1.4E: Exercises for Section 1.3

In exercises 1 - 5, convert each angle in degrees to radians. Write the answer as a multiple of π.

1) $240°$

Answer

$\frac{4\pi}{3}$ rad

2) $15°$

3) $60°$

Answer

$\frac{\pi}{3}$ rad

4) $-225°$

5) $330°$

Answer

$\frac{11\pi}{6}$ rad

In exercises 6 - 10, convert each angle in radians to degrees.

6) $\frac{\pi}{2}$ rad

7) $\frac{7\pi}{6}$ rad

Answer

$210°$

8) $\frac{11\pi}{2}$ rad

9) -3π rad

Answer

$-540°$

10) $\frac{5\pi}{12}$ rad

In exercises 11 - 16, evaluate the functional values.

11) $\cos\frac{4\pi}{3}$

Answer

$\cos\frac{4\pi}{3}=-0.5$

12) $\tan\frac{19\pi}{4}$

13) $\sin\left(-\frac{3\pi}{4}\right)$

Answer

$\sin\left(-\frac{3\pi}{4}\right)=-\frac{\sqrt{2}}{2}$

14) $\sec\left(-\frac{\pi}{6}\right)$

15) $\sin\left(-\frac{\pi}{12}\right)$

Answer

$\sin\left(-\frac{\pi}{12}\right)=\dfrac{\sqrt{3}-1}{2\sqrt{2}}$

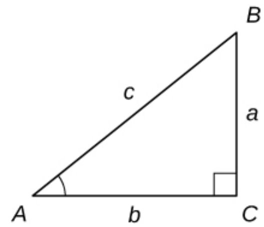
16) $\cos\left(-\frac{5\pi}{12}\right)$

In exercises 17 - 22, consider triangle ABC, a right triangle with a right angle at C.

a. Find the missing side of the triangle.

b. Find the six trigonometric function values for the angle at A.

Where necessary, round to one decimal place.



17) $a=4,\ c=7$

Answer

a. $b=5.7$

b. $\sin A=\frac{4}{7},\ \cos A=\frac{5.7}{7},\ \tan A=\frac{4}{5.7},\ \csc A=\frac{7}{4},\ \sec A=\frac{7}{5.7},\ \cot A=\frac{5.7}{4}$

18) $a=21,\ c=29$

19) $a=85.3,\ b=125.5$

Answer

a. $c=151.7$

b. $\sin A=0.5623,\ \cos A=0.8273,\ \tan A=0.6797,\ \csc A=1.778,\ \sec A=1.209,\ \cot A=1.471$

20) $b=40,\ c=41$

21) $a=84,\ b=13$

Answer

a. $c=85$

b. $\sin A=\frac{84}{85},\ \cos A=\frac{13}{85},\ \tan A=\frac{84}{13},\ \csc A=\frac{85}{84},\ \sec A=\frac{85}{13},\ \cot A=\frac{13}{84}$

22) $b=28,\ c=35$

In exercises 23 - 26, P is a point on the unit circle.

a. Find the (exact) missing coordinate value of each point and

b. find the values of the six trigonometric functions for the angle θ with a terminal side that passes through point P.

Rationalize denominators.

23) $P\left(\frac{7}{25},y\right),\quad y>0$

Answer

a. $y=\frac{24}{25}$

b. $\sin\theta=\frac{24}{25},\ \cos\theta=\frac{7}{25},\ \tan\theta=\frac{24}{7},\ \csc\theta=\frac{25}{24},\ \sec\theta=\frac{25}{7},\ \cot\theta=\frac{7}{24}$

24) $P\left(-\frac{15}{17},y\right),\quad y>0$

25) $P\left(x,\frac{\sqrt{7}}{3}\right),\quad x>0$

Answer

a. $x=-\frac{\sqrt{2}}{3}$

b. $\sin\theta=\frac{\sqrt{7}}{3},\ \cos\theta=-\frac{\sqrt{2}}{3},\ \tan\theta=-\frac{\sqrt{14}}{2},\ \csc\theta=\frac{3\sqrt{7}}{7},\ \sec\theta=-\frac{3\sqrt{2}}{2},\ \cot\theta=-\frac{\sqrt{14}}{7}$

26) $P\left(x,\ -\frac{\sqrt{15}}{4}\right),\quad y>0$

In exercises 27 - 34, simplify each expression by writing it in terms of sines and cosines, then simplify. The final answer does not have to be in terms of sine and cosine only.

27) $\tan^2 x+\sin x\csc x$

Answer

$\sec^2 x$

28) $\sec x\sin x\cot x$

29) $\dfrac{\tan^2 x}{\sec^2 x}$

Answer

$\sin^2 x$

30) $\sec x-\cos x$

31) $(1+\tan\theta)^2-2\tan\theta$

Answer

$\sec^2\theta$

32) $(\sin x)(\csc x-\sin x)$

33) $\dfrac{\cos t}{\sin t}+\dfrac{\sin t}{1+\cos t}$

Answer

$\dfrac{1}{\sin t}=\csc t$

34) $\dfrac{1+\tan^2\alpha}{1+\cot^2\alpha}$

In exercises 35 - 42, verify that each equation is an identity.

