4.3E: Exercises for Section 4.2

1) What is the linear approximation for any generic linear function $y = mx + b$?

2) Determine the necessary conditions such that the linear approximation function is constant. Use a graph to prove your result.

Answer

$f'(a) = 0$

3) Explain why the linear approximation becomes less accurate as you increase the distance between x and a. Use a graph to prove your argument.

4) When is the linear approximation exact?

Answer

The linear approximation exact when $y = f(x)$ is linear or constant.

In exercises 5 - 10, find the linear approximation $L(x)$ to $y = f(x)$ near $x = a$ for the function.

5) [T] $f(x) = x + x^4, \quad a = 0$

6) [T] $f(x) = \dfrac{1}{x}, \quad a = 2$

Answer

$L(x) = \frac{1}{2} - \frac{1}{4}(x-2)$

7) [T] $f(x) = \tan x, \quad a = \frac{\pi}{4}$

8) [T] $f(x) = \sin x, \quad a = \frac{\pi}{2}$

Answer

$L(x) = 1$

9) [T] $f(x) = x \sin x, \quad a = 2\pi$

10) [T] $f(x) = \sin^2 x, \quad a = 0$

Answer

$L(x) = 0$

In exercises 11 - 16, compute the values given within 0.01 by deciding on the appropriate $f(x)$ and a, and evaluating $L(x) = f(a) + f'(a)(x-a)$. Check your answer using a calculator.

11) [T] $(2.001)^6$

12) [T] $\sin(0.02)$

Answer

$\sin(0.02) \approx 0.02$

13) [T] $\cos(0.03)$

14) [T] $(15.99)^{1/4}$

Answer

$(15.99)^{1/4} \approx 1.9996875$

15) [T] $\dfrac{1}{0.98}$

16) [T] $\sin(3.14)$

Answer

$\sin(3.14) \approx 0.001593$

In exercises 17 - 22, determine the appropriate $f(x)$ and a, and evaluate $L(x) = f(a) + f'(a)(x-a)$. Calculate the numerical error in the linear approximations that follow.

17) $(1.01)^3$

18) $\cos(0.01)$

Answer

$\cos(0.01) \approx L(0.01) = f(0) + f'(0)(0-0.01) = 1;$ error, 0.00005

19) $(\sin(0.01))^2$

20) $(1.01)^{-3}$

Answer

$(1.01)^{-3} \approx L(1.01) = f(1) + f'(1)(1.01-1) = 0.97;$ error, 0.0006

21) $\left(1+\frac{1}{10}\right)^{10}$

22) $\sqrt{8.99}$

Answer

$\sqrt{8.99} \approx L(8.99) = f(9) + f'(9)(8.99-9) = 3 - \frac{1}{600};$ error, 4.632×10^{-7}

In exercises 23 - 26, find the differential of the function.

23) $y = 3x^4 + x^2 - 2x + 1$

24) $y = x\cos x$

Answer

$dy = (\cos x - x\sin x)\,dx$

25) $y = \sqrt{1+x}$

26) $y = \dfrac{x^2+2}{x-1}$

Answer

$$dy = \left(\frac{x^2-2x-2}{(x-1)^2}\right)dx$$

In exercises 27 - 32, find the differential and evaluate for the given x and dx.

27) $y = 3x^2 - x + 6,\ x = 2,\ dx = 0.1$

28) $y = \dfrac{1}{x+1},\ x = 1,\ dx = 0.25$

Answer

$$dy = -\frac{1}{(x+1)^2}dx, \quad dy = -\frac{1}{16}$$

29) $y = \tan x,\ x = 0,\ dx = \frac{\pi}{10}$

30) $y = \dfrac{3x^2+2}{\sqrt{x+1}},\ x = 0,\ dx = 0.1$

Answer

$$dy = \frac{9x^2 + 12x - 2}{2(x+1)^{3/2}} dx, \quad dy = -0.1$$

31) $y = \dfrac{\sin(2x)}{x},\ x = \pi,\ dx = 0.25$

32) $y = x^3 + 2x + \dfrac{1}{x},\ x = 1,\ dx = 0.05$

Answer

$$dy = \left(3x^2 + 2 - \frac{1}{x^2}\right) dx, \quad dy = 0.2$$

In exercises 33 - 38, find the change in volume dV or in surface area dA.

33) dV if the sides of a cube change from 10 to 10.1.

34) dA if the sides of a cube change from x to $x + dx$.

Answer

$$dA = 12x\, dx$$

35) dA if the radius of a sphere changes from r by dr.

36) dV if the radius of a sphere changes from r by dr.

Answer

$$dV = 4\pi r^2 dr$$

37) dV if a circular cylinder with $r = 2$ changes height from 3 cm to $3.05 cm$.

38) dV if a circular cylinder of height 3 changes from $r = 2$ to $r = 1.9$ cm.

Answer

$$dV = -1.2\pi\ \mathrm{cm}^3$$

In exercises 39 - 41, use differentials to estimate the maximum and relative error when computing the surface area or volume.

39) A spherical golf ball is measured to have a radius of 5 mm, with a possible measurement error of 0.1 mm. What is the possible change in volume?

40) A pool has a rectangular base of 10 ft by 20 ft and a depth of 6 ft. What is the change in volume if you only fill it up to 5.5 ft?

Answer

$$-100\ \mathrm{ft}^3$$

41) An ice cream cone has height 4 in. and radius 1 in. If the cone is 0.1 in. thick, what is the difference between the volume of the cone, including the shell, and the volume of the ice cream you can fit inside the shell?

In exercises 42 - 44, confirm the approximations by using the linear approximation at $x = 0$.

42) $\sqrt{1 - x} \approx 1 - \frac{1}{2}x$

43) $\dfrac{1}{\sqrt{1 - x^2}} \approx 1$

44) $\sqrt{c^2 + x^2} \approx c$
