4.6E: Exercises for Section 4.5

1) If c is a critical point of $f(x)$, when is there no local maximum or minimum at c? Explain.

2) For the function $y = x^3$, is $x = 0$ both an inflection point and a local maximum/minimum?

Answer

It is not a local maximum/minimum because f' does not change sign

3) For the function $y = x^3$, is $x = 0$ an inflection point?

4) Is it possible for a point c to be both an inflection point and a local extremum of a twice differentiable function?

Answer

No

5) Why do you need continuity for the first derivative test? Come up with an example.

6) Explain whether a concave-down function has to cross $y = 0$ for some value of x.

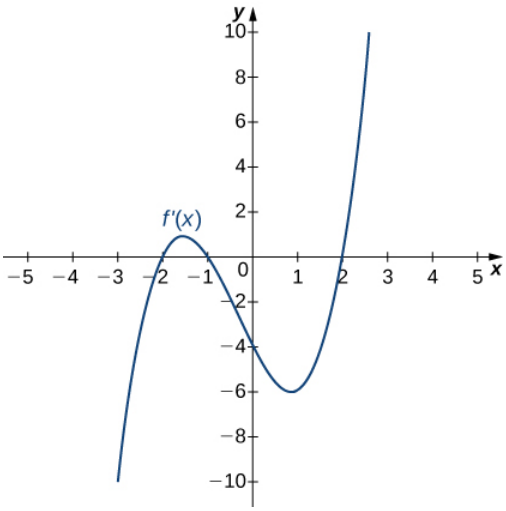
Answer

False; for example, $y = \sqrt{x}$.

7) Explain whether a polynomial of degree 2 can have an inflection point.

In exercises 8 - 12, analyze the graphs of f', then list all intervals where f is increasing or decreasing.

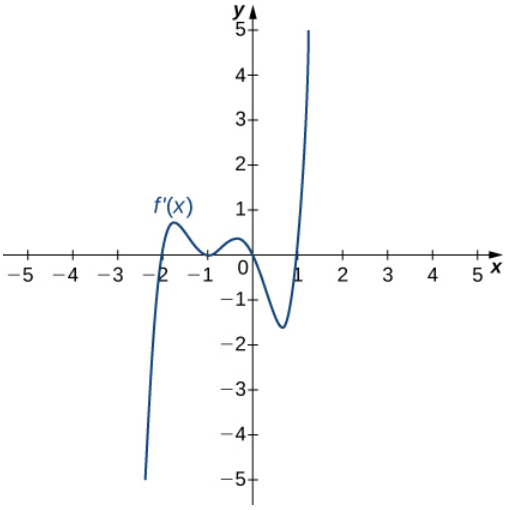
8)



Answer

Increasing for $-2 < x < -1$ and $x > 2$;
Decreasing for $x < -2$ and $-1 < x < 2$

9)



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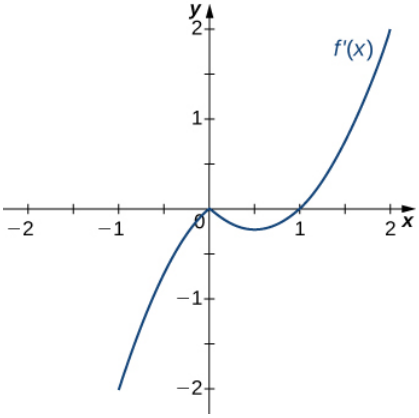
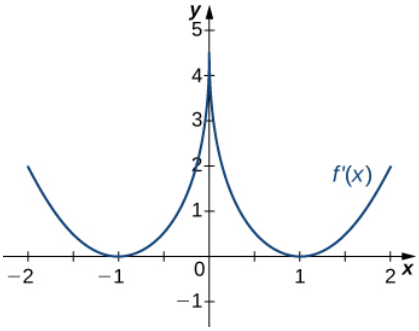


Answer

Decreasing for $x < 1$,
Increasing for $x > 1$

11)



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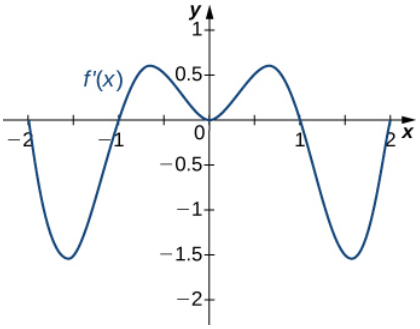


Answer

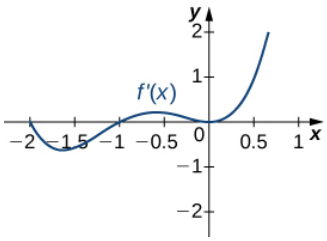
Decreasing for $-2 < x < -1$ and $1 < x < 2$;
Increasing for $-1 < x < 1$ and $x < -2$ and $x > 2$

In exercises 13 - 17, analyze the graphs of f', then list all intervals where

a. f is increasing and decreasing and

b. the minima and maxima are located.

