

4.9E: Exercises for Section 4.8

In exercises 1 - 6, evaluate the limit.

1) Evaluate the limit $\lim_{x \rightarrow \infty} \frac{e^x}{x}$.

2) Evaluate the limit $\lim_{x \rightarrow \infty} \frac{e^x}{x^k}$.

Answer

$$\lim_{x \rightarrow \infty} \frac{e^x}{x^k} = \infty$$

3) Evaluate the limit $\lim_{x \rightarrow \infty} \frac{\ln x}{x^k}$.

4) Evaluate the limit $\lim_{x \rightarrow a} \frac{x - a}{x^2 - a^2}$.

Answer

$$\lim_{x \rightarrow a} \frac{x - a}{x^2 - a^2} = \frac{1}{2a}$$

5. Evaluate the limit $\lim_{x \rightarrow a} \frac{x - a}{x^3 - a^3}$.

6. Evaluate the limit $\lim_{x \rightarrow a} \frac{x - a}{x^n - a^n}$.

Answer

$$\lim_{x \rightarrow a} \frac{x - a}{x^n - a^n} = \frac{1}{na^{n-1}}$$

In exercises 7 - 11, determine whether you can apply L'Hôpital's rule directly. Explain why or why not. Then, indicate if there is some way you can alter the limit so you can apply L'Hôpital's rule.

7) $\lim_{x \rightarrow 0^+} x^2 \ln x$

8) $\lim_{x \rightarrow \infty} x^{1/x}$

Answer

Cannot apply directly; use logarithms

9) $\lim_{x \rightarrow 0} x^{2/x}$

10) $\lim_{x \rightarrow 0} \frac{x^2}{1/x}$

Answer

Cannot apply directly; rewrite as $\lim_{x \rightarrow 0} x^3$

11) $\lim_{x \rightarrow \infty} \frac{e^x}{x}$

In exercises 12 - 40, evaluate the limits with either L'Hôpital's rule or previously learned methods.

12) $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$

Answer

$$\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3} = 6$$

$$13) \lim_{x \rightarrow 3} \frac{x^2 - 9}{x + 3}$$

$$14) \lim_{x \rightarrow 0} \frac{(1+x)^{-2} - 1}{x}$$

Answer

$$\lim_{x \rightarrow 0} \frac{(1+x)^{-2} - 1}{x} = -2$$

$$15) \lim_{x \rightarrow \pi/2} \frac{\cos x}{\frac{\pi}{2} - x}$$

$$16) \lim_{x \rightarrow \pi} \frac{x - \pi}{\sin x}$$

Answer

$$\lim_{x \rightarrow \pi} \frac{x - \pi}{\sin x} = -1$$

$$17) \lim_{x \rightarrow 1} \frac{x - 1}{\sin x}$$

$$18) \lim_{x \rightarrow 0} \frac{(1+x)^n - 1}{x}$$

Answer

$$\lim_{x \rightarrow 0} \frac{(1+x)^n - 1}{x} = n$$

$$19) \lim_{x \rightarrow 0} \frac{(1+x)^n - 1 - nx}{x^2}$$

$$20) \lim_{x \rightarrow 0} \frac{\sin x - \tan x}{x^3}$$

Answer

$$\lim_{x \rightarrow 0} \frac{\sin x - \tan x}{x^3} = -\frac{1}{2}$$

$$21) \lim_{x \rightarrow 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{x}$$

