

4.5: Intersection and Union of Events and Venn Diagrams

Sometimes, when the probability problems are complex, it can be helpful to graph the situation. Tree diagrams and Venn diagrams are two tools that can be used to visualize and solve conditional probabilities.

Tree Diagrams

A *tree diagram* is a special type of graph used to determine the outcomes of an experiment. It consists of "branches" that are labeled with either frequencies or probabilities. Tree diagrams can make some probability problems easier to visualize and solve. The following example illustrates how to use a tree diagram.

Example 4.5.1: Probabilities from Sampling with replacement

In an urn, there are 11 balls. Three balls are red (R) and eight balls are blue (B). Draw two balls, one at a time, with replacement (remember that "with replacement" means that you put the first ball back in the urn before you select the second ball). The tree diagram using frequencies that show all the possible outcomes follows.

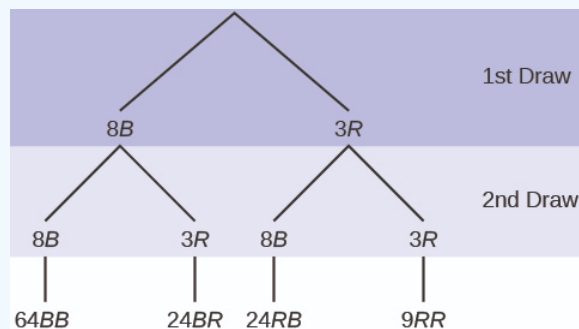


Figure 4.5.1: Total = $64 + 24 + 24 + 9 = 121$

The first set of branches represents the first draw. The second set of branches represents the second draw. Each of the outcomes is distinct. In fact, we can list each red ball as R_1 , R_2 , and R_3 and each blue ball as B_1 , B_2 , B_3 , B_4 , B_5 , B_6 , B_7 , and B_8 . Then the nine RR outcomes can be written as:

R_1R_1 R_1R_2 R_1R_3 R_2R_1 R_2R_2 R_2R_3 R_3R_1 R_3R_2 R_3R_3

The other outcomes are similar.

There are a total of 11 balls in the urn. Draw two balls, one at a time, with replacement. There are $11(11) = 121$ outcomes, the size of the sample space.

Example 4.5.2: Probabilities from Sampling without replacement

An urn has three red marbles and eight blue marbles in it. Draw two marbles, one at a time, this time without replacement, from the urn. (remember that "without replacement" means that you do not put the first ball back before you select the second marble). Following is a tree diagram for this situation. The branches are labeled with probabilities instead of frequencies. The numbers at the ends of the branches are calculated by multiplying the numbers on the two corresponding branches, for example, $\left(\frac{3}{11}\right)\left(\frac{2}{10}\right) = \frac{6}{110}$.

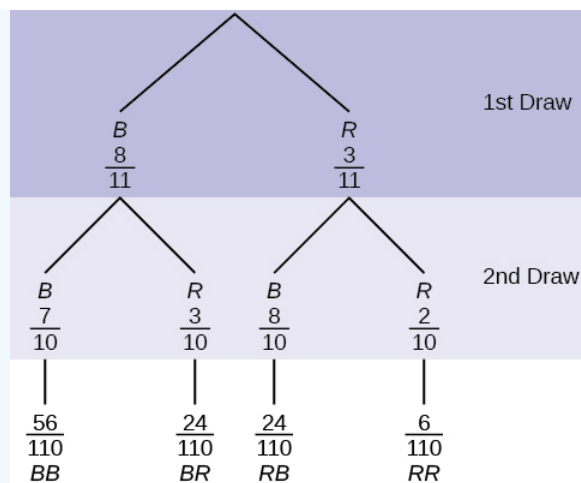


Figure 4.5.3: Total = $\frac{56+24+24+6}{110} = \frac{110}{110} = 1$

If you draw a red on the first draw from the three red possibilities, there are two red marbles left to draw on the second draw. You do not put back or replace the first marble after you have drawn it. You draw **without replacement**, so that on the second draw there are ten marbles left in the urn.