13)



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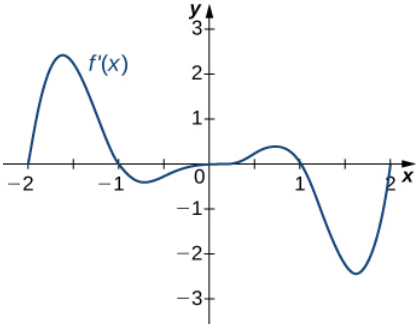
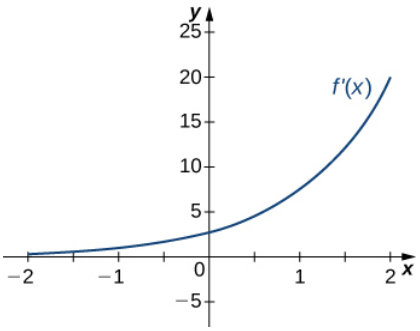


Answer

a. Increasing over $-2 < x < -1,\ 0 < x < 1, x > 2$, Decreasing over $x < -2,\ -1 < x < 0,\ 1 < x < 2$;
b. Maxima at $x = -1$ and $x = 1$, Minima at $x = -2$ and $x = 0$ and $x = 2$

15)



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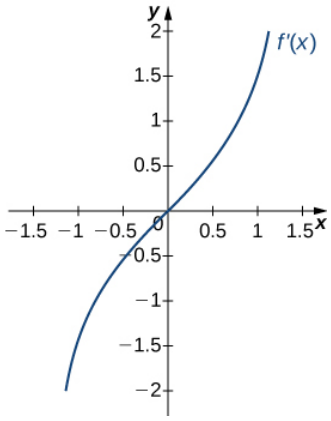
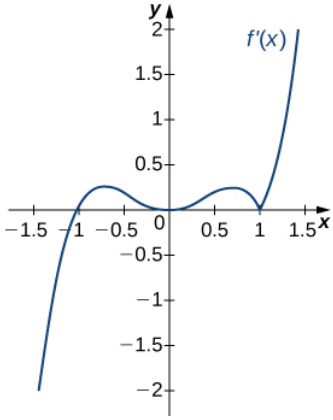


Answer

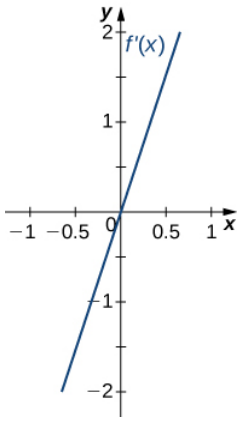
a. Increasing over $x > 0$, Decreasing over $x < 0$;
b. Minimum at $x = 0$

17)



In exercises 18 - 22, analyze the graphs of f', then list all inflection points and intervals f that are concave up and concave down.

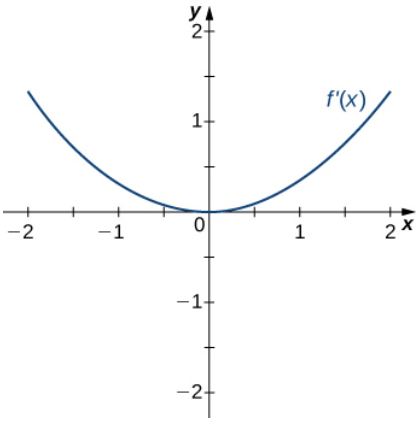
18)



Answer

Concave up for all x,
No inflection points

19)



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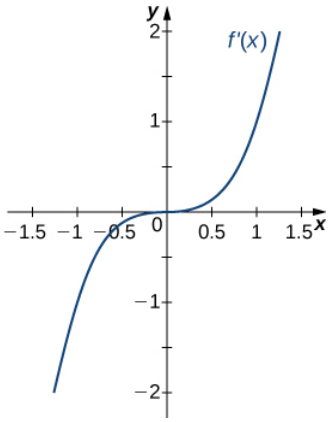
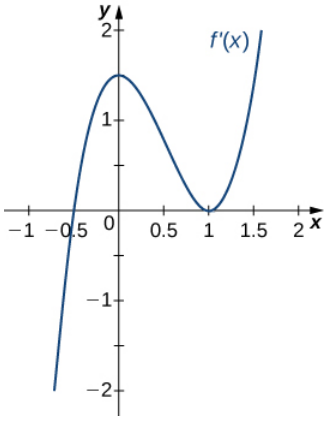


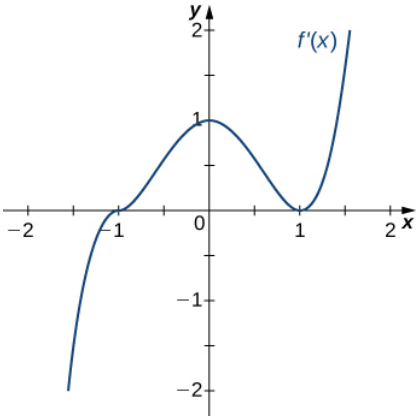
Answer

Concave up for all x,
No inflection points

21)



22)



Answer

Concave up for $x < 0$ and $x > 1$,
Concave down for $0 < x < 1$,
Inflection points at $x = 0$ and $x = 1$

For exercises 23 - 27, draw a graph that satisfies the given specifications for the domain $x = [-3, 3]$. The function does not have to be continuous or differentiable.

23) $f(x) > 0,\ f'(x) > 0$ over $x > 1,\ -3 < x < 0,\ f'(x) = 0$ over $0 < x < 1$

24) $f'(x) > 0$ over $x > 2,\ -3 < x < -1,\ f'(x) < 0$ over $-1 < x < 2,\ f''(x) < 0$ for all x

Answer

Answers will vary

25) $f''(x) < 0$ over $-1 < x < 1,\ f''(x) > 0,\ -3 < x < -1,\ 1 < x < 3,$ local maximum at $x = 0$, local minima at $x = \pm 2$

26) There is a local maximum at $x = 2$, local minimum at $x = 1$, and the graph is neither concave up nor concave down.

Answer

Answers will vary

27) There are local maxima at $x = \pm 1$, the function is concave up for all x, and the function remains positive for all x.

For the following exercises, determine

a. intervals where f is increasing or decreasing and

b. local minima and maxima of f.

28) $f(x) = \sin x + \sin^3 x$ over $-\pi < x < \pi$

Answer

a. Increasing over $-\frac{\pi}{2} < x < \frac{\pi}{2}$, decreasing over $x < -\frac{\pi}{2}$, $x > \frac{\pi}{2}$

b. Local maximum at $x = \frac{\pi}{2}$; local minimum at $x = -\frac{\pi}{2}$

29) $f(x) = x^2 + \cos x$

For exercise 30, determine

a. intervals where f is concave up or concave down, and

b. the inflection points of f.

30) $f(x) = x^3 - 4x^2 + x + 2$

Answer

a. Concave up for $x > \frac{4}{3}$, concave down for $x < \frac{4}{3}$

b. Inflection point at $x = \frac{4}{3}$

For exercises 31 - 37, determine

a. intervals where f is increasing or decreasing,

b. local minima and maxima of f,

c. intervals where f is concave up and concave down, and

d. the inflection points of f.

31) $f(x) = x^2 - 6x$

32) $f(x) = x^3 - 6x^2$

Answer

a. Increasing over $x < 0$ and $x > 4$, decreasing over $0 < x < 4$

b. Maximum at $x = 0$, minimum at $x = 4$

c. Concave up for $x > 2$, concave down for $x < 2$

d. Inflection point at $x = 2$

33) $f(x) = x^4 - 6x^3$

34) $f(x) = x^{11} - 6x^{10}$

Answer

a. Increasing over $x < 0$ and $x > \frac{60}{11}$, decreasing over $0 < x < \frac{60}{11}$

b. Maximum at $x = 0$, minimum at $x = \frac{60}{11}$

c. Concave down for $x < \frac{54}{11}$, concave up for $x > \frac{54}{11}$

d. Inflection point at $x = \frac{54}{11}$

35) $f(x) = x + x^2 - x^3$

36) $f(x) = x^2 + x + 1$

Answer

a. Increasing over $x > -\frac{1}{2}$, decreasing over $x < -\frac{1}{2}$

b. Minimum at $x = -\frac{1}{2}$

c. Concave up for all x

d. No inflection points

37) $f(x) = x^3 + x^4$

For exercises 38 - 47, determine

a. intervals where f is increasing or decreasing,

b. local minima and maxima of f,

c. intervals where f is concave up and concave down, and

d. the inflection points of f. Sketch the curve, then use a calculator to compare your answer. If you cannot determine the exact answer analytically, use a calculator.

38) [T] $f(x) = \sin(\pi x) - \cos(\pi x)$ over $x = [-1, 1]$

Answer

a. Increases over $-\frac{1}{4} < x < \frac{3}{4}$, decreases over $x > \frac{3}{4}$ and $x < -\frac{1}{4}$
b. Minimum at $x = -\frac{1}{4}$, maximum at $x = \frac{3}{4}$
c. Concave up for $-\frac{3}{4} < x < \frac{1}{4}$, concave down for $x < -\frac{3}{4}$ and $x > \frac{1}{4}$
d. Inflection points at $x = -\frac{3}{4}$, $x = \frac{1}{4}$

39) [T] $f(x) = x + \sin(2x)$ over $x = [-\frac{\pi}{2}, \frac{\pi}{2}]$

40) [T] $f(x) = \sin x + \tan x$ over $(-\frac{\pi}{2}, \frac{\pi}{2})$

Answer

a. Increasing for all x
b. No local minimum or maximum
c. Concave up for $x > 0$, concave down for $x < 0$
d. Inflection point at $x = 0$

41) [T] $f(x) = (x-2)^2(x-4)^2$

42) [T] $f(x) = \dfrac{1}{1-x}, \quad x \neq 1$

Answer

a. Increasing for all x where defined
b. No local minima or maxima
c. Concave up for $x < 1$; concave down for $x > 1$
d. No inflection points in domain

43) [T] $f(x) = \dfrac{\sin x}{x}$ over $x = [-2\pi, 0) \cup (0, 2\pi]$

44) $f(x) = \sin(x)e^x$ over $x = [-\pi, \pi]$

Answer

a. Increasing over $-\frac{\pi}{4} < x < \frac{3\pi}{4}$, decreasing over $x > \frac{3\pi}{4}$, $x < -\frac{\pi}{4}$
b. Minimum at $x = -\frac{\pi}{4}$, maximum at $x = \frac{3\pi}{4}$
c. Concave up for $-\frac{\pi}{2} < x < \frac{\pi}{2}$, concave down for $x < -\frac{\pi}{2}$, $x > \frac{\pi}{2}$
d. Inflection points at $x = \pm\frac{\pi}{2}$

45) $f(x) = \ln x\sqrt{x}, \quad x > 0$

46) $f(x) = \frac{1}{4}\sqrt{x} + \frac{1}{x}, \quad x > 0$

Answer

a. Increasing over $x > 4$, decreasing over $0 < x < 4$
b. Minimum at $x = 4$

c. Concave up for $0 < x < 8\sqrt[3]{2}$, concave down for $x > 8\sqrt[3]{2}$
d. Inflection point at $x = 8\sqrt[3]{2}$

47) $f(x) = \dfrac{e^x}{x}, \quad x \neq 0$

In exercises 48 - 52, interpret the sentences in terms of f, f', and f''.

48) The population is growing more slowly. Here f is the population.

Answer

$f > 0,\ f' > 0,\ f'' < 0$

49) A bike accelerates faster, but a car goes faster. Here $f =$ Bike's position minus Car's position.

50) The airplane lands smoothly. Here f is the plane's altitude.

Answer

$f > 0,\ f' < 0,\ f'' > 0$

51) Stock prices are at their peak. Here $f$is the stock price.

52) The economy is picking up speed. Here f is a measure of the economy, such as GDP.

Answer

$f > 0,\ f' > 0,\ f'' > 0$

For exercises 53 - 57, consider a third-degree polynomial $f(x)$, which has the properties $f'(1) = 0$ and $f'(3) = 0$.

Determine whether the following statements are true or false. Justify your answer.

53) $f(x) = 0$ for some $1 \leq x \leq 3$.

54) $f''(x) = 0$ for some $1 \leq x \leq 3$.

Answer

True, by the Mean Value Theorem

55) There is no absolute maximum at $x = 3$.

56) If $f(x)$ has three roots, then it has 1 inflection point.

Answer

True, examine derivative

57) If $f(x)$ has one inflection point, then it has three real roots.