$$22) \lim_{x \rightarrow 0} \frac{e^x - x - 1}{x^2}$$

Answer

$$\lim_{x \rightarrow 0} \frac{e^x - x - 1}{x^2} = \frac{1}{2}$$

$$23) \lim_{x \rightarrow 0} \frac{\tan x}{\sqrt{x}}$$

$$24) \lim_{x \rightarrow 1} \frac{x - 1}{\ln x}$$

Answer

$$\lim_{x \rightarrow 1} \frac{x - 1}{\ln x} = 1$$

$$25) \lim_{x \rightarrow 0} (x+1)^{1/x}$$

$$26) \lim_{x \rightarrow 1} \frac{\sqrt{x} - \sqrt[3]{x}}{x-1}$$

Answer

$$\lim_{x \rightarrow 1} \frac{\sqrt{x} - \sqrt[3]{x}}{x-1} = \frac{1}{6}$$

$$27) \lim_{x \rightarrow 0^+} x^{2x}$$

$$28) \lim_{x \rightarrow \infty} x \sin\left(\frac{1}{x}\right)$$

Answer

$$\lim_{x \rightarrow \infty} x \sin\left(\frac{1}{x}\right) = 1$$

$$29) \lim_{x \rightarrow 0} \frac{\sin x - x}{x^2}$$

$$30) \lim_{x \rightarrow 0^+} x \ln(x^4)$$

Answer

$$\lim_{x \rightarrow 0^+} x \ln(x^4) = 0$$

$$31) \lim_{x \rightarrow \infty} (x - e^x)$$

$$32) \lim_{x \rightarrow \infty} x^2 e^{-x}$$

Answer

$$\lim_{x \rightarrow \infty} x^2 e^{-x} = 0$$

$$33) \lim_{x \rightarrow 0} \frac{3^x - 2^x}{x}$$

$$34) \lim_{x \rightarrow 0} \frac{1 + 1/x}{1 - 1/x}$$

Answer

$$\lim_{x \rightarrow 0} \frac{1 + 1/x}{1 - 1/x} = -1$$

$$35) \lim_{x \rightarrow \pi/4} (1 - \tan x) \cot x$$

$$36) \lim_{x \rightarrow \infty} x e^{1/x}$$

Answer

$$\lim_{x \rightarrow \infty} x e^{1/x} = \infty$$

$$37) \lim_{x \rightarrow 0} x^{1/\cos x}$$

$$38) \lim_{x \rightarrow 0^+} x^{1/x}$$

Answer

$$\lim_{x \rightarrow 0^+} x^{1/x} = 0$$

$$39) \lim_{x \rightarrow 0} \left(1 - \frac{1}{x}\right)^x$$

$$40) \lim_{x \rightarrow \infty} \left(1 - \frac{1}{x}\right)^x$$

Answer

$$\lim_{x \rightarrow \infty} \left(1 - \frac{1}{x}\right)^x = \frac{1}{e}$$

For exercises 41 - 50, use a calculator to graph the function and estimate the value of the limit, then use L'Hôpital's rule to find the limit directly.

$$41) [T] \lim_{x \rightarrow 0} \frac{e^x - 1}{x}$$

$$42) [T] \lim_{x \rightarrow 0} x \sin\left(\frac{1}{x}\right)$$

Answer

$$\lim_{x \rightarrow 0} x \sin\left(\frac{1}{x}\right) = 0$$

$$43) [T] \lim_{x \rightarrow 1} \frac{x - 1}{1 - \cos(\pi x)}$$

$$44) [T] \lim_{x \rightarrow 1} \frac{e^{x-1} - 1}{x - 1}$$

Answer

$$\lim_{x \rightarrow 1} \frac{e^{x-1} - 1}{x - 1} = 1$$

$$45) [T] \lim_{x \rightarrow 1} \frac{(x-1)^2}{\ln x}$$

$$46) [T] \lim_{x \rightarrow \pi} \frac{1 + \cos x}{\sin x}$$

Answer

$$\lim_{x \rightarrow \pi} \frac{1 + \cos x}{\sin x} = 0$$

$$47) [T] \lim_{x \rightarrow 0} \left(\csc x - \frac{1}{x}\right)$$

$$48) [T] \lim_{x \rightarrow 0^+} \tan(x^x)$$

Answer

$$\lim_{x \rightarrow 0^+} \tan(x^x) = \tan 1$$

$$49) [T] \lim_{x \rightarrow 0^+} \frac{\ln x}{\sin x}$$

$$50) [T] \lim_{x \rightarrow 0} \frac{e^x - e^{-x}}{x}$$

Answer

$$\lim_{x \rightarrow 0} \frac{e^x - e^{-x}}{x} = 2$$

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