

2.2: Organizing and Graphing Quantitative Data

For most of the work you do in this course, you will be working with quantitative data, and you will use a frequency table and frequency histogram to organize and graph the data. An advantage of a frequency table and frequency histogram is that they can be used to organize and display large data sets. A rule of thumb is to use a histogram when the data set consists of 100 values or more.

Frequency

Twenty students were asked how many hours they worked per day. Their responses, in hours, are as follows:

5; 6; 3; 3; 2; 4; 7; 5; 2; 3; 5; 6; 5; 4; 4; 3; 5; 2; 5; 3.

Table lists the different data values in ascending order and their frequencies.

Table 2.2.1: Frequency Table of Student Work Hours

DATA VALUE	FREQUENCY
2	3
3	5
4	3
5	6
6	2
7	1

Definition: Relative Frequency

A frequency is the number of times a value of the data occurs. According to Table 2.2.1, there are three students who work two hours, five students who work three hours, and so on. The sum of the values in the frequency column, 20, represents the total number of students included in the sample.

Definition: Relative frequencies

A *relative frequency* is the ratio (fraction or proportion) of the number of times a value of the data occurs in the set of all outcomes to the total number of outcomes. To find the relative frequencies, divide each frequency by the total number of students in the sample—in this case, 20. Relative frequencies can be written as fractions, percents, or decimals.

Table 2.2.2: Frequency Table of Student Work Hours with Relative Frequencies

DATA VALUE	FREQUENCY	RELATIVE FREQUENCY
2	3	$\frac{3}{20}$ or 0.15
3	5	$\frac{5}{20}$ or 0.25
4	3	$\frac{3}{20}$ or 0.15
5	6	$\frac{6}{20}$ or 0.30
6	2	$\frac{2}{20}$ or 0.10
7	1	$\frac{1}{20}$ or 0.05

The sum of the values in the relative frequency column of Table 2.2.2 is $\frac{20}{20}$, or 1.

Definition: Cumulative Relative Frequency

Cumulative relative frequency is the accumulation of the previous relative frequencies. To find the cumulative relative frequencies, add all the previous relative frequencies to the relative frequency for the current row, as shown in Table 2.2.3.

Table 2.2.3: Frequency Table of Student Work Hours with Relative and Cumulative Relative Frequencies

DATA VALUE	FREQUENCY	RELATIVE FREQUENCY	CUMULATIVE RELATIVE FREQUENCY
2	3	$\frac{3}{20}$ or 0.15	0.15
3	5	$\frac{5}{20}$ or 0.25	$0.15 + 0.25 = 0.40$
4	3	$\frac{3}{20}$ or 0.15	$0.40 + 0.15 = 0.55$
5	6	$\frac{6}{20}$ or 0.30	$0.55 + 0.30 = 0.85$
6	2	$\frac{2}{20}$ or 0.10	$0.85 + 0.10 = 0.95$
7	1	$\frac{1}{20}$ or 0.05	$0.95 + 0.05 = 1.00$

The last entry of the cumulative relative frequency column is one, indicating that one hundred percent of the data has been accumulated.

Table 2.2.4 represents the heights, in inches, of a sample of 100 male semiprofessional soccer players.

Table 2.2.4: Frequency Table of Soccer Player Height

HEIGHTS (INCHES)	FREQUENCY	RELATIVE FREQUENCY	CUMULATIVE RELATIVE FREQUENCY
59.95–61.95	5	$\frac{5}{100} = 0.05$	0.05
61.95–63.95	3	$\frac{3}{100} = 0.03$	$0.05 + 0.03 = 0.08$
63.95–65.95	15	$\frac{15}{100} = 0.15$	$0.08 + 0.15 = 0.23$
65.95–67.95	40	$\frac{40}{100} = 0.40$	$0.23 + 0.40 = 0.63$
67.95–69.95	17	$\frac{17}{100} = 0.17$	$0.63 + 0.17 = 0.80$
69.95–71.95	12	$\frac{12}{100} = 0.12$	$0.80 + 0.12 = 0.92$
71.95–73.95	7	$\frac{7}{100} = 0.07$	$0.92 + 0.07 = 0.99$
73.95–75.95	1	$\frac{1}{100} = 0.01$	$0.99 + 0.01 = 1.00$
	Total = 100	Total = 1.00	

The data in this table have been **grouped** into the following intervals:

- 61.95 to 63.95 inches
- 63.95 to 65.95 inches
- 65.95 to 67.95 inches
- 67.95 to 69.95 inches
- 69.95 to 71.95 inches
- 71.95 to 73.95 inches
- 73.95 to 75.95 inches

In this sample, there are **five** players whose heights fall within the interval 59.95–61.95 inches, **three** players whose heights fall within the interval 61.95–63.95 inches, **15** players whose heights fall within the interval 63.95–65.95 inches, **40** players whose heights fall within the interval 65.95–67.95 inches, **17** players whose heights fall within the interval 67.95–69.95 inches, **12** players

whose heights fall within the interval 69.95–71.95, **seven** players whose heights fall within the interval 71.95–73.95, and **one** player whose heights fall within the interval 73.95–75.95. All heights fall between the endpoints of an interval and not at the endpoints.

Collaborative Exercise 2.2.7

In your class, have someone conduct a survey of the number of siblings (brothers and sisters) each student has. Create a frequency table. Add to it a relative frequency column and a cumulative relative frequency column. Answer the following questions:

- What percentage of the students in your class have no siblings?
- What percentage of the students have from one to three siblings?
- What percentage of the students have fewer than three siblings?

Example 2.2.7

Nineteen people were asked how many miles, to the nearest mile, they commute to work each day. The data are as follows: 2; 5; 7; 3; 2; 10; 18; 15; 20; 7; 10; 18; 5; 12; 13; 12; 4; 5; 10. Table 2.2.6 was produced:

Table 2.2.6: Frequency of Commuting Distances

DATA	FREQUENCY	RELATIVE FREQUENCY	CUMULATIVE RELATIVE FREQUENCY
3	3	$\frac{3}{19}$	0.1579
4	1	$\frac{1}{19}$	0.2105
5	3	$\frac{3}{19}$	0.1579
7	2	$\frac{2}{19}$	0.2632
10	3	$\frac{3}{19}$	0.4737
12	2	$\frac{2}{19}$	0.7895
13	1	$\frac{1}{19}$	0.8421
15	1	$\frac{1}{19}$	0.8948
18	1	$\frac{1}{19}$	0.9474
20	1	$\frac{1}{19}$	1.0000

- Is the table correct? If it is not correct, what is wrong?
- True or False: Three percent of the people surveyed commute three miles. If the statement is not correct, what should it be? If the table is incorrect, make the corrections.
- What fraction of the people surveyed commute five or seven miles?
- What fraction of the people surveyed commute 12 miles or more? Less than 12 miles? Between five and 13 miles (not including five and 13 miles)?

Answer

- No. The frequency column sums to 18, not 19. Not all cumulative relative frequencies are correct.
- False. The frequency for three miles should be one; for two miles (left out), two. The cumulative relative frequency column should read: 0.1052, 0.1579, 0.2105, 0.3684, 0.4737, 0.6316, 0.7368, 0.7895, 0.8421, 0.9474, 1.0000.
- $\frac{5}{19}$
- $\frac{7}{19}$, $\frac{12}{19}$, $\frac{7}{19}$

A histogram consists of contiguous (adjoining) boxes. It has both a horizontal axis and a vertical axis. The horizontal axis is labeled with what the data represents (for instance, distance from your home to school). The vertical axis is labeled either frequency or relative frequency (or percent frequency or probability). The graph will have the same shape with either label. The histogram (like the stemplot) can give you the shape of the data, the center, and the spread of the data.

The relative frequency is equal to the frequency for an observed value of the data divided by the total number of data values in the sample. (Remember, frequency is defined as the number of times an answer occurs.) If:

- f is frequency
- n is total number of data values (or the sum of the individual frequencies), and
- RF is relative frequency,

then:

$$RF = \frac{f}{n} \quad (2.2.1)$$

For example, if three students in Mr. Ahab's English class of 40 students received from 90% to 100%, then, $f = 3$, $n = 40$, and $RF = \frac{3}{40} = 0.075$. 7.5% of the students received 90–100%. 90–100% are quantitative measures.

