10.1: Chi-Square Distribution

A χ^2 -distribution (chi-square, pronounced “ki-square”) is another special type of distribution for a continuous random variable. The sampling distribution for a variance and standard deviation follows a chi-square distribution.

Properties of the χ^2 -distribution density curve:

1. Right skewed starting at zero.
2. The center and spread of a χ^2 -distribution are determined by the degrees of freedom with a mean = *df* and standard deviation = \(\sqrt{2*df*}\).
3. Chi-square variables cannot be negative.
4. As the degrees of freedom increase, the χ^2 -distribution becomes normally distributed for df > 50. Figure 10-1 shows χ^2 -distributions for *df* of 2, 4, 10, and 30.
5. The total area under the curve is equal to 1, or 100%.

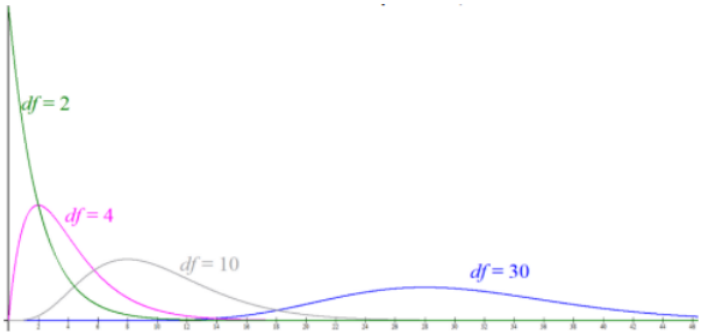

Figure 10-1

We will use the χ^2 -distribution for hypothesis testing later in this chapter. For now, we are just learning how to find a critical value χ^2_α.

The symbol χ^2_α is the critical value on the χ^2 -distribution curve with area 1 – α below the critical value and area α above the critical value, as shown below in Figure 10-2.

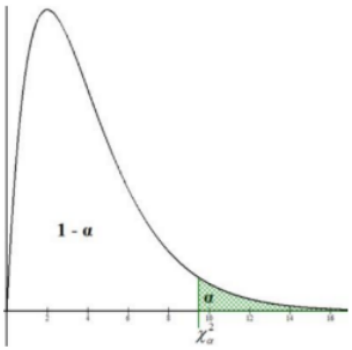

Figure 10-2

Use technology to compute the critical value for the χ^2 -distribution.

TI-84: Use the INVCHI2 program downloaded at Rachel Webb’s website: http://MostlyHarmlessStatistics.com. Start the program and enter the area α and the *df* when prompted.

TI-89: Go to the [Apps] **Stat/List Editor**, then select F5 [DISTR]. This will get you a menu of probability distributions. Arrow down to **Inverse > Inverse Chi-Square** and press [ENTER]. Enter the area 1 – α to the left of the χ value and the *df* into each cell. Press [ENTER].

Excel: =CHISQ.INV(1 – α, *df*) or =CHISQ.INV.RT(α, *df*)

Alternatively, use the following online calculator: https://homepage.divms.uiowa.edu/~mbognar/applets/chisq.html.

Compute the critical value χ^2_α for a $\alpha = 0.05$ and $df = 6$.

Solution

Start by drawing the curve and determining the area in the right-tail as shown in Figure 10-3. Then use technology to find the critical value.

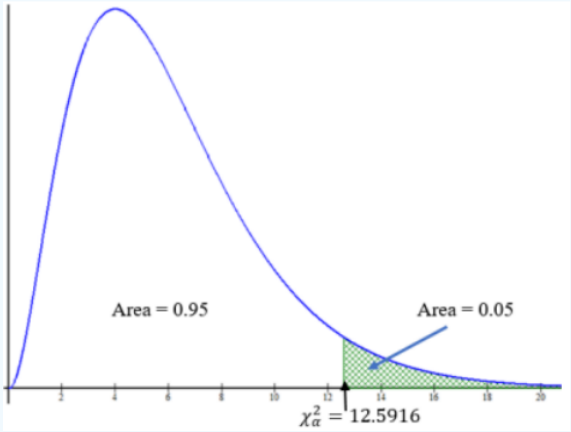

Figure 10-3

In Excel there are two options. Use =CHISQ.INV(area in left-tail,*df*) or right-tail =CHISQ.INV.RT(area in righttail,*df*). For this example, then we would have χ^2_α =CHISQ.INV(0.95,6) or =CHISQ.INV.RT(0.05,6) = 12.5916.

TI-89 use Distr > Inverse Chi-square with area 0.95 and *df* = 6