# Mostly Harmless Statistics Formula Packet

## Chapter 3 Formulas

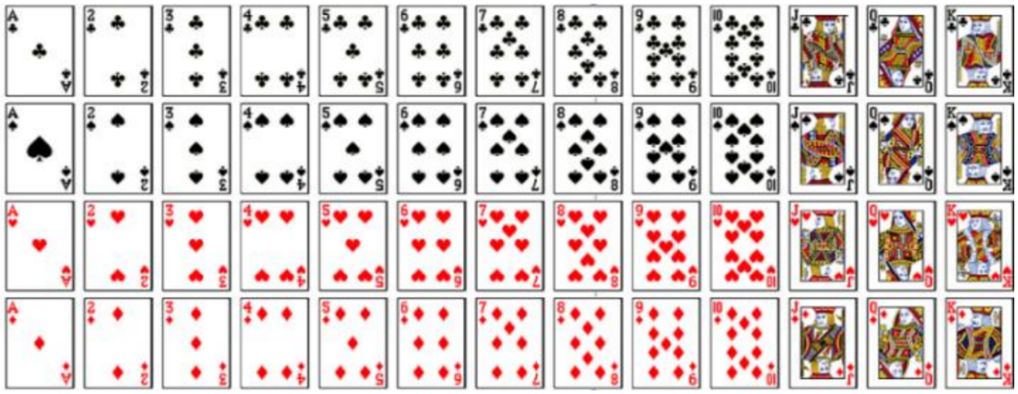
| | |
|---|---|
| **Sample Mean:** $\bar{x} = \frac{\sum x}{n}$le | **Population Mean:** $\mu = \frac{\sum x}{N}$ |
| **Weighted Mean:** $\bar{x} = \frac{\sum (xw)}{\sum w}$ | **Range** = Max − Min |
| **Sample Standard Deviation:** $s = \sqrt{\frac{\sum (x-\bar{x})^2}{n-1}}$ | **Population Standard Deviation** = $\sigma$ |
| **Sample Variance:** $s^2 = \frac{\sum (x-\bar{x})^2}{n-1}$ | **Population Variance** = $\sigma^2$ |
| **Coefficient of Variation:** $\text{CVar} = \left(\frac{s}{\bar{x}} \cdot 100\right)$ | ***Z*-Score:** $z = \frac{x-\bar{x}}{s}$ |
| **Percentile Index:** $i = \frac{(n+1) \cdot p}{100}$ | **Interquartile Range:** $\text{IQR} = Q_3 - Q_1$ |
| **Empirical Rule:** $z = 1, 2, 3 \Rightarrow 68$ | **Outlier Lower Limit:** $Q_1 - (1.5 \cdot \text{IQR})$ |
| **Chebyshev's Inequality:** $\left(\left(1 - \frac{1}{(z)^2}\right) \cdot 100r\right)$ | **Outlier Upper Limit:** $Q_3 + (1.5 \cdot \text{IQR})$ |

**TI-84:** Enter the data in a list and then press [STAT]. Use cursor keys to highlight CALC. Press 1 or [ENTER] to select **1:1-Var Stats**. Press [2nd], then press the number key corresponding to your data list. Press [Enter] to calculate the statistics. Note: the calculator always defaults to $L_1$ if you do not specify a data list.


$s_x$ is the sample standard deviation. You can arrow down and find more statistics. Use the min and max to calculate the range by hand. To find the variance simply square the standard deviation.

## Chapter 4 Formulas

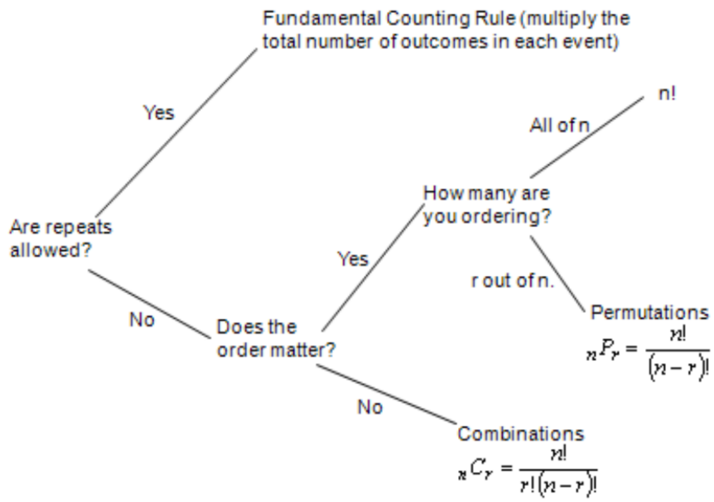
| | |
|---|---|
| **Complement Rules:** $P(A) + P(A^C) = 1$<br>$P(A) = 1 - P(A^C)$<br>$P(A^C) = 1 - P(A)$ | **Mutually Exclusive Events:** $P(A \cup B) = 0$ |
| **Union Rule:** $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ | **Independent Events:** $P(A \cup B) = P(A) \cdot P(B)$ |
| **Intersection Rule:** $P(A \cap B) = P(A) \cdot P(A \mid B)$ | **Conditional Probability Rule:** $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$ |
| **Fundamental Counting Rule:** $m_1 \cdot m_2 \cdots m_n$ | **Factorial Rule:** $n! = n \cdot (n-1) \cdot (n-2) \cdots 3 \cdot 2 \cdot 1$ |
| **Combination Rule:** ${}_nC_r = \frac{n!}{(r!(n-r)!)}$ | **Permutation Rule:** ${}_nP_r = \frac{n!}{(n-r)!}$ |



clubs = ♣, spades = ♠, hearts = ♥, diamonds = ♦

| Sum of 2 Dice | | Second Die | | | | | |
|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 |
| First Die | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| | 6 | 7 | 8 | 9 | 10 | 11 | 12 |

