

3.3: Complement Rule

Learning Objectives

- Find the probability of the complement of an event
- Use a Venn diagram to find or visualize the probability of an experiment

✓ Example 3.3.1

A random sample of 500 records from the 2020 United States Census were downloaded to Excel and the following pivot table was found for biological sex and marital status. Select one member at random and find the following probabilities.

	Female	Male	Grand Total
Divorced	21	17	38
Married/spouse absent	5	9	14
Married/spouse present	92	100	192
Never married/single	93	129	222
Separated	1	2	3
Widowed	20	11	31
Grand Total	232	268	500

- Find the probability that a person is divorced.
- Find the probability that a person is not divorced.

Solution

- Take the row total of all divorced which is 38, and then divide by the grand total of 500. Thus, $P(\text{Divorced}) = \frac{38}{500} = \frac{19}{250} = 0.076$.
- We can add up all the other category totals besides divorced, $14 + 192 + 222 + 3 + 31 = 462$, then divide by the grand total. Thus, $P(\text{Not Divorced}) = \frac{462}{500} = \frac{231}{250} = 0.924$.

There is a faster way to compute these probabilities that will be important for more complicated probabilities. It is called the complement rule. The table contains 100% ($100\% = 1$ as a proportion) of our data so we can assume that the probability of the divorced is the opposite (complement) to the probability of not being divorced.

Notice that the $P(\text{Divorced}) + P(\text{Not Divorced}) = 1$. This is because these two events have no outcomes in common, and together they make up the entire sample space. Events that have this property are called **complementary events**. Notice $P(\text{Not Divorced}) = 1 - P(\text{Divorced}) = 1 - 0.076 = 0.924$, which is the same answer from part b in the previous example.

Formulas

If two events are **complementary events**, then to find the probability of one event, just subtract the probability from 1. Notation for the complement of A also called “A prime” is A' .

$$P(A) + P(A') = 1 \text{ or } P(A) = 1 - P(A') \text{ or } P(A') = 1 - P(A)$$

The complement of A or A' is comprised of all of the outcomes in the sample space that are not in A. Some texts will use the notation for the complement of A as A^C or \bar{A} , instead of A' .

✓ Example 3.3.2

Suppose you know that the probability of it raining today is 80%. What is the probability of it not raining today?



Solution

Since "not raining" is the complement of "raining", then $P(\text{not raining}) = 1 - P(\text{raining}) = 1 - 0.8 = 0.2$.

Venn Diagrams

Figure 3.3.1 is an example of a **Venn diagram** and is a visual way to represent sets and probability. The rectangle represents all the possible outcomes in the entire sample space (the population). The shapes inside the rectangle represent each event in the sample space. Usually these are ovals, but they can be any shape you want. If there are any shared elements between the events, then the circles should overlap one another.

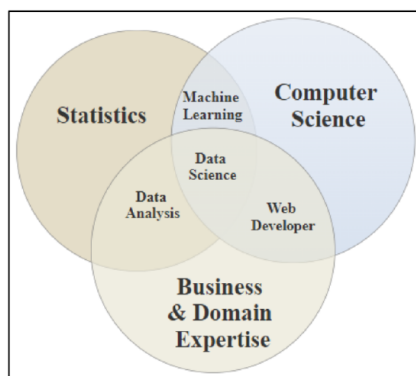


Figure 3.3.1

The field of statistics includes machine learning, data analysis, and data science. The field of computer science includes machine learning, data science and web development. The field of business and domain expertise includes data analysis, data science and web development. If you know machine learning, then you will need a background in both statistics and computer science. If you are a data scientist, then you will need a background in statistics, computer science, business and domain expertise.

✓ Example 3.3.3

Suppose you know the probability of not getting the flu is 0.24. Draw a Venn diagram and find the probability of getting the flu.

Solution

Since getting the flu is the complement of not getting the flu, the $P(\text{getting the flu}) = 1 - P(\text{not getting the flu}) = 1 - 0.24 = 0.76$.

Label each space in the Venn diagram as in Figure 3.3.2.

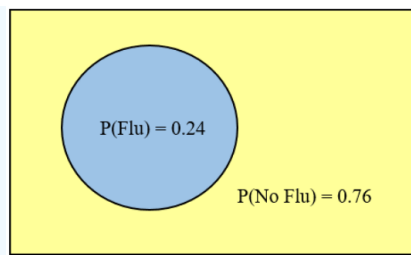


Figure 3.3.2

The complement is useful when you are trying to find the probability of an event that involves the words “at least” or an event that involves the words “at most.” As an example of an “at least” event, suppose you want to find the probability of making at least \$50,000 when you graduate from college. That means you want the probability of your salary being greater than or equal to \$50,000.

An example of an “at most” event is supposing you want to find the probability of rolling a die and getting at most a 4. That means that you want to get less than or equal to a 4 on the die, namely the numbers 1, 2, 3, or 4.

The reason to use the complement is that sometimes it is easier to find the probability of the complement and then subtract from 1. We will use this idea again in section 3.5.

This page titled [3.3: Complement Rule](#) is shared under a [CC BY-SA 4.0](#) license and was authored, remixed, and/or curated by [Rachel Webb](#) via [source content](#) that was edited to the style and standards of the LibreTexts platform.

- [4.3: Complement Rule](#) by [Rachel Webb](#) is licensed [CC BY-SA 4.0](#). Original source: <https://mostlyharmlessstat.wixsite.com/webpage>.