3.3: Multiplication Rule for Independent Events

On days that your favorite basketball team plays, does it affect your ability to find parking on campus? If not, then we call these two events independent.

Independent Events

Two events are independent if the following are true:

- For the probably of event A given event B, $P(\text{A}|\text{B}) = P(\text{A})$
- For the probably of event B given event A, $P(\text{B}|\text{A}) = P(\text{B})$
- $P(\text{A AND B}) = P(\text{A})P(\text{B})$

Two events A and B are independent if the knowledge that one occurred does not affect the chance the other occurs. For example, the outcomes of two roles of a fair die are independent events. The outcome of the first roll does not change the probability for the outcome of the second roll. To show two events are independent, you must show only one of the above conditions. If two events are NOT independent, then we say that they are dependent.

Note, *independent* and *mutually exclusive* do not mean the same thing. For example, campus parking availability is not affected by your favorite basketball team game days (i.e., independent events). However, that does not mean they cannot happen at the same time (i.e., not mutually exclusive).

Example 3.3.1

Let event $\text{A} =$ learning Spanish. Let event B = learning German. Then A AND B = learning Spanish and German. Suppose $P(\text{A}) = 0.4$ and $P(\text{B}) = 0.2$. $P(\text{A AND B}) = 0.08$. Are events A and B independent? Hint: You must show ONE of the following:

- $P(\text{A}|\text{B}) = P(\text{A})$
- $P(\text{B}|\text{A})$
- $P(\text{A AND B}) = P(\text{A})P(\text{B})$

Answer

$$P(\text{A}|\text{B}) = \frac{\text{P(A AND B)}}{P(\text{B})} = \frac{0.08}{0.2} = 0.4 = P(\text{A}) \tag{3.3.1}$$

The events are independent because $P(\text{A}|\text{B}) = P(\text{A})$.

Example 3.3.2

Let event $\text{G} =$ taking a math class. Let event $\text{H} =$ taking a science class. Then, $\text{G AND H} =$ taking a math class and a science class. Suppose $P(\text{G}) = 0.6$, $P(\text{H}) = 0.5$, and $P(\text{G AND H}) = 0.3$. Are G and H independent?

If G and H are independent, then you must show **ONE** of the following:

- $P(\text{G}|\text{H}) = P(\text{G})$
- $P(\text{H}|\text{G}) = P(\text{H})$
- $P(\text{G AND H}) = P(\text{G})P(\text{H})$

The choice you make depends on the information you have. You could choose any of the methods here because you have the necessary information.

a. Show that $P(\text{G}|\text{H}) = P(\text{G})$.
b. Show $P(\text{G AND H}) = P(\text{G})P(\text{H})$.

Solution

a. $P(\text{G}|\text{H}) = \dfrac{P(\text{G AND H})}{P(\text{H})} = \dfrac{0.3}{0.5} = 0.6 = P(\text{G})$
b. $P(\text{G})P(\text{H}) = (0.6)(0.5) = 0.3 = P(\text{G AND H})$

Since G and H are independent, knowing that a person is taking a science class does not change the chance that he or she is taking a math class. If the two events had not been independent (that is, they are dependent) then knowing that a person is taking a science class would change the chance he or she is taking math. For practice, show that $P(\text{H}|\text{G}) = P(\text{H})$ to show that G and H are independent events.

Multiplication Rule for Independent Events

Given events A and B are independent, then

$$P(\text{A AND B}) = P(\text{A}) \cdot P(\text{B})$$

Example 3.3.3

In a bag of colored blocks, 5 are red, 4 are green, and 3 are blue. Draw three blocks. After each draw, you replace the block into the bag. For parts (b)-(c), find the probabilities of the events.

a. Is event F described below independent?
b. Let F = the event drawing a red, green, and blue.
c. Let G = the event drawing two red and one blue.
d. Let H = the event of drawing all red.

Answer a

Yes. After each draw, you replace the block into the bag. So, after drawing a red block, you replace it into the bag, which does not affect the probability of drawing any subsequent blocks. This is also true for events G and H.

Answer b

$$P(\text{G}) = P(\text{red AND green AND blue}) = P(\text{red}) \cdot P(\text{green}) \cdot P(\text{blue}) = \frac{5}{12} \cdot \frac{4}{12} \cdot \frac{3}{12} = 0.0347$$

Answer c

$$P(\text{G}) = P(\text{red AND red AND blue}) = P(\text{red}) \cdot P(\text{red}) \cdot P(\text{blue}) = \frac{5}{12} \cdot \frac{5}{12} \cdot \frac{3}{12} = 0.0434$$

Answer d

$$P(\text{H}) = P(\text{red AND red AND red}) = P(\text{red}) \cdot P(\text{red}) \cdot P(\text{red}) = \frac{5}{12} \cdot \frac{5}{12} \cdot \frac{5}{12} = \left(\frac{5}{12}\right)^3 = 0.0723$$

Suppose you flip a coin three times. The sample space $S = \{HHH, HHT, HTT, HTH, THT, THH, TTH, TTT\}$. We can see that there are seven outcomes with *at least one* head. (The only outcome that does not have at least one head is "TTT".)

