7.3: The Area of a Triangle

For each of the triangles in Figure $7.3.1$, side AB is called the base and CD is called the **height** or **altitude** drawn to this base. The base can be any state of the triangle though it is usually chosen to be the side on which the triangle appears to be resting. The height is the line drawn perpendicular to the base from the opposite vertex. Note that the height may fall outside the triangle, If the triangle is obtuse, and that the height may be one of the legs, if the triangle is a right triangle.

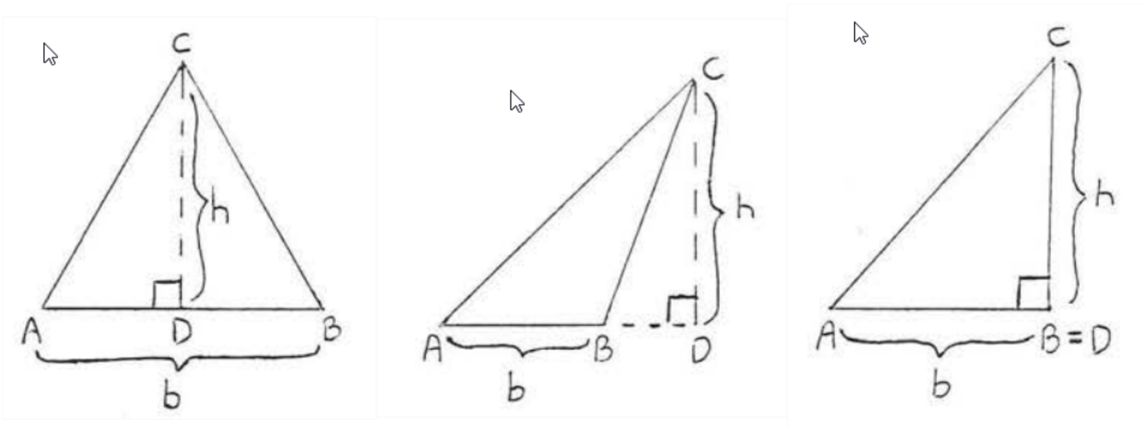


Figure $7.3.1$: Triangles with base b and height h.

Theorem 7.3.1

The area of a triangle is equal to one-half of its base times its height.

$$A = \frac{1}{2}bh \tag{7.3.1}$$

Proof

For each of the triangles illustrated in Figure $7.3.1$, draw AE and CE so that $ABCE$ is a parallelogram (Figure $PageIndex2$). $\triangle ABC \cong \triangle CEA$ so area of $\triangle ABC =$ area of $\triangle CEA$. Therefore area of $\triangle ABC = \frac{1}{2}$ area of parallelogram $ABCE = \frac{1}{2}bh$.

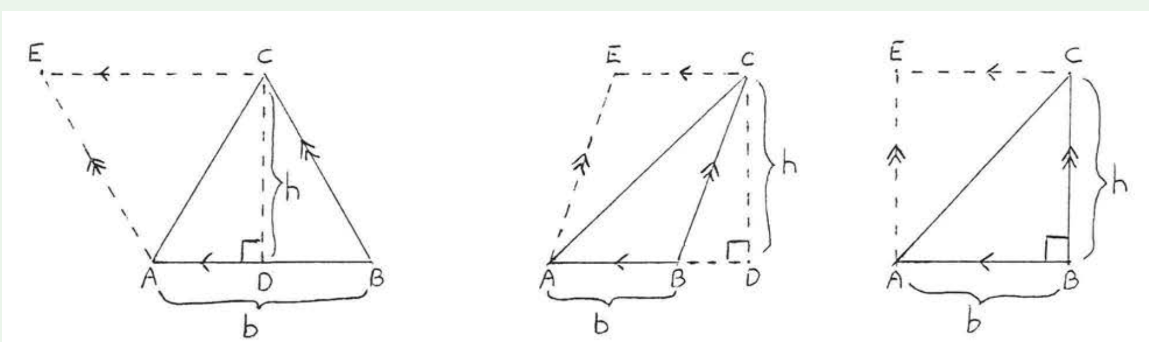
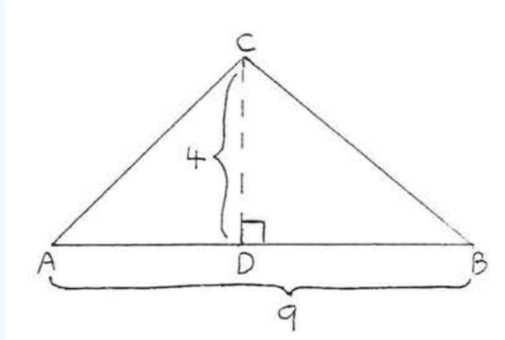


Figure $7.3.2$ Draw AE and CE so that $ABCE$ is a parallelogram.

