2.3E: Exercises for Section 2.2

Intuitive Definition of Limits

For exercises 1 - 2, consider the function $f(x) = \dfrac{x^2-1}{|x-1|}$.

1) [T] Complete the following table for the function. Round your solutions to four decimal places.

x	$f(x)$	x	$f(x)$
0.9	a.	1.1	e.
0.99	b.	1.01	f.
0.999	c.	1.001	g.
0.9999	d.	1.0001	h.

2) What do your results in the preceding exercise indicate about the two-sided limit $\lim\limits_{x\to 1} f(x)$? Explain your response.

Answer

$\lim\limits_{x\to 1} f(x)$ does not exist because $\lim\limits_{x\to 1^-} f(x) = -2 \neq \lim\limits_{x\to 1^+} f(x) = 2$.

For exercises 3 - 5, consider the function $f(x) = (1+x)^{1/x}$.

3) [T] Make a table showing the values of f for $x = -0.01,\ -0.001,\ -0.0001,\ -0.00001$ and for $x = 0.01,\ 0.001,\ 0.0001,\ 0.00001$ Round your solutions to five decimal places.

x	$f(x)$	x	$f(x)$
-0.01	a.	0.01	e.
-0.001	b.	0.001	f.
-0.0001	c.	0.0001	g.
-0.00001	d.	0.00001	h.

4) What does the table of values in the preceding exercise indicate about the function $f(x) = (1+x)^{1/x}$?

Answer

$\lim\limits_{x\to 0}(1+x)^{1/x} \approx 2.7183.$

5) To which mathematical constant do the values in the preceding exercise appear to be approaching? This is the actual limit here.

In exercises 6 - 8, use the given values to set up a table to evaluate the limits. Round your solutions to eight decimal places.

6) [T] $\lim\limits_{x\to 0} \dfrac{\sin 2x}{x}$; $\quad \pm 0.1,\ \pm 0.01,\ \pm 0.001,\ \pm .0001$

x	$\frac{\sin 2x}{x}$	x	$\frac{\sin 2x}{x}$
-0.1	a.	0.1	e.
-0.01	b.	0.01	f.
-0.001	c.	0.001	g.
-0.0001	d.	0.0001	h.

Answer

a. 1.98669331; b. 1.99986667; c. 1.99999867; d. 1.99999999; e. 1.98669331; f. 1.99986667; g. 1.99999867; h. 1.99999999;

$\lim_{x\to 0} \frac{\sin 2x}{x} = 2$

7) [T] $\lim_{x\to 0} \frac{\sin 3x}{x}$ $\pm 0.1,\ \pm 0.01,\ \pm 0.001,\ \pm 0.0001$

x	$\frac{\sin 3x}{x}$	x	$\frac{\sin 3x}{x}$
-0.1	a.	0.1	e.
-0.01	b.	0.01	f.
-0.001	c.	0.001	g.
-0.0001	d.	0.0001	h.

8) Use the preceding two exercises to conjecture (guess) the value of the following limit: $\lim_{x\to 0} \frac{\sin ax}{x}$ for a, a positive real value.

Answer

$\lim_{x\to 0} \frac{\sin ax}{x} = a$

[T] In exercises 9 - 14, set up a table of values to find the indicated limit. Round to eight significant digits.

9) $\lim_{x\to 2} \frac{x^2-4}{x^2+x-6}$

x	$\frac{x^2-4}{x^2+x-6}$	x	$\frac{x^2-4}{x^2+x-6}$
1.9	a.	2.1	e.
1.99	b.	2.01	f.
1.999	c.	2.001	g.
1.9999	d.	2.0001	h.

10) $\lim_{x\to 1}(1-2x)$

x	$1-2x$	x	$1-2x$
0.9	a.	1.1	e.
0.99	b.	1.01	f.
0.999	c.	1.001	g.
0.9999	d.	1.0001	h.