A probability is always a number between 0 and 1.
$0 \leq P(X) \leq 1$




## Chapter 5 Formulas

| | |
|---|---|
| Discrete Distribution Table:<br>$0 \leq \mathrm{P}(x_i) \leq 1 \qquad \sum \mathrm{P}(x_i) = 1$ | Discrete Distribution Mean: $\mu = \sum (x_i \cdot \mathrm{P}(x_i))$ |
| Discrete Distribution Variance:<br>$\sigma^2 = \sum (x_i^2 \cdot \mathrm{P}(x_i)) - \mu^2$ | Discrete Distribution Standard Deviation: $\sigma = \sqrt{\sigma^2}$ |
| Geometric Distribution:<br>$\mathrm{P}(X = x) = p \cdot q^{x-1}, x = 1, 2, 3, \ldots$ | Geometric Distribution Mean: $\mu = \frac{1}{p}$<br>Variance: $\sigma^2 = \frac{1-p}{p^2}$<br>Standard Deviation: $\sigma = \sqrt{\frac{1-p}{p^2}}$ |
| Binomial Distribution:<br>$\mathrm{P}(X = x) = {}_nC_x p^x \cdot q^{(n-x)}, x = 0, 1, 2, \ldots, n$ | Binomial Distribution Mean: $\mu = n \cdot p$<br>Variance: $sigma^2 = n \cdot p \cdot q$<br>Standard Deviation: $\sigma = \sqrt{n \cdot p \cdot q}$ |
| Hypergeometric Distribution:<br>$\mathrm{P}(X = x) = \frac{{}_aC_x \cdot {}_bC_{n-x}}{{}_NC_n}$ | $p = \mathrm{P}(\text{success}) \qquad p = \mathrm{P}(\text{failure}) = 1 - p$<br>$n = \text{sample size} \qquad N = \text{population size}$ |
| Unit Change for Poisson Distribution:<br>New $\mu$ = old $\mu \left(\frac{\text{new units}}{\text{old units}}\right)$ | Poisson Distribution:<br>$\mathrm{P}(X = x) = \frac{e^{-\mu}\mu^x}{x!}$ |

| $\mathrm{P}(X = x)$ | $\mathrm{P}(X \leq x)$ | $\mathrm{P}(X \geq x)$ |
|---|---|---|
| Is the same as | Is less than or equal to | Is greater than or equal to |

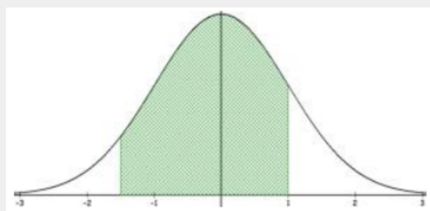
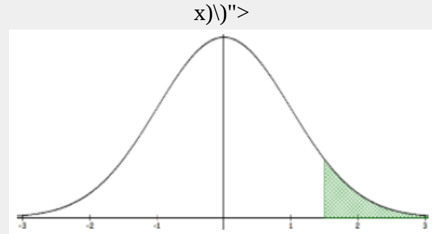
| $\mathrm{P}(X=x)$ | $\mathrm{P}(X \leq x)$ | $\mathrm{P}(X \geq x)$ |
|---|---|---|
| Is equal to | Is at most | Is at least |
| Is exactly the same as | Is not greater than | Is not less than |
| Has not changed from | Within | Is more than or equal to |
| | | |
| **Excel**<br>$=\mathrm{binom.dist}(x, n, p, 0)$<br>$=\mathrm{HYPGEOM.DIST}(x, n, a, N, 0)$<br>$=\mathrm{POISSON.DIST}(x, \mu, 0)$ | **Excel**<br>$=\mathrm{binom.dist}(x, n, p, 1)$<br>$=\mathrm{HYPGEOM.DIST}(x, n, a, N, 1)$<br>$=\mathrm{POISSON.DIST}(x, \mu, 1)$ | **Excel**<br>$=1-\mathrm{binom.dist}(x-1, n, p, 1)$<br>$=1-\mathrm{HYPGEOM.DIST}(x-1, n, a, N, 1)$<br>$=1-\mathrm{POISSON.DIST}(x-1, \mu, 1)$ |
| **TI Calculator**<br>$\mathrm{geometpdf}(p, x)$<br>$\mathrm{binompdf}(n, p, x)$<br>$\mathrm{poissonpdf}(\mu, x)$ | **TI Calculator**<br>$\mathrm{binomcdf}(n, p, x)$<br>$\mathrm{poissoncdf}(\mu, x)$ | **TI Calculator**<br>$1-\mathrm{binomcdf}(n, p, x-1)$<br>$1-\mathrm{poissoncdf}(\mu, x-1)$ |

| | $\mathrm{P}(X>x)$ | $\mathrm{P}(X<x)$ |
|---|---|---|
| How do you tell them apart?<br>• Geometric – A percent or proportion is given. There is no set sample size until a success is achieved.<br>• Binomial – A percent or proportion is given. A sample size is given.<br>• Hypergeometric – Usually frequencies of successes are given instead of percentages. A sample size is given.<br>• Poisson – An average or mean is given. There is no set sample size until a success is achieved. | x)\)">More than | Less than |
| | x)\)">Greater than | Below |
| | x)\)">Above | Lower than |
| | x)\)">Higher than | Shorter than |
| | x)\)">Longer than | Smaller than |
| | x)\)">Bigger than | Decreased |
| | x)\)">Increased | Reduced |
| | x)\)"> | |
| | x)\)">**Excel**<br>$=1-\mathrm{binom.dist}(x, n, p, 1)$<br>$=1-\mathrm{HYPGEOM.DIST}(x, n, a, N, 1)$<br>$=1-\mathrm{POISSON.DIST}(x, \mu, 1)$ | **Excel**<br>$=\mathrm{binom.dist}(x-1, n, p, 1)$<br>$=\mathrm{HYPGEOM.DIST}(x-1, n, a, N, 1)$<br>$=\mathrm{POISSON.DIST}(x-1, \mu, 1)$ |
| | x)\)">**TI Calculator**<br>$1-\mathrm{binomcdf}(n, p, x)$<br>$1-\mathrm{poissoncdf}(\mu, x)$ | **TI Calculator**<br>$\mathrm{binomcdf}(n, p, x-1)$<br>$\mathrm{poissoncdf}(\mu, x-1)$ |

