3.9: Chapter 3 Exercises

3.1 Introduction

1. An experiment is to flip a fair coin 3 times. Write out the sample space for this experiment.
2. An experiment is to flip a fair coin then roll a fair die. Write out the sample space for this experiment.
3. An experiment is to draw 2 cards from a standard deck of 52 cards with replacement and note the color of the card (red/black). Write out the sample space for this experiment.
4. An experiment is to roll 2 dice and record the number on each die. Write out the sample space for this experiment using ordered pairs for each outcome in a table.

3.2 Three Types of Probability

5. An experiment is to flip a fair coin 3 times. What is the probability of getting exactly 2 heads?
6. An experiment is to flip a fair coin then roll a fair die. What is the probability of getting heads with an odd number on the die?
7. An experiment is to draw 2 cards from a standard deck of 52 cards with replacement. Find the probability of getting at least one black card.
8. An experiment is to roll 2 dice and record the number on each die. Find the probability of getting a number less than 3 on the first die and a number greater than 3 on the second die.
9. The number of M&M candies for each color found in a case were recorded in the table below. What is the probability of selecting a red M&M?

Blue	Brown	Green	Orange	Red	Yellow	Total
481	371	483	544	372	369	2620

10. In the game of roulette, there is a wheel with spaces marked 0 through 36 and a space marked 00. A ball rolls across the spinning wheel until it lands in a number. The numbers are colored black, red or green. Find the probability of winning if you bet on the number 30 and it comes up on the wheel. See Figure 3.9.1.

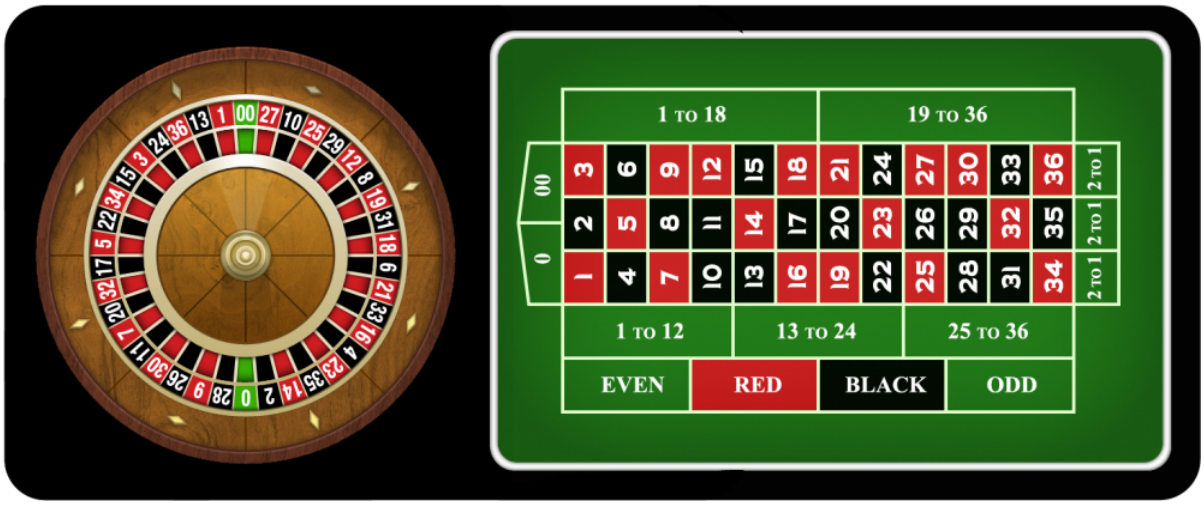


Figure 3.9.1 (Image from https://www.kindpng.com)

11. In the game of roulette, you can place a bet on a color, number, or range of numbers. Find the probability of winning on the following bets in roulette. Note: 0 and 00 are not counted as even or odd. See Figure 3.9.1.
    a. P(red)
    b. P(odd)
    c. P(13 to 24)
12. A raffle sells 1000 tickets for $35 each to win a new car. What is the probability of winning the car?
13. "If there is a 60% chance of rain on Saturday and a 70% chance of rain on Sunday, then there is a 130% chance of rain over the weekend." Explain, in complete sentences, what is wrong with the previous statement. (from *Introductory Statistics*)

3.3 Complement Rule

14. A card is drawn from a standard deck of cards. What is the probability that it is not a face card?
15. A poll showed that 48.7% of Americans say they believe that Marilyn Monroe had an affair with JFK. What is the probability of randomly selecting someone who does not believe that Marilyn Monroe had an affair with JFK?
16. Your favorite basketball player is an 81% free throw shooter. Find the probability that they do not make their next free throw shot.
17. In the game of roulette, is getting an odd number the complement of getting an even number? Explain. See Figure 3.9.1 and question 11. (from *Introductory Statistics*)

3.4 Union and Intersection

18. A card is drawn from a standard deck of cards. Find the probability that it is a diamond or an even-numbered card.
19. A card is drawn from a standard deck of cards. Find the probability that it is a diamond and an even-numbered card.
20. Find the probability of rolling a sum of two dice that is more than 7.
21. Find the probability of rolling a sum of two dice that is a 7 or a 12.
22. A and B are mutually exclusive events. $P(A) = 0.26$ and $P(B) = 0.37$. Find $P(A \cap B)$ and $P(A \cup B)$. (from *Introductory Statistics*)
23. The probability that a consumer entering a retail outlet for microcomputers and software packages will buy a computer of a certain type is 0.15. The probability that the consumer will buy a particular software package is 0.10. There is a 0.05 probability that the consumer will buy both the computer and the software package. What is the probability that the consumer will buy the computer or the software package?
24. 40% of the students at a local college belong to a club and 50% work part time. 5% of the students work part time and belong to a club. Draw a Venn diagram showing the relationships and find the probability that a student does neither. (from *Introductory Statistics*)

3.5 Independent Events

25. Give an example of an experiment that has 2 or more independent events.
26. Let event A = learning Spanish. Let event B = learning German. P(A) = 0.4, P(B) = 0.2, and P(A and B) = 0.08. Are events A and B independent? Explain. (from *Introductory Statistics*)
27. In a particular college class, 60% of the students are female. 50% of all students in the class have long hair. 45% of the students are female and have long hair. Of the female students, 75% have long hair. One student is picked randomly. Are the events of being female and having long hair independent? Explain. (from *Introductory Statistics*)
28. A high school has 200 graduating seniors. 140 will go to college. 40 will get a full-time job. The rest are taking a year off before going to school or work. 50 will play college sports. 30 of those who will work full-time play sports. 5 of those taking a year off play sports. (from *Introductory Statistics*)
    a. What is the probability a senior will take the year off?
    b. What is the probability a senior plays sports?
    c. What is the probability a senior will take the year off and plays sports?
    d. Are taking the year off and playing sports independent?
    e. Are taking the year off and playing sports mutually exclusive?
    f. What is the probability a senior is not taking the year off?
    g. What is the probability that a senior plays sports and is not taking the year off?
    h. What is the probability that a senior plays sports or is taking the year off?

3.6 Conditional Probability

29. Two cards are drawn from a standard deck without replacement.
    a. Find the probability that both are face cards.
    b. Find the probability of drawing at most one face card.
    c. Find the probability of drawing at least one face card.

30. A random sample of 500 people's marital status and biological sex from the 2020 United States Census are recorded in the following contingency table.

	Female	Male	Grand Total
Divorced	21	17	38
Married/spouse absent	5	9	14
Married/spouse present	92	100	192
Never married/single	93	129	222
Separated	1	2	3
Widowed	20	11	31
Grand Total	232	268	500

    a. Find the probability that a randomly selected person is single.
    b. Find the probability that a randomly selected person is not single.
    c. Find the probability that a randomly selected person is single or male.
    d. Find the probability that a randomly selected person is divorced or widowed.
    e. Given that randomly selected person is male, what is the probability they are single?
    f. Are the events divorced and male mutually exclusive?
    g. Are the events divorced and male independent? Verify using the formula.

31. A fitness center owner kept track of members over the last year. They recorded if the person stretched before they exercised, and whether they sustained an injury. The following contingency table shows the results. Select one member at random and find the following.

	Injury	No Injury	Total
Stretched	52	270	322
Did Not Stretch	21	57	78
Total	73	327	400

    a. P(No Injury)
    b. P(Injury ∩ Stretch)
    c. Find the probability that a randomly selected member stretched or sustained an injury.
    d. Find the probability that a randomly selected member stretched given that they sustained an injury.
    e. P(Injury | Did Not Stretch)

32. A certain virus infects one in every 400 people. A test used to detect the virus in a person is positive 90% of the time if the person has the virus, and 8% of the time if the person does not have the virus (false positive). Let A be the event "the person is infected" and B be the event "the person tests positive."

    a. Find the probability that a person has the virus given that they have tested positive, i.e. find $P(A|B)$.
    b. Find the probability that a person does not have the virus given that they test negative, i.e. find $P(A'|B')$.

33. A store purchases baseball hats from three different manufacturers. In manufacturer A's box there are 12 blue hats, 6 red hats, and 6 green hats. In manufacturer B's box there are 10 blue hats, 10 red hats, and 4 green hats. In manufacturer C's box, there are 8 blue hats, 8 red hats, and 8 green hats. A hat is randomly selected. Given that the hat selected is green, what is the probability that it came from manufacturer B's box? (Hint: Make a table with the colors as the columns and the manufacturers as the rows.)

34. The following table represents food purchase amounts and whether the customer used cash or a credit/debit card. One customer is chosen at random. Give answers as a decimal rounded to 4 places.

	Less than $10	$10 - $49	$50 or More	Total
Cash Purchase	11	10	18	39

Card Purchase	17	6	19	42
Total	28	16	37	81

a. Find the probability that the customer's purchasing method was a cash purchase or the customer spent $10 - $49.
b. Find the probability that the customer's purchasing method was a cash purchase and the customer spent $10 - $49.
c. Find the probability that the customer's purchasing method was a cash purchase given they spent $10 - $49.
d. Find the probability that the customer spent less than $50.
e. What percent of cash purchases were for $50 or more?

3.7 Counting Rules

35. You are going to a Humane Society benefit dinner, and need to decide before the dinner what you want for salad, main dish, and dessert. You have 2 salads to choose from, 3 main dishes, and 5 desserts. How many different meals are available?
36. How many different 10-digit phone numbers are possible in the area code 714 if the first number cannot start with a 0 or 1?
37. You are opening a screen-printing business. You can have long sleeves or short sleeves, 3 different colors, 5 different designs, and 4 different sizes. How many different shirts can you make?
38. Calculate the following.
    a. ${}_9P_4$
    b. ${}_{10}P_6$
    c. ${}_{10}C_5$
    d. ${}_{20}C_4$
    e. 8!
    f. 5!
39. The Circle K International Club has 30 members. They need to pick a president, vice president of administration, vice president of service, secretary and treasurer from the 30 members. How many different ways can this be done?
40. How many different 4-digit personal identification numbers (PIN) are there if repeats are not allowed?
41. A baseball team has a 20-person roster. A batting order has 9 people. How many different batting orders are there?
42. How many ways can you choose 4 cookies from a cookie jar containing 25 cookies of all the same type?
43. A typical PE locker is opened with correct sequence of 3 numbers between 0 and 49 inclusive. A number can be used more than once, for example 8-8-8 is valid. How many possible locker combinations are there?
44. In the game of Megabucks, you get 6 numbers from 48 possible numbers without replacement. Megabucks jackpots start at $1 million and grow until someone wins. What is the probability of matching all 6 numbers in any order?

Mixed Practice

45. A report for a school's computer web visits for the past month obtained the following information. Draw a Venn Diagram and find the percentage that visited none of these three sites last month.

    37% visited Facebook

    42% visited LinkedIn

    29% visited Google

    27% visited Facebook and LinkedIn

    19% visited Facebook and Google

    19% visited LinkedIn and Google

    14% visited all three sites

46. At a college, 72% of courses have final exams and 46% of courses require research papers. Suppose that 32% of courses have a research paper and a final exam. (from *Introductory Statistics*)
    a. Find the probability that a course has a final exam or a research paper.
    b. Find the probability that a course has neither of these two requirements.
47. Carlos plays on the school's soccer team. He makes a goal 65% of the time he shoots. Carlos is going to attempt 2 goals in a row in the next game. Let event A = Carlos is successful on his first attempt. $P(A) = 0.65$. Let event B = Carlos is successful

on his second attempt. $P(B) = 0.65$. Carlos tends to shoot in streaks. The probability that he makes the second goal given that he made the first goal is 0.90. (from *Introductory Statistics*)

a. What is the probability that he makes both goals?
b. What is the probability that Carlos makes either the first or the second goal?
c. Are events A and B independent? Explain.
d. Are events A and B mutually exclusive? Explain.

48. Giving a test to a group of students, the grades and if they were business majors are summarized in the following table. One student is chosen at random. Give answers as a decimal rounded to 4 places.

	A	B	C	Total
Business Majors	4	5	13	22
Non-business Majors	18	10	19	47
Total	22	15	32	69

a. Find the probability that the student was a non-business major or got a C.
b. Find the probability that the student was a non-business major and got a C.
c. Find the probability that the student was a non-business major given they got a C.
d. Find the probability that the student did not get a B.
e. Find $P(B \cup \text{Business Major})$.
f. Find $P(C|\text{ Business Major})$.

49. The smallpox data set provides a sample of 6224 individuals from the year 1721 who were exposed to smallpox in Boston.

	Inoculated	Not Inoculated	Total
Lived	238	5136	5374
Died	6	844	850
Total	244	5980	6224

Fenner F. 1988. Smallpox and Its Eradication (History of International Public Health, No. 6). Geneva: World Health Organization. ISBN 92-4-156110-6.

a. Find the relative frequencies for each cell in the table.

	Inoculated	Not Inoculated	Total
Lived			
Died			
Total			1

b. Find the probability that a person was inoculated.
c. Find the probability that a person lived.
d. Find the probability that a person died or was inoculated.
e. Find the probability that a person died if they were inoculated.
f. Given that a person was not inoculated, what is the probability that they died?

50. California's standard car license plates have one number followed by 3 letters followed by 3 numbers. If the first number before the letters cannot be 0 and repeats are allowed, how many different license plates are possible?
51. A computer generates a random password for your account (the password is not case sensitive). The password must consist of 8 characters, each of which can be any letter or number. How many different passwords could be generated?
52. How many unique tests can be made from a test bank of 20 questions if the test consists of 8 questions, order does not matter?
53. Rachel wants to place billboard advertisements throughout the county for her new business. How many ways can Rachel choose 15 neighborhoods to advertise in if there are 30 neighborhoods in the county? (from *Precalculus*)

54. A store has 8 cellular phones, and 3 of them are defective. What is the probability that a couple purchasing 2 phones receives 2 phones that are not defective? (from *Precalculus*)

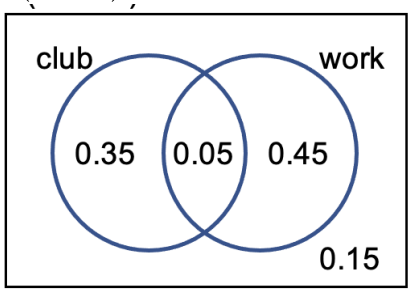
Answers to Exercises

1. S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}
2. S = {H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6}
3. S = {RR, RB, BR, BB}
4.

(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

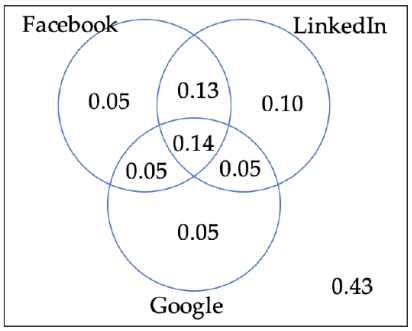
5. $P(A) = \frac{3}{8}$
6. $P(A) = \frac{1}{4}$
7. $P(A) = \frac{3}{4}$
8. $P(A) = \frac{1}{6}$
9. $P(\text{red}) = \frac{93}{655} \approx 0.1420$
10. $P(30) = \frac{1}{38}$
11. a. $P(\text{red}) = \frac{9}{19} \approx 0.4737$
    b. $P(\text{odd}) = \frac{9}{19} \approx 0.4737$
    c. $P(13 \text{ to } 24) = \frac{6}{19} \approx 0.3158$
12. $P(\text{win}) = \frac{1}{1000} = 0.001$
13. It is not possible for probabilties to be greater than 100% (or 1). You should not add the probabilities.
14. $P(F') = \frac{10}{13}$
15. $P(\text{no affair}) = 0.513 = 51.3\%$
16. $P(\text{miss}) = 0.19$
17. No, odd and even are not complementary events in roulette since there are also the 0 and 00 spaces (thus they do not make up the entire sample space).
18. $P(\text{diamond} \cup \text{even}) = \frac{7}{13}$
19. $P(\text{diamond} \cap \text{even}) = \frac{5}{52}$
20. $P(\text{sum} > 7) = \frac{5}{12}$
21. $P(7 \cup 12) = \frac{7}{36}$
22. $P(A \cap B) = 0; P(A \cup B) = 0.63$
23. $P(C \cup SP) = 0.20$

24. $P(\text{neither}) = 0.15 = 15\%$



25. flipping a coin 2 or more times
26. Yes, A and B are independent since $P(A \cap B) = P(A) \cdot P(B)$.
27. Female and having long hair are not independent since $P(F \cap L) \neq P(F) \cdot P(L)$.
28. a. $P(A) = \frac{1}{10} = 0.1$
    b. $P(B) = \frac{17}{40} = 0.425$
    c. $P(A \cap B) = \frac{1}{40} = 0.025$
    d. Since $P(A \cap B) \neq P(A) \cdot P(B)$, taking the year off and playing sports are not independent.
    e. They are not mutually exclusive since $P(A \cap B) \neq 0$.
    f. $P(A') = 0.9$
    g. $P(B \cap A') = \frac{2}{5} = 0.4$
    h. $P(A \cup B) = 0.5$
29. a. $P(F_1 \cap F_2) = \frac{11}{221}$
    b. $P(\text{at most 1 } F) = \frac{210}{221}$
    c. $P(\text{at least 1 } F) = \frac{7}{17}$
30. a. $P(S) = \frac{222}{500} = 0.444$
    b. $P(S') = \frac{278}{500} = 0.556$
    c. $P(S \cup M) = \frac{361}{500} = 0.722$
    d. $P(D \cup W) = \frac{69}{500} = 0.138$
    e. $P(S|M) = \frac{129}{268} \approx 0.4813$
    f. No, divorced and male are not mutually exclusive since they can occur at the same time.
    g. No, divorced and male are not independent. (check formula!)
31. a. $P(\text{No Injury}) = \frac{327}{400} = 0.8175$
    b. $P(\text{Injury} \cap \text{Stretch}) = \frac{52}{400} = 0.13$
    c. $P(S \cup I) = \frac{343}{400} = 0.8575$
    d. $P(S|I) = \frac{52}{73} \approx 0.7123$
    e. $P(\text{Injury}|\text{Did Not Stretch}) = \frac{21}{78} \approx 0.2692$
32. a. $P(A|B) \approx 0.0274$
    b. $P(A'|B') \approx 0.9997$
33. $P(B|\text{green}) = \frac{2}{9} \approx 0.2222$

34. a. $P(A \cup D) = \frac{5}{9} \approx 0.5556$
    b. $P(A \cap D) = \frac{10}{81} \approx 0.1235$
    c. $P(A|D) = \frac{5}{8} = 0.6250$
    d. $P(C \cup D) = \frac{44}{81} \approx 0.5432$
    e. $P(E|A) = \frac{6}{13} \approx 0.4615$ = 46.15%
35. 30 different meals
36. 8,000,000 phone numbers
37. 120 different shirts
38. a. 3024
    b. 151,200
    c. 252
    d. 4845
    e. 40,320
    f. 120
39. 17,100,720
40. 5040
41. 60,949,324,800 or 6.09×10^{10}
42. 12,650
43. 117,649
44. $P(\text{win}) = \frac{1}{12,271,512} \approx 0.000000081$
45. 43% visited none of the 3 sites last month



46. a. $P(F \text{ or } R) = 0.86$
    b. $P(\text{neither}) = 0.14$
47. a. $P(A \text{ and } B) = 0.585$
    b. $P(A \text{ or } B) = 0.715$
    c. No, A and B are not independent since $P(A \text{and} B) \neq P(A) \cdot P(B)$.
    d. No, A and B are not mutually exclusive since $P(A \text{and} B) \neq 0$.
48. a. $P(M' \cup C) = \frac{60}{69} \approx 0.8696$
    b. $P(M' \cap C) = \frac{19}{69} \approx 0.2754$
    c. $P(M'|C) = \frac{19}{32} \approx 0.5938$
    d. $P(B') = \frac{54}{69} \approx 0.7826$
    e. $P(B \cup M) = \frac{32}{69} \approx 0.4638$

f. $P(C|M) = \frac{13}{22} \approx 0.5909$

49. a.

	Inoculated	Not Inoculated	Total
Lived	0.0382	0.8252	0.8634
Died	0.0010	0.1356	0.1366
Total	0.0392	0.9608	1

b. $P(I)$ = 0.0392
c. $P(L)$ = 0.8634
d. $P(D \text{ or } I)$ = 0.1748
e. $P(D|I) \approx$ 0.0255
f. $P(D|I') \approx$ 0.1411

50. 158,184,000
51. $36^8 = 2.82 \times 10^{12}$ passwords
52. 125,970 tests
53. 155,117,520 ways
54. $P(2 \text{ not defective}) = \frac{5}{14} \approx 0.3571$
