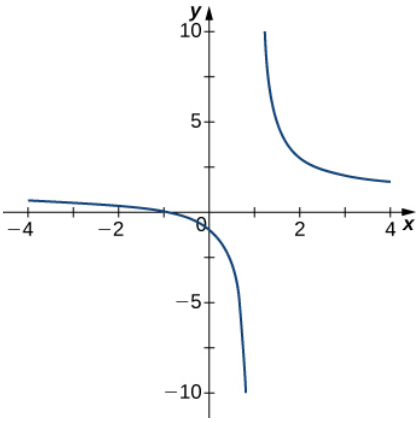
4.7E: Exercises for Section 4.6

For exercises 1 - 5, examine the graphs. Identify where the vertical asymptotes are located.

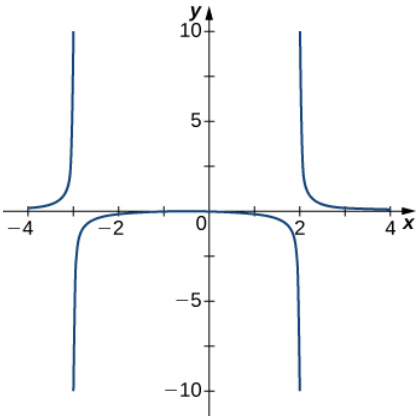
1)



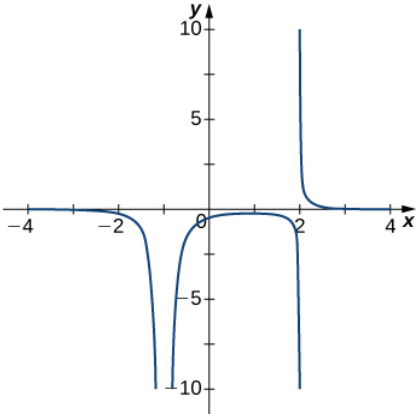
Answer

$x = 1$

2)



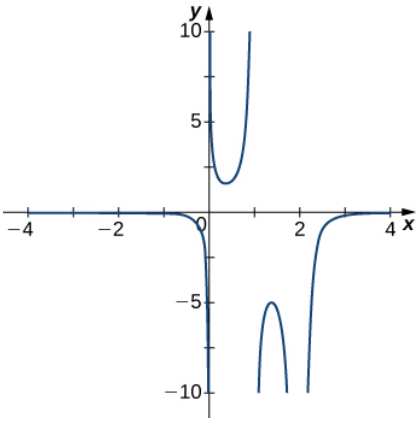
3)



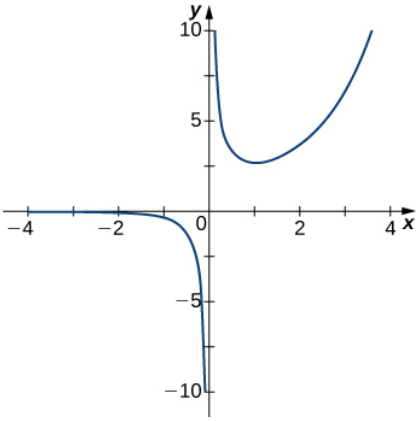
Answer

$x = -1,\ x = 2$

4)



5)



Answer

$x = 0$

For the functions $f(x)$ in exercises 6 - 10, determine whether there is an asymptote at $x = a$. Justify your answer without graphing on a calculator.

6) $f(x) = \dfrac{x+1}{x^2+5x+4}, \quad a = -1$

7) $f(x) = \dfrac{x}{x-2}, \quad a = 2$

Answer

Yes, there is a vertical asymptote at $x = 2$.

8) $f(x) = (x+2)^{3/2}, \quad a = -2$

9) $f(x) = (x-1)^{-1/3}, \quad a = 1$

Answer

Yes, there is vertical asymptote at $x = 1$.

10) $f(x) = 1 + x^{-2/5}, \quad a = 1$

In exercises 11 - 20, evaluate the limit.

11) $\lim\limits_{x \to \infty} \dfrac{1}{3x+6}$

Answer

$$\lim_{x\to\infty}\frac{1}{3x+6}=0$$

12) $\lim_{x\to\infty}\frac{2x-5}{4x}$

13) $\lim_{x\to\infty}\frac{x^2-2x+5}{x+2}$

Answer

$$\lim_{x\to\infty}\frac{x^2-2x+5}{x+2}=\infty$$

14) $\lim_{x\to-\infty}\frac{3x^3-2x}{x^2+2x+8}$

15) $\lim_{x\to-\infty}\frac{x^4-4x^3+1}{2-2x^2-7x^4}$

Answer

$$\lim_{x\to-\infty}\frac{x^4-4x^3+1}{2-2x^2-7x^4}=-\frac{1}{7}$$

16) $\lim_{x\to\infty}\frac{3x}{\sqrt{x^2+1}}$

17) $\lim_{x\to-\infty}\frac{\sqrt{4x^2-1}}{x+2}$

Answer

$$\lim_{x\to-\infty}\frac{\sqrt{4x^2-1}}{x+2}=-2$$

18) $\lim_{x\to\infty}\frac{4x}{\sqrt{x^2-1}}$

19) $\lim_{x\to-\infty}\frac{4x}{\sqrt{x^2-1}}$

Answer

$$\lim_{x\to-\infty}\frac{4x}{\sqrt{x^2-1}}=-4$$

20) $\lim_{x\to\infty}\frac{2\sqrt{x}}{x-\sqrt{x}+1}$

For exercises 21 - 25, find the horizontal and vertical asymptotes.

21) $f(x)=x-\frac{9}{x}$

Answer

Horizontal: none,
Vertical: $x=0$

22) $f(x)=\frac{1}{1-x^2}$

23) $f(x)=\frac{x^3}{4-x^2}$

Answer

Horizontal: none,
Vertical: $x = \pm 2$

24) $f(x) = \dfrac{x^2+3}{x^2+1}$

25) $f(x) = \sin(x)\sin(2x)$

Answer

Horizontal: none,
Vertical: none

26) $f(x) = \cos x + \cos(3x) + \cos(5x)$

27) $f(x) = \dfrac{x\sin(x)}{x^2-1}$

Answer

Horizontal: $y = 0$,
Vertical: $x = \pm 1$

28) $f(x) = \dfrac{x}{\sin(x)}$

29) $f(x) = \dfrac{1}{x^3+x^2}$

Answer

Horizontal: $y = 0$,
Vertical: $x = 0$ and $x = -1$

30) $f(x) = \dfrac{1}{x-1} - 2x$

31) $f(x) = \dfrac{x^3+1}{x^3-1}$

Answer

Horizontal: $y = 1$,
Vertical: $x = 1$

32) $f(x) = \dfrac{\sin x + \cos x}{\sin x - \cos x}$

33) $f(x) = x - \sin x$

Answer

Horizontal: none,
Vertical: none

34) $f(x) = \dfrac{1}{x} - \sqrt{x}$

For exercises 35 - 38, construct a function $f(x)$ that has the given asymptotes.

35) $x = 1$ and $y = 2$

Answer

Answers will vary, for example: $y = \dfrac{2x}{x-1}$

36) $x = 1$ and $y = 0$

37) $y = 4,\ x = -1$

Answer

Answers will vary, for example: $y = \dfrac{4x}{x+1}$

38) $x = 0$

In exercises 39 - 43, graph the function on a graphing calculator on the window $x = [-5, 5]$ and estimate the horizontal asymptote or limit. Then, calculate the actual horizontal asymptote or limit.

39) [T] $f(x) = \dfrac{1}{x+10}$

Answer

$\lim\limits_{x\to\infty} \dfrac{1}{x+10} = 0$ so f has a horizontal asymptote of $y = 0$.

40) [T] $f(x) = \dfrac{x+1}{x^2+7x+6}$

41) [T] $\lim\limits_{x\to-\infty} x^2+10x+25$

Answer

$\lim\limits_{x\to-\infty} x^2+10x+25 = \infty$

42) [T] $\lim\limits_{x\to-\infty} \dfrac{x+2}{x^2+7x+6}$

43) [T] $\lim\limits_{x\to\infty} \dfrac{3x+2}{x+5}$

Answer

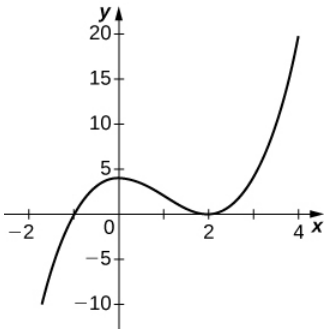
$\lim\limits_{x\to\infty} \dfrac{3x+2}{x+5} = 3$ so this function has a horizontal asymptote of $y = 3$.

In exercises 44 - 55, draw a graph of the functions without using a calculator. Be sure to notice all important features of the graph: local maxima and minima, inflection points, and asymptotic behavior.