## Chapter 6 Formulas

| | |
|---|---|
| **Uniform Distribution**<br>$f(x)=\frac{1}{b-a}$, for $a \leq x \leq b$<br>$\mathrm{P}(X \geq x)=\mathrm{P}(X>x)=\left(\frac{1}{b-a}\right) \cdot(b-x)$<br>$\mathrm{P}(X \leq x)=\mathrm{P}(X<x)=\left(\frac{1}{b-a}\right) \cdot(x-a)$<br>$\mathrm{P}\left(x_1 \leq X \leq x_2\right)=\mathrm{P}\left(x_1<X<x_2\right)=\left(\frac{1}{b-a}\right) \cdot\left(x_2-x_1\right)$ | **Exponential Distribution**<br>$f(x)=\frac{1}{\mu} e^{(-x / \mu)}$, for $x \geq 0$<br>$\mathrm{P}(X \geq x)=\mathrm{P}(X>x)=e^{-x / \mu}$<br>$\mathrm{P}(X \leq x)=\mathrm{P}(X<x)=1-e^{-x / \mu}$<br>$\mathrm{P}\left(x_1 \leq X \leq x_2\right)=\mathrm{P}\left(x_1<X<x_2\right)=e^{\left(-x_1 / \mu\right)}-e^{\left(-x_2 / \mu\right)}$ |
| **Standard Normal Distribution**<br>$\mu=0, \sigma=1$<br>$z$-score: $z=\frac{x-\mu}{\sigma}$<br>$x=z \sigma+\mu$ | **Central Limit Theorem**<br>Z-score: $z=\frac{\bar{x}-\mu}{\left(\frac{\sigma}{\sqrt{n}}\right)}$ |

In the table below, note that when $\mu = 0$ and $\sigma = 1$ use the NORM.S. DIST or NORM.S.INV function in Excel for a standard normal distribution.

| $\mathrm{P}(X \leq x)$ or $\mathrm{P}(X < x)$ | $\mathrm{P}\left(x_1 < X < x_2\right)$ or $\mathrm{P}\left(x_1 \leq X \leq x_2\right)$ | $\mathrm{P}(X \geq x)$ or $\mathrm{P}(X > x)$ |
|---|---|---|
| Is less than or equal to | Between | x)\)">Is greater than or equal to |
| Is at most | | x)\)">Is at least |
| Is not greater than | | x)\)">Is not less than |
| Within | | x)\)">More than |
| Less than | | x)\)">Greater than |
| Below | | x)\)">Above |
| Lower than | | x)\)">Higher than |
| Shorter than | | x)\)">Longer than |
| Smaller than | | x)\)">Bigger than |
| Decreased | | x)\)">Increased |
| Reduced | | x)\)">Larger |
|  |  | x)\)">  |
| **Excel**<br>**Finding a Probability:**<br>$= \mathrm{NORM.DIST}(x, \mu, \sigma, \mathrm{true})$<br>**Finding a Percentile:**<br>$= \mathrm{NORM.INV}(\mathrm{area}, \mu, \sigma)$ | **Excel**<br>**Finding a Probability:**<br>$= \mathrm{NORM.DIST}(x_2, \mu, \sigma, \mathrm{true}) - \mathrm{NORM.DIST}(x_1, \mu, \sigma, \mathrm{true})$<br>**Finding a Percentile:**<br>$x_1 = \mathrm{NORM.INV}((1-\mathrm{area})/2, \mu, \sigma)$<br>$x_2 = \mathrm{NORM.INV}(1-((1-\mathrm{area})/2), \mu, \sigma)$ | x)\)">**Excel**<br>**Finding a Probability:**<br>$= 1 - \mathrm{NORM.DIST}(x, \mu, \sigma, \mathrm{true})$<br>**Finding a Percentile:**<br>$= \mathrm{NORM.INV}(1-\mathrm{area}, \mu, \sigma)$ |
| **TI Calculator**<br>**Finding a Probability:**<br>$= \mathrm{normalcdf}(-1\mathrm{E}99, x, \mu, \sigma)$<br>**Finding a Percentile:**<br>$= \mathrm{invNorm}(\mathrm{area}, \mu, \sigma)$ | **TI Calculator**<br>**Finding a Probability:**<br>$= \mathrm{normalcdf}(x_1, x_2, \mu, \sigma)$<br>**Finding a Percentile:**<br>$x_1 = \mathrm{invNorm}((1-\mathrm{area})/2, \mu, \sigma)$<br>$x_2 = \mathrm{invNorm}(1-((1-\mathrm{area})/2), \mu, \sigma)$ | x)\)">**TI Calculator**<br>**Finding a Probability:**<br>$= \mathrm{normalcdf}(x, 1\mathrm{E}99, \mu, \sigma)$<br>**Finding a Percentile:**<br>$= \mathrm{invNorm}(1-\mathrm{area}, \mu, \sigma)$ |

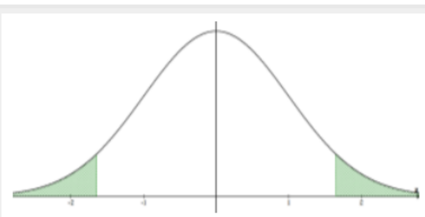
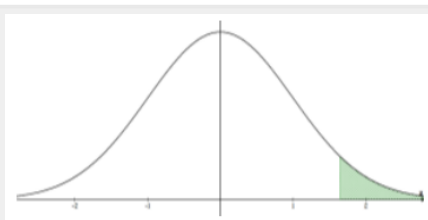
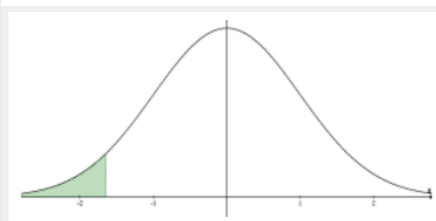
## Chapter 7 Formulas

| | |
|---|---|
| **Confidence Interval for One Proportion**<br>$\hat{p} \pm z_{\alpha/2}\sqrt{\left(\frac{\hat{p}\hat{q}}{n}\right)}$<br>$\hat{p} = \frac{x}{n}$<br>$\hat{q} = 1 - \hat{p}$<br>TI-84: 1 − PropZInt | **Sample Size for Proportion**<br>$n = p^* \cdot q^* \left(\frac{z_{\alpha/2}}{E}\right)^2$<br>Always round up to whole number.<br>If $p$ is not given use $p^* = 0.5$.<br>$E$ = Margin of Error |
| **Confidence Interval for One Mean**<br>Use z-interval when $\sigma$ is given.<br>Use t-interval when $s$ is given.<br>If $n < 30$, population needs to be normal. | **Z-Confidence Interval**<br>$\bar{x} \pm z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$<br>TI-84: ZInterval |

