4.1: Probability Experiments and Sample Spaces

Probability is a measure that is associated with how certain we are of outcomes of a particular experiment or activity. An **experiment** is a planned operation carried out under controlled conditions. If the result is not predetermined, then the experiment is said to be a **chance** experiment. Flipping one fair coin twice is an example of an experiment.

A result of an experiment is called an **outcome**. The **sample space** of an experiment is the set of all possible outcomes. Three ways to represent a sample space are: to list the possible outcomes, to create a tree diagram, or to create a Venn diagram. The uppercase letter *S* is used to denote the sample space. For example, if you flip one fair coin, $S = \{\text{H, T}\}$ where $\text{H} =$ heads and $\text{T} =$ tails are the outcomes.

An **event** is any combination of outcomes. Upper case letters like A and B represent events. For example, if the experiment is to flip one fair coin, event A might be getting at most one head. The probability of an event A is written $P(\text{A})$.

Definition: Probability

The *probability* of any outcome is the long-term relative frequency of that outcome. Probabilities are between zero and one, inclusive (that is, zero and one and all numbers between these values).

- $P(\text{A}) = 0$ means the event A can never happen.
- $P(\text{A}) = 1$ means the event A always happens.
- $P(\text{A}) = 0.5$ means the event A is equally likely to occur or not to occur. For example, if you flip one fair coin repeatedly (from 20 to 2,000 to 20,000 times) the relative frequency of heads approaches 0.5 (the probability of heads).

The "OR" Event

An outcome is in the event A OR B if the outcome is in A or is in B or is in both A and B. For example, let $\text{A} = \{1, 2, 3, 4, 5\}$ and $\text{B} = \{4, 5, 6, 7, 8\}$. $\text{A OR B} = \{1, 2, 3, 4, 5, 6, 7, 8\}$. Notice that 4 and 5 are NOT listed twice.

The "AND" Event

An outcome is in the event A AND B if the outcome is in both A and B at the same time. For example, let A and B be {1, 2, 3, 4, 5} and {4, 5, 6, 7, 8}, respectively. Then $\text{A AND B} = 4, 5$.

The **complement** of event A is denoted A' (read "A prime"). A' consists of all outcomes that are **NOT** in A. Notice that

$$P(\text{A}) + P(\text{A}') = 1.$$

For example, let $\text{S} = \{1, 2, 3, 4, 5, 6\}$ and let $\text{A} = 1, 2, 3, 4$. Then, $\text{A}' = 5, 6$ and $P(A) = \frac{4}{6}$, $P(\text{A}') = \frac{2}{6}$, and

$$P(\text{A}) + P(\text{A}') = \frac{4}{6} + \frac{2}{6} = 1.$$

The conditional probability of A given B is written $P(\text{A}|\text{B})$. $P(\text{A}|\text{B})$ is the probability that event A will occur given that the event B has already occurred. **A conditional reduces the sample space**. We calculate the probability of A from the reduced sample space B. The formula to calculate $P(\text{A}|\text{B})$ is

$$P(\text{A}|\text{B}) = \frac{\text{P(A AND B)}}{\text{P(B)}}$$

where $P(\text{B})$ is greater than zero.

For example, suppose we toss one fair, six-sided die. The sample space $\text{S} = \{1, 2, 3, 4, 5, 6\}$. Let $\text{A} =$ face is 2 or 3 and $\text{B} =$ face is even (2, 4, 6). To calculate $P(\text{A}|\text{B})$, we count the number of outcomes 2 or 3 in the sample space $\text{B} = \{2, 4, 6\}$. Then we divide that by the number of outcomes B (rather than S).

We get the same result by using the formula. Remember that S has six outcomes.

$$P(\text{A}|\text{B}) = \frac{\text{P(A AND B)}}{P(\text{B})}$$
$$= \frac{\frac{\text{the number of outcomes that are 2 or 3 and even in S}}{6}}{\frac{\text{the number of outcomes that are even in S}}{6}}$$
$$= \frac{\frac{1}{6}}{\frac{3}{6}} = \frac{1}{3}$$

Understanding Terminology and Symbols

It is important to read each problem carefully to think about and understand what the events are. Understanding the wording is the first very important step in solving probability problems. Reread the problem several times if necessary. Clearly identify the event of interest. Determine whether there is a condition stated in the wording that would indicate that the probability is conditional; carefully identify the condition, if any.

