

## 2.5: Measures of Position- Percentiles and Quartiles

The common measures of location are **quartiles** and **percentiles**. Quartiles are special percentiles. The first quartile,  $Q_1$ , is the same as the 25<sup>th</sup> percentile, and the third quartile,  $Q_3$ , is the same as the 75<sup>th</sup> percentile. The median,  $M$ , is called both the second quartile and the 50<sup>th</sup> percentile.

To calculate quartiles and percentiles, the data must be ordered from smallest to largest. Quartiles divide ordered data into quarters. Percentiles divide ordered data into hundredths. To score in the 90<sup>th</sup> percentile of an exam does not mean, necessarily, that you received 90% on a test. It means that 90% of test scores are the same or less than your score and 10% of the test scores are the same or greater than your test score.

Percentiles are useful for comparing values. For this reason, universities and colleges use percentiles extensively. One instance in which colleges and universities use percentiles is when SAT results are used to determine a minimum testing score that will be used as an acceptance factor. For example, suppose Duke accepts SAT scores at or above the 75<sup>th</sup> percentile. That translates into a score of at least 1220.

Percentiles are mostly used with very large populations. Therefore, if you were to say that 90% of the test scores are less (and not the same or less) than your score, it would be acceptable because removing one particular data value is not significant.

The **median** is a number that measures the "center" of the data. You can think of the median as the "middle value," but it does not actually have to be one of the observed values. It is a number that separates ordered data into halves. Half the values are the same number or smaller than the median, and half the values are the same number or larger. For example, consider the following data.

1; 11.5; 6; 7.2; 4; 8; 9; 10; 6.8; 8.3; 2; 2; 10; 1

Ordered from smallest to largest:

1; 1; 2; 2; 4; 6; 6.8; 7.2; 8; 8.3; 9; 10; 10; 11.5

Since there are 14 observations, the median is between the seventh value, 6.8, and the eighth value, 7.2. To find the median, add the two values together and divide by two.

$$\frac{6.8 + 7.2}{2} = 7 \quad (2.5.1)$$

The median is seven. Half of the values are smaller than seven and half of the values are larger than seven.

**Quartiles** are numbers that separate the data into quarters. Quartiles may or may not be part of the data. To find the quartiles, first find the median or second quartile. The first quartile,  $Q_1$ , is the middle value of the lower half of the data, and the third quartile,  $Q_3$ , is the middle value, or median, of the upper half of the data. To get the idea, consider the same data set:

1; 1; 2; 2; 4; 6; 6.8; 7.2; 8; 8.3; 9; 10; 10; 11.5

The median or **second quartile** is seven. The lower half of the data are 1, 1, 2, 2, 4, 6, 6.8. The middle value of the lower half is two.

1; 1; 2; 2; 4; 6; 6.8

The number two, which is part of the data, is the **first quartile**. One-fourth of the entire sets of values are the same as or less than two and three-fourths of the values are more than two.

The upper half of the data is 7.2, 8, 8.3, 9, 10, 10, 11.5. The middle value of the upper half is nine.

The **third quartile**,  $Q_3$ , is nine. Three-fourths (75%) of the ordered data set are less than nine. One-fourth (25%) of the ordered data set are greater than nine. The third quartile is part of the data set in this example.

The **interquartile range** is a number that indicates the spread of the middle half or the middle 50% of the data. It is the difference between the third quartile ( $Q_3$ ) and the first quartile ( $Q_1$ ).

$$IQR = Q_3 - Q_1 \quad (2.4.1)$$

The *IQR* can help to determine potential **outliers**. A value is suspected to be a potential outlier if it is less than  $(1.5)(IQR)$  below the first quartile or more than  $(1.5)(IQR)$  above the third quartile. Potential outliers always require further investigation.

### Definition: Outliers

A potential outlier is a data point that is significantly different from the other data points. These special data points may be errors or some kind of abnormality or they may be a key to understanding the data.

