9.1: Growth and Yield Models

Forest and natural resource management decisions are often based on information collected on past and present resource conditions. This information provides us with not only current details on the timber we manage (e.g., volume, diameter distribution) but also allows us to track changes in growth, mortality, and ingrowth over time. We use this information to make predictions of future growth and yield based on our management objectives. Techniques for forecasting stand dynamics are collectively referred to as growth and yield models. Growth and yield models are relationships between the amount of yield or growth and the many different factors that explain or predict this growth.

Before we continue our examination of growth and yield models, let's review some basic terms.

- **Yield**: total volume available for harvest at a given time
- **Growth**: difference in volume between the beginning and end of a specified period of time (V2 – V1)
- **Annual growth**: when growth is divided by number of years in the growing period
- **Model:** a mathematical function used to relate observed growth rates or yield to measured tree, stand, and site variables
- **Estimation:** a statistical process of obtaining coefficients for models that describe the growth rates or yield as a function of measured tree, stand, and site variables
- **Evaluation**: considering how, where, and by whom the model should be used, how the model and its components operate, and the quality of the system design and its biological reality
- **Verification**: the process of confirming that the model functions correctly with respect to the conceptual model. In other words, verification makes sure that there are no flaws in the programming logic or algorithms, and no bias in computation (systematic errors).
- **Validation**: checks the accuracy and consistency of the model and tests the model to see how well it reflects the real system, if possible, using an independent data set
- **Simulation**: using a computer program to simulate an abstract model of a particular system. We use a growth model to estimate stand development through time under alternative conditions or silvicultural practices.
- **Calibration**: the process of modifying the model to account for local conditions that may differ from those on which the model was based
- **Monitoring**: continually checking the simulation output of the system to identify any shortcomings of the model
- **Deterministic model**: a model in which the outcomes are determined through known relationships among states and events, without any room for random variation. In forestry, a deterministic model provides an estimate of average stand growth, and given the same initial conditions, a deterministic model will always predict the same result.
- **Stochastic model**: a model that attempts to illustrate the natural variation in a system by providing different predictions (each with a specific probability of occurrence) given the same initial conditions. A stochastic model requires multiple runs to provide estimates of the variability of predictions.
- **Process model**: a model that attempts to simulate biological processes that convert carbon dioxide, nutrients, and moisture into biomass through photosynthesis
- **Succession model**: a model that attempts to model species succession, but is generally unable to provide reliable information on timber yield

Models

Growth and yield models are typically stated as mathematical equations and can be implicit or explicit in form. An implicit model defines the variables in the equation but the specific relationship is not quantified. For example,

$$V = f(BA, H_t) \tag{9.1.1}$$

where *V* is volume (ft3/ac), *BA* is density (basal area in ft2), *Ht* is total tree height. This model says that volume is a function of (depends on) density and height, but it does not put a numerical value on the volume for specific values of basal area and height. This equation becomes explicit when we specify the relationship such as

\[ln(*V*) = -0.723 + 0.781*ln(*BA*)+ 0.922 ln(*H_t*)\]

Growth and yield models can be linear or nonlinear equations. In this linear model, all the independent variables of *X*1 and *X*2 are only raised to the first power.

$$y = 1.29 + 7.65X1 - 27.02X2 \tag{9.1.2}$$

A nonlinear model has independent variables with exponents different from one.

$$y = b_o e^{b_1 X} \tag{9.1.3}$$

In this example, *b*0 and *b*1 are parameters to be estimated and *X* is the independent variable.

Classification of Growth and Yield Models

Growth and yield models have long been part of forestry but development and use has greatly increased in the last 25 years due to the accessibility of computers. There are many different approaches to modeling, each with their own advantages and disadvantages. Selecting a specific type of modeling approach often depends on the type of data used. Growth and yield models are categorized depending on whether they model the whole stand, the diameter classes, or individual trees.

Whole Stand Models

Whole stand models may or may not contain density as an independent variable. Density-free whole stand models provide the basis for traditional normal yield tables since "normal" implies nature's maximum density, and empirical yield tables assume nature's average density. In both of these cases, stand volume at a specific age is typically a function of stand age and site index. Variable-density whole stand models use density as an explicit independent variable to predict current or future volume. Buckman (1962) published the first study in the United States that directly predicted growth from current stand variables, then integrated the growth function to obtain yield:

$$Y = 1.6689 + 0.041066BA - 0.00016303BA_2 - 0.076958A + 0.00022741A_2 + 0.06441S \tag{9.1.4}$$

where *Y* = periodic net annual basal area increment
BA = basal area, in square feet per acre
A = age, in years
S = site index

Diameter distribution models are a refinement of whole stand models. This type of model disaggregates the results at each age and then adds additional information about diameter class structure such as height and volume. The number of stems in each class is a function of the stand variables and all growth functions are for the stand. This type of whole stand model provides greater detail of the stand conditions in terms of volume, tree size, and value.

Diameter Class Models

Diameter class models (not to be confused with diameter distribution models) simulate growth and volume for each diameter class based on the average tree in each class. The number of trees in each class is empirically determined. The diameter class volumes are computed separately for each diameter class, then summed up to obtain stand values. Stand table projection is a common diameter class method used to predict short-term future conditions based on observable diameter growth for that stand. Mortality, harvest, and ingrowth must be computed separately. Differences in projection methods are based on the distribution of the number of stems in each class and how the growth rate is applied. For example, the simplest projection method is based on two assumptions: 1) that all tree diameters in a diameter class equal the midpoint diameter for that class, and 2) that they all grow at the same average rate. An improvement upon this method is to use a movement ratio that defines the proportion of trees which move into a higher DBH class.

$$m = \frac{g}{i} x100 \tag{9.1.5}$$

where *m* is the movement ratio, *g* is the average periodic diameter increment for that specific class, and *i* is the diameter class interval. Let's look at an example.

Assume for a specific DBH class that *g* is 1.2 in. and *i* (class interval) is 2.0 in.

$$m = \frac{1.2}{2.0} x1000 = 60 \tag{9.1.6}$$

This means that 60% of the trees in that diameter class will move up to the next diameter class, and 40% will remain in this class. If the diameter class interval was one inch, the movement ratio would be different.

$$m = \frac{1.2}{1.0} x100 = 120 \tag{9.1.7}$$

In this case, all the trees in this diameter class would move up at least one size class and 20% of them would move up two size classes.

Individual Tree Models

Individual tree models simulate the growth of each individual tree in the tree list. These models are more complex but have become more common as computing power has increased. Individual tree models typically simulate the height, diameter, and survival of each tree while calculating its growth. Individual tree data are aggregated *after* the model grows each tree, while stand models aggregate individual tree data into stand variables *before* the growth model is applied. Additionally, this type of model allows the user to include a measure of competition for each tree. Because of this, individual tree models are typically divided into two groups based on how competition is treated.

Distance-independent models define the competitive neighborhood for a subject tree by its own diameter, height, and condition to stand characteristics such as basal area, number of trees per area, and average diameter, however, the distances between trees are not required for computing the competition for each tree. Distance-dependent models include distance and bearing to all neighboring trees, along with their diameter. This way, the competitive neighborhood for each subject tree is precisely and uniquely defined. While this approach seems logically superior to distance-independent methods, there has not been any clear documented evidence to support the use of distance-dependent competition measures over distance-independent measures.

There are many growth and yield models and simulators available and it can be difficult to select the most appropriate model. There are advantages and disadvantages to many of these options and foresters must be concerned with the reliability of the estimates, the flexibility of the model to deal with management alternatives, the level of required detail, and the efficiency for providing information in a clear and useable fashion. Many models have been created using a broad range of available data. These models are best used for comparative purposes only. In other words, they are most appropriate when comparing the outcomes from different management options instead of predicting results for a specific stand. It is important to review and understand the foundations for any model or simulator before using it.

Forest Vegetation Simulator

The Forest Vegetation Simulator (FVS, Wykoff et al. 1982; Dixon 2002) is a distance-independent, individual-tree forest growth model commonly used in the United States to support forest management decisions. Projections are typically made at the stand level, but FVS has the ability to expand the spatial scope to much larger management units. FVS began as the Prognosis Model for Stand Development (Stage 1973) with the objective to predict stand dynamics in the mixed forests of Idaho and Montana. This model became the common modeling platform for the USDA Forest Service and was renamed FVS.

Stands are the basic unit of management and projections are dependent on the interactions among trees within stands using key variables such as density, species, diameter, height, crown ratio, diameter growth, and height growth. