4.2: The Base Rate Fallacy in Medical Testing

There has been some controversy over the use of mammograms in screening breast cancer. Some argue that the dangers of false positive results, such as unnecessary biopsies, surgery and chemotherapy, outweigh the benefits of early cancer detection. This is a statistical question. Let's evaluate it.

Suppose 0.8% of women who get mammograms have breast cancer. In 90% of women with breast cancer, the mammogram will correctly detect it. (That's the statistical power of the test. This is an estimate, since it's hard to tell how many cancers are missed if we don't know they're there.) However, among women with no breast cancer at all, about 7% will get a positive reading on the mammogram, leading to further tests and biopsies and so on. If you get a positive mammogram result, what are the chances you have breast cancer?

Ignoring the chance that you, the reader, are male,[1] the answer is 9%.[35]

Despite the test only giving false positives for 7% of cancer-free women, analogous to testing for $p < 0.07$, 91% of positive tests are false positives.

How did I calculate this? It's the same method as the cancer drug example. Imagine $1,000$ randomly selected women who choose to get mammograms. Eight of them (0.8%) have breast cancer. The mammogram correctly detects 90% of breast cancer cases, so about seven of the eight women will have their cancer discovered. However, there are 992 women without breast cancer, and 7% will get a false positive reading on their mammograms, giving us 70 women incorrectly told they have cancer.

In total, we have 77 women with positive mammograms, 7 of whom actually have breast cancer. Only 9% of women with positive mammograms have breast cancer.

If you administer questions like this one to statistics students and scientific methodology instructors, more than a third fail.[35] If you ask doctors, two thirds fail.[10] They erroneously conclude that a $p < 0.05$ result implies a 95% chance that the result is true – but as you can see in these examples, the likelihood of a positive result being true depends on *what proportion of hypotheses tested are true*. And we are very fortunate that only a small proportion of women have breast cancer at any given time.

Examine introductory statistical textbooks and you will often find the same error. P values are counterintuitive, and the base rate fallacy is everywhere.

Footnotes

[1] Interestingly, being male doesn't exclude you from getting breast cancer; it just makes it exceedingly unlikely.