| | |
|---|---|
| **Z-Critical Values**<br>Excel: $z_{\alpha/2} = \text{NORM.INV}(1 - \text{area}/2, 0, 1)$<br>TI-84: $z_{\alpha/2} = \text{invNorm}(1 - \text{area}/2, 0, 1)$ | **t-Critical Values**<br>Excel: $t_{\alpha/2} = \text{T.INV}(1 - \text{area}/2, df)$<br>TI-84: $t_{\alpha/2} = \text{invT}(1 - \text{area}/2, df)$ |
| **t-Confidence Interval**<br>$\bar{x} \pm t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$<br>$df = n - 1$<br>TI-84: TInterval | **Sample Size for Mean**<br>$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E}\right)^2$<br>Always round up to whole number.<br>$E$ = Margin of Error |

## Chapter 8 Formulas

| | |
|---|---|
| **Hypothesis Test for One Mean**<br>Use z-test when $\sigma$ is given.<br>Use t-test when $s$ is given.<br>If $n < 30$, population needs to be normal. | **Type I Error -**<br>Reject $H_0$ when $H_0$ is true.<br>**Type II Error -**<br>Fail to reject $H_0$ when $H_0$ is false. |
| **Z-Test:**<br>$H_0 : \mu = \mu_0$<br>$H_1 : \mu \neq \mu_0$<br>$z = \frac{\bar{x} - \mu_0}{\left(\frac{\sigma}{\sqrt{n}}\right)}$ TI-84: Z-Test | **t-Test:**<br>$H_0 : \mu = \mu_0$<br>$H_1 : \mu \neq \mu_0$<br>$t = \frac{\bar{x} - \mu_0}{\left(\frac{s}{\sqrt{n}}\right)}$ TI-84: T-Test |
| **z-Critical Values**<br>Excel:<br>Two-tail: $z_{\alpha/2} = \text{NORM.INV}(1 - \alpha/2, 0, 1)$<br>Right-tail: $z_{1-\alpha} = \text{NORM.INV}(1 - \alpha, 0, 1)$<br>Left-tail: $z_\alpha = NORM.INV(\alpha, 0, 1)$<br><br>TI-84:<br>Two-tail: $z_{\alpha/2} = \text{invNorm}(1 - \alpha/2, 0, 1)$<br>Right-tail: $z_{1-\alpha} = \text{invNorm}(1 - \alpha, 0, 1)$<br>Left-tail: $z_\alpha = \text{invNorm}(\alpha, 0, 1)$ | **t-Critical Values**<br>Excel:<br>Two-tail: $t_{\alpha/2} = \text{T.INV}(1 - \alpha/2, df)$<br>Right-tail: $t_{1-\alpha} = \text{T.INV}(1 - \alpha, df)$<br>Left-tail: $t_\alpha = \text{T.INV}(\alpha, df)$<br><br>TI-84:<br>Two-tail: $t_{\alpha/2} = \text{invT}(1 - \alpha/2, df)$<br>Right-tail: $t_{1-\alpha} = \text{invT}(1 - \alpha, df)$<br>Left-tail: $t_\alpha = \text{invT}(\alpha, df)$ |
| **Hypothesis Test for One Proportion**<br>$H_0 : p = p_0$<br>$H_1 : p \neq p_0$<br>$z = \frac{\hat{p} - p_0}{\sqrt{\left(\frac{p_0 q_0}{n}\right)}}$<br>TI-84: 1-PropZTest | **Rejection Rules:**<br>P-value method: reject $H_0$ when the p-value $\leq \alpha$.<br>Critical value method: reject $H_0$ when the test statistic is in the critical region (shaded tails). |

| Two-tailed Test | Right-tailed Test | Left-tailed Test |
|---|---|---|
| $H_0 : \mu = \mu_0$ or $H_0 : p = p_0$<br>$H_1 : \mu \neq \mu_0$ or $H_0 : p \neq p_0$ | $H_0 : \mu = \mu_0$ or $H_0 : p = p_0$<br>$H_1 : \mu > \mu_0$ or $H_0 : p > p_0$ | $H_0 : \mu = \mu_0$ or $H_0 : p = p_0$<br>$H_1 : \mu < \mu_0$ or $H_0 : p < p_0$ |
|  |  |  |

| **Claim is in the Null Hypothesis** | | |
|---|---|---|
| = | $\leq$ | $\geq$ |

| Claim is in the Null Hypothesis | | |
|---|---|---|
| Is equal to | Is less than or equal to | Is greater than or equal to |
| Is exactly the same as | Is at most | Is at least |
| Has not changed from | Is not more than | Is not less than |
| Is the same as | Within | Is more than or equal to |

| Claim is in the Alternative Hypothesis | | |
|---|---|---|
| $\neq$ | > | < |
| Is not | More than | Less than |
| Is not equal to | Greater than | Below |
| Is different from | Above | Lower than |
| Has changed from | Higher than | Shorter than |
| Is not the same as | Longer than | Smaller than |
| | Bigger than | Decreased |
| | Increased | Reduced |

## Chapter 9 Formulas

| | |
|---|---|
| **Hypothesis Test for Two Dependent Means**<br>$H_0 : \mu_D = 0$<br>$H_1 : \mu_D \neq 0$<br>$t = \frac{\bar{D} - \mu_D}{\left(\frac{s_D}{\sqrt{n}}\right)}$<br>TI-84: T-Test | **Confidence Interval for Two Dependent Means**<br>$\bar{D} \pm t_{\alpha/2}\left(\frac{s_D}{\sqrt{n}}\right)$<br>TI-84: TInterval |
| **Hypothesis Test for Two Independent Means**<br>Z-Test: $H_0 : \mu_1 = \mu_2$<br>$H_1 : \mu_1 \neq \mu_2$<br>$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)_0}{\sqrt{\left(\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)}}$<br>TI-84: 2-SampZTest | **Confidence Interval for Two Independent Means Z-Interval**<br>$(\bar{x}_1 - \bar{x}_2) \pm z_{\alpha/2}\sqrt{\left(\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)}$<br>TI-84: 2-SampZInt |