Example 7.3.1

Find the area:



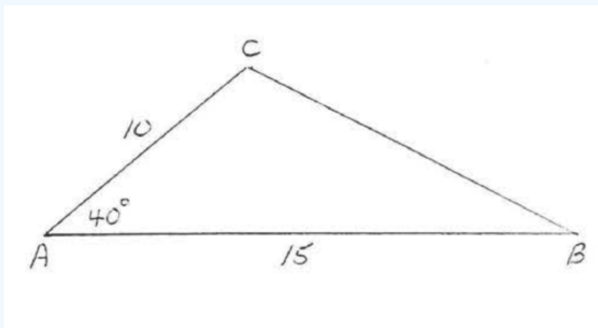
Solution

$A = \frac{1}{2}bh = \frac{1}{2}(9)(4) = \frac{1}{2}(36) = 18.$

Answer: 18.

Example 7.3.2

Find the area to the nearest tenth:



Solution

Draw the height h as shown in Figure $7.3.3$

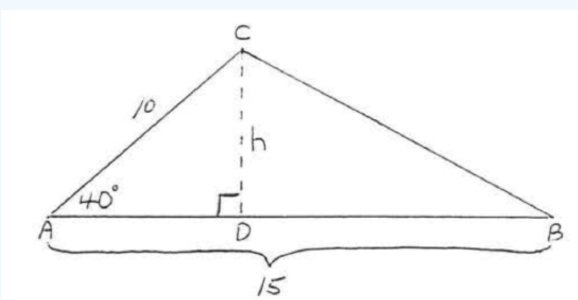


Figure $7.3.3$ Draw height h.

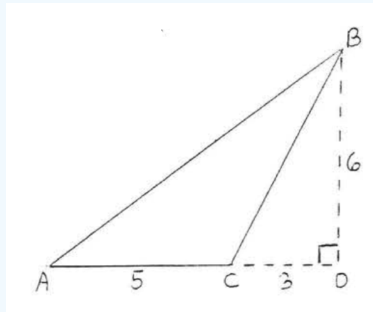
$$\begin{aligned} \sin 40^{\circ} &= \frac{h}{10} \\ .6428 &= \frac{h}{10} \\ (10)(.6428) &= \frac{h}{10}(10) \\ 6.428 &= h \end{aligned} \tag{7.3.2}$$

Area = $\frac{1}{2}bh = \frac{1}{2}(15)(6.428) = \frac{1}{2}(96.420) = 48.21 = 48.2$

Answer: $A = 48.2$

Example 7.3.3

Find the area and perimeter:



Solution

$A = \frac{1}{2}bh = \frac{1}{2}(5)(6) = \frac{1}{2}(30) = 15.$

To find AB and BC we use the Pythagorean theorem:

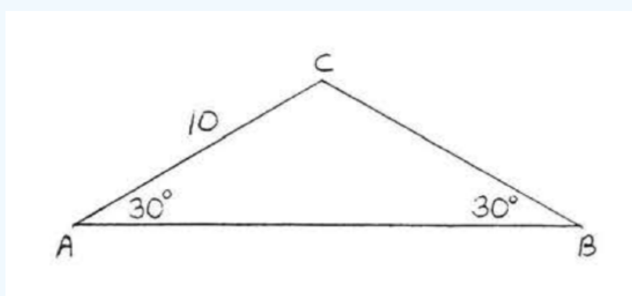
$$\begin{aligned} \mathrm{AD}^2 + \mathrm{BD}^2 &= \mathrm{AB}^2 & \mathrm{CD}^2 + \mathrm{BD}^2 &= \mathrm{BC}^2 \\ 8^2 + 6^2 &= \mathrm{AB}^2 & 3^2 + 6^2 &= \mathrm{BC}^2 \\ 64 + 36 &= \mathrm{AB}^2 & 9 + 36 &= \mathrm{BC}^2 \\ 100 &= \mathrm{AB}^2 & 45 &= \mathrm{BC}^2 \\ 10 &= \mathrm{AB} & \mathrm{BC} = \sqrt{45} &= \sqrt{9}\sqrt{5} = 3\sqrt{5} \end{aligned}$$

Perimeter = $AB + AC + BC = 10 + 5 + 3\sqrt{5} = 15 + 3\sqrt{5}$

Answer: $A = 15, P = 15 + 3\sqrt{5}$.

Example 7.3.4

Find the area and perimeter:



Solution

$\angle A = \angle B = 30^\circ$ so $\triangle ABC$ is isosceles with $BC = AC = 10$. Draw height h as in Figure $7.3.4$.

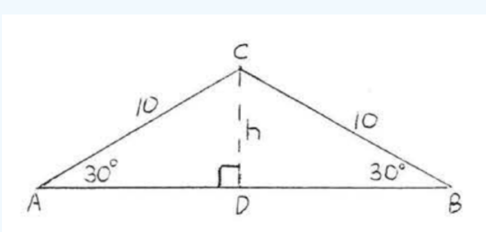


Figure $7.3.4$ Draw height h.

