2.4E: Exercises for Section 2.3

In exercises 1 - 4, use the limit laws to evaluate each limit. Justify each step by indicating the appropriate limit law(s).

1) $\lim_{x\to 0}(4x^2-2x+3)$

Answer

Use constant multiple law and difference law:

$$\lim_{x\to 0}(4x^2-2x+3)=4\lim_{x\to 0}x^2-2\lim_{x\to 0}x+\lim_{x\to 0}3=0+0+3=3$$

2) $\lim_{x\to 1}\frac{x^3+3x^2+5}{4-7x}$

3) $\lim_{x\to -2}\sqrt{x^2-6x+3}$

Answer

Use root law: $\lim_{x\to -2}\sqrt{x^2-6x+3}=\sqrt{\lim_{x\to -2}(x^2-6x+3)}=\sqrt{19}$

4) $\lim_{x\to -1}(9x+1)^2$

In exercises 5 - 10, use direct substitution to evaluate the limit of each continuous function.

5) $\lim_{x\to 7}x^2$

Answer

$$\lim_{x\to 7}x^2 = 49$$

6) $\lim_{x\to -2}(4x^2-1)$

7) $\lim_{x\to 0}\frac{1}{1+\sin x}$

Answer

$$\lim_{x\to 0}\frac{1}{1+\sin x} = 1$$

8) $\lim_{x\to 2}e^{2x-x^2}$

9) $\lim_{x\to 1}\frac{2-7x}{x+6}$

Answer

$$\lim_{x\to 1}\frac{2-7x}{x+6} = -\frac{5}{7}$$

10) $\lim_{x\to 3}\ln e^{3x}$

In exercises 11 - 20, use direct substitution to show that each limit leads to the indeterminate form $0/0$. Then, evaluate the limit analytically.

11) $\lim_{x\to 4}\frac{x^2-16}{x-4}$

Answer

When $x=4$, $\quad \frac{x^2-16}{x-4}=\frac{16-16}{4-4}=\frac{0}{0}$;

then, $\lim\limits_{x\to 4}\frac{x^2-16}{x-4}=\lim\limits_{x\to 4}\frac{(x+4)(x-4)}{x-4}=\lim\limits_{x\to 4}(x+4)=4+4=8$

12) $\lim\limits_{x\to 2}\frac{x-2}{x^2-2x}$

13) $\lim\limits_{x\to 6}\frac{3x-18}{2x-12}$

Answer

When $x=6$, $\quad \frac{3x-18}{2x-12}=\frac{18-18}{12-12}=\frac{0}{0}$;

then, $\lim\limits_{x\to 6}\frac{3x-18}{2x-12}=\lim\limits_{x\to 6}\frac{3(x-6)}{2(x-6)}=\lim\limits_{x\to 6}\frac{3}{2}=\frac{3}{2}$

14) $\lim\limits_{h\to 0}\frac{(1+h)^2-1}{h}$

15) $\lim\limits_{t\to 9}\frac{t-9}{\sqrt{t}-3}$

Answer

When $t=9$, $\quad \frac{t-9}{\sqrt{t}-3}=\frac{9-9}{3-3}=\frac{0}{0}$;

then, $\lim\limits_{t\to 9}\frac{t-9}{\sqrt{t}-3}=\lim\limits_{t\to 9}\frac{t-9}{\sqrt{t}-3}\frac{\sqrt{t}+3}{\sqrt{t}+3}=\lim\limits_{t\to 9}\frac{(t-9)(\sqrt{t}+3)}{t-9}=\lim\limits_{t\to 9}(\sqrt{t}+3)=\sqrt{9}+3=6$

16) $\lim\limits_{h\to 0}\frac{\frac{1}{a+h}-\frac{1}{a}}{h}$, where a is a real-valued constant

17) $\lim\limits_{\theta\to \pi}\frac{\sin\theta}{\tan\theta}$

Answer

When $\theta=\pi$, $\quad \frac{\sin\theta}{\tan\theta}=\frac{\sin\pi}{\tan\pi}=\frac{0}{0}$;

then, $\lim\limits_{\theta\to \pi}\frac{\sin\theta}{\tan\theta}=\lim\limits_{\theta\to \pi}\frac{\sin\theta}{\frac{\sin\theta}{\cos\theta}}=\lim\limits_{\theta\to \pi}\cos\theta=\cos\pi=-1$

