# 5.7: The Poisson Distribution

The Poisson distribution is popular for modelling the number of times an event occurs in an interval of time or space. It is a discrete probability distribution that expresses the probability of a given number of events occurring in a fixed interval of time and/or space if these events occur with a known average rate and independently of the time since the last event.

**two main characteristics of a Poisson experiment**

1. The Poisson probability distribution gives the probability of a number of events occurring in a **fixed interval** of time or space if these events happen with a known average rate and independently of the time since the last event. For example, a book editor might be interested in the number of words spelled incorrectly in a particular book. It might be that, on the average, there are five words spelled incorrectly in 100 pages. The interval is the 100 pages.
2. The Poisson distribution may be used to approximate the binomial if the probability of success is "small" (such as 0.01) and the number of trials is "large" (such as 1,000). You will verify the relationship in the homework exercises. $n$ is the number of trials, and $p$ is the probability of a "success."

The random variable $X =$ the number of occurrences in the interval of interest.

**Example $5.7.1$**

The average number of loaves of bread put on a shelf in a bakery in a half-hour period is 12. Of interest is the number of loaves of bread put on the shelf in five minutes. The time interval of interest is five minutes. What is the probability that the number of loaves, selected randomly, put on the shelf in five minutes is three?

**Solution**

Let $X =$ the number of loaves of bread put on the shelf in five minutes. If the average number of loaves put on the shelf in 30 minutes (half-hour) is 12, then the average number of loaves put on the shelf in five minutes is $\left(\frac{5}{30}\right)(12) = 2$ loaves of bread.

The probability question asks you to find $P(x = 3)$.

**Example $5.7.2$**

A bank expects to receive six bad checks per day, on average. What is the probability of the bank getting fewer than five bad checks on any given day? Of interest is the number of checks the bank receives in one day, so the time interval of interest is one day. Let $X =$ the number of bad checks the bank receives in one day. If the bank expects to receive six bad checks per day then the average is six checks per day. Write a mathematical statement for the probability question.

**Answer**

$P(x < 5)$

**Example $5.7.3$**

You notice that a news reporter says "uh," on average, two times per broadcast. What is the probability that the news reporter says "uh" more than two times per broadcast. This is a Poisson problem because you are interested in knowing the number of times the news reporter says "uh" during a broadcast.

a. What is the interval of interest?
b. What is the average number of times the news reporter says "uh" during one broadcast?
c. Let $X =$ ____________. What values does $X$ take on?
d. The probability question is $P($______$)$.

**Solutions**

a. one broadcast
b. 2
c. Let $X =$ the number of times the news reporter says "uh" during one broadcast.

$$x = 0, 1, 2, 3, \ldots \tag{5.7.1}$$

d. $P(x > 2)$

## Notation for the Poisson: $P =$ Poisson Probability Distribution Function

$$X \sim P(\mu) \tag{5.7.2}$$

Read this as "$X$ is a random variable with a Poisson distribution." The parameter is $\mu$ (or $\lambda$); $\mu$ (or $\lambda$) $=$ the mean for the interval of interest.

### Example 5.7.4

Leah's answering machine receives about six telephone calls between 8 a.m. and 10 a.m. What is the probability that Leah receives more than one call in the next 15 minutes?

**Solution**

Let $X$ = the number of calls Leah receives in 15 minutes. (The *interval of interest* is 15 minutes or $\frac{1}{4}$ hour.)

$$x = 0, 1, 2, 3, \ldots \tag{5.7.3}$$

If Leah receives, on the average, six telephone calls in two hours, and there are eight 15 minute intervals in two hours, then Leah receives

$\left(\frac{1}{8}\right)(6) = 0.75$ calls in 15 minutes, on average. So, $\mu = 0.75$ for this problem.

$X \sim P(0.75)$

Find $P(x > 1)$. $P(x > 1) = 0.1734$ (calculator or computer)

- Press 1 – and then press 2$^{\text{nd}}$ DISTR.
- Arrow down to poissoncdf. Press ENTER.
- Enter (.75,1).
- The result is $P(x > 1) = 0.1734$.

> The TI calculators use $\lambda$ (lambda) for the mean.

The probability that Leah receives more than one telephone call in the next 15 minutes is about 0.1734:

$P(x > 1) = 1 - \text{poissoncdf}(0.75, 1)$.

The graph of $X \sim P(0.75)$ is:

This graphs shows a poisson probability distribution. It has 5 bars that decrease in height from left to right. The x-axis shows values in increments of 1 starting with 0, representing the number of calls Leah receives within 15 minutes. The y-axis ranges from 0 to 0.5 in increments of 0.1.

Figure 5.7.1

The *y*-axis contains the probability of $x$ where $X =$ the number of calls in 15 minutes.