Answer

a. −0.80000000; b. −0.98000000; c. −0.99800000; d. −0.99980000; e. −1.2000000; f. −1.0200000; g. −1.0020000; h. −1.0002000;

$\lim_{x\to 1}(1-2x) = -1$

11) $\lim_{x\to 0} \frac{5}{1-e^{1/x}}$

x	$\frac{5}{1-e^{1/x}}$	x	$\frac{5}{1-e^{1/x}}$
-0.1	a.	0.1	e.
-0.01	b.	0.01	f.
-0.001	c.	0.001	g.
-0.0001	d.	0.0001	h.

12) $\lim\limits_{z \to 0} \frac{z-1}{z^2(z+3)}$

z	$\frac{z-1}{z^2(z+3)}$	z	$\frac{z-1}{z^2(z+3)}$
-0.1	a.	0.1	e.
-0.01	b.	0.01	f.
-0.001	c.	0.001	g.
-0.0001	d.	0.0001	h.

Answer

a. −37.931034; b. −3377.9264; c. −333,777.93; d. −33,337,778; e. −29.032258; f. −3289.0365; g. −332,889.04; h. −33,328,889

$$\lim_{x \to 0} \frac{z-1}{z^2(z+3)} = -\infty$$

13) $\lim\limits_{t \to 0^+} \frac{\cos t}{t}$

t	$\frac{\cos t}{t}$
0.1	a.
0.01	b.
0.001	c.
0.0001	d.

14) $\lim\limits_{x \to 2} \frac{1-\frac{2}{x}}{x^2-4}$

x	$\frac{1-\frac{2}{x}}{x^2-4}$	x	$\frac{1-\frac{2}{x}}{x^2-4}$
1.9	a.	2.1	e.
1.99	b.	2.01	f.
1.999	c.	2.001	g.
1.9999	d.	2.0001	h.

Answer

a. 0.13495277; b. 0.12594300; c. 0.12509381; d. 0.12500938; e. 0.11614402; f. 0.12406794; g. 0.12490631; h. 0.12499063;

$$\therefore \lim_{x \to 2} \frac{1-\frac{2}{x}}{x^2-4} = 0.1250 = \frac{1}{8}$$

[T] In exercises 15 - 16, set up a table of values and round to eight significant digits. Based on the table of values, make a guess about what the limit is. Then, use a calculator to graph the function and determine the limit. Was the conjecture correct? If not, why does the method of tables fail?

15) $\lim_{\theta \to 0} \sin\left(\frac{\pi}{\theta}\right)$

θ	$\sin\left(\frac{\pi}{\theta}\right)$	θ	$\sin\left(\frac{\pi}{\theta}\right)$
-0.1	a.	0.1	e.
-0.01	b.	0.01	f.
-0.001	c.	0.001	g.
-0.0001	d.	0.0001	h.

16) $\lim_{\alpha \to 0^+} \frac{1}{\alpha}\cos\left(\frac{\pi}{\alpha}\right)$

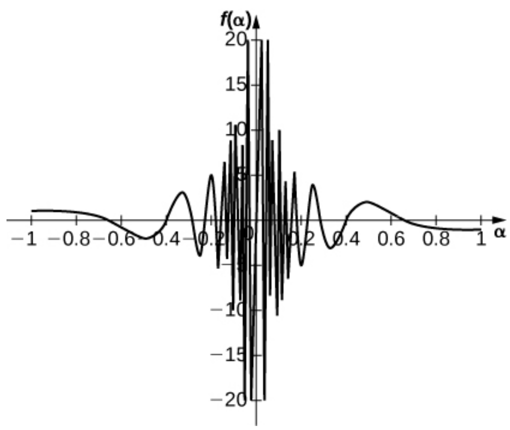
a	$\frac{1}{\alpha}\cos\left(\frac{\pi}{\alpha}\right)$
0.1	a.
0.01	b.
0.001	c.
0.0001	d.