Example 4.1.1

The sample space S is the whole numbers starting at one and less than 20.

a. $S =$ _____

Let event $A =$ the even numbers and event $B =$ numbers greater than 13.

b. $A =$ _____, $B =$ _____
c. $P(\text{A}) =$ _____, $P(\text{B}) =$ _____
d. A AND B = _____, A OR B = _____
e. $P(\text{A AND B}) =$ _____, $P(\text{A OR B}) =$ _____
f. $\text{A}' =$ _____, $P(\text{A}') =$ _____
g. $P(\text{A}) + P(\text{A}') =$ _____
h. $P(\text{A}|\text{B}) =$ _____, $P(\text{B}|\text{A}) =$ _____; are the probabilities equal?

Answer

a. $\text{S} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19\}$
b. $\text{A} = \{2, 4, 6, 8, 10, 12, 14, 16, 18\}, \text{B} = \{14, 15, 16, 17, 18, 19\}$
c. $P(\text{A}) = \frac{9}{19}$, $P(\text{B}) = \frac{6}{19}$
d. A AND B $= \{14, 16, 18\}$, A OR B $= \{2, 4, 6, 8, 10, 12, 14, 15, 16, 17, 18, 19\}$
e. $P(\text{A AND B}) = \frac{3}{19}$, $P(\text{A OR B}) = \frac{12}{19}$
f. $\text{A}' = 1, 3, 5, 7, 9, 11, 13, 15, 17, 19$, $P(\text{A}') = \frac{10}{19}$
g. $P(\text{A}) + P(\text{A}') = 1$ $\left(\left(\frac{9}{19} + \frac{10}{19} = 1\right)\right.$
h. $P(\text{A}|\text{B}) = \frac{\text{P(A AND B)}}{\text{P(B)}} = \frac{3}{6}$, $P(\text{B}|\text{A}) = \frac{\text{P(A AND B)}}{\text{P(A)}} = \frac{3}{9}$, No

Example 4.1.2A

A fair, six-sided die is rolled. Describe the sample space *S*, identify each of the following events with a subset of *S* and compute its probability (an outcome is the number of dots that show up).

a. Event T = the outcome is two.
b. Event A = the outcome is an even number.
c. Event B = the outcome is less than four.
d. The complement of A.
e. A GIVEN B
f. B GIVEN A
g. A AND B
h. A OR B

i. A OR B′
j. Event N = the outcome is a prime number.
k. Event I = the outcome is seven.

Solution

a. $\mathrm{T} = \{2\}$, $P(\mathrm{T}) = \frac{1}{6}$
b. $A = \{2, 4, 6\}$, $P(\mathrm{A}) = \frac{1}{2}$
c. $\mathrm{B} = \{1, 2, 3\}$, $P(\mathrm{B}) = \frac{1}{2}$
d. $\mathrm{A}' = \{1, 3, 5\}$, $P(\mathrm{A}') = \frac{1}{2}$
e. $\mathrm{A}|\mathrm{B} = \{2\}$, $P(\mathrm{A}|\mathrm{B}) = \frac{1}{3}$
f. $\mathrm{B}|\mathrm{A} = \{2\}$, $P(\mathrm{B}|\mathrm{A}) = \frac{1}{3}$
g. $\mathrm{A\ AND\ B} = 2, P(\mathrm{A\ AND\ B}) = \frac{1}{6}$
h. $\mathrm{A\ OR\ B} = \{1, 2, 3, 4, 6\}$, $P(\mathrm{A\ OR\ B}) = \frac{5}{6}$
i. $\mathrm{A\ OR\ B}' = \{2, 4, 5, 6\}$, $P(\mathrm{A\ OR\ B}') = \frac{2}{3}$
j. $\mathrm{N} = \{2, 3, 5\}$, $P(\mathrm{N}) = \frac{1}{2}$
k. A six-sided die does not have seven dots. $P(7) = 0$.

Example 4.1.2B

Table describes the distribution of a random sample S of 100 individuals, organized by gender and whether they are right- or left-handed.

	Right-handed	Left-handed
Males	43	9
Females	44	4

Let's denote the events $M =$ the subject is male, $F =$ the subject is female, $R =$ the subject is right-handed, $L =$ the subject is left-handed. Compute the following probabilities:

a. $P(\mathrm{M})$
b. $P(\mathrm{F})$
c. $P(\mathrm{R})$
d. $P(\mathrm{L})$
e. $P(\mathrm{M\ AND\ R})$
f. $P(\mathrm{F\ AND\ L})$
g. $P(\mathrm{M\ OR\ F})$
h. $P(\mathrm{M\ OR\ R})$
i. $P(\mathrm{F\ OR\ L})$
j. $P(\mathrm{M'})$
k. $P(\mathrm{R}|\mathrm{M})$
l. $P(\mathrm{F}|\mathrm{L})$
m. $P(\mathrm{L}|\mathrm{F})$

Answer

a. $P(\mathrm{M}) = 0.52$
b. $P(\mathrm{F}) = 0.48$
c. $P(\mathrm{R}) = 0.87$
d. $P(\mathrm{L}) = 0.13$
e. $P(\mathrm{M\ AND\ R}) = 0.43$
f. $P(\mathrm{F\ AND\ L}) = 0.04$
g. $P(\mathrm{M\ OR\ F}) = 1$
h. $P(\mathrm{M\ OR\ R}) = 0.96$

i. $P(\text{F OR L}) = 0.57$
j. $P(\text{M'}) = 0.48$
k. $P(\text{R}|\text{M}) = 0.8269$ (rounded to four decimal places)
l. $P(\text{F}|\text{L}) = 0.3077$ (rounded to four decimal places)
m. $P(\text{L}|\text{F}) = 0.0833$

WeBWorK Problems

References

1. "Countries List by Continent." Worldatlas, 2013. Available online at http://www.worldatlas.com/cntycont.htm (accessed May 2, 2013).

Review

In this module we learned the basic terminology of probability. The set of all possible outcomes of an experiment is called the sample space. Events are subsets of the sample space, and they are assigned a probability that is a number between zero and one, inclusive.

Formula Review

A and B are events

$P(\text{S}) = 1$ where S is the sample space

$0 \leq P(\text{A}) \leq 1$

$P(\text{A}|\text{B}) = \frac{\text{P(A AND B)}}{\text{P(B)}}$

Glossary

Conditional Probability
the likelihood that an event will occur given that another event has already occurred

Equally Likely
Each outcome of an experiment has the same probability.

Event
a subset of the set of all outcomes of an experiment; the set of all outcomes of an experiment is called a **sample space** and is usually denoted by S. An event is an arbitrary subset in S. It can contain one outcome, two outcomes, no outcomes (empty subset), the entire sample space, and the like. Standard notations for events are capital letters such as A, B, C, and so on.

Experiment
a planned activity carried out under controlled conditions

Outcome
a particular result of an experiment

Probability
a number between zero and one, inclusive, that gives the likelihood that a specific event will occur; the foundation of statistics is given by the following 3 axioms (by A.N. Kolmogorov, 1930's): Let S denote the sample space and A and B are two events in S. Then:

- $0 \leq P(\text{A}) \leq 1$
- If A and B are any two mutually exclusive events, then $\text{P(A OR B)} = P(\text{A}) + P(\text{B})$.
- $P(\text{S}) = 1$

Sample Space
the set of all possible outcomes of an experiment

The AND Event

An outcome is in the event A AND B if the outcome is in both A AND B at the same time.

The Complement Event

The complement of event A consists of all outcomes that are NOT in A.

The Conditional Probability of *A* GIVEN *B*

$P(\mathrm{A}|\mathrm{B})$ is the probability that event A will occur given that the event B has already occurred.

The Or Event

An outcome is in the event A OR B if the outcome is in A or is in B or is in both A and B.

Contributors and Attributions

- Barbara Illowsky and Susan Dean (De Anza College) with many other contributing authors. Content produced by OpenStax College is licensed under a Creative Commons Attribution License 4.0 license. Download for free at http://cnx.org/contents/30189442-699...b91b9de@18.114.