18) $\lim\limits_{x\to 1}\frac{x^3-1}{x^2-1}$

19) $\lim\limits_{x\to 1/2}\frac{2x^2+3x-2}{2x-1}$

Answer

When $x=1/2$, $\quad \frac{2x^2+3x-2}{2x-1}=\frac{\frac{1}{2}+\frac{3}{2}-2}{1-1}=\frac{0}{0}$;

then, $\lim\limits_{x\to 1/2}\frac{2x^2+3x-2}{2x-1}=\lim\limits_{x\to 1/2}\frac{(2x-1)(x+2)}{2x-1}=\lim\limits_{x\to 1/2}(x+2)=\frac{1}{2}+2=\frac{5}{2}$

20) $\lim\limits_{x\to -3} \frac{\sqrt{x+4}-1}{x+3}$

In exercises 21 - 24, use direct substitution to obtain an undefined expression. Then, use the method used in Example 9 of this section to simplify the function and determine the limit.

21) $\lim\limits_{x\to -2^-} \frac{2x^2+7x-4}{x^2+x-2}$

Answer

$-\infty$

22) $\lim\limits_{x\to -2^+} \frac{2x^2+7x-4}{x^2+x-2}$

23) $\lim\limits_{x\to 1^-} \frac{2x^2+7x-4}{x^2+x-2}$

Answer

$-\infty$

24) $\lim\limits_{x\to 1^+} \frac{2x^2+7x-4}{x^2+x-2}$

In exercises 25 - 32, assume that $\lim\limits_{x\to 6} f(x)=4$, $\lim\limits_{x\to 6} g(x)=9$, and $\lim\limits_{x\to 6} h(x)=6$. Use these three facts and the limit laws to evaluate each limit.

25) $\lim\limits_{x\to 6} 2f(x)g(x)$

Answer

$$\lim_{x\to 6} 2f(x)g(x) = 2\left(\lim_{x\to 6} f(x)\right)\left(\lim_{x\to 6} g(x)\right) = 2(4)(9) = 72$$

26) $\lim\limits_{x\to 6} \frac{g(x)-1}{f(x)}$

27) $\lim\limits_{x\to 6} \left(f(x)+\frac{1}{3}g(x)\right)$

Answer

$$\lim_{x\to 6}\left(f(x)+\frac{1}{3}g(x)\right) = \lim_{x\to 6} f(x)+\frac{1}{3}\lim_{x\to 6} g(x) = 4+\frac{1}{3}(9) = 7$$

28) $\lim\limits_{x\to 6} \frac{\left(h(x)\right)^3}{2}$

29) $\lim\limits_{x\to 6} \sqrt{g(x)-f(x)}$

Answer

$$\lim_{x\to 6}\sqrt{g(x)-f(x)} = \sqrt{\lim_{x\to 6} g(x)-\lim_{x\to 6} f(x)} = \sqrt{9-4} = \sqrt{5}$$

30) $\lim\limits_{x\to 6} x\cdot h(x)$

31) $\lim\limits_{x\to 6} [(x+1)\cdot f(x)]$

Answer

$$\lim_{x\to 6}[(x+1)f(x)] = \left(\lim_{x\to 6}(x+1)\right)\left(\lim_{x\to 6} f(x)\right) = 7(4) = 28$$

32) $\lim_{x \to 6}(f(x) \cdot g(x) - h(x))$

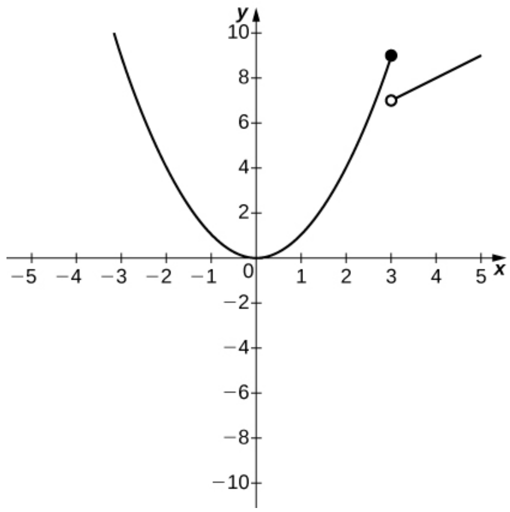
[T] In exercises 33 - 35, use a calculator to draw the graph of each piecewise-defined function and study the graph to evaluate the given limits.