44) $y = 3x^2+2x+4$

45) $y = x^3-3x^2+4$

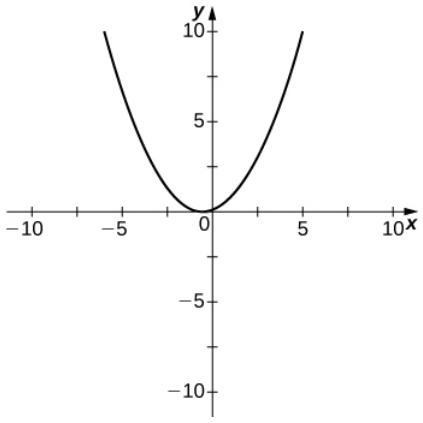
Answer



46) $y = \dfrac{2x+1}{x^2+6x+5}$

47) $y = \dfrac{x^3 + 4x^2 + 3x}{3x + 9}$

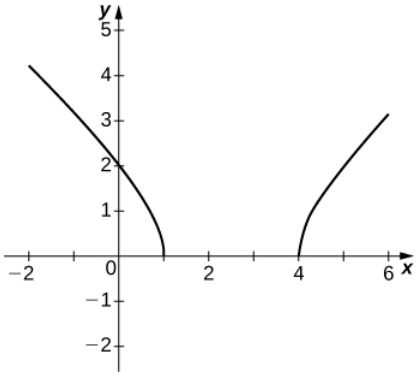
Answer



48) $y = \dfrac{x^2 + x - 2}{x^2 - 3x - 4}$

49) $y = \sqrt{x^2 - 5x + 4}$

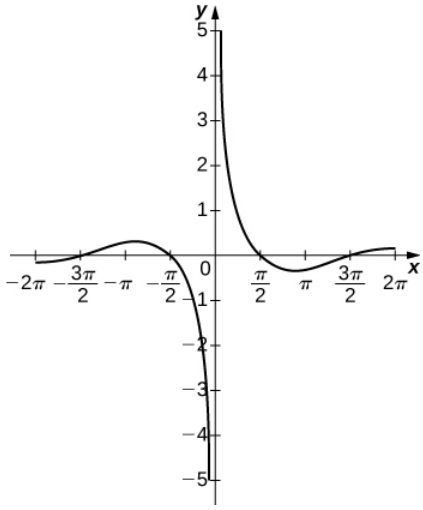
Answer



50) $y = 2x\sqrt{16 - x^2}$

51) $y = \dfrac{\cos x}{x}$, on $x = [-2\pi, 2\pi]$

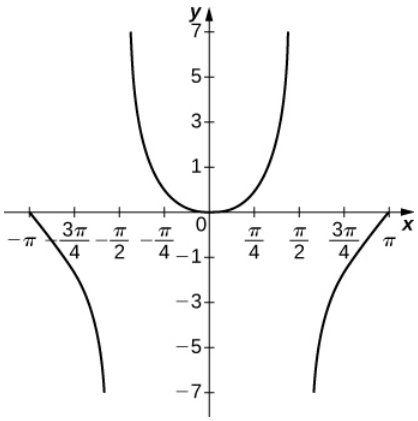
Answer



52) $y = e^x - x^3$

53) $y = x \tan x, \quad x = [-\pi, \pi]$

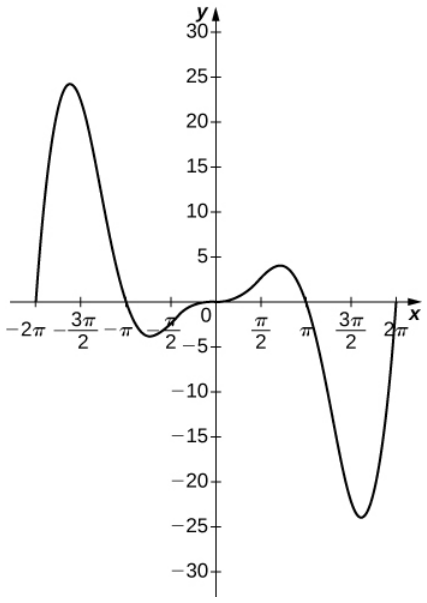
Answer



54) $y = x \ln(x), \quad x > 0$

55) $y = x^2 \sin(x), \quad x = [-2\pi, 2\pi]$

Answer



56) For $f(x) = \dfrac{P(x)}{Q(x)}$ to have an asymptote at $y = 2$ then the polynomials $P(x)$ and $Q(x)$ must have what relation?

57) For $f(x) = \dfrac{P(x)}{Q(x)}$ to have an asymptote at $x = 0$, then the polynomials $P(x)$ and $Q(x)$. must have what relation?

Answer

$Q(x)$. must have have x^{k+1} as a factor, where $P(x)$ has x^k as a factor.

58) If $f\,'(x)$ has asymptotes at $y = 3$ and $x = 1$, then $f(x)$ has what asymptotes?

59) Both $f(x) = \dfrac{1}{x-1}$ and $g(x) = \dfrac{1}{(x-1)^2}$ have asymptotes at $x = 1$ and $y = 0$. What is the most obvious difference between these two functions?

Answer

$\lim\limits_{x \to 1^-} f(x) = -\infty$ and $\lim\limits_{x \to 1^-} g(x) = \infty$

60) True or false: Every ratio of polynomials has vertical asymptotes.