**Hypothesis Test for Two Independent Means**

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$

**T-Test: Assume variances are unequal**

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)_0}{\sqrt{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)}}$$

TI-84: 2-SampTTest

$$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\left(\left(\frac{s_1^2}{n_1}\right)^2 \left(\frac{1}{n_1-1}\right) + \left(\frac{s_2^2}{n_2}\right)^2 \left(\frac{1}{n_2-1}\right)\right)}$$

**T-Test: Assume variances are equal**

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\left(\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{(n_1+n_2-2)}\right)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$df = n_1 - n_2 - 2$

**Confidence Interval for Two Independent Means**

**T-Interval: Assume variances are unequal**

$$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2} \sqrt{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)}$$

TI-84: 2-SampTInt

$$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\left(\left(\frac{s_1^2}{n_1}\right)^2 \left(\frac{1}{n_1-1}\right) + \left(\frac{s_2^2}{n_2}\right)^2 \left(\frac{1}{n_2-1}\right)\right)}$$

**T-Interval: Assume variances are equal**

$$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2} \sqrt{\left(\left(\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{(n_1-n_2-2)}\right)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)\right)}$$

$df = n_1 - n_2 - 2$

**Hypothesis Test for Two Proportions**

$H_0 : p_1 = p_2$

$H_1 : p_1 \neq p_2$

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\left(\hat{p} \cdot \hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)\right)}}$$

$\hat{p} = \frac{(x_1+x_2)}{(n_1+n_2)} = \frac{(\hat{p}_1 \cdot n_1 + \hat{p}_2 \cdot n_2)}{(n_1+n_2)}$

$\hat{q} = 1 - \hat{p}$

$\hat{p}_1 = \frac{x_1}{n_1}, \qquad \hat{p}_2 = \frac{x_2}{n_2}$

TI-84: 2-PropZTest

**Confidence Interval for Two Proportions**

$$(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2} \sqrt{\left(\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}\right)}$$

$\hat{p}_1 = \frac{x_1}{n_1} \qquad \hat{p}_2 = \frac{x_2}{n_2}$

$\hat{q}_1 = 1 - \hat{p}_1 \qquad \hat{q}_2 = 1 - \hat{p}_2$

TI-84: 2-PropZInt

**Hypothesis Test for Two Variances**

$H_0 : \sigma_1^2 = \sigma_2^2$

$H_1 : \sigma_1^2 \neq \sigma_2^2$

$F = \frac{s_1^2}{s_2^2}$

$df_N = n_1 - 1, \qquad df_D = n_2 - 1$

TI-84: 2\text{-SampFTest}}

**Hypothesis Test for Two Standard Deviations**

$H_0 : \sigma_1 = \sigma_2$

$H_1 : \sigma_1 \neq \sigma_2$

$F = \frac{s_1^2}{s_2^2}$

$df_N = n_1 - 1, \qquad df_D = n_2 - 1$

TI-84: 2\text{-SampFTest}}

**F-Critical Values**

Excel:

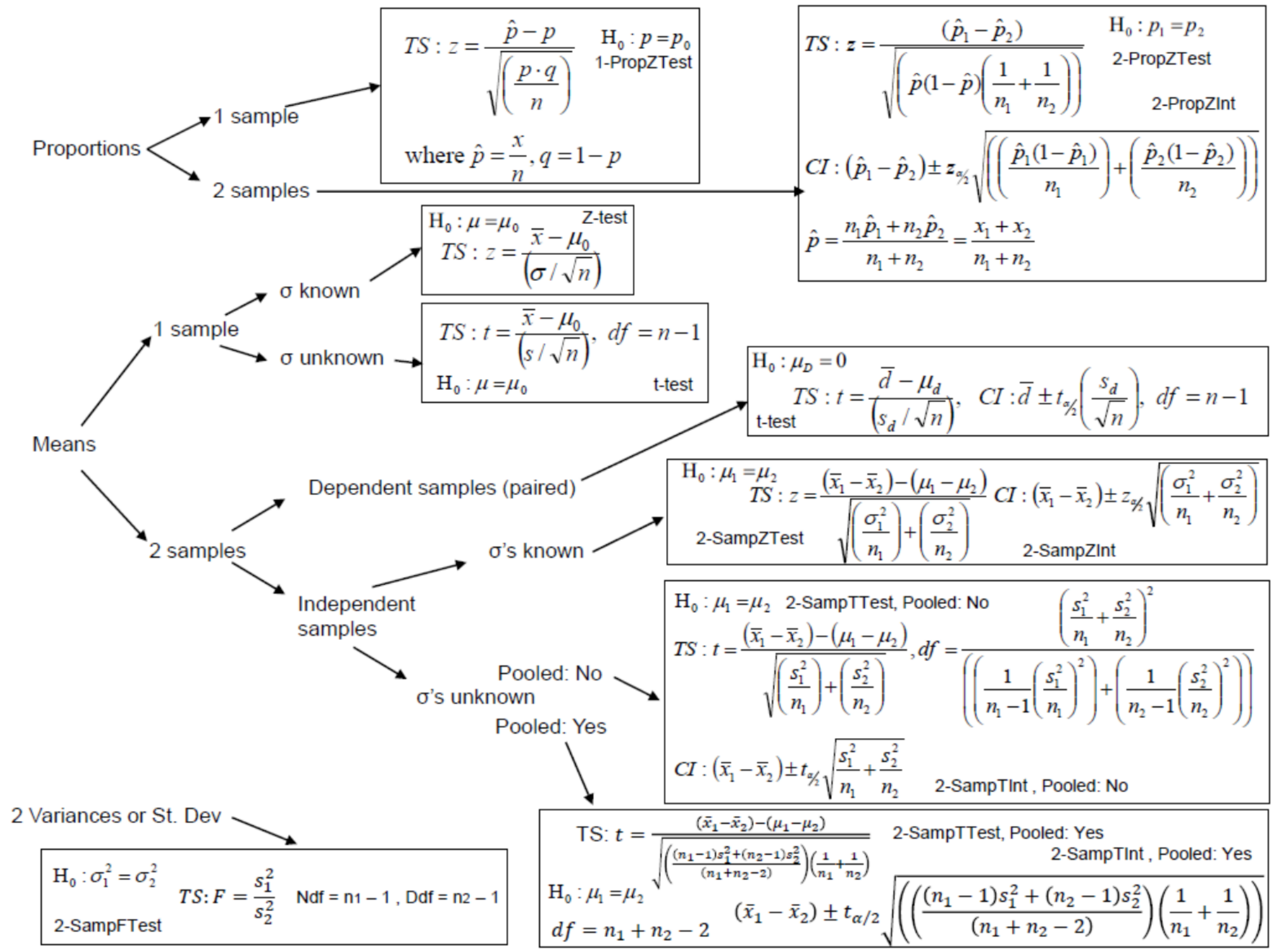
Two-tail: $F_{\alpha/2} = \text{F.INV}(1 - \alpha/2, 0, 1)$