Answer

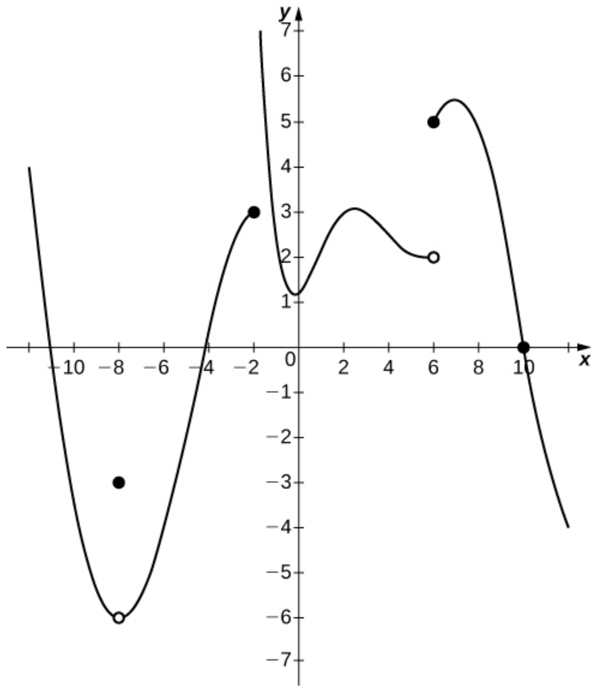
a. 10.00000; b. 100.00000; c. 1000.0000; d. 10,000.000;

Guess: $\lim_{\alpha \to 0^+} \frac{1}{\alpha}\cos\left(\frac{\pi}{\alpha}\right) = \infty$;

Actual: DNE , since the graph shows the function oscillates wildly between values approaching positive infinity and values approaching negative infinity, as the value of α approaches 0 from the positive side.



In exercises 17 - 20, consider the graph of the function $y = f(x)$ shown here. Which of the statements about $y = f(x)$ are true and which are false? Explain why a statement is false.



17) $\lim_{x\to 10} f(x) = 0$

18) $\lim_{x\to -2^+} f(x) = 3$

Answer

False; $\lim_{x\to -2^+} f(x) = +\infty$

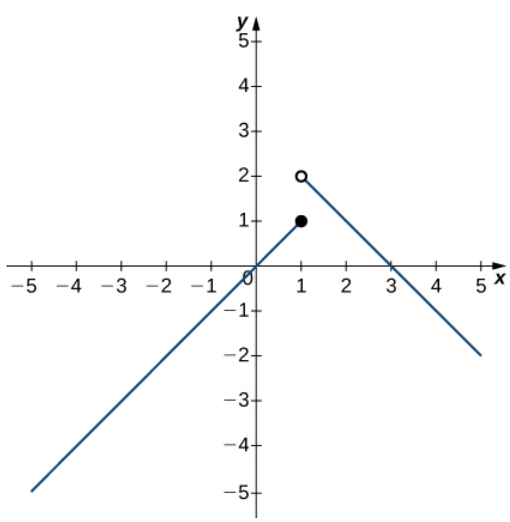
19) $\lim_{x\to -8} f(x) = f(-8)$

20) $\lim_{x\to 6} f(x) = 5$

Answer

False; $\lim_{x\to 6} f(x)$ DNE since $\lim_{x\to 6^-} f(x) = 2$ and $\lim_{x\to 6^+} f(x) = 5$.

In exercises 21 - 25, use the following graph of the function $y = f(x)$ to find the values, if possible. Estimate when necessary.