What is the probability of getting at least one head when a coin is flipped three times?

\(P(\text{at least one head when a coin is flipped three times}) = \dfrac{7}{8}=0.875\)

Luckily for us, this example required us to flip a coin three times which made finding the probability above easier by counting the outcomes. What if we were asked to flip the coin 5 times? or 10 times? or 20 times? This would be troublesome as listing the sample space would take a while.

Turns out, there is a formula for such situations.

"At least one" Probability

$$P(\text{at least one}) = 1 - P(\text{none})$$

Example 3.3.4

You are asked to flip a coin three times. What is the probability that you flip at least one head?

Answer

Using the formula above, we get

$$P(\text{at least one head}) = 1 - P(\text{none are heads})$$

On the right side of the formula, we need to find the probability that none of the three flips are heads. This is the same thing as saying find the probability that all three flips were tails. So,

$$
\begin{aligned}
&P(\text{none are heads}) \\
&= P(\text{tail AND tail AND tail}) \\
&= P(\text{tail}) \cdot P(\text{tail}) \cdot P(\text{tail}) \\
&= \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \\
&= 0.125
\end{aligned}
$$

Now,

$$P(\text{at least one head}) = 1 - P(\text{none are heads}) = 1 - 0.125 = 0.875$$

Which, you'll notice is the same answer as above when this problem was done by counting.

Example 3.3.5

You are asked to flip a coin ten times. What is the probability that you flip at least one head?

Answer

Using the formula above, we get

$$P(\text{at least one head}) = 1 - P(\text{none are heads})$$

On the right side of the formula, we need to find the probability that none of the ten flips are heads. This is the same thing as saying find the probability that all ten flips were tails. So,

$$
\begin{aligned}
&P(\text{none are heads}) \\
&= P(\text{tail AND tail AND tail AND tail AND tail AND tail AND tail AND tail AND tail AND tail}) \\
&= P(\text{tail}) \cdot P(\text{tail}) \cdot P(\text{tail}) \cdot P(\text{tail}) \cdot P(\text{tail}) \cdot P(\text{tail}) \cdot P(\text{tail}) \cdot P(\text{tail}) \cdot P(\text{tail}) \cdot P(\text{tail}) \\
&= \left(\frac{1}{2}\right)^{10} = 0.001
\end{aligned}
$$

Now,

$$P(\text{at least one head}) = 1 - P(\text{none are heads}) = 1 - 0.001 = 0.999$$

Example 3.3.6

At the local pet shop, the manager has noticed a that a fish food manufacture has been sending expired fish food in the big bulk orders to the store. On average, they send 5 out of 100 bags that are expired. Before putting the product on the store floor, she will randomly select three bags of fish food. If at least one of the randomly selected three bags is expired, she will return the entire order to the manufacturer. What is the probability that the entire order will be returned?

Answer

Using the formula above, we get

$$P(\text{at least one of the randomly selected bags is bad}) = 1 - P(\text{none of the randomly selected bags are bad})$$

On the right side of the formula, we need to find the probability that none of the randomly selected bags are bad. This is the same thing as saying find the probability that all three bags are good. So,

$$P(\text{none are heads}) = P(\text{good AND good AND good}) = P(\text{good}) \cdot P(\text{good}) \cdot P(\text{good}) = \frac{95}{100} \cdot \frac{95}{100} \cdot \frac{95}{100} = 0.8574$$

Now,

$P(\text{at least one of the randomly selected bags is bad}) = 1 - P(\text{none of the randomly selected bags are bad})$ = 1-0.8574=0.1426\)

The probability that the manager will send back the entire order to the manufacture is 0.1426

Glossary

Independent Events

If one event does not affect the other, then the two events are independent.

Dependent Events

If two events are NOT independent, then we say that they are dependent.

Sampling with Replacement

If each member of a population is replaced after it is picked, then that member has the possibility of being chosen more than once.

Sampling without Replacement

When sampling is done without replacement, each member of a population may be chosen only once.

Contributors and Attributions

- Barbara Illowsky and Susan Dean (De Anza College) with many other contributing authors. Content produced by OpenStax College is licensed under a Creative Commons Attribution License 4.0 license. Download for free at http://cnx.org/contents/30189442-699...b91b9de@18.114.