Right-tail: $F_{1-\alpha} = \text{F.INV}(1 - \alpha, 0, 1)$

Left-tail: $F_{\alpha} = \text{F.INV}(\alpha, 0, 1)$

**For z and t-Critical Values refer back to Chapter 8**

TI-84: invF program can be downloaded at http://www.MostlyHarmlessStatistics.com.




## Chapter 10 Formulas

| | |
|---|---|
| **Goodness of Fit Test**<br>$H_0: p_1=p_0, p_2=p_0, \ldots, p_k=p_0$<br>$H_1$ : At least one proportion is different.<br>$\chi^2=\sum\frac{(O-E)^2}{E}$<br>$df=k-1, p_0=1/k$ or given %<br>TI-84: $\chi^2$ GOF-Test | **Test for Independence**<br>$H_0$ : Variable 1 and Variable 2 are independent.<br>$H_1$ : Variable 1 and Variable 2 are dependent.<br>$\chi^2=\sum\frac{(O-E)^2}{E}$<br>$df=(R-1)(C-1)$<br>TI-84: $\chi^2$-Test |

## Chapter 11 Formulas

One-Way ANOVA:

$H_0: \mu_1 = \mu_2 = \mu_3 = \ldots = \mu_k$ $\quad k =$ number of groups

$H_1$ : At least one mean is different.

| Source | SS = Sum of Squares | $df$ | MS = Mean Square | F |
|---|---|---|---|---|
| Between (Factor) | $\sum n_i(\bar{x}_i - \bar{x}_{GM})^2$ | k – 1 | $MSB = \frac{SSB}{k-1}$ | $F = \frac{MSB}{MSW}$ |
| Within (Error) | $\sum(n_i - 1)s_i^2$ | $N$ – k | $MSW = \frac{SSW}{N-k}$ | |
| Total | SST | $N$ – 1 | | |

$\bar{x}_i$ = sample mean from the $i^{th}$ group

$n_i$ = sample size of the $i^{th}$ group

$s_i^2$ = sample variance from the $i^{th}$ group

$N = n_1 + n_2 + \cdots + n_k$

$\bar{x}_{GM} = \frac{\sum x_i}{N}$

**Bonferroni** test statistic: $t = \dfrac{\bar{x}_i - \bar{x}_j}{\sqrt{\left(MSW\left(\frac{1}{n_i} + \frac{1}{n_j}\right)\right)}}$

$H_0: \mu_i = \mu_j$

$H_1: \mu_i \neq \mu_j$

Multiply p-value by $m = {}_kC_2$ , divide area for critical value by $m = {}_kC_2$

**Two-Way ANOVA:**

Row Effect (Factor A): $H_0$ : The row variable has no effect on the average _______________.

$H_1$ : The row variable has an effect on the average _______________.

Column Effect (Factor B): $H_0$ : The column variable has no effect on the average _______________.

$H_1$ : The column variable has an effect on the average _______________.

Interaction Effect (A $\times$ B\):

$H_0$ : There is no interaction effect between row variable and column variable on the average _______________.

$H_1$ : There is an interaction effect between row variable and column variable on the average _______________.

| **Source** | **SS** | ***df*** | **MS** | **F** |
|---|---|---|---|---|
| **A (row factor)** | $SS_A$ | $a$ – 1 | $MS_A = \frac{SS_A}{df_A}$ | $F_A = \frac{MS_A}{MSE}$ |
| **B (column factor)** | $SS_B$ | $b$ – 1 | $MS_B = \frac{SS_B}{df_B}$ | $F_B = \frac{MS_B}{MSE}$ |
| **A×B (interaction)** | $SS_{A\times B}$ | ($a$ – 1) ($b$ – 1) | $MS_{A\times B} = \frac{SS_{A\times B}}{df_{A\times B}}$ | $F_{A\times B} = \frac{MS_{A\times B}}{MSE}$ |
| **Error (within)** | SSE | $ab(n$ – 1) | $MSE = \frac{SSE}{df_E}$ | |
| **Total** | SST | $N$ – 1 | | |

## Chapter 12 Formulas

$SS_{xx} = (n-1)s_x^2$

$SS_{yy} = (n-1)s_y^2$

$SS_{xy} = \sum(xy) - n \cdot \bar{x} \cdot \bar{y}$

**Correlation Coefficient**

$r = \frac{SS_{xy}}{\sqrt{(SS_{xx} \cdot SS_{yy})}}$

Slope = $b_1 = \frac{SS_{xy}}{SS_{xx}}$

y-intercept = $b_0 = \bar{y} - b_1\bar{x}$

**Regression Equation** (Line of Best Fit): $\hat{y} = b_0 + b_1 x$

**Correlation t-test**

$H_0: \rho = 0;\ H_1: \rho \neq 0 \qquad t = r\sqrt{\left(\frac{n-2}{1-r^2}\right)} \qquad df = n-2$

**Slope t-test**

$H_0: \beta_1 = 0;\ H_1: \beta_1 \neq 0 \qquad t = \frac{b_1}{\sqrt{\left(\frac{MSE}{SS_{xx}}\right)}} \qquad df = n-p-1 = n-$

**Residual**

$e_i = y_i - \hat{y}_i$ (Residual plots should have no patterns.)

**Standard Error of Estimate**

$s_{est} = \sqrt{\frac{\sum (y_i - \hat{y}_i)^2}{n-2}} = \sqrt{MSE}$

**Prediction Interval**

$\hat{y} = t_{\alpha/2} \cdot s_{est}\sqrt{\left(1 + \frac{1}{n} + \frac{(x-\bar{x})^2}{SS_{xx}}\right)}$

**Slope/Model F-test**

$H_0: \beta_1 = 0;\ H_1: \beta_1 \neq 0$

| Source | SS = Sum of Squares | *df* | MS = Mean Square | F |
|---|---|---|---|---|
| Regression | SSR = $\frac{(SS_{xy})^2}{SS_{xx}}$ | p | $MSR = \frac{SSR}{p}$ | $F = \frac{MSR}{MSE}$ |
| Error | SSE = $SS_{yy}$ – SSR | $n$ – p – 1 | $MSE = \frac{SSE}{n-p-1}$ | |
| Total | SST = $SS_{yy}$ | $n$ – 1 | | |

**Multiple Linear Regression Equation**

$\hat{y} = b_0 + b_1x_1 + b_2x_2 + \cdots + b_px_p$

**Coefficient of Determination**

$R^2 = (r)^2 = \frac{SSR}{SST}$

**Model F-Test for Multiple Regression**

$H_0: \beta_1 = \beta_2 = \cdots \beta_p = 0$

$H_1$ : At least one slope is not zero.

**Adjusted Coefficient of Determination**

$R^2_{adj} = 1 - \left(\frac{(1-R^2)(n-1)}{(n-p-1)}\right)$

## Chapter 13 Formulas

**Ranking Data**

- Order the data from smallest to largest.
- The smallest value gets a rank of 1.
- The next smallest gets a rank of 2, etc.
- If there are any values that tie, then each of the tied values gets the average of the corresponding ranks.

**Sign Test**

$H_0$ : Median $= MD_0$

$H_1$ : Median $\neq MD_0$

p-value uses binomial distribution with $p = 0.5$ and $n$ is the sample size not including ties with the median or differences of 0.

- For a 2-tailed test, the test statistic, $x$, is the smaller of the plus or minus signs. If $x$ is the test statistic, the p-value for a two-tailed test is $2 \cdot \mathrm{P}(X \leq x)$ .
- For a right-tailed test, the test statistic, $x$, is the number of plus signs. For a left-tailed test, the test statistic, $x$, is the number of minus signs. The p-value for a one-tailed test is $\mathrm{P}(X \geq x)$ or $\mathrm{P}(X \leq x)$ .

**Wilcoxon Signed-Rank Test**

$n$ is the sample size not including a difference of 0. When $n < 30$, use test statistic $w_s$, which is the absolute value of the smaller of the sum of ranks. CV uses table below.

If critical value is not in table then use an online calculator: http://www.socscistatistics.com/tests/signedranks

When $n \geq 30$, use z-test statistic: $z = \frac{\left(w_s - \left(\frac{n(n+1)}{4}\right)\right)}{\sqrt{\left(\frac{n(n+1)(2n+1)}{24}\right)}}$

**Mann-Whitney U Test**

When $n_1 \leq 20$ and $n_2 \leq 20$

$U_1 = R_1 - \frac{n_1(n_1+1)}{2},\ U_2 = R_2 - \frac{n_2(n_2+1)}{2}$ .

$U = \text{Min}\,(U_1, U_2)$

CV uses tables below. If critical value is not in tables then use an online calculator:
https://www.socscistatistics.com/tests/mannwhitney/default.aspx

When $n_1 > 20$ and $n_2 > 20$, use z-test statistic:

$z = \frac{\left(U - \left(\frac{n_1 \cdot n_2}{2}\right)\right)}{\sqrt{\left(\frac{n_1 \cdot n_2(n_1+n_2+1)}{12}\right)}}$

Wilcoxon Signed-Rank Critical Values

| | 1-Tailed α | | | 2-Tailed α | | |
|---|---|---|---|---|---|---|
| n | 0.01 | 0.05 | 0.10 | 0.01 | 0.05 | 0.10 |
| 5 | - | 0 | 2 | - | - | 0 |
| 6 | - | 2 | 3 | - | 0 | 2 |
| 7 | 0 | 3 | 5 | - | 2 | 3 |
| 8 | 1 | 5 | 8 | 0 | 3 | 5 |
| 9 | 3 | 8 | 10 | 1 | 5 | 8 |
| 10 | 5 | 10 | 14 | 3 | 8 | 10 |
| 11 | 7 | 13 | 17 | 5 | 10 | 13 |
| 12 | 9 | 17 | 21 | 7 | 13 | 17 |
| 13 | 12 | 21 | 26 | 9 | 17 | 21 |
| 14 | 15 | 25 | 31 | 12 | 21 | 25 |
| 15 | 19 | 30 | 36 | 15 | 25 | 30 |
| 16 | 23 | 35 | 42 | 19 | 29 | 35 |
| 17 | 27 | 41 | 48 | 23 | 34 | 41 |
| 18 | 32 | 47 | 55 | 27 | 40 | 47 |
| 19 | 37 | 53 | 62 | 32 | 46 | 53 |
| 20 | 43 | 60 | 69 | 37 | 52 | 60 |
| 21 | 49 | 67 | 77 | 42 | 58 | 67 |
| 22 | 55 | 75 | 86 | 48 | 65 | 75 |
| 23 | 62 | 83 | 94 | 54 | 73 | 83 |
| 24 | 69 | 91 | 104 | 61 | 81 | 91 |
| 25 | 76 | 100 | 113 | 68 | 89 | 100 |
| 26 | 84 | 110 | 124 | 75 | 98 | 110 |
| 27 | 92 | 119 | 134 | 83 | 107 | 119 |
| 28 | 101 | 130 | 145 | 91 | 116 | 130 |
| 29 | 110 | 140 | 157 | 100 | 126 | 140 |