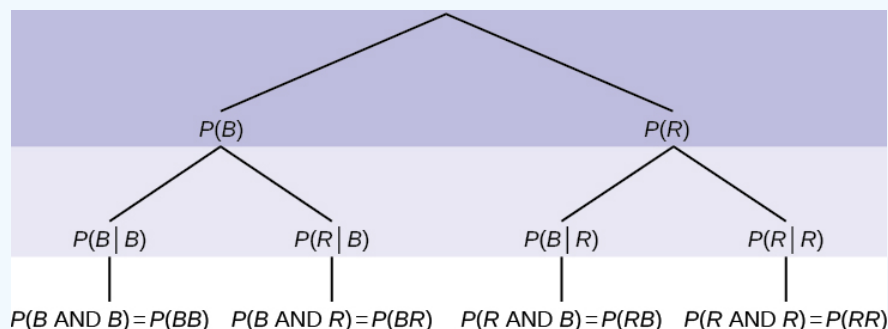
Calculate the following probabilities using the tree diagram.

- $P(RR) = \underline{\hspace{2cm}}$
- Fill in the blanks: $P(RB \text{ OR } BR) = \left(\frac{3}{11}\right)\left(\frac{8}{10}\right) + (\underline{\hspace{1cm}})(\underline{\hspace{1cm}}) = \frac{48}{110}$
- $P(R \text{ on 2nd} | B \text{ on 1st}) = \underline{\hspace{2cm}}$
- Fill in the blanks: $P(R \text{ on 1st AND } B \text{ on 2nd}) = P(RB) = (\underline{\hspace{1cm}})(\underline{\hspace{1cm}}) = \frac{24}{110}$
- Find $P(BB)$.
- Find $P(B \text{ on 2nd} | R \text{ on 1st})$.

Answers

- $P(RR) = \left(\frac{3}{11}\right)\left(\frac{2}{10}\right) = \frac{6}{110}$
- $P(RB \text{ OR } BR) = \left(\frac{3}{11}\right)\left(\frac{8}{10}\right) + \left(\frac{8}{11}\right)\left(\frac{3}{10}\right) = \frac{48}{110}$
- $P(R \text{ on 2nd} | B \text{ on 1st}) = \frac{3}{10}$
- $P(R \text{ on 1st AND } B \text{ on 2nd}) = P(RB) = \left(\frac{3}{11}\right)\left(\frac{8}{10}\right) = \frac{24}{110}$
- $P(BB) = \left(\frac{8}{11}\right)\left(\frac{7}{10}\right)$
- Using the tree diagram, $P(B \text{ on 2nd} | R \text{ on 1st}) = P(R|B) = \frac{8}{10}$.

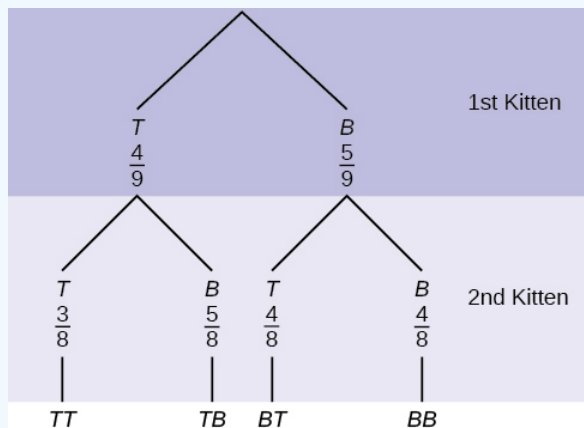
If we are using probabilities, we can label the tree in the following general way.



- $P(R|R)$ here means $P(R \text{ on 2nd} | R \text{ on 1st})$
- $P(B|R)$ here means $P(B \text{ on 2nd} | R \text{ on 1st})$
- $P(R|B)$ here means $P(R \text{ on 2nd} | B \text{ on 1st})$
- $P(B|B)$ here means $P(B \text{ on 2nd} | B \text{ on 1st})$

Example 4.5.3

A litter of kittens available for adoption at the Humane Society has four tabby kittens and five black kittens. A family comes in and randomly selects two kittens (without replacement) for adoption.