$\triangle ACD$ is a $30^\circ - 60^\circ - 90^\circ$ triangle hence

$$\begin{aligned}\text{hypotenuse} &= 2(\text{short leg})\\ 10 &= 2h\\ 5 &= h\\ \text{long leg} &= (\text{short leg})(\sqrt{3})\\ AD &= h\sqrt{3} = 5\sqrt{3}.\end{aligned} \tag{7.3.3}$$

Similarly $BD = 5\sqrt{3}$.

Area = $\frac{1}{2}bh = \frac{1}{2}(5\sqrt{3}+5\sqrt{3})(5) = \frac{1}{2}(10\sqrt{3})(5) = \frac{1}{2}(50\sqrt{3}) = 25\sqrt{3}$.

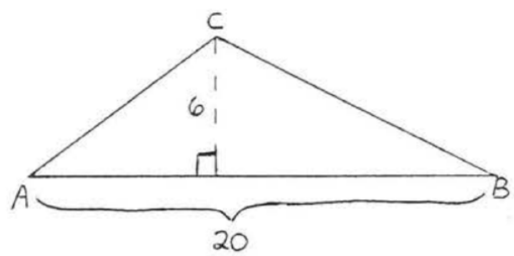
Perimeter = $10+10+5\sqrt{3}+5\sqrt{3} = 20+10\sqrt{3}$.

Answer: $A = 25\sqrt{3}, P = 20+10\sqrt{3}$.

Problems

1 - 4. Find the area of $\triangle ABC$:

1.



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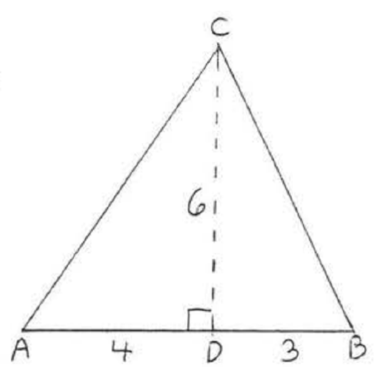

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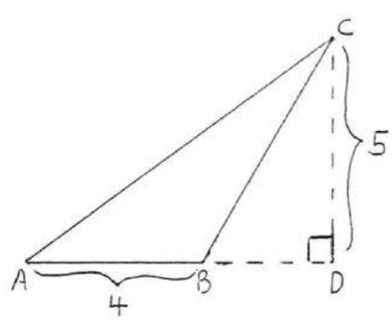

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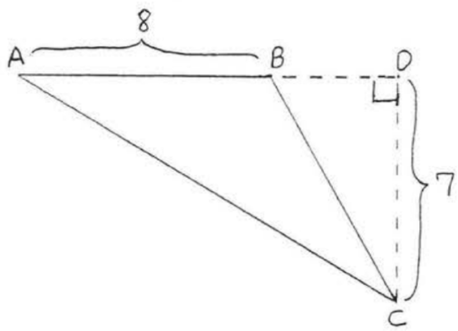
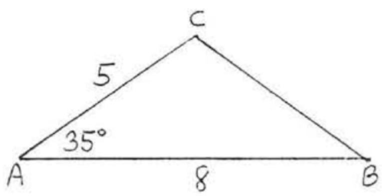


5 - 6. Find the area to the nearest tenth:

5.



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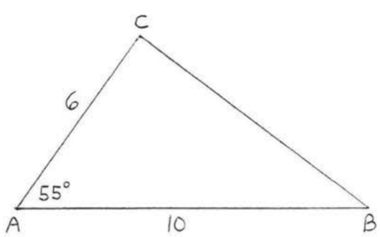
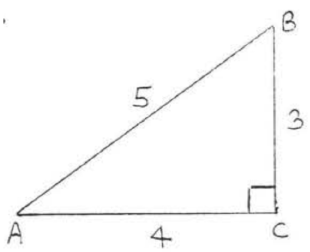


7 - 20. Find the area and perimeter of $\triangle ABC$:

7.