21) $\lim_{x \to 1^-} f(x)$

22) $\lim_{x \to 1^+} f(x)$

Answer

2

23) $\lim_{x \to 1} f(x)$

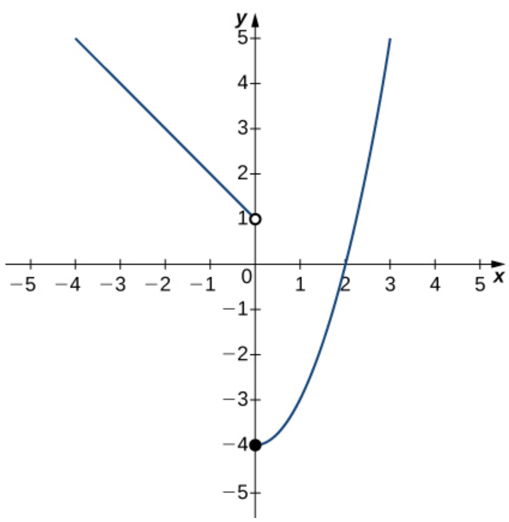
24) $\lim_{x \to 2} f(x)$

Answer

1

25) $f(1)$

In exercises 26 - 29, use the graph of the function $y = f(x)$ shown here to find the values, if possible. Estimate when necessary.



26) $\lim_{x \to 0^-} f(x)$

Answer

1

27) $\lim_{x \to 0^+} f(x)$

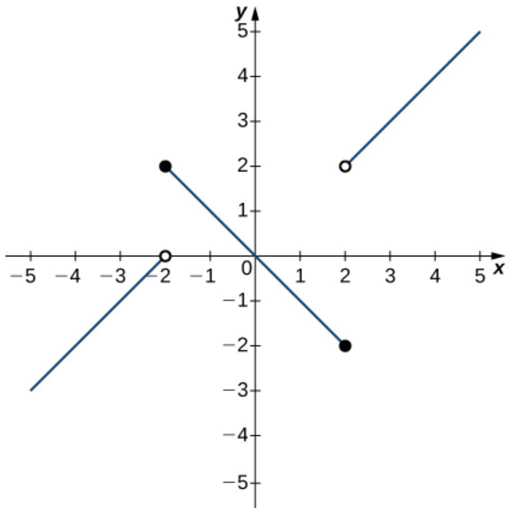
28) $\lim_{x \to 0} f(x)$

Answer

DNE

29) $\lim_{x \to 2} f(x)$

In exercises 30 - 35, use the graph of the function $y = f(x)$ shown here to find the values, if possible. Estimate when necessary.



30) $\lim_{x \to -2^-} f(x)$

Answer

0

31) $\lim_{x \to -2^+} f(x)$

32) $\lim_{x \to -2} f(x)$

Answer

DNE

33) $\lim_{x \to 2^-} f(x)$

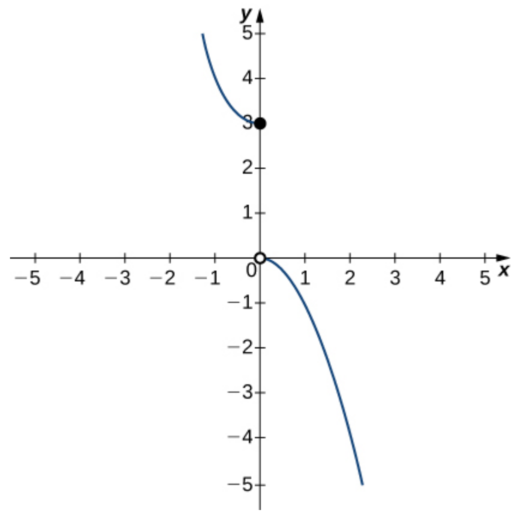
34) $\lim_{x \to 2^+} f(x)$

Answer

2

35) $\lim_{x \to 2} f(x)$

In exercises 36 - 38, use the graph of the function $y = g(x)$ shown here to find the values, if possible. Estimate when necessary.