To construct a histogram, first decide how many bars or intervals, also called classes, represent the data. Many histograms consist of five to 15 bars or classes for clarity. The number of bars needs to be chosen. Choose a starting point for the first interval to be less than the smallest data value. A convenient starting point is a lower value carried out to one more decimal place than the value with the most decimal places. For example, if the value with the most decimal places is 6.1 and this is the smallest value, a convenient starting point is 6.05 ($6.1 - 0.05 = 6.05$). We say that 6.05 has more precision. If the value with the most decimal places is 2.23 and the lowest value is 1.5, a convenient starting point is 1.495 ($1.5 - 0.005 = 1.495$). If the value with the most decimal places is 3.234 and the lowest value is 1.0, a convenient starting point is 0.9995 ($1.0 - 0.0005 = 0.9995$). If all the data happen to be integers and the smallest value is two, then a convenient starting point is 1.5 ($2 - 0.5 = 1.5$). Also, when the starting point and other boundaries are carried to one additional decimal place, no data value will fall on a boundary. The next two examples go into detail about how to construct a histogram using continuous data and how to create a histogram using discrete data.

Example 2.2.1

The following data are the heights (in inches to the nearest half inch) of 100 male semiprofessional soccer players. The heights are **continuous** data, since height is measured.

60; 60.5; 61; 61; 61.5

63.5; 63.5; 63.5

64; 64; 64; 64; 64; 64; 64; 64; 64.5; 64.5; 64.5; 64.5; 64.5; 64.5; 64.5

66; 66; 66; 66; 66; 66; 66; 66; 66; 66; 66.5; 66.5; 66.5; 66.5; 66.5; 66.5; 66.5; 66.5; 66.5; 66.5; 66.5; 66.5; 67; 67; 67; 67; 67; 67; 67; 67; 67; 67; 67.5; 67.5; 67.5; 67.5; 67.5; 67.5; 67.5

68; 68; 69; 69; 69; 69; 69; 69; 69; 69; 69; 69.5; 69.5; 69.5; 69.5

70; 70; 70; 70; 70; 70; 70.5; 70.5; 71; 71; 71

72; 72; 72; 72.5; 72.5; 73; 73.5

74

The smallest data value is 60. Since the data with the most decimal places has one decimal (for instance, 61.5), we want our starting point to have two decimal places. Since the numbers 0.5, 0.05, 0.005, etc. are convenient numbers, use 0.05 and subtract it from 60, the smallest value, for the convenient starting point.

$60 - 0.05 = 59.95$ which is more precise than, say, 61.5 by one decimal place. The starting point is, then, 59.95.

The largest value is 74, so $74 + 0.05 = 74.05$ is the ending value.

Next, calculate the width of each bar or class interval. To calculate this width, subtract the starting point from the ending value and divide by the number of bars (you must choose the number of bars you desire). Suppose you choose eight bars.

$$\frac{74.05 - 59.95}{8} = 1.76 \quad (2.2.2)$$

We will round up to two and make each bar or class interval two units wide. Rounding up to two is one way to prevent a value from falling on a boundary. Rounding to the next number is often necessary even if it goes against the standard rules of rounding. For this example, using 1.76 as the width would also work. A guideline that is followed by some for the width of a bar or class interval is to take the square root of the number of data values and then round to the nearest whole number, if necessary. For example, if there are 150 values of data, take the square root of 150 and round to 12 bars or intervals.

The boundaries are:

- 59.95
- $59.95 + 2 = 61.95$
- $61.95 + 2 = 63.95$
- $63.95 + 2 = 65.95$
- $65.95 + 2 = 67.95$
- $67.95 + 2 = 69.95$
- $69.95 + 2 = 71.95$
- $71.95 + 2 = 73.95$
- $73.95 + 2 = 75.95$

The heights 60 through 61.5 inches are in the interval 59.95–61.95. The heights that are 63.5 are in the interval 61.95–63.95. The heights that are 64 through 64.5 are in the interval 63.95–65.95. The heights 66 through 67.5 are in the interval 65.95–67.95. The heights 68 through 69.5 are in the interval 67.95–69.95. The heights 70 through 71 are in the interval 69.95–71.95. The heights 72 through 73.5 are in the interval 71.95–73.95. The height 74 is in the interval 73.95–75.95.

The following histogram displays the heights on the x-axis and relative frequency on the y-axis.

