8.4: Hypothesis Test on a Single Standard Deviation

A test of a single standard deviation assumes that the underlying distribution is **normal**. The null and alternative hypotheses are stated in terms of the population standard deviation (or population variance). The test statistic is:

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} \tag{8.4.1}$$

where:

- n is the the total number of data
- s^2 is the sample variance
- σ^2 is the population variance

The requirements to be able to perform a hypothesis test on a population standard deviation are:

- the sample must be obtained from a simple random sample or from a randomized experiment
- the population has a normal distribution

You may think of s as the random variable in this test. The number of degrees of freedom is $df = n-1$. **A test of a single standard deviation may be right-tailed, left-tailed, or two-tailed.** The next example will show you how to set up the null and alternative hypotheses. The null and alternative hypotheses contain statements about the population variance.

Example 8.4.1

Math instructors are not only interested in how their students do on exams, on average, but how the exam scores vary. To many instructors, the standard deviation may be more important than the average.

Suppose a math instructor believes that the standard deviation for his final exam is five points. One of his best students thinks otherwise. The student claims that the standard deviation is more than five points. If the student were to conduct a hypothesis test, what would the null and alternative hypotheses be?

Answer

- $H_0 : \sigma = 5$
- $H_a : \sigma > 5$

Exercise 8.4.2

A SCUBA instructor wants to record the collective depths each of his students dives during their checkout. He is interested in how the depths vary, even though everyone should have been at the same depth. He believes the standard deviation is three feet. His assistant thinks the standard deviation is less than three feet. Suppose the instructor finds a random sample of 25 SCUBA students and found that the sample standard deviation is 2.8 feet.

With a significance level of 5%, test the claim that the diving depths is less than 3 feet.

Answer

$H_0 : \sigma = 3$

$H_a : \sigma < 3$

The word **"less"** tells you this is a left-tailed test.

Distribution for the test: χ^2_{24}, where $n =$ the number of customers sampled $df = n-1 = 25-1 = 24$

Calculate the test statistic (Equation 8.4.1):

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(25-1)(2.8)^2}{3^2} = 20.91$$

where $n = 25$, $s = 2.8$, and $\sigma = 3$.

Graph:

Figure 8.4.1 .

Probability statement: p-value $= P(\chi^2 < 20.91) = 0.356$

In `2nd DISTR` , use `7:χ2cdf` . The syntax is `(lower, upper, df)` for the parameter list. For Example, `χ2cdf(-1E99,20.91,24)` . The p-value $= 0.356$.

Compare α and the p-value:

Given $\alpha = 0.05, p\text{-value} = 0.356\}$, then $\alpha < p\text{-value}$

Make a decision: Since $\alpha < p$-value, fail to reject H_0. This means that you are not reject $\sigma = 3$. In other words, you do think the standard deviation of diving depth less than 3 feet.

Conclusion: At a 5% level of significance, from the data, there is not sufficient evidence to conclude that a SCUBA students' collective depth is less than 3 feet.

References

1. “AppleInsider Price Guides.” Apple Insider, 2013. Available online at http://appleinsider.com/mac_price_guide (accessed May 14, 2013).
2. Data from the World Bank, June 5, 2012.

Review

To test variability, use the chi-square test of a single variance. The test may be left-, right-, or two-tailed, and its hypotheses are always expressed in terms of the variance (or standard deviation).

Formula Review

$\chi^2 = \frac{(n-1)\cdot s^2}{\sigma^2}$ Test of a single variance statistic where:

n : sample size

s : sample standard deviation

σ : population standard deviation

$df = n – 1 \text{Degrees of freedom}$

Test of a Single Standard Deviation

- Use the test to determine standard deviation.
- The degrees of freedom is the $\text{number of samples} - 1$.
- The test statistic is $\frac{(n-1) \cdot s^2}{\sigma^2}$, where $n = \text{the total number of data}$, $s^2 = \text{sample variance}$, and $\sigma^2 = \text{population variance}$.
- The test may be left-, right-, or two-tailed.