- What is the probability that both kittens are tabby?
a. $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$ b. $\left(\frac{4}{9}\right)\left(\frac{4}{9}\right)$ c. $\left(\frac{4}{9}\right)\left(\frac{3}{8}\right)$ d. $\left(\frac{4}{9}\right)\left(\frac{5}{9}\right)$
- What is the probability that one kitten of each coloring is selected?
a. $\left(\frac{4}{9}\right)\left(\frac{5}{9}\right)$ b. $\left(\frac{4}{9}\right)\left(\frac{5}{8}\right)$ c. $\left(\frac{4}{9}\right)\left(\frac{5}{9}\right) + \left(\frac{5}{9}\right)\left(\frac{4}{9}\right)$ d. $\left(\frac{4}{9}\right)\left(\frac{5}{8}\right) + \left(\frac{5}{9}\right)\left(\frac{4}{8}\right)$
- What is the probability that a tabby is chosen as the second kitten when a black kitten was chosen as the first?
- What is the probability of choosing two kittens of the same color?

Answer

- a. c, b. d, c. $\frac{4}{8}$, d. $\frac{32}{72}$

Venn Diagram

A Venn diagram is a picture that represents the outcomes of an experiment. It generally consists of a box that represents the sample space S together with circles or ovals. The circles or ovals represent events.

Example 4.5.4

Suppose an experiment has the outcomes 1, 2, 3, ..., 12 where each outcome has an equal chance of occurring. Let event $A = \{1, 2, 3, 4, 5, 6\}$ and event $B = \{6, 7, 8, 9\}$. Then $A \text{ AND } B = \{6\}$ and $A \text{ OR } B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. The Venn diagram is as follows:

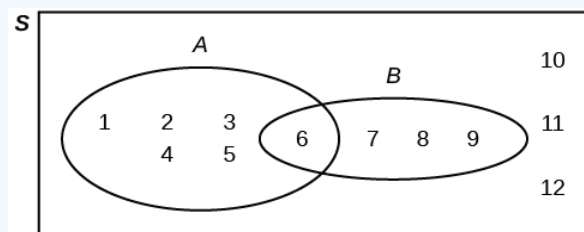


Figure 4.5.5:

Example 4.5.5

Flip two fair coins. Let $A =$ tails on the first coin. Let $B =$ tails on the second coin. Then $A = \{TT, TH\}$ and $B = \{TT, HT\}$. Therefore, $A \text{ AND } B = \{TT\}$. $A \text{ OR } B = \{TH, TT, HT\}$.

The sample space when you flip two fair coins is $X = \{HH, HT, TH, TT\}$. The outcome HH is in NEITHER A NOR B . The Venn diagram is as follows:

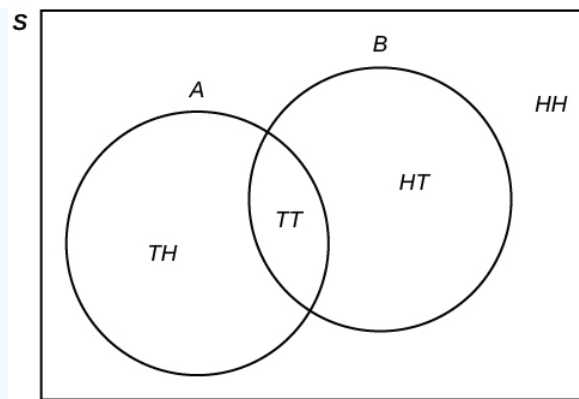


Figure 4.5.7:

Example 4.5.6: Probability and Venn Diagrams

Forty percent of the students at a local college belong to a club and 50% work part time. Five percent of the students work part time and belong to a club. Draw a Venn diagram showing the relationships. Let C = student belongs to a club and PT = student works part time.