36) $\lim\limits_{x \to 0^-} g(x)$

Answer

3

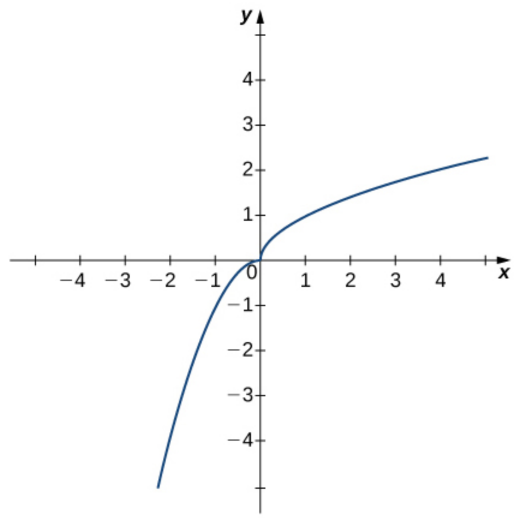
37) $\lim\limits_{x \to 0^+} g(x)$

38) $\lim\limits_{x \to 0} g(x)$

Answer

DNE

In exercises 39 - 41, use the graph of the function $y = h(x)$ shown here to find the values, if possible. Estimate when necessary.



39) $\lim\limits_{x \to 0^-} h(x)$

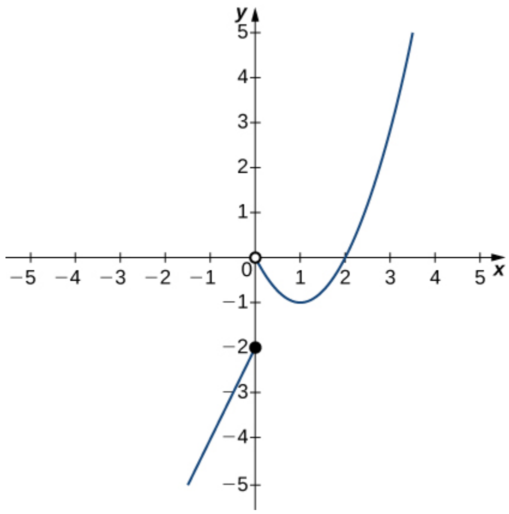
40) $\lim\limits_{x \to 0^+} h(x)$

Answer

0

41) $\lim_{x\to 0} h(x)$

In exercises 42 - 46, use the graph of the function $y=f(x)$ shown here to find the values, if possible. Estimate when necessary.



42) $\lim_{x\to 0^-} f(x)$

Answer

-2

43) $\lim_{x\to 0^+} f(x)$

44) $\lim_{x\to 0} f(x)$

Answer

DNE

45) $\lim_{x\to 1} f(x)$

46) $\lim_{x\to 2} f(x)$

Answer

0

Infinite Limits

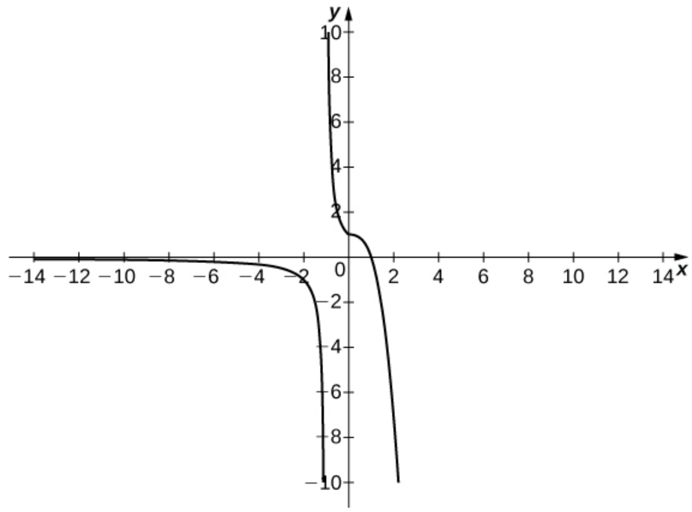
In exercises 47 - 51, sketch the graph of a function with the given properties.

47) $\lim_{x\to 2} f(x)=1, \quad \lim_{x\to 4^-} f(x)=3, \quad \lim_{x\to 4^+} f(x)=6, \quad x=4$ is not defined.

48) $\lim_{x\to -\infty} f(x)=0, \quad \lim_{x\to -1^-} f(x)=-\infty, \quad \lim_{x\to -1^+} f(x)=\infty, \quad \lim_{x\to 0} f(x)=f(0), \quad f(0)=1, \quad \lim_{x\to \infty} f(x)=-\infty$

Answer

Answers may vary

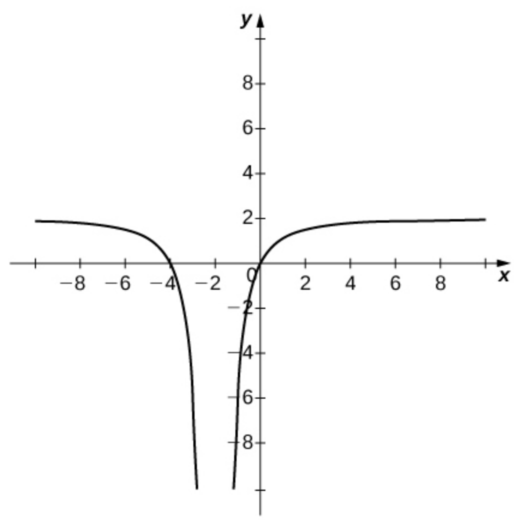


49) $\lim\limits_{x\to-\infty} f(x)=2, \quad \lim\limits_{x\to3^-} f(x)=-\infty, \quad \lim\limits_{x\to3^+} f(x)=\infty, \quad \lim\limits_{x\to\infty} f(x)=2, \quad f(0)=-\frac{1}{3}$

50) $\lim\limits_{x\to-\infty} f(x)=2, \quad \lim\limits_{x\to-2} f(x)=-\infty, \quad \lim\limits_{x\to\infty} f(x)=2, \quad f(0)=0$

Answer

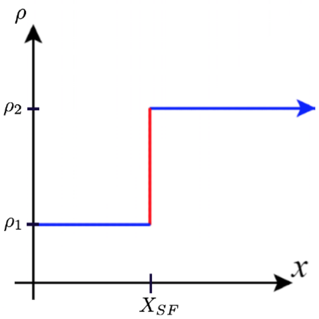
Answer may vary



51) $\lim\limits_{x\to-\infty} f(x)=0, \quad \lim\limits_{x\to-1^-} f(x)=\infty, \quad \lim\limits_{x\to-1^+} f(x)=-\infty, \quad f(0)=-1, \quad \lim\limits_{x\to1^-} f(x)=-\infty, \quad \lim\limits_{x\to1^+} f(x)=\infty,$

$\lim\limits_{x\to\infty} f(x)=0$

52) Shock waves arise in many physical applications, ranging from supernovas to detonation waves. A graph of the density of a shock wave with respect to distance, x, is shown here. We are mainly interested in the location of the front of the shock, labeled X_{SF} in the diagram.



a. Evaluate $\lim\limits_{x \to X_{SF}^+} \rho(x)$.

b. Evaluate $\lim\limits_{x \to X_{SF}^-} \rho(x)$.

c. Evaluate $\lim\limits_{x \to X_{SF}} \rho(x)$. Explain the physical meanings behind your answers.

Answer

a. ρ_2 b. ρ_1 c. DNE unless $\rho_1 = \rho_2$. As you approach X_{SF} from the right, you are in the high-density area of the shock. When you approach from the left, you have not experienced the "shock" yet and are at a lower density.

53) A track coach uses a camera with a fast shutter to estimate the position of a runner with respect to time. A table of the values of position of the athlete versus time is given here, where x is the position in meters of the runner and t is time in seconds. What is $\lim\limits_{t \to 2} x(t)$? What does it mean physically?

$t(sec)$	$x(m)$
1.75	4.5
1.95	6.1
1.99	6.42
2.01	6.58
2.05	6.9
2.25	8.5