7.2: Confidence Interval for a Proportion

During an election year, we see articles in the newspaper that state confidence intervals in terms of proportions or percentages. For example, a poll for a particular candidate running for president might show that the candidate has 40% of the vote within three percentage points (if the sample is large enough). Often, election polls are calculated with 95% confidence, so, the pollsters would be 95% confident that the true proportion of voters who favored the candidate would be between 0.37 and 0.43: (0.40 – 0.03,0.40 + 0.03).

Investors in the stock market are interested in the true proportion of stocks that go up and down each week. Businesses that sell personal computers are interested in the proportion of households in the United States that own personal computers. Confidence intervals can be calculated for the true proportion of stocks that go up or down each week and for the true proportion of households in the United States that own personal computers.

The build a confidence interval for population proportion p, we use:

$$\hat{p} - z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} < p < \hat{p} + z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}. \tag{7.2.1}$$

where

- $\hat{p} = \dfrac{x}{n}$, the estimated proportion of successes $\hat{p}$ is a point estimate for p, the true proportion.)
- $x =$ the number of successes
- $n =$ the size of the sample
- $z_{\frac{\alpha}{2}} \cdot \sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$ is called the **margin of error**

In the margin of error formula, the sample proportions $\hat{p}$ and 1-{\hat p} are estimates of the unknown population proportions p and 1-p. The estimated proportions $\hat{p}$ and $1-\hat{p}$ are used because p and $1-\hat{p}$ are not known. The sample proportions $\hat{p}$ and $1-\hat{p}$ are calculated from the data: $\hat{p}$ is the estimated proportion of successes, and $1-\hat{p}$ is the estimated proportion of failures.

Example 7.2.1

Suppose that a market research firm is hired to estimate the percent of adults living in a large city who have cell phones. Five hundred randomly selected adult residents in this city are surveyed to determine whether they have cell phones. Of the 500 people surveyed, 421 responded yes - they own cell phones. Using a 95% confidence level, compute a confidence interval estimate for the true proportion of adult residents of this city who have cell phones.

Answer

- The first solution is step-by-step (Solution A).
- The second solution uses a function of the TI-83, 83+, or 84 calculators (Solution B).

Solution A

- $n = 500$
- $x =$ the number of successes $= 421$

$$\hat{p} = \frac{x}{n} = \frac{421}{500} = 0.842$$

- $\hat{p} = 0.842$ is the sample proportion; this is the point estimate of the population proportion.

$$1 - \hat{p} = 1 - 0.842 = 0.158$$

Since the confidence level $CL = 0.95$, then

$$\alpha = 1 - CL = 1 - 0.95 = 0.05$$

$$\text{So, } \frac{\alpha}{2} = 0.025.$$

Then

$$z_{\frac{\alpha}{2}} = z_{0.025} = 1.96$$

Use the TI-83, 83+, or 84+ calculator command invNorm(0.975,0,1) to find $z_{0.025}$. Remember that the area to the right of $z_{0.025}$ is 0.025 and the area to the left of $z_{0.025}$ is 0.975. This can also be found using appropriate commands on other calculators, using a computer, or using a Standard Normal probability table.

$$\text{margin of error} = z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$= 1.96 \cdot \sqrt{\frac{(0.842)(0.158)}{500}} = 0.032$$

$$\hat{p} - \text{margin of error} = 0.842 - 0.032 = 0.81$$

$$\hat{p} + \text{margin of error} = 0.842 + 0.032 = 0.874$$

The confidence interval for the true binomial population proportion is $(\hat{p} - \text{margin of error}, \hat{p} + \text{margin of error}) = (0.810, 0.874)$.

Interpretation

We estimate with 95% confidence that between 81% and 87.4% of all adult residents of this city have cell phones.

Explanation of 95% Confidence Level

Ninety-five percent of the confidence intervals constructed in this way would contain the true value for the population proportion of all adult residents of this city who have cell phones.