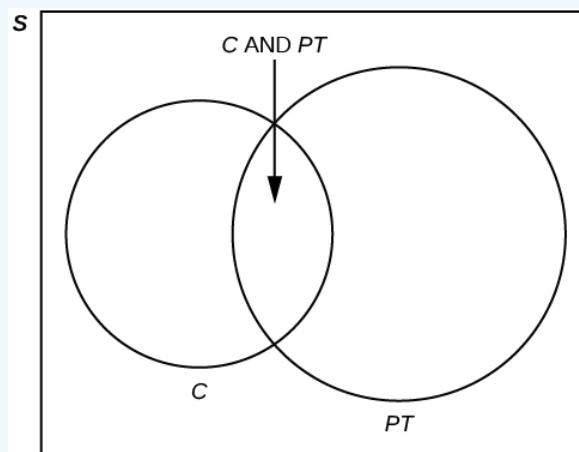


Figure 4.5.8:

If a student is selected at random, find

- the probability that the student belongs to a club. $P(C) = 0.40$
- the probability that the student works part time. $P(PT) = 0.50$
- the probability that the student belongs to a club AND works part time. $P(C \text{ AND } PT) = 0.05$
- the probability that the student belongs to a club **given** that the student works part time.

$$P(C|PT) = \frac{P(C \text{ AND } PT)}{P(PT)} = \frac{0.05}{0.50} = 0.1$$

- the probability that the student belongs to a club **OR** works part time.

$$P(C \text{ OR } PT) = P(C) + P(PT) - P(C \text{ AND } PT) = 0.40 + 0.50 - 0.05 = 0.85$$

Example 4.5.7

A person with type O blood and a negative Rh factor (Rh-) can donate blood to any person with any blood type. Four percent of African Americans have type O blood and a negative RH factor, 5–10% of African Americans have the Rh- factor, and 51% have type O blood.

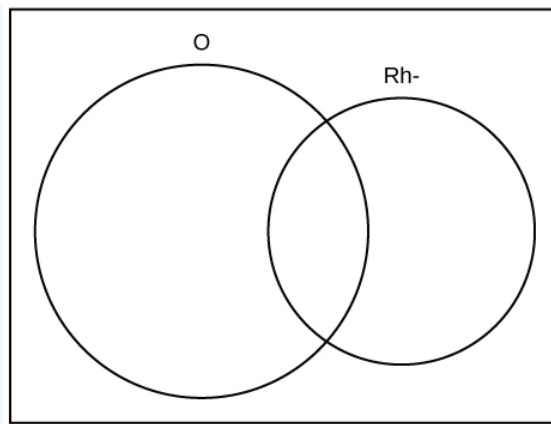


Figure 4.5.10:

The “O” circle represents the African Americans with type O blood. The “Rh-“ oval represents the African Americans with the Rh- factor.

We will take the average of 5% and 10% and use 7.5% as the percent of African Americans who have the Rh- factor. Let O = African American with Type O blood and R = African American with Rh- factor.

- $P(O) =$ _____
- $P(R) =$ _____
- $P(O \text{ AND } R) =$ _____
- $P(O \text{ OR } R) =$ _____
- In the Venn Diagram, describe the overlapping area using a complete sentence.
- In the Venn Diagram, describe the area in the rectangle but outside both the circle and the oval using a complete sentence.

Answer

- 0.51; b. 0.075; c. 0.04; d. 0.545; e. The area represents the African Americans that have type O blood and the Rh- factor. f. The area represents the African Americans that have neither type O blood nor the Rh- factor.

WeBWork Problems

References

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Review

A tree diagram use branches to show the different outcomes of experiments and makes complex probability questions easy to visualize. A Venn diagram is a picture that represents the outcomes of an experiment. It generally consists of a box that represents the sample space S together with circles or ovals. The circles or ovals represent events. A Venn diagram is especially helpful for visualizing the OR event, the AND event, and the complement of an event and for understanding conditional probabilities.

Glossary

Tree Diagram

the useful visual representation of a sample space and events in the form of a “tree” with branches marked by possible outcomes together with associated probabilities (frequencies, relative frequencies)

Venn Diagram

the visual representation of a sample space and events in the form of circles or ovals showing their intersections

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