### Example 2.4.1

For the following 13 real estate prices, calculate the *IQR* and determine if any prices are potential outliers. Prices are in dollars.  
389,950; 230,500; 158,000; 479,000; 639,000; 114,950; 5,500,000; 387,000; 659,000; 529,000; 575,000; 488,800; 1,095,000

#### Answer

Order the data from smallest to largest.

114,950; 158,000; 230,500; 387,000; 389,950; 479,000; 488,800; 529,000; 575,000; 639,000; 659,000; 1,095,000; 5,500,000

$$M = 488,800$$

$$Q_1 = \frac{230,500 + 387,000}{2} = 308,750$$

$$Q_3 = \frac{639,000 + 659,000}{2} = 649,000$$

$$IQR = 649,000 - 308,750 = 340,250$$

$$(1.5)(IQR) = (1.5)(340,250) = 510,375$$

$$Q_1 - (1.5)(IQR) = 308,750 - 510,375 = -201,625$$

$$Q_3 + (1.5)(IQR) = 649,000 + 510,375 = 1,159,375$$

No house price is less than  $-201,625$ . However, 5,500,000 is more than 1,159,375. Therefore, 5,500,000 is a potential **outlier**.

### Example 2.4.2

For the two data sets in the [test scores example](#), find the following:

- The interquartile range. Compare the two interquartile ranges.
- Any outliers in either set.

#### Answer

The five number summary for the day and night classes is

	Minimum	$Q_1$	Median	$Q_3$	Maximum
Day	32	56	74.5	82.5	99
Night	25.5	78	81	89	98

- The *IQR* for the day group is  $Q_3 - Q_1 = 82.5 - 56 = 26.5$

The *IQR* for the night group is  $Q_3 - Q_1 = 89 - 78 = 11$

The interquartile range (the spread or variability) for the day class is larger than the night class *IQR*. This suggests more variation will be found in the day class's class test scores.

- Day class outliers are found using the *IQR* times 1.5 rule. So,

- $Q_1 - IQR(1.5) = 56 - 26.5(1.5) = 16.25$
- $Q_3 + IQR(1.5) = 82.5 + 26.5(1.5) = 122.25$

Since the minimum and maximum values for the day class are greater than 16.25 and less than 122.25, there are no outliers.

Night class outliers are calculated as:

- $Q_1 - IQR(1.5) = 78 - 11(1.5) = 61.5$
- $Q_3 + IQR(1.5) = 89 + 11(1.5) = 105.5$

For this class, any test score less than 61.5 is an outlier. Therefore, the scores of 45 and 25.5 are outliers. Since no test score is greater than 105.5, there is no upper end outlier.

### Example 2.4.3

Fifty statistics students were asked how much sleep they get per school night (rounded to the nearest hour). The results were:

AMOUNT OF SLEEP PER SCHOOL NIGHT (HOURS)	FREQUENCY	RELATIVE FREQUENCY	CUMULATIVE RELATIVE FREQUENCY
4	2	0.04	0.04
5	5	0.10	0.14
6	7	0.14	0.28
7	12	0.24	0.52
8	14	0.28	0.80
9	7	0.14	0.94
10	3	0.06	1.00

**Find the 28<sup>th</sup> percentile.** Notice the 0.28 in the "cumulative relative frequency" column. Twenty-eight percent of 50 data values is 14 values. There are 14 values less than the 28<sup>th</sup> percentile. They include the two 4s, the five 5s, and the seven 6s. The 28<sup>th</sup> percentile is between the last six and the first seven. **The 28<sup>th</sup> percentile is 6.5.**

**Find the median.** Look again at the "cumulative relative frequency" column and find 0.52. The median is the 50<sup>th</sup> percentile or the second quartile. 50% of 50 is 25. There are 25 values less than the median. They include the two 4s, the five 5s, the seven 6s, and eleven of the 7s. The median or 50<sup>th</sup> percentile is between the 25<sup>th</sup>, or seven, and 26<sup>th</sup>, or seven, values. **The median is seven.**

**Find the third quartile.** The third quartile is the same as the 75<sup>th</sup> percentile. You can "eyeball" this answer. If you look at the "cumulative relative frequency" column, you find 0.52 and 0.80. When you have all the fours, fives, sixes and sevens, you have 52% of the data. When you include all the 8s, you have 80% of the data. **The 75<sup>th</sup> percentile, then, must be an eight.** Another way to look at the problem is to find 75% of 50, which is 37.5, and round up to 38. The third quartile,  $Q_3$ , is the 38<sup>th</sup> value, which is an eight. You can check this answer by counting the values. (There are 37 values below the third quartile and 12 values above.)

### Example 2.4.4

Using [Table](#):

- Find the 80<sup>th</sup> percentile.
- Find the 90<sup>th</sup> percentile.
- Find the first quartile. What is another name for the first quartile?

#### Solution

Using the data from the frequency table, we have:

- The 80<sup>th</sup> percentile is between the last eight and the first nine in the table (between the 40<sup>th</sup> and 41<sup>st</sup> values). Therefore, we need to take the mean of the 40<sup>th</sup> and 41<sup>st</sup> values. The 80<sup>th</sup> percentile =  $\frac{8+9}{2} = 8.5$
- The 90<sup>th</sup> percentile will be the 45<sup>th</sup> data value (location is  $0.90(50) = 45$ ) and the 45<sup>th</sup> data value is nine.
- $Q_1$  is also the 25<sup>th</sup> percentile. The 25<sup>th</sup> percentile location calculation:  $P_{25} = 0.25(50) = 12.5 \approx 13$  the 13<sup>th</sup> data value. Thus, the 25<sup>th</sup> percentile is six.

## COLLABORATIVE STATISTICS

Your instructor or a member of the class will ask everyone in class how many sweaters they own. Answer the following questions:

- How many students were surveyed?
- What kind of sampling did you do?
- Construct two different histograms. For each, starting value = \_\_\_\_\_ ending value = \_\_\_\_\_.
- Find the median, first quartile, and third quartile.
- Construct a table of the data to find the following:
  - the 10<sup>th</sup> percentile
  - the 70<sup>th</sup> percentile
  - the percent of students who own less than four sweaters

### A Formula for Finding the $k$ th Percentile

If you were to do a little research, you would find several formulas for calculating the  $k$ th percentile. Here is one of them.

- $k$  = the  $k$ th percentile. It may or may not be part of the data.
- $i$  = the index (ranking or position of a data value)
- $n$  = the total number of data

Order the data from smallest to largest.

$$\text{Calculate } i = \frac{k}{100}(n+1) \quad i = k100(n+1)$$

If  $i$  is an integer, then the  $k^{\text{th}}$  percentile is the data value in the  $i^{\text{th}}$  position in the ordered set of data.

If  $i$  is not an integer, then round  $i$  up and round  $i$  down to the nearest integers. Average the two data values in these two positions in the ordered data set. This is easier to understand in an example.

#### Example 2.4.5

Listed are 29 ages for Academy Award winning best actors *in order from smallest to largest*.

18; 21; 22; 25; 26; 27; 29; 30; 31; 33; 36; 37; 41; 42; 47; 52; 55; 57; 58; 62; 64; 67; 69; 71; 72; 73; 74; 76; 77

- Find the 70<sup>th</sup> percentile.
- Find the 83<sup>rd</sup> percentile.

#### Solution

- $k = 70$
  - $i$  = the index
  - $n = 29$

$i = \frac{k}{100}(n+1) = \frac{70}{100}(29+1) = 21$  . Twenty-one is an integer, and the data value in the 21<sup>st</sup> position in the ordered data set is 64. The 70<sup>th</sup> percentile is 64 years.

- $k = 83^{\text{rd}}$  percentile
  - $i = \text{the index}$
  - $n = 29$

$i = \frac{k}{100}(n+1) = (\frac{83}{100})(29+1) = 24.9$  , which is NOT an integer. Round it down to 24 and up to 25. The age in the 24<sup>th</sup> position is 71 and the age in the 25<sup>th</sup> position is 72. Average 71 and 72. The 83<sup>rd</sup> percentile is 71.5 years.

