

## 3.4.2: Linear Equations

Linear regression for two variables is based on a linear equation with one independent variable. The equation has the form:

$$y = a + bx$$

where  $a$  and  $b$  are constant numbers. The variable  $x$  is the *independent variable*, and  $y$  is the *dependent variable*. Typically, you choose a value to substitute for the independent variable and then solve for the dependent variable.

### ✓ Example 3.4.2.1

The following examples are linear equations.

$$y = 3 + 2x$$

$$y = -0.01 + 1.2x$$

### ? Exercise 3.4.2.1

Is the following an example of a linear equation?

$$y = -0.125 - 3.5x$$

**Answer**

yes

The graph of a linear equation of the form  $y = a + bx$  is a **straight line**. Any line that is not vertical can be described by this equation.

### ✓ Example 3.4.2.2

Graph the equation  $y = -1 + 2x$ .

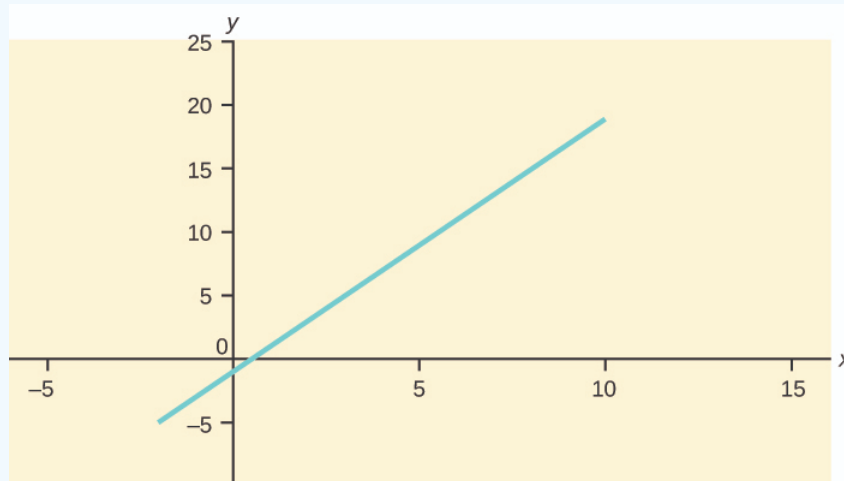


Figure 3.4.2.1.

### ? Exercise 3.4.2.2

Is the following an example of a linear equation? Why or why not?

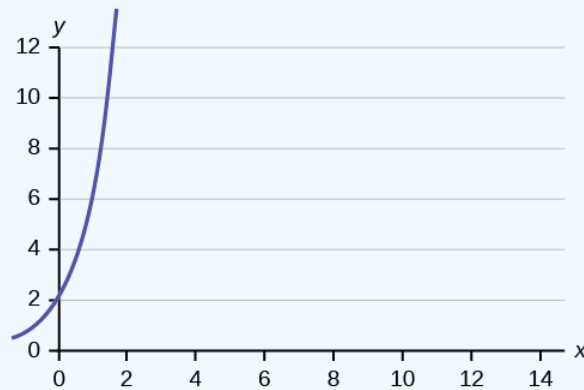


Figure 3.4.2.2.

**Answer**

No, the graph is not a straight line; therefore, it is not a linear equation.

✓ **Example 3.4.2.3**

Aaron's Word Processing Service (AWPS) does word processing. The rate for services is \$32 per hour plus a \$31.50 one-time charge. The total cost to a customer depends on the number of hours it takes to complete the job.

Find the equation that expresses the **total cost** in terms of the **number of hours** required to complete the job.

**Answer**

Let  $x$  = the number of hours it takes to get the job done.

Let  $y$  = the total cost to the customer.

The \$31.50 is a fixed cost. If it takes  $x$  hours to complete the job, then  $(32)(x)$  is the cost of the word processing only. The total cost is:  $y = 31.50 + 32x$

? **Exercise 3.4.2.3**

Emma's Extreme Sports hires hang-gliding instructors and pays them a fee of \$50 per class as well as \$20 per student in the class. The total cost Emma pays depends on the number of students in a class. Find the equation that expresses the total cost in terms of the number of students in a class.

**Answer**

$$y = 50 + 20x$$

## Slope and Y-Intercept of a Linear Equation

For the linear equation  $y = a + bx$ ,  $b$  = slope and  $a$  =  $y$ -intercept. From algebra recall that the slope is a number that describes the steepness of a line, and the  $y$ -intercept is the  $y$  coordinate of the point  $(0, a)$  where the line crosses the  $y$ -axis.

