3.1: Introduction

Learning Objectives

- Find the sample space of an experiment
- Use a tree diagram to find the outcomes of an experiment

One story about how probability theory was developed is that a gambler wanted to know when to bet more and when to bet less. He talked to a couple of friends of his that happened to be mathematicians. Their names were Pierre de Fermat and Blaise Pascal. Since then many other mathematicians have worked to develop probability theory.

Figure 3.1.1: Fermat and Pascal

Understanding probability is important in life. Examples of mundane questions that probability can answer for you are: do I need to carry an umbrella? Do you wear a heavy coat on a given day? More important questions that probability can help with are your chances that the car you are buying will need more maintenance, your chances of passing a class, your chances of winning the lottery, or your chances of catching a deadly virus. The chance of you winning the lottery is very small, yet many people will spend the money on lottery tickets. In general, events that have a low probability (under 5%) are unlikely to occur. Whereas if an event has a high probability of happening (over 80%), then there is a good chance that the event will happen. This chapter will present some of the theory that you need to help decide if an event is likely to happen or not.

First, some definitions:

Definition: Experiment

An activity or process that has a set of well-defined results and can be repeated indefinitely.

Definition: Outcomes

The results of an experiment.

Definition: Sample Space

Collection of all possible outcomes of the experiment. Usually denoted as S.

Definition: Event

The set of outcomes that are a subset of the sample space. The symbol used for an event is usually a capital letter, often at the beginning of the alphabet like A, B or C.

Here are some examples of sample spaces and events.

Figure 3.1.2

Experiment	Sample Space	Example of Event

Experiment	Sample Space	Example of Event
Toss a coin twice	S = {HH, HT, TH, TT}	A = Getting exactly two heads = {HH}
Toss a coin twice	S = {HH, HT, TH, TT}	B = Getting at least one heads = {HH, HT, TH}
Roll a die	S = {1, 2, 3, 4, 5, 6}	C = Roll an odd number = {1, 3, 5}
Roll a die	S = {1, 2, 3, 4, 5, 6}	D = Roll a prime number = {2, 3, 5}
Roll a die	S = {1, 2, 3, 4, 5, 6}	E = Roll an even number = {2, 4, 6}

A **tree diagram** is a graphical way of representing a random experiment with multiple steps.

Example 3.1.1

A bag contains 10 colored marbles: 7 red and 3 blue. A random experiment consists of drawing a marble from the bag, then drawing another marble without replacement (without putting the first marble back in the bag). Create a tree diagram for this experiment and write out the sample space.

Solution

The first marble that is drawn can be either red or blue and is represented with the first sideways V split. The next marble that is drawn is represented by the two sideways V splits on the right. The top V assumes that a red marble was drawn first, and then the second marble drawn can be either red or blue. The bottom V assumes that a blue marble was drawn first, and then the second marble drawn can be either red or blue. Then combine the colors as you trace up the four pathways from left to right.

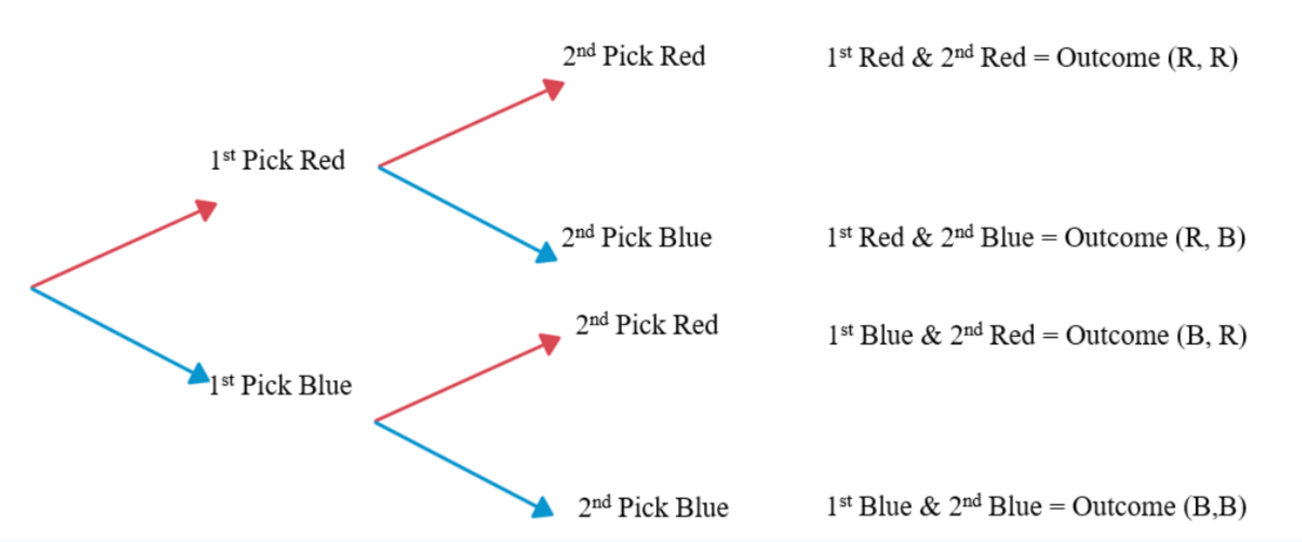

Figure 3.1.3

The sample space would be S = {RR, RB, BR, BB}. Note that the event RB and BR are considered different outcomes since they are picked in a different order and are considered distinct events.