

### 3.9E: Exercises for Section 3.8

In exercises 1 - 10, use implicit differentiation to find  $\frac{dy}{dx}$ .

1)  $x^2 - y^2 = 4$

2)  $6x^2 + 3y^2 = 12$

**Answer**

$$\frac{dy}{dx} = \frac{-2x}{y}$$

3)  $x^2y = y - 7$

4)  $3x^3 + 9xy^2 = 5x^3$

**Answer**

$$\frac{dy}{dx} = \frac{x}{3y} - \frac{y}{2x}$$

5)  $xy - \cos(xy) = 1$

6)  $y\sqrt{x+4} = xy + 8$

**Answer**

$$\frac{dy}{dx} = \frac{y - \frac{y}{2\sqrt{x+4}}}{\sqrt{x+4} - x}$$

7)  $-xy - 2 = \frac{x}{7}$

8)  $y \sin(xy) = y^2 + 2$

**Answer**

$$\frac{dy}{dx} = \frac{y^2 \cos(xy)}{2y - \sin(xy) - xy \cos(xy)}$$

9)  $(xy)^2 + 3x = y^2$

10)  $x^3y + xy^3 = -8$

**Answer**

$$\frac{dy}{dx} = \frac{-3x^2y - y^3}{x^3 + 3xy^2}$$

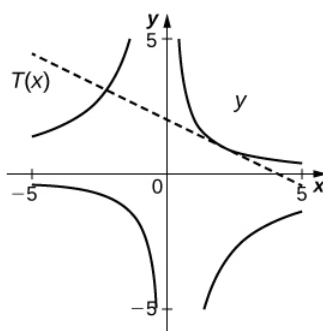
For exercises 11 - 16, find the equation of the tangent line to the graph of the given equation at the indicated point. Use a calculator or computer software to graph the function and the tangent line.

11) [T]  $x^4y - xy^3 = -2$ ,  $(-1, -1)$

12) [T]  $x^2y^2 + 5xy = 14$ ,  $(2, 1)$

**Answer**

$$y = -\frac{1}{2}x + 2$$

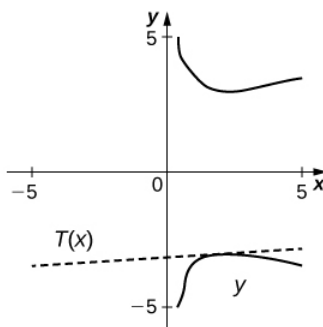


13) [T]  $\tan(xy) = y, \quad \left(\frac{\pi}{4}, 1\right)$

14) [T]  $xy^2 + \sin(\pi y) - 2x^2 = 10, \quad (2, -3)$

**Answer**

$$y = \frac{1}{\pi+12}x - \frac{3\pi+38}{\pi+12}$$

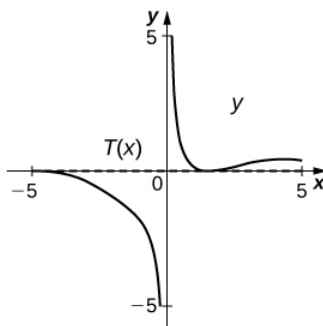


15) [T]  $\frac{x}{y} + 5x - 7 = -\frac{3}{4}y, \quad (1, 2)$

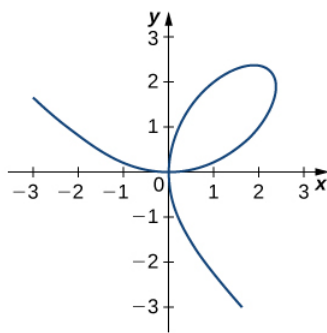
16) [T]  $xy + \sin(x) = 1, \quad \left(\frac{\pi}{2}, 0\right)$

**Answer**

$$y = 0$$



17) [T] The graph of a folium of Descartes with equation  $2x^3 + 2y^3 - 9xy = 0$  is given in the following graph.



- a. Find the equation of the tangent line at the point  $(2, 1)$ . Graph the tangent line along with the folium.
  - b. Find the equation of the normal line to the tangent line in a. at the point  $(2, 1)$ .
- 18) For the equation  $x^2 + 2xy - 3y^2 = 0$ ,
- a. Find the equation of the normal to the tangent line at the point  $(1, 1)$ .
  - b. At what other point does the normal line in a. intersect the graph of the equation?

**Answer**

- a.  $y = -x + 2$
- b.  $(3, -1)$

- 19) Find all points on the graph of  $y^3 - 27y = x^2 - 90$  at which the tangent line is vertical.
- 20) For the equation  $x^2 + xy + y^2 = 7$ ,
- a. Find the  $x$ -intercept(s).
  - b. Find the slope of the tangent line(s) at the  $x$ -intercept(s).
  - c. What does the value(s) in part b. indicate about the tangent line(s)?

**Answer**

- a.  $(\pm\sqrt{7}, 0)$
- b.  $-2$
- c. They are parallel since the slope is the same at both intercepts.

- 21) Find the equation of the tangent line to the graph of the equation  $\sin^{-1} x + \sin^{-1} y = \frac{\pi}{6}$  at the point  $(0, \frac{1}{2})$ .
- 22) Find the equation of the tangent line to the graph of the equation  $\tan^{-1}(x + y) = x^2 + \frac{\pi}{4}$  at the point  $(0, 1)$ .

**Answer**

$$y = -x + 1$$

- 23) Find  $y'$  and  $y''$  for  $x^2 + 6xy - 2y^2 = 3$ .
- 24) [T] The number of cell phones produced when  $x$  dollars is spent on labor and  $y$  dollars is spent on capital invested by a manufacturer can be modeled by the equation  $60x^{3/4}y^{1/4} = 3240$ .
- a. Find  $\frac{dy}{dx}$  and evaluate at the point  $(81, 16)$ .
  - b. Interpret the result of a.

**Answer**

- a.  $\frac{dy}{dx} = -0.5926$
- b. When \$81 is spent on labor and \$16 is spent on capital, the amount spent on capital is decreasing by \$0.5926 per \$1 spent on labor.

25) [T] The number of cars produced when  $x$  dollars is spent on labor and  $y$  dollars is spent on capital invested by a manufacturer can be modeled by the equation  $30x^{1/3}y^{2/3} = 360$ .

(Both  $x$  and  $y$  are measured in thousands of dollars.)

a. Find  $\frac{dy}{dx}$  and evaluate at the point  $(27, 8)$ .

b. Interpret the result of part a.

26) The volume of a right circular cone of radius  $x$  and height  $y$  is given by  $V = \frac{1}{3}\pi x^2 y$ . Suppose that the volume of the cone is  $85\pi \text{ cm}^3$ . Find  $\frac{dy}{dx}$  when  $x = 4$  and  $y = 16$ .

**Answer**

$$\frac{dy}{dx} = -8$$

**For exercises 27 - 28, consider a closed rectangular box with a square base with side  $x$  and height  $y$ .**

27) Find an equation for the surface area of the rectangular box,  $S(x, y)$ .

28) If the surface area of the rectangular box is 78 square feet, find  $\frac{dy}{dx}$  when  $x = 3$  feet and  $y = 5$  feet.

**Answer**

$$\frac{dy}{dx} = -2.67$$

**In exercises 29 - 31, use implicit differentiation to determine  $y'$ . Does the answer agree with the formulas we have previously determined?**

29)  $x = \sin y$

30)  $x = \cos y$

**Answer**

$$y' = -\frac{1}{\sqrt{1-x^2}}$$

31)  $x = \tan y$

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