3.6: Powers and Roots

Learning Outcomes

1. Raise a number to a power using technology.
2. Take the square root of a number using technology.
3. Apply the order of operations when there is root or a power.

It can be a challenge when we first try to use technology to raise a number to a power or take a square root of a number. In this section, we will go over some pointers on how to successfully take powers and roots of a number. We will also continue our practice with the order of operations, remembering that as long as there are no parentheses, exponents always come before all other operations. We will see that taking a power of a number comes up in probability and taking a root comes up in finding standard deviations.

Powers

Just about every calculator, computer, and smartphone can take powers of a number. We just need to remember that the symbol "^" is used to mean "to the power of". We also need to remember to use parentheses if we need to force other arithmetic to come before the exponentiation.

Example 3.6.1

Evaluate: 1.04^5 and round to two decimal places.

Solution

This definitely calls for the use of technology. Most calculators, whether hand calculators or computer calculators, use the symbol "^" (shift 6 on the keyboard) for exponentiation. We type in:

$$1.04^5 = 1.2166529$$

We are asked to round to two decimal places. Since the third decimal place is a 6 which is 5 or greater, we round up to get:

$$1.04^5 \approx 1.22$$

Example 3.6.2

Evaluate: $2.8^{5.3\times 0.17}$ and round to two decimal places.

Solution

First note that on a computer we use "*" (shift 8) to represent multiplication. If we were to put in 2.8 ^ 5.3 * 0.17 into the calculator, we would get the wrong answer, since it will perform the exponentiation before the multiplication. Since the original question has the multiplication inside the exponent, we have to force the calculator to perform the multiplication first. We can ensure that multiplication occurs first by including parentheses:

$$2.8^{5.3\times 0.17} = 2.52865$$

Now round to decimal places to get:

$$2.8^{5.3\times 0.17} \approx 2.53$$

Example 3.6.3

If we want to find the probability that if we toss a six sided die five times that the first two rolls will each be a 1 or a 2 and the last three die rolls will be even, then the probability is:

$$\left(\frac{1}{3}\right)^2 \times \left(\frac{1}{2}\right)^3$$

What is this probability rounded to three decimal places?

Solution

We find:

$$(1/3)^2(1/2)^3 \approx 0.013888889$$

Now round to three decimal places to get

$$\left(\frac{1}{3}\right)^2 \times \left(\frac{1}{2}\right)^3 \approx 0.014$$

Square Roots

Square roots come up often in statistics, especially when we are looking at standard deviations. We need to be able to use a calculator or computer to compute a square root of a number. There are two approaches that usually work. The first approach is to use the $\sqrt{\ }$ symbol on the calculator if there is one. For a computer, using sqrt() usually works. For example if you put 10*sqrt(2) in the Google search bar, it will show you 14.1421356. A second way that works for pretty much any calculator, whether it is a hand held calculator or a computer calculator, is to realize that the square root of a number is the same thing as the number to the 1/2 power. In order to not have to wrap 1/2 in parentheses, it is easier to type in the number to the 0.5 power.

Example 3.6.3

Evaluate $\sqrt{42}$ and round your answer to two decimal places.

Solution

Depending on the technology you are using you will either enter the square root symbol and then the number 42 and then close the parentheses if they are presented and then hit enter. If you are using a computer, you can use sqrt(42). The third way that will work for both is to enter:

$$42^{0.5} \approx 6.4807407$$

You must then round to two decimal places. Since 0 is less than 5, we round down to get:

$$\sqrt{42} \approx 6.48$$

Example 3.6.4

The "z-score" is for the value of 28 for a sampling distribution with sample size 60 coming from a population with mean 28.3 and standard deviation 5 is defined by:

$$z = \frac{28 - 28.3}{\frac{5}{\sqrt{60}}}$$

Find the z-score rounded to two decimal places.

Solution

We have to be careful about the order of operations when putting it into the calculator. We enter:

$$(28 - 28.3)/(5/60\hat{}0.5) = -0.464758$$

Finally, we round to 2 decimal places. Since 4 is smaller than 5, we round down to get:

$$z = \frac{28 - 28.3}{\frac{5}{\sqrt{60}}} = -0.46$$

Exercise

The standard error, which is an average of how far sample means are from the population mean is defined by:

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

where $\sigma_{\bar{x}}$ is the standard error, σ is the standard deviation, and n is the sample size. Find the standard error if the population standard deviation, σ, is 14 and the sample size, n, is 11.

- Square Root on the TI-83plus and TI-84 family of Calculators
- Square Roots with a Computer