Solution B

Press `STAT` and arrow over to `TESTS` .

Arrow down to `A:1-PropZint` . Press `ENTER` .
Arrow down to xx and enter 421.
Arrow down to nn and enter 500.
Arrow down to `C-Level` and enter .95.
Arrow down to `Calculate` and press `ENTER` .
The confidence interval is (0.81003, 0.87397).

Example 7.2.2

For a class project, a political science student at a large university wants to estimate the percent of students who are registered voters. He surveys 500 students and finds that 300 are registered voters. Compute a 90% confidence interval for the true percent of students who are registered voters, and interpret the confidence interval.

Answer

- The first solution is step-by-step (Solution A).
- The second solution uses a function of the TI-83, 83+, or 84 calculators (Solution B).

Solution A

- $x = 300$ and
- $n = 500$

$$\hat{p} = \frac{x}{n} = \frac{300}{500} = 0.600$$

$$1 - \hat{p} = 1 - 0.600 = 0.400$$

Since $CL = 0.90$, then

$$\alpha = 1 - CL = 1 - 0.90 = 0.10$$

$$\text{So, } \frac{\alpha}{2} = 0.05.$$

$$z\frac{\alpha}{2} = z_{0.05} = 1.645$$

Use the TI-83, 83+, or 84+ calculator command invNorm(0.95,0,1) to find $z_{0.05}$. Remember that the area to the right of $z_{0.05}$ is 0.05 and the area to the left of $z_{0.05}$ is 0.95. This can also be found using appropriate commands on other calculators, using a computer, or using a standard normal probability table.

$$\text{margin of error} = z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$= 1.645 \cdot \sqrt{\frac{(0.60)(0.40)}{500}} = 0.036$$

$$\hat{p} - \text{ margin of error } = 0.60 - 0.036 = 0.564$$

$$\hat{p} + \text{margin of error} = 0.60 + 0.036 = 0.636$$

The confidence interval for the true binomial population proportion is $(\hat{p} - \text{margin of error}, \hat{p} + \text{margin of error}) = (0.564, 0.636)$.

Interpretation

- We estimate with 90% confidence that the true percent of all students that are registered voters is between 56.4% and 63.6%.
- Alternate Wording: We estimate with 90% confidence that between 56.4% and 63.6% of ALL students are registered voters.

Explanation of 90% Confidence Level

Ninety percent of all confidence intervals constructed in this way contain the true value for the population percent of students that are registered voters.

Solution B

Press `STAT` and arrow over to `TESTS` .

Arrow down to `A:1-PropZint` . Press `ENTER` .
Arrow down to xx and enter 300.
Arrow down to nn and enter 500.
Arrow down to `C-Level` and enter 0.90.
Arrow down to `Calculate` and press `ENTER` .

The confidence interval is (0.564, 0.636).

Example 7.2.3

To estimate the proportion of students at a large college who are female, a random sample of 120 students is selected. There are 69 female students in the sample. Construct a 90% confidence interval for the proportion of all students at the college who are female.

Solution A

The proportion of students in the sample who are female is

$$\hat{p} = 69/120 = 0.575$$

Confidence level 90% means that $\alpha = 1 - 0.90 = 0.10$ so $\alpha/2 = 0.05$. From the last line of Figure 7.1.6 we obtain $z_{0.05} = 1.645$.

Thus

$$\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.575 \pm 1.645\sqrt{\frac{(0.575)(0.425)}{120}} = 0.575 \pm 0.074$$

One may be 90% confident that the true proportion of all students at the college who are female is contained in the interval $(0.575 - 0.074, 0.575 + 0.074) = (0.501, 0.649)$.

Solution B

Press STAT and arrow over to TESTS.

Arrow down to A:1-PropZint. Press ENTER.

Arrow down to x and enter 69.

Arrow down to n and enter 120.
Arrow down to C-Level and enter 0.90.
Arrow down to Calculate and press ENTER.

The confidence interval is (0.501,0.649).

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