Mann-Whitney U Critical Values

### Critical Values for 2-Tailed Mann-Whitney U Test for α = 0.05

| | $n_2$ | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $n_1$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 2 | - | - | - | - | - | - | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 |
| 3 | - | - | - | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 | 5 | 6 | 6 | 7 | 7 | 8 |
| 4 | - | - | 0 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 11 | 12 | 13 | 13 |
| 5 | - | 0 | 1 | 2 | 3 | 5 | 6 | 7 | 8 | 9 | 11 | 12 | 13 | 14 | 15 | 17 | 18 | 19 | 20 |
| 6 | - | 1 | 2 | 3 | 5 | 6 | 8 | 10 | 11 | 13 | 14 | 16 | 17 | 19 | 21 | 22 | 24 | 25 | 27 |
| 7 | - | 1 | 3 | 5 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 | 26 | 28 | 30 | 32 | 34 |
| 8 | 0 | 2 | 4 | 6 | 8 | 10 | 13 | 15 | 17 | 19 | 22 | 24 | 26 | 29 | 31 | 34 | 36 | 38 | 41 |
| 9 | 0 | 2 | 4 | 7 | 10 | 12 | 15 | 17 | 21 | 23 | 26 | 28 | 31 | 34 | 37 | 39 | 42 | 45 | 48 |
| 10 | 0 | 3 | 5 | 8 | 11 | 14 | 17 | 20 | 23 | 26 | 29 | 33 | 36 | 39 | 42 | 45 | 48 | 52 | 55 |
| 11 | 0 | 3 | 6 | 9 | 13 | 16 | 19 | 23 | 26 | 30 | 33 | 37 | 40 | 44 | 47 | 51 | 55 | 58 | 62 |
| 12 | 1 | 4 | 7 | 11 | 14 | 18 | 22 | 26 | 29 | 33 | 37 | 41 | 45 | 49 | 53 | 57 | 61 | 65 | 69 |
| 13 | 1 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 33 | 37 | 41 | 45 | 50 | 54 | 59 | 63 | 67 | 72 | 76 |
| 14 | 1 | 5 | 9 | 13 | 17 | 22 | 26 | 31 | 36 | 40 | 45 | 50 | 55 | 59 | 64 | 67 | 74 | 78 | 83 |
| 15 | 1 | 5 | 10 | 14 | 19 | 24 | 29 | 34 | 39 | 44 | 49 | 54 | 59 | 64 | 70 | 75 | 80 | 85 | 90 |
| 16 | 1 | 6 | 11 | 15 | 21 | 26 | 31 | 37 | 42 | 47 | 53 | 59 | 64 | 70 | 75 | 81 | 86 | 92 | 98 |
| 17 | 2 | 6 | 11 | 17 | 22 | 28 | 34 | 39 | 45 | 51 | 57 | 63 | 67 | 75 | 81 | 87 | 93 | 99 | 105 |
| 18 | 2 | 7 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 55 | 61 | 67 | 74 | 80 | 86 | 93 | 99 | 106 | 112 |
| 19 | 2 | 7 | 13 | 19 | 25 | 32 | 38 | 45 | 52 | 58 | 65 | 72 | 78 | 85 | 92 | 99 | 106 | 113 | 119 |
| 20 | 2 | 8 | 14 | 20 | 27 | 34 | 41 | 48 | 55 | 62 | 69 | 76 | 83 | 90 | 98 | 105 | 112 | 119 | 127 |

### Critical Values for 2-Tailed Mann-Whitney U Test for α = 0.01

| | $n_2$ | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $n_1$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 2 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | 0 | 0 |
| 3 | - | - | - | - | - | - | - | 0 | 0 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 | 3 |
| 4 | - | - | - | - | 0 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 5 | 5 | 6 | 6 | 7 | 8 |
| 5 | - | - | - | 0 | 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| 6 | - | - | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 9 | 10 | 11 | 12 | 13 | 15 | 16 | 17 | 18 |
| 7 | - | - | 0 | 1 | 3 | 4 | 6 | 7 | 9 | 10 | 12 | 13 | 15 | 16 | 18 | 19 | 21 | 22 | 24 |
| 8 | - | - | 1 | 2 | 4 | 6 | 7 | 9 | 11 | 13 | 15 | 17 | 18 | 20 | 22 | 24 | 26 | 28 | 30 |
| 9 | - | 0 | 1 | 3 | 5 | 7 | 9 | 11 | 13 | 16 | 18 | 20 | 22 | 24 | 27 | 29 | 31 | 33 | 36 |
| 10 | - | 0 | 2 | 4 | 6 | 9 | 11 | 13 | 16 | 18 | 21 | 24 | 26 | 29 | 31 | 34 | 37 | 39 | 42 |
| 11 | - | 0 | 2 | 5 | 7 | 10 | 13 | 16 | 18 | 21 | 24 | 27 | 30 | 33 | 36 | 39 | 42 | 45 | 46 |
| 12 | - | 1 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 31 | 34 | 37 | 41 | 44 | 47 | 51 | 54 |
| 13 | - | 1 | 3 | 7 | 10 | 13 | 17 | 20 | 24 | 27 | 31 | 34 | 38 | 42 | 45 | 49 | 53 | 56 | 60 |
| 14 | - | 1 | 4 | 7 | 11 | 15 | 18 | 22 | 26 | 30 | 34 | 38 | 42 | 46 | 50 | 54 | 58 | 63 | 67 |
| 15 | - | 2 | 5 | 8 | 12 | 16 | 20 | 24 | 29 | 33 | 37 | 42 | 46 | 51 | 55 | 60 | 64 | 69 | 73 |
| 16 | - | 2 | 5 | 9 | 13 | 18 | 22 | 27 | 31 | 36 | 41 | 45 | 50 | 55 | 60 | 65 | 70 | 74 | 79 |
| 17 | - | 2 | 6 | 10 | 15 | 19 | 24 | 29 | 34 | 39 | 44 | 49 | 54 | 60 | 65 | 70 | 75 | 81 | 86 |
| 18 | - | 2 | 6 | 11 | 16 | 21 | 26 | 31 | 37 | 42 | 47 | 53 | 58 | 64 | 70 | 75 | 81 | 87 | 92 |
| 19 | 0 | 3 | 7 | 12 | 17 | 22 | 28 | 33 | 39 | 45 | 51 | 56 | 63 | 69 | 74 | 81 | 87 | 93 | 99 |
| 20 | 0 | 3 | 8 | 13 | 18 | 24 | 30 | 36 | 42 | 46 | 54 | 60 | 67 | 73 | 79 | 86 | 92 | 99 | 105 |