2.7: Chapter 2 Review Exercises

***True or False*. In exercises 1 - 4, justify your answer with a proof or a counterexample.**

1) A function has to be continuous at $x = a$ if the $\lim_{x \to a} f(x)$ exists.

2) You can use the quotient rule to evaluate $\lim_{x \to 0} \frac{\sin x}{x}$.

Answer

False, since we cannot have $\lim_{x \to 0} x = 0$ in the denominator.

3) If there is a vertical asymptote at $x = a$ for the function $f(x)$, then f is undefined at the point $x = a$.

4) If $\lim_{x \to a} f(x)$ does not exist, then f is undefined at the point $x = a$.

Answer

False. A jump discontinuity is possible.

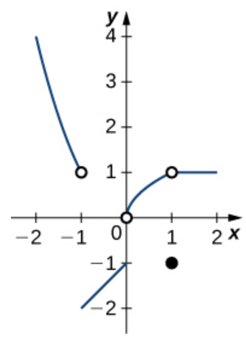
5) Using the graph, find each limit or explain why the limit does not exist.

a. $\lim_{x \to -1} f(x)$

b. $\lim_{x \to 1} f(x)$

c. $\lim_{x \to 0^+} f(x)$

d. $\lim_{x \to 2} f(x)$



In exercises 6 - 15, evaluate the limit algebraically or explain why the limit does not exist.

6) $\lim_{x \to 2} \frac{2x^2 - 3x - 2}{x - 2}$

Answer

5

7) $\lim_{x \to 0} 3x^2 - 2x + 4$

8) $\lim_{x \to 3} \frac{x^3 - 2x^2 - 1}{3x - 2}$

Answer

$8/7$

9) $\lim_{x \to \pi/2} \frac{\cot x}{\cos x}$

10) $\lim\limits_{x \to -5} \frac{x^2+25}{x+5}$

Answer

DNE

11) $\lim\limits_{x \to 2} \frac{3x^2-2x-8}{x^2-4}$

12) $\lim\limits_{x \to 1} \frac{x^2-1}{x^3-1}$

Answer

$2/3$

13) $\lim\limits_{x \to 1} \frac{x^2-1}{\sqrt{x}-1}$

14) $\lim\limits_{x \to 4} \frac{4-x}{\sqrt{x}-2}$

Answer

-4

15) $\lim\limits_{x \to 4} \frac{1}{\sqrt{x}-2}$

In exercises 16 - 17, use the squeeze theorem to prove the limit.

16) $\lim\limits_{x \to 0} x^2 \cos(2\pi x) = 0$

Answer

Since $-1 \le \cos(2\pi x) \le 1$, then $-x^2 \le x^2 \cos(2\pi x) \le x^2$. Since $\lim\limits_{x \to 0} x^2 = 0 = \lim\limits_{x \to 0} -x^2$, it follows that $\lim\limits_{x \to 0} x^2 \cos(2\pi x) = 0$.

17) $\lim\limits_{x \to 0} x^3 \sin\left(\frac{\pi}{x}\right) = 0$

18) Determine the domain such that the function $f(x) = \sqrt{x-2} + xe^x$ is continuous over its domain.

Answer

$[2, \infty]$

In exercises 19 - 20, determine the value of c such that the function remains continuous. Draw your resulting function to ensure it is continuous.

19) $f(x) = \begin{cases} x^2+1, & \text{if } x > c \\ 2^x, & \text{if } x \le c \end{cases}$

20) $f(x) = \begin{cases} \sqrt{x+1}, & \text{if } x > -1 \\ x^2+c, & \text{if } x \le -1 \end{cases}$

In exercises 21 - 22, use the precise definition of limit to prove the limit.

21) $\lim\limits_{x \to 1} (8x+16) = 24$

22) $\lim\limits_{x \to 0} x^3 = 0$

Answer

$\delta = \sqrt[3]{\varepsilon}$

23) A ball is thrown into the air and the vertical position is given by $x(t) = -4.9t^2 + 25t + 5$. Use the Intermediate Value Theorem to show that the ball must land on the ground sometime between 5 sec and 6 sec after the throw.

24) A particle moving along a line has a displacement according to the function $x(t) = t^2 - 2t + 4$, where x is measured in meters and t is measured in seconds. Find the average velocity over the time period $t = [0, 2]$.

Answer

0 m/sec

25) From the previous exercises, estimate the instantaneous velocity at $t = 2$ by checking the average velocity within $t = 0.01$ sec.