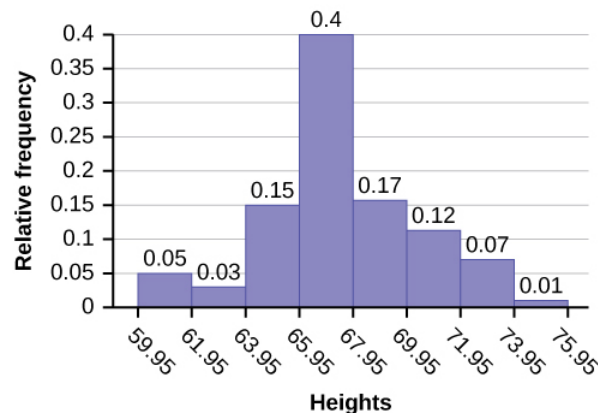


Figure 2.2.1: Histogram of something

Example 2.2.2

The following data are the number of books bought by 50 part-time college students at ABC College. The number of books is **discrete data**, since books are counted.

1; 1; 1; 1; 1; 1; 1; 1; 1; 1
 2; 2; 2; 2; 2; 2; 2; 2; 2; 2
 3; 3; 3; 3; 3; 3; 3; 3; 3; 3; 3; 3; 3; 3; 3
 4; 4; 4; 4; 4
 5; 5; 5; 5
 6; 6

Eleven students buy one book. Ten students buy two books. Sixteen students buy three books. Six students buy four books. Five students buy five books. Two students buy six books.

Because the data are integers, subtract 0.5 from 1, the smallest data value and add 0.5 to 6, the largest data value. Then the starting point is 0.5 and the ending value is 6.5.

Next, calculate the width of each bar or class interval. If the data are discrete and there are not too many different values, a width that places the data values in the middle of the bar or class interval is the most convenient. Since the data consist of the numbers 1, 2, 3, 4, 5, 6, and the starting point is 0.5, a width of one places the 1 in the middle of the interval from 0.5 to 1.5, the 2 in the middle of the interval from 1.5 to 2.5, the 3 in the middle of the interval from 2.5 to 3.5, the 4 in the middle of the interval from _____ to _____, the 5 in the middle of the interval from _____ to _____, and the _____ in the middle of the interval from _____ to _____.

Answer

Calculate the number of bars as follows:

$$\frac{6.5 - 0.5}{\text{number of bars}} = 1$$

where 1 is the width of a bar. Therefore, bars = 6.

The following histogram displays the number of books on the x-axis and the frequency on the y-axis.

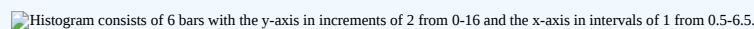


Figure 2.2.2.

Example 2.2.3

Using this data set, construct a histogram.

Number of Hours My Classmates Spent Playing Video Games on Weekends

9.95	10	2.25	16.75	0
19.5	22.5	7.5	15	12.75
5.5	11	10	20.75	17.5
23	21.9	24	23.75	18
20	15	22.9	18.8	20.5

Answer

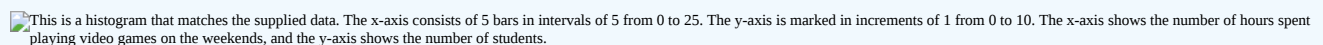


Figure 2.2.3.

Some values in this data set fall on boundaries for the class intervals. A value is counted in a class interval if it falls on the left boundary, but not if it falls on the right boundary. Different researchers may set up histograms for the same data in different ways. There is more than one correct way to set up a histogram.

Frequency Polygons

Frequency polygons are analogous to line graphs, and just as line graphs make continuous data visually easy to interpret, so too do frequency polygons. To construct a frequency polygon, first examine the data and decide on the number of intervals, or class intervals, to use on the x-axis and y-axis. After choosing the appropriate ranges, begin plotting the data points. After all the points are plotted, draw line segments to connect them.

Example 2.2.4

A frequency polygon was constructed from the frequency table below.

Frequency Distribution for Calculus Final Test Scores

Lower Bound	Upper Bound	Frequency	Cumulative Frequency
49.5	59.5	5	5

Lower Bound	Upper Bound	Frequency	Cumulative Frequency
59.5	69.5	10	15
69.5	79.5	30	45
79.5	89.5	40	85
89.5	99.5	15	100

 A frequency polygon was constructed from the frequency table below.

Figure 2.2.4.

The first label on the x -axis is 44.5. This represents an interval extending from 39.5 to 49.5. Since the lowest test score is 54.5, this interval is used only to allow the graph to touch the x -axis. The point labeled 54.5 represents the next interval, or the first “real” interval from the table, and contains five scores. This reasoning is followed for each of the remaining intervals with the point 104.5 representing the interval from 99.5 to 109.5. Again, this interval contains no data and is only used so that the graph will touch the x -axis. Looking at the graph, we say that this distribution is skewed because one side of the graph does not mirror the other side.

Suppose that we want to study the temperature range of a region for an entire month. Every day at noon we note the temperature and write this down in a log. A variety of statistical studies could be done with this data. We could find the mean or the median temperature for the month. We could construct a histogram displaying the number of days that temperatures reach a certain range of values. However, all of these methods ignore a portion of the data that we have collected.

One feature of the data that we may want to consider is that of time. Since each date is paired with the temperature reading for the day, we don’t have to think of the data as being random. We can instead use the times given to impose a chronological order on the data. A graph that recognizes this ordering and displays the changing temperature as the month progresses is called a time series graph.

Constructing a Time Series Graph

To construct a time series graph, we must look at both pieces of our **paired data set**. We start with a standard Cartesian coordinate system. The horizontal axis is used to plot the date or time increments, and the vertical axis is used to plot the values of the variable that we are measuring. By doing this, we make each point on the graph correspond to a date and a measured quantity. The points on the graph are typically connected by straight lines in the order in which they occur.

Example 2.2.6

The following data shows the Annual Consumer Price Index, each month, for ten years. Construct a time series graph for the Annual Consumer Price Index data only.

Year	Jan	Feb	Mar	Apr	May	Jun	Jul
2003	181.7	183.1	184.2	183.8	183.5	183.7	183.9
2004	185.2	186.2	187.4	188.0	189.1	189.7	189.4
2005	190.7	191.8	193.3	194.6	194.4	194.5	195.4
2006	198.3	198.7	199.8	201.5	202.5	202.9	203.5
2007	202.416	203.499	205.352	206.686	207.949	208.352	208.299
2008	211.080	211.693	213.528	214.823	216.632	218.815	219.964
2009	211.143	212.193	212.709	213.240	213.856	215.693	215.351
2010	216.687	216.741	217.631	218.009	218.178	217.965	218.011
2011	220.223	221.309	223.467	224.906	225.964	225.722	225.922

Year	Jan	Feb	Mar	Apr	May	Jun	Jul
2012	226.665	227.663	229.392	230.085	229.815	229.478	229.104

Year	Aug	Sep	Oct	Nov	Dec	Annual
2003	184.6	185.2	185.0	184.5	184.3	184.0
2004	189.5	189.9	190.9	191.0	190.3	188.9
2005	196.4	198.8	199.2	197.6	196.8	195.3
2006	203.9	202.9	201.8	201.5	201.8	201.6
2007	207.917	208.490	208.936	210.177	210.036	207.342
2008	219.086	218.783	216.573	212.425	210.228	215.303
2009	215.834	215.969	216.177	216.330	215.949	214.537
2010	218.312	218.439	218.711	218.803	219.179	218.056
2011	226.545	226.889	226.421	226.230	225.672	224.939
2012	230.379	231.407	231.317	230.221	229.601	229.594

Answer

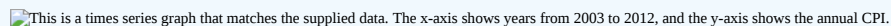


Figure 2.2.7.

Uses of a Time Series Graph

Time series graphs are important tools in various applications of statistics. When recording values of the same variable over an extended period of time, sometimes it is difficult to discern any trend or pattern. However, once the same data points are displayed graphically, some features jump out. Time series graphs make trends easy to spot.

Review

A **histogram** is a graphic version of a frequency distribution. The graph consists of bars of equal width drawn adjacent to each other. The horizontal scale represents classes of quantitative data values and the vertical scale represents frequencies. The heights of the bars correspond to frequency values. Histograms are typically used for large, continuous, quantitative data sets. A frequency polygon can also be used when graphing large data sets with data points that repeat. The data usually goes on y-axis with the frequency being graphed on the x-axis. Time series graphs can be helpful when looking at large amounts of data for one variable over a period of time.

WebWork Problems

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Frequency

the number of times a value of the data occurs

Histogram

a graphical representation in $x - y$ form of the distribution of data in a data set; x represents the data and y represents the frequency, or relative frequency. The graph consists of contiguous rectangles.

Relative Frequency

the ratio of the number of times a value of the data occurs in the set of all outcomes to the number of all outcomes

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