35) $\dfrac{\tan\theta\cot\theta}{\csc\theta}=\sin\theta$

36) $\dfrac{\sec^2\theta}{\tan\theta}=\sec\theta\csc\theta$

37) $\dfrac{\sin t}{\csc t}+\dfrac{\cos t}{\sec t}=1$

38) $\dfrac{\sin x}{\cos x+1}+\dfrac{\cos x-1}{\sin x}=0$

39) $\cot\gamma+\tan\gamma=\sec\gamma\csc\gamma$

40) $\sin^2\beta+\tan^2\beta+\cos^2\beta=\sec^2\beta$

41) $\dfrac{1}{1-\sin\alpha}+\dfrac{1}{1+\sin\alpha}=2\sec^2\alpha$

42) $\dfrac{\tan\theta-\cot\theta}{\sin\theta\cos\theta}=\sec^2\theta-\csc^2\theta$

In exercises 43 - 50, solve the trigonometric equations on the interval $0 \leq \theta < 2\pi$.

43) $2\sin\theta - 1 = 0$

Answer

$\left\{\frac{\pi}{6}, \frac{5\pi}{6}\right\}$

44) $1 + \cos\theta = \frac{1}{2}$

45) $2\tan^2\theta = 2$

Answer

$\left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$

46) $4\sin^2\theta - 2 = 0$

47) $\sqrt{3}\cot\theta + 1 = 0$

Answer

$\left\{\frac{2\pi}{3}, \frac{5\pi}{3}\right\}$

48) $3\sec\theta - 2\sqrt{3} = 0$

49) $2\cos\theta\sin\theta = \sin\theta$

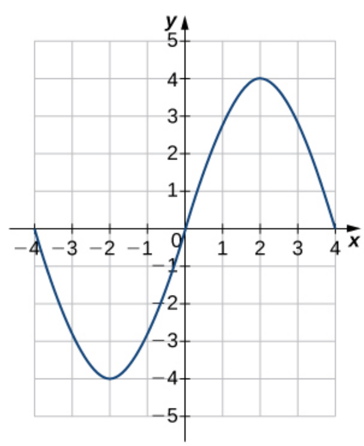
Answer

$\left\{0, \pi, \frac{\pi}{3}, \frac{5\pi}{3}\right\}$

50) $\csc^2\theta + 2\csc\theta + 1 = 0$

In exercises 51 - 54, each graph is of the form $y = A\sin Bx$ or $y = A\cos Bx$, where $B > 0$. Write the equation of the graph.

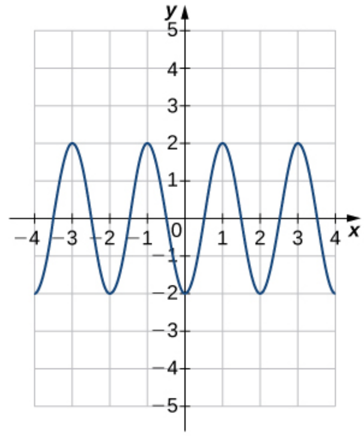
51)



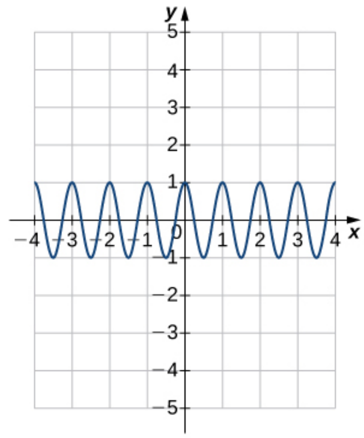
Answer

$y = 4\sin\left(\frac{\pi}{4}x\right)$

52)



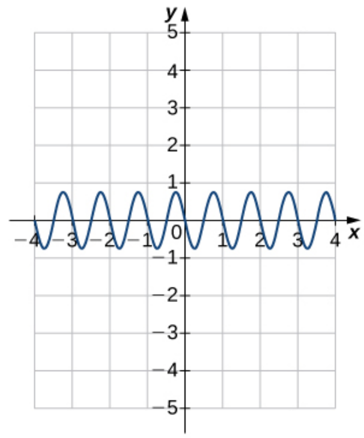
53)



Answer

$y = \cos 2\pi x$

54)



In exercises 55 - 60, find

a. the amplitude,

b. the period, and

c. the phase shift with direction for each function.

55) $y = \sin\left(x - \frac{\pi}{4}\right)$

Answer

a. 1
b. 2π
c. $\frac{\pi}{4}$ units to the right

56) $y = 3\cos(2x + 3)$

57) $y = -\frac{1}{2}\sin\left(\frac{1}{4}x\right)$

Answer

a. $\frac{1}{2}$
b. 8π
c. No phase shift

58) $y = 2\cos\left(x - \frac{\pi}{3}\right)$

59) $y = -3\sin(\pi x + 2)$

Answer

a. 3
b. 2
c. $\frac{2}{\pi}$ units to the left

60) $y = 4\cos\left(2x - \frac{\pi}{2}\right)$

61) [T] The diameter of a wheel rolling on the ground is 40 in. If the wheel rotates through an angle of $120°$, how many inches does it move? Approximate to the nearest whole inch.

Answer

Approximately 42 in.

62) [T] Find the length of the arc intercepted by central angle θ in a circle of radius r. Round to the nearest hundredth.

a. $r = 12.8$ cm, $\theta = \frac{5\pi}{6}$ rad b. $r = 4.378$ cm, $\theta = \frac{7\pi}{6}$ rad c. $r = 0.964$ cm, $\theta = 50°$ d. $r = 8.55$ cm, $\theta = 325°$

63) [T] As a point P moves around a circle, the measure of the angle changes. The measure of how fast the angle is changing is called angular speed, ω, and is given by $\omega = \theta/t$, where θ is in radians and t is time. Find the angular speed for the given data. Round to the nearest thousandth.

a. $\theta = \frac{7\pi}{4}$ rad, $t = 10$ sec b. $\theta = \frac{3\pi}{5}$ rad, $t = 8$ sec c. $\theta = \frac{2\pi}{9}$ rad, $t = 1$ min d. $\theta = 23.76$ rad, $t = 14$ min

Answer

a. 0.550rad/sec
b. 0.236rad/sec
c. 0.698rad/min
d. 1.697rad/min

64) [T] A total of $250,000\ \mathrm{m}^2$ of land is needed to build a nuclear power plant. Suppose it is decided that the area on which the power plant is to be built should be circular.

a)Find the radius of the circular land area.

b)If the land area is to form a $45°$ sector of a circle instead of a whole circle, find the length of the curved side.

65) [T] The area of an isosceles triangle with equal sides of length x is $\frac{1}{2}x^2\sin\theta$,

where θ is the angle formed by the two sides. Find the area of an isosceles triangle with equal sides of length 8 in. and angle $\theta = \frac{5\pi}{12}$ rad.

Answer

$\approx 30.9 \text{ in}^2$

66) [T] A particle travels in a circular path at a constant angular speed ω. The angular speed is modeled by the function $\omega = 9|\cos(\pi t - \pi/12)|$. Determine the angular speed at $t = 9$ sec.

67) [T] An alternating current for outlets in a home has voltage given by the function $V(t) = 150\cos 368t$,

where V is the voltage in volts at time t in seconds.

a) Find the period of the function and interpret its meaning.

b) Determine the number of periods that occur when 1 sec has passed.

Answer

a. $\frac{\pi}{184}$; the voltage repeats every $\frac{\pi}{184}$ sec
b. Approximately 59 periods

68) [T] The number of hours of daylight in a northeast city is modeled by the function

$$N(t) = 12 + 3\sin\left[\frac{2\pi}{365}(t - 79)\right],$$

where t is the number of days after January 1.

a) Find the amplitude and period.

b) Determine the number of hours of daylight on the longest day of the year.

c) Determine the number of hours of daylight on the shortest day of the year.

d) Determine the number of hours of daylight 90 days after January 1.

e) Sketch the graph of the function for one period starting on January 1.

69) [T] Suppose that $T = 50 + 10\sin\left[\frac{\pi}{12}(t - 8)\right]$ is a mathematical model of the temperature (in degrees Fahrenheit) at t hours after midnight on a certain day of the week.

a) Determine the amplitude and period.

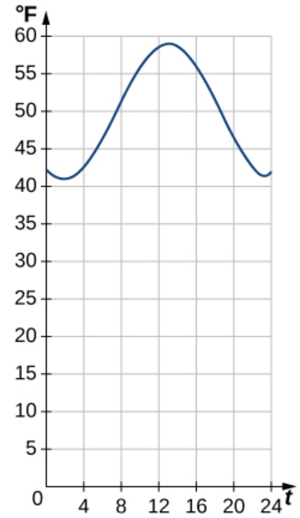
b) Find the temperature 7 hours after midnight.

c) At what time does $T = 60°$?

d) Sketch the graph of T over $0 \leq t \leq 24$.

Answer

a. Amplitude = 10; Period=24
b. $47.4°$F
c. 14 hours later, or 2 p.m.
d.



70) [T] The function $H(t)=8\sin\left(\frac{\pi}{6}t\right)$ models the height H (in feet) of the tide t hours after midnight. Assume that $t=0$ is midnight.

a) Find the amplitude and period.

b) Graph the function over one period.

c) What is the height of the tide at 4:30 a.m.?

Contributors