

12.3: A Test of Independence or Homogeneity

Tests of independence involve using a contingency table of observed (data) values.

The test statistic for a *test of independence* is similar to that of a goodness-of-fit test:

$$\sum_{(i,j)} \frac{(O - E)^2}{E} \quad (12.3.1)$$

where:

- O = observed values
- E = expected values
- i = the number of rows in the table
- j = the number of columns in the table

There are $i \cdot j$ terms of the form $\frac{(O-E)^2}{E}$.

The expected value for each cell needs to be at least five in order for you to use this test.

A test of independence determines whether two factors are independent or not. You first encountered the term independence in [Probability Topics](#). As a review, consider the following example.

Example 12.3.1

Suppose A = a speeding violation in the last year and B = a cell phone user while driving. If A and B are independent then $P(A \text{ AND } B) = P(A)P(B)$. $A \text{ AND } B$ is the event that a driver received a speeding violation last year and also used a cell phone while driving. Suppose, in a study of drivers who received speeding violations in the last year, and who used cell phone while driving, that 755 people were surveyed. Out of the 755, 70 had a speeding violation and 685 did not; 305 used cell phones while driving and 450 did not.

Let y = expected number of drivers who used a cell phone while driving and received speeding violations.

If A and B are independent, then $P(A \text{ AND } B) = P(A)P(B)$. By substitution,

$$\frac{y}{755} = \left(\frac{70}{755} \right) \left(\frac{305}{755} \right)$$

Solve for y :

$$y = \frac{(70)(305)}{755} = 28.3$$

About 28 people from the sample are expected to use cell phones while driving and to receive speeding violations.

In a test of independence, we state the null and alternative hypotheses in words. Since the contingency table consists of **two factors**, the null hypothesis states that the factors are **independent** and the alternative hypothesis states that they are **not independent (dependent)**. If we do a test of independence using the example, then the null hypothesis is:

H_0 : Being a cell phone user while driving and receiving a speeding violation are independent events.

If the null hypothesis were true, we would expect about 28 people to use cell phones while driving and to receive a speeding violation.

The test of independence is always right-tailed because of the calculation of the test statistic. If the expected and observed values are not close together, then the test statistic is very large and way out in the right tail of the chi-square curve, as it is in a goodness-of-fit.

The number of degrees of freedom for the test of independence is:

$$df = (\text{number of columns} - 1)(\text{number of rows} - 1)$$

The following formula calculates the **expected number** (E):

$$E = \frac{(\text{row total})(\text{column total})}{\text{total number surveyed}}$$

Example 12.3.2

In a volunteer group, adults 21 and older volunteer from one to nine hours each week to spend time with a disabled senior citizen. The program recruits among community college students, four-year college students, and nonstudents. In Table 12.3.1 is a **sample** of the adult volunteers and the number of hours they volunteer per week.

Table 12.3.1: Number of Hours Worked Per Week by Volunteer Type (Observed). The table contains **observed (O)** values (data).

Type of Volunteer	1–3 Hours	4–6 Hours	7–9 Hours	Row Total
Community College Students	111	96	48	255
Four-Year College Students	96	133	61	290
Nonstudents	91	150	53	294
Column Total	298	379	162	839

Is the number of hours volunteered **independent** of the type of volunteer?

Answer

The **observed table** and the question at the end of the problem, "Is the number of hours volunteered independent of the type of volunteer?" tell you this is a test of independence. The two factors are **number of hours volunteered** and **type of volunteer**. This test is always right-tailed.

- H_0 : The number of hours volunteered is **independent** of the type of volunteer.
- H_a : The number of hours volunteered is **dependent** on the type of volunteer.

The expected results are in Table 12.3.2

Table 12.3.2: Number of Hours Worked Per Week by Volunteer Type (Expected). The table contains **expected (E)** values (data).

Type of Volunteer	1-3 Hours	4-6 Hours	7-9 Hours
Community College Students	90.57	115.19	49.24
Four-Year College Students	103.00	131.00	56.00
Nonstudents	104.42	132.81	56.77

For example, the calculation for the expected frequency for the top left cell is

$$E = \frac{(\text{row total})(\text{column total})}{\text{total number surveyed}} = \frac{(255)(298)}{839} = 90.57$$

Calculate the test statistic: $\chi^2 = 12.99$ (calculator or computer)

Distribution for the test: χ^2_4

$$df = (3 \text{ columns} - 1)(3 \text{ rows} - 1) = (2)(2) = 4$$

Graph:


 Nonsymmetrical chi-square curve with values of 0 and 12.99 on the x-axis representing the test statistic of number of hours worked by volunteers of different types. A vertical upward line extends from 12.99 to the curve and the area to the right of this is equal to the p-value.

Figure 12.3.1.

Probability statement: $p\text{-value} = P(\chi^2 > 12.99) = 0.0113$

Compare α and the p -value: Since no α is given, assume $\alpha = 0.05$. $p\text{-value} = 0.0113$. $\alpha > p\text{-value}$.

Make a decision: Since $\alpha > p\text{-value}$, reject H_0 . This means that the factors are not independent.

Conclusion: At a 5% level of significance, from the data, there is sufficient evidence to conclude that the number of hours volunteered and the type of volunteer are dependent on one another.

For the example in Table, if there had been another type of volunteer, teenagers, what would the degrees of freedom be?

Example 12.3.3

De Anza College is interested in the relationship between anxiety level and the need to succeed in school. A random sample of 400 students took a test that measured anxiety level and need to succeed in school. Table shows the results. De Anza College wants to know if anxiety level and need to succeed in school are independent events.

Need to Succeed in School vs. Anxiety Level

Need to Succeed in School	High Anxiety	Med-high Anxiety	Medium Anxiety	Med-low Anxiety	Low Anxiety	Row Total
High Need	35	42	53	15	10	155
Medium Need	18	48	63	33	31	193
Low Need	4	5	11	15	17	52
Column Total	57	95	127	63	58	400

- How many high anxiety level students are expected to have a high need to succeed in school?
- If the two variables are independent, how many students do you expect to have a low need to succeed in school and a med-low level of anxiety?
- $E = \frac{(\text{row total})(\text{column total})}{\text{total surveyed}} = \underline{\hspace{2cm}}$
- The expected number of students who have a med-low anxiety level and a low need to succeed in school is about $\underline{\hspace{2cm}}$.

Solution

a. The column total for a high anxiety level is 57. The row total for high need to succeed in school is 155. The sample size or total surveyed is 400.

$$E = \frac{(\text{row total})(\text{column total})}{\text{total surveyed}} = \frac{155 \cdot 57}{400} = 22.09 \quad (12.3.2)$$

The expected number of students who have a high anxiety level and a high need to succeed in school is about 22.

b. The column total for a med-low anxiety level is 63. The row total for a low need to succeed in school is 52. The sample size or total surveyed is 400.

$$c. E = \frac{(\text{row total})(\text{column total})}{\text{total surveyed}} = 8.19$$

d. 8

WeBWork Problems

References

- DiCamilo, Mark, Mervin Field, "Most Californians See a Direct Linkage between Obesity and Sugary Sodas. Two in Three Voters Support Taxing Sugar-Sweetened Beverages If Proceeds are Tied to Improving School Nutrition and Physical Activity

- Programs.” The Field Poll, released Feb. 14, 2013. Available online at field.com/fieldpollonline/sub...rs/RIs2436.pdf (accessed May 24, 2013).
2. Harris Interactive, “Favorite Flavor of Ice Cream.” Available online at <http://www.statisticbrain.com/favori...r-of-ice-cream> (accessed May 24, 2013)
3. “Youngest Online Entrepreneurs List.” Available online at <http://www.statisticbrain.com/younge...repreneur-list> (accessed May 24, 2013).

Review

To assess whether two factors are independent or not, you can apply the test of independence that uses the chi-square distribution. The null hypothesis for this test states that the two factors are independent. The test compares observed values to expected values. The test is right-tailed. Each observation or cell category must have an expected value of at least 5.

Formula Review

Test of Independence

- The number of degrees of freedom is equal to $(\text{number of columns} - 1)(\text{number of rows} - 1)$.
- The test statistic is $\sum_{(i,j)} \frac{(O-E)^2}{E}$ where O = observed values, E = expected values, i = the number of rows in the table, and j = the number of columns in the table.
- If the null hypothesis is true, the expected number $E = \frac{(\text{row total})(\text{column total})}{\text{total surveyed}}$.

Glossary

Contingency Table

a table that displays sample values for two different factors that may be dependent or contingent on one another; it facilitates determining conditional probabilities.

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