3.2: The Addition Rules of Probability

Your favorite professional basketball team either won or lost their last game. Winning and losing are mutually exclusive events.

Mutually Exclusive Events

A and B are mutually exclusive events (or disjoint events) if they **cannot** occur at the same time. This means that A and B do not share any outcomes and $P(\mathrm{A\ AND\ B}) = 0$.

For example, suppose the sample space

$$S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}.$$

Let $\mathrm{X} = \{1, 2, 3, 4, 5\}$, $\mathrm{Y} = \{4, 5, 6, 7, 8\}$, and $\mathrm{Z} = \{7, 9\}$. Events X and Y both have 4 and 5. Thus, $\mathrm{X\ AND\ Y} = \{4, 5\}$.

Because there are two shared outcomes from the sample space S, the probability of X AND Y is

$$P(\mathrm{X\ AND\ Y}) = \frac{2}{10}$$

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Since $P(\mathrm{X\ AND\ Y})$ is not equal to zero, X and Y **are not** mutually exclusive.

However, events X and Z have no outcomes (numbers) in common. So, $P(\mathrm{X\ AND\ Z}) = \frac{0}{10} = 0$. Therefore, X and Y **are** mutually exclusive.

The Addition Rule of Probability

The probability of two mutually exclusive events A OR B (two events that share no outcomes) is

$$P(A \text{ OR } B) = P(A) + P(B)$$

The probability of two **non**-mutually exclusive events A OR B (two events that share outcomes) is

$$P(A \text{ OR } B) = P(A) + P(B) - P(A \text{ AND } B)$$

Using the example from above, where the sample space $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and events $\mathrm{X} = \{1, 2, 3, 4, 5\}$, $\mathrm{Y} = \{4, 5, 6, 7, 8\}$, and $\mathrm{Z} = \{7, 9\}$.

Since events X and Z are mutually exclusive then the probability of X OR Z

$$P(X \text{ OR } Z) = P(X) + P(Z) = \frac{5}{10} + \frac{2}{10} = \frac{7}{10}. \tag{3.2.1}$$

Since events X and Y are **not** mutually exclusive then the probability of X OR Y

$$P(X \text{ OR } Y) = P(X) + P(Y) - P(X \text{ AND } Y) = \frac{5}{10} + \frac{5}{10} - \frac{2}{10} = \frac{8}{10}. \tag{3.2.2}$$

Below we will see our first **contingency table**, which is a table with categories in both the horizontal and vertical direction. As you see below, the contingency table describes cell phone users versus speeding violations.

Example 3.2.1

Suppose a study of speeding violations and drivers who use cell phones produced the following fictional data:

	Speeding violation in the last year	**No speeding violation in the last year**	**Total**
Cell phone user	25	280	305
Not a cell phone user	45	405	450

	Speeding violation in the last year	No speeding violation in the last year	Total
Total	70	685	755

The total number of people in the sample is 755. The row totals are 305 and 450. The column totals are 70 and 685. Notice that 305 + 450 = 755 and 70 + 685 = 755.

Calculate the following probabilities using the table.

a. Find $P(\text{Person is a cell phone user})$.
b. Find $P(\text{person had no violation in the last year})$.
c. Find $P(\text{Person had no violation in the last year AND was a cell phone user})$.
d. Find $P(\text{Person is a cell phone user OR person had no violation in the last year})$.

Answer

a. $\frac{\text{number of cell phone users}}{\text{total number in study}} = \frac{305}{755}$
b. $\frac{\text{number that had no violation}}{\text{total number in study}} = \frac{685}{755}$
c. $\frac{280}{755}$
d. $\left(\frac{305}{755} + \frac{685}{755}\right) - \frac{280}{755} = \frac{710}{755}$

Exercise 3.2.2

Table shows the number of athletes who stretch before exercising and how many had injuries within the past year.

	Injury in last year	No injury in last year	Total
Stretches	55	295	350
Does not stretch	231	219	450
Total	286	514	800

a. What is $P(\text{athlete stretches before exercising})$?
b. What is $P(\text{athlete stretches before exercising and no injury in the last year})$?
c. What is $P(\text{athlete stretches before exercising or no injury in the last year})$?

