1.7: Chapter 1 Review Exercises

True or False? Justify your answer with a proof or a counterexample.

1) A function is always one-to-one.

2) $f \circ g = g \circ f$, assuming f and g are functions.

Answer

False

3) A relation that passes the horizontal and vertical line tests is a one-to-one function.

4) A relation passing the horizontal line test is a function.

Answer

False

State the domain and range of the given functions:

$f = x^2 + 2x - 3$, $g = \ln(x - 5)$, $h = \dfrac{1}{x+4}$

5) h

6) g

Answer

Domain: $x > 5$, Range: all real numbers

7) $h \circ f$

8) $g \circ f$

Answer

Domain: $x > 2$ and $x < -4$, Range: all real numbers

Find the degree, y-intercept, and zeros for the following polynomial functions.

9) $f(x) = 2x^2 + 9x - 5$

10) $f(x) = x^3 + 2x^2 - 2x$

Answer

Degree of 3, y-intercept: $(0, 0)$, Zeros: $0,\ \sqrt{3} - 1,\ -1 - \sqrt{3}$

Simplify the following trigonometric expressions.

11) $\dfrac{\tan^2 x}{\sec^2 x} + \cos^2 x$

12) $\cos^2 x - \sin^2 x$

Answer

$\cos(2x)$

Solve the following trigonometric equations on the interval $\theta = [-2\pi, 2\pi]$ exactly.

13) $6\cos 2x - 3 = 0$

14) $\sec^2 x - 2\sec x + 1 = 0$

Answer

$0, \pm 2\pi$

Solve the following logarithmic equations.

15) $5^x = 16$

16) $\log_2(x+4) = 3$

Answer

4

Are the following functions one-to-one over their domain of existence? Does the function have an inverse? If so, find the inverse $f^{-1}(x)$ of the function. Justify your answer.

17) $f(x) = x^2 + 2x + 1$

18) $f(x) = \dfrac{1}{x}$

Answer

One-to-one; yes, the function has an inverse; inverse: $f^{-1}(x) = \dfrac{1}{x}$

For the following problems, determine the largest domain on which the function is one-to-one and find the inverse on that domain.

19) $f(x) = \sqrt{9-x}$

20) $f(x) = x^2 + 3x + 4$

Answer

$x \geq -\frac{3}{2}, \quad f^{-1}(x) = -\frac{3}{2} + \frac{1}{2}\sqrt{4x-7}$

21) A car is racing along a circular track with diameter of 1 mi. A trainer standing in the center of the circle marks his progress every 5 sec. After 5 sec, the trainer has to turn 55° to keep up with the car. How fast is the car traveling?

For the following problems, consider a restaurant owner who wants to sell T-shirts advertising his brand. He recalls that there is a fixed cost and variable cost, although he does not remember the values. He does know that the T-shirt printing company charges $440 for 20 shirts and $1000 for 100 shirts.

22) a. Find the equation $C = f(x)$ that describes the total cost as a function of number of shirts and

b. determine how many shirts he must sell to break even if he sells the shirts for $10 each.

Answer

a. $C(x) = 300 + 7x$
b. $100\,\text{shirts}$

23) a. Find the inverse function $x = f^{-1}(C)$ and describe the meaning of this function.

b. Determine how many shirts the owner can buy if he has $8000 to spend.

For the following problems, consider the population of Ocean City, New Jersey, which is cyclical by season.

24) The population can be modeled by $P(t) = 82.5 - 67.5\cos[(\pi/6)t]$, where t is time in months ($t = 0$ represents January 1) and P is population (in thousands). During a year, in what intervals is the population less than 20,000? During what intervals is the population more than 140,000?

Answer

The population is less than 20,000 from December 8 through January 23 and more than 140,000 from May 29 through August 2

25) In reality, the overall population is most likely increasing or decreasing throughout each year. Let’s reformulate the model as $P(t)=82.5-67.5\cos[(\pi/6)t]+t$, where t is time in months ($t=0$ represents January 1) and P is population (in thousands). When is the first time the population reaches 200,000?

For the following problems, consider radioactive dating. A human skeleton is found in an archeological dig. Carbon dating is implemented to determine how old the skeleton is by using the equation $y=e^{rt}$, where y is the percentage of radiocarbon still present in the material, t is the number of years passed, and $r=-0.0001210$ is the decay rate of radiocarbon.

26) If the skeleton is expected to be 2000 years old, what percentage of radiocarbon should be present?

Answer

78.51%

27) Find the inverse of the carbon-dating equation. What does it mean? If there is 25% radiocarbon, how old is the skeleton?

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