### Example 5.7.5

According to Baydin, an email management company, an email user gets, on average, 147 emails per day. Let $X =$ the number of emails an email user receives per day. The discrete random variable $X$ takes on the values $x = 0, 1, 2 \ldots$. The random variable $X$ has a Poisson distribution: $X \sim P(147)$. The mean is 147 emails.

a. What is the probability that an email user receives exactly 160 emails per day?
b. What is the probability that an email user receives at most 160 emails per day?
c. What is the standard deviation?

**Solutions**

a. $P(x = 160) = \text{poissonpdf}(147, 160) \approx 0.0180$

b. $P(x \leq 160) = \text{poissoncdf}(147, 160) \approx 0.8666$
c. Standard Deviation $= \sigma = \sqrt{\mu} = \sqrt{147} \approx 12.1244$

Example 5.7.6

Text message users receive or send an average of 41.5 text messages per day.

a. How many text messages does a text message user receive or send per hour?
b. What is the probability that a text message user receives or sends two messages per hour?
c. What is the probability that a text message user receives or sends more than two messages per hour?

**Solutions**

a. Let $X =$ the number of texts that a user sends or receives in one hour. The average number of texts received per hour is $\frac{41.5}{24} \approx 1.7292$.
b. $X \sim P(1.7292)$, so $P(x = 2) = \text{poissonpdf}(1.7292, 2) \approx 0.2653$
c. $P(x > 2) = 1 - P(x \leq 2) = 1 - \text{poissoncdf}(1.7292, 2) \approx 1 - 0.7495 = 0.2505$

The Poisson distribution can be used to approximate probabilities for a binomial distribution. This next example demonstrates the relationship between the Poisson and the binomial distributions. Let $n$ represent the number of binomial trials and let $p$ represent the probability of a success for each trial. If $n$ is large enough and $p$ is small enough then the Poisson approximates the binomial very well. In general, $n$ is considered "large enough" if it is greater than or equal to 20. The probability $p$ from the binomial distribution should be less than or equal to 0.05. When the Poisson is used to approximate the binomial, we use the binomial mean $\mu = np$. The variance of $X$ is $\sigma^2 = \sqrt{\mu}$ and the standard deviation is $\sigma = \sqrt{\mu}$. The Poisson approximation to a binomial distribution was commonly used in the days before technology made both values very easy to calculate.

Example 5.7.7

On May 13, 2013, starting at 4:30 PM, the probability of low seismic activity for the next 48 hours in Alaska was reported as about 1.02%. Use this information for the next 200 days to find the probability that there will be low seismic activity in ten of the next 200 days. Use both the binomial and Poisson distributions to calculate the probabilities. Are they close?

**Answer**

Let $X$ = the number of days with low seismic activity.

Using the binomial distribution:

- $P(x = 10) = \text{binompdf}(200, .0102, 10) \approx\ 0.000039$

Using the Poisson distribution:

- Calculate $\mu = np = 200(0.0102) \approx 2.04$
- $P(x = 10) = \text{poissonpdf}(2.04, 10) \approx 0.000045$

We expect the approximation to be good because $n$ is large (greater than 20) and $p$ is small (less than 0.05). The results are close—both probabilities reported are almost 0.

## WeBWorK Problems

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## Review

A Poisson probability distribution of a discrete random variable gives the probability of a number of events occurring in a fixed interval of time or space, if these events happen at a known average rate and independently of the time since the last event. The Poisson distribution may be used to approximate the binomial, if the probability of success is "small" (less than or equal to 0.05) and the number of trials is "large" (greater than or equal to 20).

## Formula Review

$X \sim P(\mu)$ means that $X$ has a Poisson probability distribution where $X =$ the number of occurrences in the interval of interest.

$X$ takes on the values $x = 0, 1, 2, 3, \ldots$

The mean $\mu$ is typically given.

The variance is $\sigma = \mu$, and the standard deviation is

$$\sigma = \sqrt{\mu} \tag{5.7.4}$$

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When $P(\mu)$ is used to approximate a binomial distribution, $\mu = np$ where $n$ represents the number of independent trials and $p$ represents the probability of success in a single trial.

## Contributors and Attributions

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*Use the following information to answer the next six exercises:* On average, a clothing store gets 120 customers per day.

## Glossary

**Poisson Probability Distribution**

a discrete random variable (RV) that counts the number of times a certain event will occur in a specific interval; characteristics of the variable:

- The probability that the event occurs in a given interval is the same for all intervals.
- The events occur with a known mean and independently of the time since the last event.

The distribution is defined by the mean $\mu$ of the event in the interval. Notation: $X \sim P(\mu)$. The mean is $\mu = np$. The standard deviation is $\sigma = \sqrt{\mu}$. The probability of having exactly $x$ successes in $r$ trials is $P(X = x) =$

$$\left(e^{-\mu}\right)\frac{\mu^{x}}{x!} \tag{5.7.5}$$

. The Poisson distribution is often used to approximate the binomial distribution, when $n$ is "large" and $p$ is "small" (a general rule is that $n$ should be greater than or equal to 20 and $p$ should be less than or equal to 0.05).

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