3.5: Independent Events

Learning Objectives

- Understand the difference between independent and dependent events
- Use the Multiplication Rule to find the probability of independent events

Two events are **independent** if the outcome of one event does not influence the outcome of the second event. If two events are not independent, they are **dependent** events. For instance, if two coins are flipped, they are independent since flipping one coin does not affect the outcome of the second coin.

Formula: Multiplication Rule

Multiplication Rule: If A and B are independent events, then $P(A \cap B) = P(A) \cdot P(B)$.

Be careful with this rule. You cannot just multiply probabilities to find an intersection unless you know they are independent. Also, do not confuse independent events with mutually exclusive events. Two events are mutually exclusive when $P(A \cap B) = 0$.

Example 3.5.1

If a random experiment consists of flipping a coin twice, find the probability of getting heads twice in a row.

Solution

The event of getting heads on the first flip is independent of getting heads on the second flip since the probability does not change with each flip of the coin. Thus, using the Multiplication Rule of independent events: P(heads and heads) = P(1st coin heads)·P(2nd coin heads) = $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$ = 0.25.

Example 3.5.2

The probability of Apple stock rising is 0.3, the probability of Boeing stock rising is 0.4. Assume Apple and Boeing stocks are independent. What is the probability that neither stock rises?

Solution

Let A = Apple stock and B = Boeing stock. Since A and B are independent, the probability of both stocks rising at the same time is $P(A \cap B) = 0.3 \cdot 0.4 = 0.12$. "Neither stock" is the complement to "either stock," which is the union of both stocks. Using the Complement Rule with the Addition Rule, P(not either) = $1 - P(A \cup B) = 1 - [P(A) + P(B) - P(A \cap B)]$ = 1 – (0.3 + 0.4 - 0.12) = 1 – 0.58 = 0.42.

Example 3.5.3

The probability that a student has their own laptop is 0.78. If three students are randomly selected, what is the probability that at least one owns a laptop?

Solution

There is an assumption that the three students are not related and that the probability of one owning a laptop is independent of the other students owning a laptop. Since the sample space of the number of students owning a laptop is S = {0, 1, 2, 3}, “at least one owns a laptop” means that 1, 2 or 3 students in the sample own a laptop. The probability of “at least one owns a laptop” is the complement of “none owns a laptop” (0 students) since the two events make up the total sample space. From the Complement Rule, the probability of one student not owning a laptop is P(no laptop) = 1 – 0.78 = 0.22. Using the Multiplication Rule of independent events, the probability of none owning a laptop (all three students have no laptop) is P(none owns a laptop) = $(0.22)^3$ = 0.0106. Using the Complement Rule again, P(at least one owns a laptop) = 1 – P(none owns a laptop) = 1 – 0.0106 = 0.9894.

Often, the fastest way to calculate a probability of “at least one” is to use the Complement Rule as in the previous example.

Formula

$$P(\text{at least one}) = 1 - P(\text{none})$$

Another important thing to note is that when two events are dependent, you cannot simply multiply their corresponding probabilities to find their intersection. You will need to use the General Multiplication Rule discussed in the next section.

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