#### Note 2.4.2

You can calculate percentiles using calculators and computers. There are a variety of online calculators.

## A Formula for Finding the Percentile of a Value in a Data Set

- Order the data from smallest to largest.
- $x$  = the number of data values counting from the bottom of the data list up to but not including the data value for which you want to find the percentile.
- $y$  = the number of data values equal to the data value for which you want to find the percentile.
- $n$  = the total number of data.
- Calculate  $\frac{x + 0.5y}{n}(100)$ . Then round to the nearest integer.

### Example 2.4.6

Listed are 29 ages for Academy Award winning best actors *in order from smallest to largest*.

18; 21; 22; 25; 26; 27; 29; 30; 31; 33; 36; 37; 41; 42; 47; 52; 55; 57; 58; 62; 64; 67; 69; 71; 72; 73; 74; 76; 77

a. Find the percentile for 58.

b. Find the percentile for 25.

### Solution

a. Counting from the bottom of the list, there are 18 data values less than 58. There is one value of 58.

$$x = 18 \text{ and } y = 1. \quad \frac{x + 0.5y}{n}(100) = \frac{18 + 0.5(1)}{29}(100) = 63.80. \text{ 58 is the 64}^{\text{th}} \text{ percentile.}$$

b. Counting from the bottom of the list, there are three data values less than 25. There is one value of 25.

$$x = 3 \text{ and } y = 1. \quad \frac{x + 0.5y}{n}(100) = \frac{3 + 0.5(1)}{29}(100) = 12.07. \text{ Twenty-five is the 12}^{\text{th}} \text{ percentile.}$$

## Interpreting Percentiles, Quartiles, and Median

A percentile indicates the relative standing of a data value when data are sorted into numerical order from smallest to largest. Percentages of data values are less than or equal to the  $p^{\text{th}}$  percentile. For example, 15% of data values are less than or equal to the 15<sup>th</sup> percentile.

- Low percentiles always correspond to lower data values.
- High percentiles always correspond to higher data values.

A percentile may or may not correspond to a value judgment about whether it is "good" or "bad." The interpretation of whether a certain percentile is "good" or "bad" depends on the context of the situation to which the data applies. In some situations, a low percentile would be considered "good;" in other contexts a high percentile might be considered "good". In many situations, there is no value judgment that applies.

Understanding how to interpret percentiles properly is important not only when describing data, but also when calculating probabilities in later chapters of this text.

### GUIDELINE

When writing the interpretation of a percentile in the context of the given data, the sentence should contain the following information.

- information about the context of the situation being considered
- the data value (value of the variable) that represents the percentile
- the percent of individuals or items with data values below the percentile
- the percent of individuals or items with data values above the percentile.

On a timed math test, the first quartile for time it took to finish the exam was 35 minutes. Interpret the first quartile in the context of this situation.

### Answer

- Twenty-five percent of students finished the exam in 35 minutes or less.

- Seventy-five percent of students finished the exam in 35 minutes or more.
- A low percentile could be considered good, as finishing more quickly on a timed exam is desirable. (If you take too long, you might not be able to finish.)

On a 20 question math test, the 70<sup>th</sup> percentile for number of correct answers was 16. Interpret the 70<sup>th</sup> percentile in the context of this situation.

**Answer**

- Seventy percent of students answered 16 or fewer questions correctly.
- Thirty percent of students answered 16 or more questions correctly.
- A higher percentile could be considered good, as answering more questions correctly is desirable.

**Example 2.4.9**

At a community college, it was found that the 30<sup>th</sup> percentile of credit units that students are enrolled for is seven units. Interpret the 30<sup>th</sup> percentile in the context of this situation.

**Answer**

- Thirty percent of students are enrolled in seven or fewer credit units.
- Seventy percent of students are enrolled in seven or more credit units.
- In this example, there is no "good" or "bad" value judgment associated with a higher or lower percentile. Students attend community college for varied reasons and needs, and their course load varies according to their needs.

**Example 2.4.10**

Sharpe Middle School is applying for a grant that will be used to add fitness equipment to the gym. The principal surveyed 15 anonymous students to determine how many minutes a day the students spend exercising. The results from the 15 anonymous students are shown.

0 minutes; 40 minutes; 60 minutes; 30 minutes; 60 minutes

10 minutes; 45 minutes; 30 minutes; 300 minutes; 90 minutes;

30 minutes; 120 minutes; 60 minutes; 0 minutes; 20 minutes

Determine the following five values.

- Min = 0
- $Q_1 = 20$
- Med = 40
- $Q_3 = 60$
- Max = 300

If you were the principal, would you be justified in purchasing new fitness equipment? Since 75% of the students exercise for 60 minutes or less daily, and since the *IQR* is 40 minutes ( $60 - 20 = 40$ ), we know that half of the students surveyed exercise between 20 minutes and 60 minutes daily. This seems a reasonable amount of time spent exercising, so the principal would be justified in purchasing the new equipment.

However, the principal needs to be careful. The value 300 appears to be a potential outlier.

$$Q_3 + 1.5(IQR) = 60 + (1.5)(40) = 120 \quad (2.5.2)$$

.

The value 300 is greater than 120 so it is a potential outlier. If we delete it and calculate the five values, we get the following values:

- Min = 0
- $Q_1 = 20$
- $Q_3 = 60$

- Max = 120

We still have 75% of the students exercising for 60 minutes or less daily and half of the students exercising between 20 and 60 minutes a day. However, 15 students is a small sample and the principal should survey more students to be sure of his survey results.

## WebWork Problems

## References

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2. Data from the United States Department of Commerce: United States Census Bureau. Available online at <http://www.census.gov/> (accessed April 3, 2013).
3. "1990 Census." United States Department of Commerce: United States Census Bureau. Available online at <http://www.census.gov/main/www/cen1990.html> (accessed April 3, 2013).
4. Data from *San Jose Mercury News*.
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## Review

The values that divide a rank-ordered set of data into 100 equal parts are called percentiles. Percentiles are used to compare and interpret data. For example, an observation at the 50<sup>th</sup> percentile would be greater than 50 percent of the other observations in the set. Quartiles divide data into quarters. The first quartile ( $Q_1$ ) is the 25<sup>th</sup> percentile, the second quartile ( $Q_2$  or median) is 50<sup>th</sup> percentile, and the third quartile ( $Q_3$ ) is the 75<sup>th</sup> percentile. The interquartile range, or *IQR*, is the range of the middle 50 percent of the data values. The *IQR* is found by subtracting  $Q_1$  from  $Q_3$ , and can help determine outliers by using the following two expressions.

- $Q_3 + IQR(1.5)$
- $Q_1 - IQR(1.5)$

## Formula Review

$$i = \frac{k}{100}(n + 1)$$

where  $i$  = the ranking or position of a data value,

- $k$  = the  $k^{\text{th}}$  percentile,
- $n$  = total number of data.

Expression for finding the percentile of a data value:  $\left(\frac{x + 0.5y}{n}\right)(100)$

where  $x$  = the number of values counting from the bottom of the data list up to but not including the data value for which you want to find the percentile,

$y$  = the number of data values equal to the data value for which you want to find the percentile,

$n$  = total number of data

## Glossary

### Interquartile Range

or *IQR*, is the range of the middle 50 percent of the data values; the *IQR* is found by subtracting the first quartile from the third quartile.

### Outlier

an observation that does not fit the rest of the data

### Percentile

a number that divides ordered data into hundredths; percentiles may or may not be part of the data. The median of the data is the second quartile and the 50<sup>th</sup> percentile. The first and third quartiles are the 25<sup>th</sup> and the 75<sup>th</sup> percentiles, respectively.

### Quartiles

the numbers that separate the data into quarters; quartiles may or may not be part of the data. The second quartile is the median of the data.

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