33) $f(x) = \begin{cases} x^2, & x \leq 3 \\ x+4, & x > 3 \end{cases}$

a. $\lim_{x \to 3^-} f(x)$

b. $\lim_{x \to 3^+} f(x)$

Answer



a. 9; b.7

34) $g(x) = \begin{cases} x^3 - 1, & x \leq 0 \\ 1, & x > 0 \end{cases}$

a. $\lim_{x \to 0^-} g(x)$

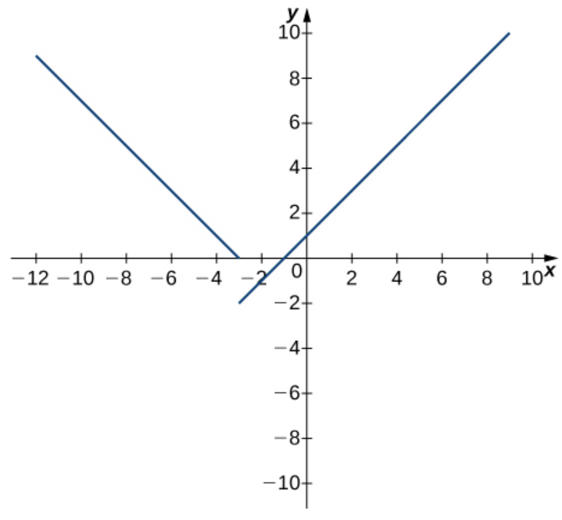
b. $\lim_{x \to 0^+} g(x)$

35) $h(x) = \begin{cases} x^2 - 2x + 1, & x < 2 \\ 3 - x, & x \geq 2 \end{cases}$

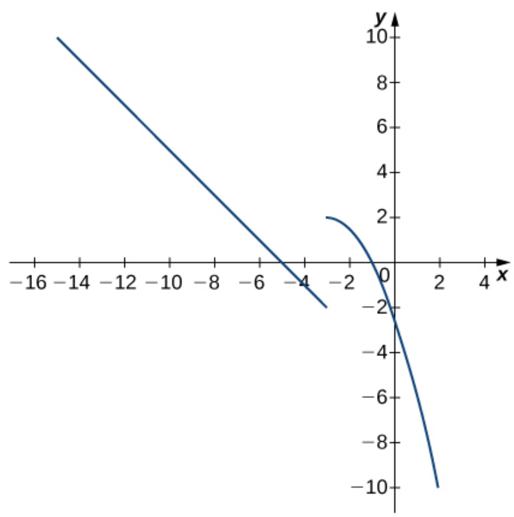
a. $\lim_{x \to 2^-} h(x)$

b. $\lim_{x \to 2^+} h(x)$

In exercises 36 - 43, use the following graphs and the limit laws to evaluate each limit.



$y = f(x)$



$y = g(x)$

36) $\lim\limits_{x\to -3^+} (f(x) + g(x))$

37) $\lim\limits_{x\to -3^-} (f(x) - 3g(x))$

Answer

$$\lim_{x\to -3^-} (f(x) - 3g(x)) = \lim_{x\to -3^-} f(x) - 3\lim_{x\to -3^-} g(x) = 0 + 6 = 6$$

38) $\lim\limits_{x\to 0} \dfrac{f(x)g(x)}{3}$

39) $\lim\limits_{x\to -5} \dfrac{2 + g(x)}{f(x)}$

Answer

$$\lim_{x\to -5} \frac{2+g(x)}{f(x)} = \frac{2 + \left(\lim\limits_{x\to -5} g(x)\right)}{\lim\limits_{x\to -5} f(x)} = \frac{2+0}{2} = 1$$

40) $\lim\limits_{x\to 1} (f(x))^2$

41) $\lim\limits_{x\to 1}\sqrt[3]{f(x)-g(x)}$

Answer

$$\lim\limits_{x\to 1}\sqrt[3]{f(x)-g(x)}=\sqrt[3]{\lim\limits_{x\to 1}f(x)-\lim\limits_{x\to 1}g(x)}=\sqrt[3]{2+5}=\sqrt[3]{7}$$

42) $\lim\limits_{x\to -7}(x\cdot g(x))$

43) $\lim\limits_{x\to -9}[x\cdot f(x)+2\cdot g(x)]$

Answer

$$\lim\limits_{x\to -9}(xf(x)+2g(x))=\left(\lim\limits_{x\to -9}x\right)\left(\lim\limits_{x\to -9}f(x)\right)+2\lim\limits_{x\to -9}g(x)=(-9)(6)+2(4)=-46$$

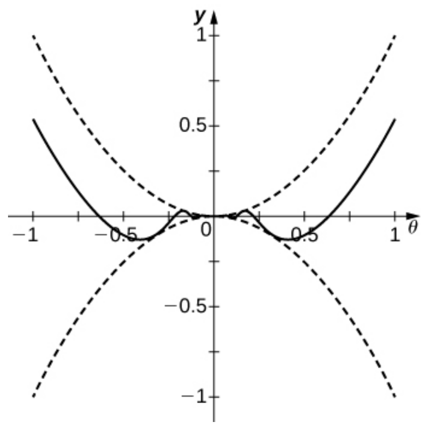
For exercises 44 - 46, evaluate the limit using the squeeze theorem. Use a calculator to graph the functions $f(x)$, $g(x)$, and $h(x)$ when possible.

44) [T] True or False? If $2x-1\le g(x)\le x^2-2x+3$, then $\lim\limits_{x\to 2}g(x)=0$.

45) [T] $\lim\limits_{\theta\to 0}\theta^2\cos\left(\frac{1}{\theta}\right)$

Answer

The limit is zero.



46) $\lim\limits_{x\to 0}f(x)$, where $f(x)=\begin{cases}0, & x \text{ rational}\\ x^2, & x \text{ irrational}\end{cases}$

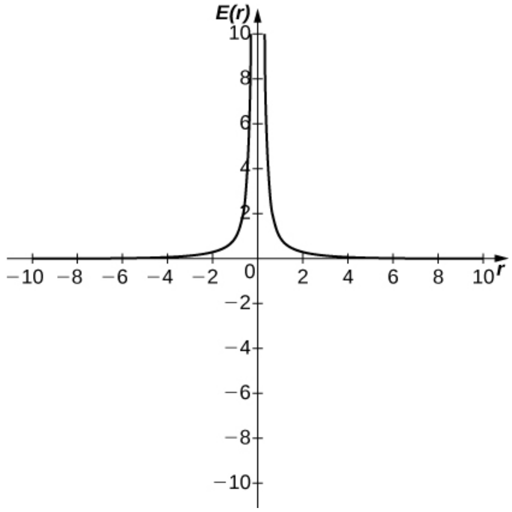
47) [T] In physics, the magnitude of an electric field generated by a point charge at a distance r in vacuum is governed by Coulomb's law: $E(r)=\frac{q}{4\pi\varepsilon_0 r^2}$, where E represents the magnitude of the electric field, q is the charge of the particle, r is the distance between the particle and where the strength of the field is measured, and $\frac{1}{4\pi\varepsilon_0}$ is Coulomb's constant: $8.988\times 109N\cdot m^2/C^2$.

a. Use a graphing calculator to graph $E(r)$ given that the charge of the particle is $q=10^{-10}$.

b. Evaluate $\lim\limits_{r\to 0^+}E(r)$. What is the physical meaning of this quantity? Is it physically relevant? Why are you evaluating from the right?

Answer

a.



b. ∞. The magnitude of the electric field as you approach the particle q becomes infinite. It does not make physical sense to evaluate negative distance.

48) [T] The density of an object is given by its mass divided by its volume: $\rho = m/V$.

a. Use a calculator to plot the volume as a function of density $(V = m/\rho)$, assuming you are examining something of mass 8 kg $(m = 8)$.

b. Evaluate $\lim\limits_{x \to 0^+} V(\rho)$ and explain the physical meaning.

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