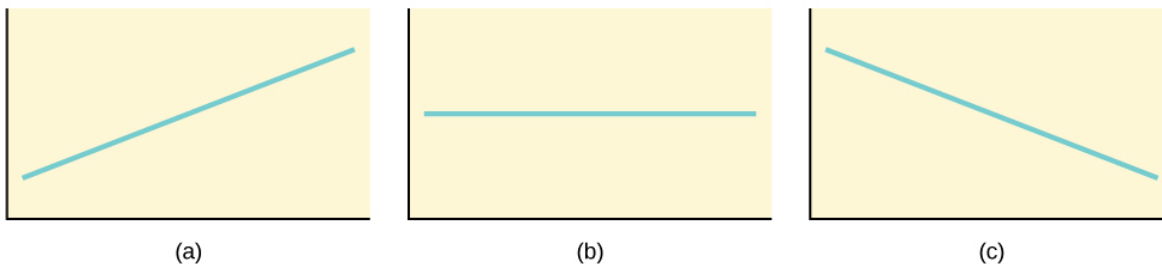


Figure 3.4.2.3: Three possible graphs of  $y = a + bx$  (a) If  $b > 0$ , the line slopes upward to the right. (b) If  $b = 0$ , the line is horizontal. (c) If  $b < 0$ , the line slopes downward to the right.

#### ✓ Example 3.4.2.4

Svetlana tutors to make extra money for college. For each tutoring session, she charges a one-time fee of \$25 plus \$15 per hour of tutoring. A linear equation that expresses the total amount of money Svetlana earns for each session she tutors is  $y = 25 + 15x$ .

What are the independent and dependent variables? What is the  $y$ -intercept and what is the slope? Interpret them using complete sentences.

#### Answer

The independent variable ( $x$ ) is the number of hours Svetlana tutors each session. The dependent variable ( $y$ ) is the amount, in dollars, Svetlana earns for each session.

The  $y$ -intercept is 25 ( $a = 25$ ). At the start of the tutoring session, Svetlana charges a one-time fee of \$25 (this is when  $x = 0$ ). The slope is 15 ( $b = 15$ ). For each session, Svetlana earns \$15 for each hour she tutors.

#### ? Exercise 3.4.2.4

Ethan repairs household appliances like dishwashers and refrigerators. For each visit, he charges \$25 plus \$20 per hour of work. A linear equation that expresses the total amount of money Ethan earns per visit is  $y = 25 + 20x$ .

What are the independent and dependent variables? What is the  $y$ -intercept and what is the slope? Interpret them using complete sentences.

#### Answer

The independent variable ( $x$ ) is the number of hours Ethan works each visit. The dependent variable ( $y$ ) is the amount, in dollars, Ethan earns for each visit.

The  $y$ -intercept is 25 ( $a = 25$ ). At the start of a visit, Ethan charges a one-time fee of \$25 (this is when  $x = 0$ ). The slope is 20 ( $b = 20$ ). For each visit, Ethan earns \$20 for each hour he works.

### Summary

The most basic type of association is a linear association. This type of relationship can be defined algebraically by the equations used, numerically with actual or predicted data values, or graphically from a plotted curve. (Lines are classified as straight curves.) Algebraically, a linear equation typically takes the form  $y = mx + b$ , where  $m$  and  $b$  are constants,  $x$  is the independent variable,  $y$  is the dependent variable. In a statistical context, a linear equation is written in the form  $y = a + bx$ , where  $a$  and  $b$  are the constants. This form is used to help readers distinguish the statistical context from the algebraic context. In the equation  $y = a + bx$ , the constant  $b$  that multiplies the  $x$  variable ( $b$  is called a coefficient) is called the **slope**. The constant  $a$  is called the  $y$ -intercept.

The **slope of a line** is a value that describes the rate of change between the independent and dependent variables. The **slope** tells us how the dependent variable ( $y$ ) changes for every one unit increase in the independent ( $x$ ) variable, on average. The  **$y$ -intercept** is used to describe the dependent variable when the independent variable equals zero.

## Formula Review

$y = a + bx$  where  $a$  is the  $y$ -intercept and  $b$  is the slope. The variable  $x$  is the independent variable and  $y$  is the dependent variable.

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