you an error message if you put in a decimal for x or n. For example, if $\hat{p}$ = 0.22 and n = 124 then 0.22*124 = 27.28, so use x = 27.