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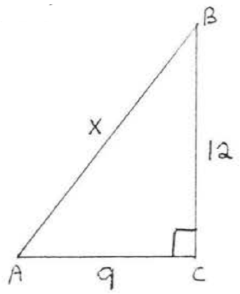

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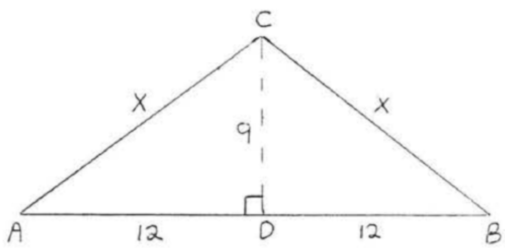

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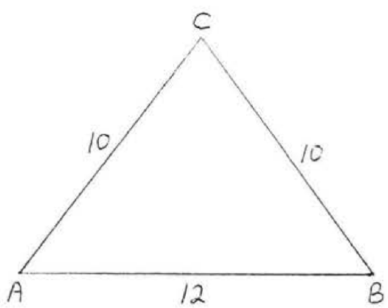

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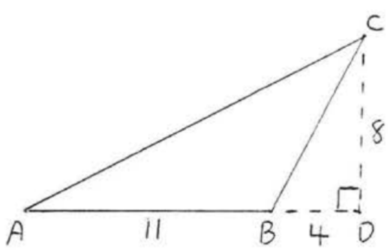

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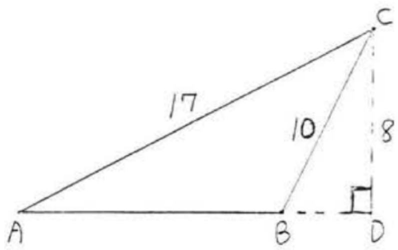

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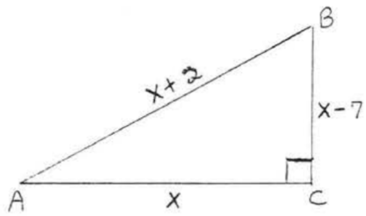

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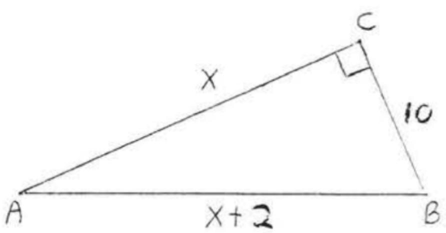

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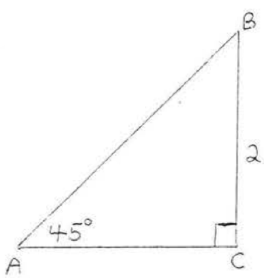

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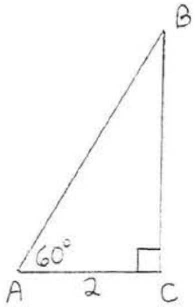

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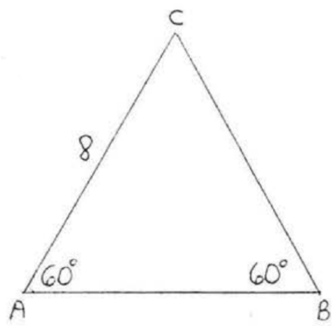

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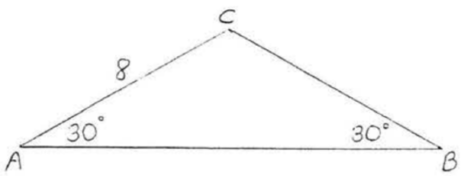
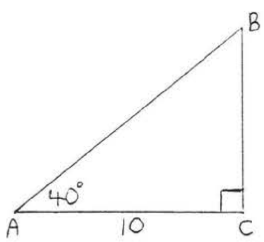


19 - 20. Find the area and perimeter to the nearest tenth:

19.



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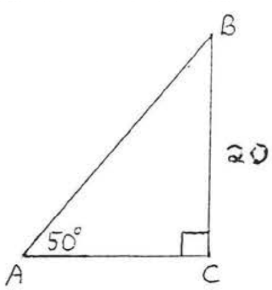
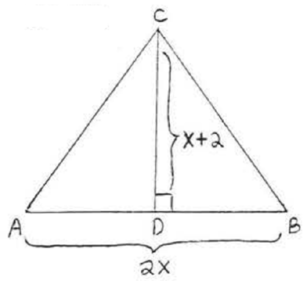
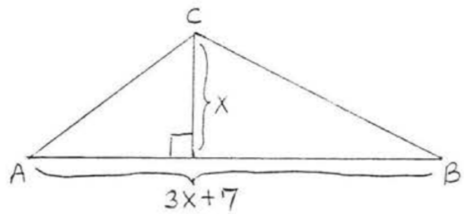


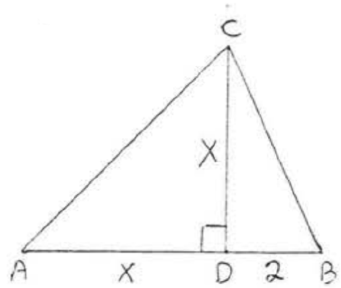
21. Find x if the area of $\triangle ABC$ is 35:



22. Find x if the area of $\triangle ABC$ is 24.



23. Find x if the area of $\triangle ABC$ is 12:



24. Find x if the area of $\triangle ABC$ is 108: