4.5: Independent Events

Two trials (or events or results of a random experiment) are independent trials if the outcome of one trial does not influence the outcome of the second trial. If two events are not independent, they are dependent events. For instance, if two coins are flipped they are independent since flipping one coin does not affect the outcome of the second coin.

Independent Events: If A and B are independent events, then $P(A \cap B) = P(A) \cdot P(B)$.

Be careful with this rule. You cannot just multiply probabilities to find an intersection unless you know they are independent. Also, do not confuse independent events with mutually exclusive events. Two events are mutually exclusive when the $P(A \cap B) = 0$.

If a random experiment consists of flipping a coin twice, find the probability of getting heads twice in a row.

Solution

The event of getting a head on the first flip is independent of getting a head on the second flip since the probability does not change with each flip of the coin. Thus, using the multiplication rule of independent events, P(Both coins are heads) = P(1st coin is a head)·P(2nd coin is a head) = $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} = 0.25$.

The probability of Apple stock rising is 0.3, the probability of Boeing stock rising is 0.4. Assume Apple and Boeing stocks are independent. What is the probability that neither stock rises?

Solution

Let A = Apple stock and B = Boeing stock. Since A and B are independent the probability of both stocks rising at the same time is $P(A \cap B) = 0.3 \cdot 0.4 = 0.12$. Neither is the complement to either. P(Not Either) = 1 – P(A U B) = 1 – (P(A) + P(B) – $P(A \cap B)$) = 1 – (0.3 + 0.4 + 0.12) = 1 – 0.58 = 0.42.

The probability that a student has their own laptop is 0.78. If three students are randomly selected, what is the probability that at least one owns a laptop?

Solution

There is an assumption that the three students are not related and that the probability of one owning a laptop is independent of the other people owning a laptop. The probability of none owning a laptop is (1 – 0.78)3 = 0.0106. The probability of at least one is the same as 1 – P(None) = 1 – 0.0106 = 0.9894.

When two events are dependent, you cannot simply multiply their corresponding probabilities to find their intersection. You will need to use the General Multiplication Rule discussed in the next section.