Answer

a. $P(\text{athlete stretches}) = \frac{350}{800} = 0.4375$
b. $P(\text{athlete stretches AND no injury in the last year}) = \frac{295}{800} = 0.3688$
c. $P(\text{athlete stretches OR no injury in the last year})$
$= P(\text{athlete stretches }) + P(\text{no injurty in the last year}) - P(\text{athlete stretches AND no injury in the last year})$
$= \frac{350}{800} + \frac{514}{800} - \frac{295}{800}$
$= \frac{569}{800} = 0.7113$

Example 3.2.3

Table shows a random sample of 100 hikers and the areas of hiking they prefer.

Hiking Area Preference

Sex	The Coastline	Near Lakes and Streams	On Mountain Peaks	Total
Female	18	16	___	45
Male	___	___	14	55
Total	___	41	___	___

a. Complete the table.
b. Find the probability that a person is female or prefers hiking on mountain peaks. Let F = being female, and let P = prefers mountain peaks.
   1. Find $P(\mathrm{F})$.
   2. Find $P(\mathrm{M})$.
   3. Find $P(\mathrm{F\ AND\ M})$.
   4. Find $P(\mathrm{F\ OR\ M})$.

Answers

a.

Hiking Area Preference

Sex	The Coastline	Near Lakes and Streams	On Mountain Peaks	Total
Female	18	16	**11**	45
Male	**16**	**25**	14	55
Total	**34**	41	**25**	**100**

b.

1. $P(\mathrm{F}) = \frac{45}{100}$
2. $P(\mathrm{M}) = \frac{25}{100}$
3. $P(\mathrm{F\ AND\ M}) = \frac{11}{100}$
4. $P(\mathrm{F\ OR\ M}) = P(\mathrm{F}) + P(\mathrm{M}) - P(\mathrm{F\ AND\ M}) = \frac{45}{100} + \frac{25}{100} - \frac{11}{100} = \frac{59}{100}$

Complement of an event

The **complement** of event A is denoted A' (read "A prime"). A' consists of all outcomes that are **NOT** in A. Notice that

$$P(\mathrm{A}) + P(\mathrm{A}') = 1.$$

In other words,

$$P(\mathrm{A}') = 1 - P(\mathrm{A})$$

For example, let $\mathrm{S} = \{1, 2, 3, 4, 5, 6\}$and let $\mathrm{A} = 1, 2, 3, 4$. Then, $\mathrm{A}' = 5, 6$ and $P(A) = \frac{4}{6}$, $P(\mathrm{A}') = \frac{2}{6}$, and

$$P(\mathrm{A}) + P(\mathrm{A}') = \frac{4}{6} + \frac{2}{6} = 1.$$

Review

There are several tools you can use to help organize and sort data when calculating probabilities. Contingency tables help display data and are particularly useful when calculating probabilities that have multiple dependent variables.

Use the following information to answer the next four exercises. Table shows a random sample of musicians and how they learned to play their instruments.

Gender	Self-taught	Studied in School	Private Instruction	Total
Female	12	38	22	72
Male	19	24	15	58
Total	31	62	37	130

Exercise 3.2.1

Find P(musician is a female).

Exercise 3.2.2

Find $P(\text{musician is a male AND had private instruction})$.

Answer

$$P(\text{musician is a male AND had private instruction}) = \frac{15}{130} = \frac{3}{26} = 0.12$$

Exercise 3.2.3

Find $P(\text{musician is a female OR is self taught})$.

Answer

\(P(\text{musician is a female OR is self taught}) \\

= P(\text{musician is a female}) + P(\text{self taught}) - P(\text{musician is a female OR is self taught})\\

=\frac{72}{130} + \frac{31}{130} - \frac{12}{130}\\

=\frac{91}{130}\)

Exercise 3.2.4

Are the events “being a female musician” and “learning music in school” mutually exclusive events?

Answer

The events are not mutually exclusive. It is possible to be a female musician who learned music in school.

References

1. “United States: Uniform Crime Report – State Statistics from 1960–2011.” The Disaster Center. Available online at http://www.disastercenter.com/crime/ (accessed May 2, 2013).

Glossary

mutually exclusive (or disjoint) events

events that cannot happen at the same time

contingency table

the method of displaying a frequency distribution as a table with rows and columns to show how two variables may be dependent (contingent) upon each other; the table provides an easy way to calculate conditional probabilities.

complement of an event

The complement of event A consists of all outcomes that are NOT in A.
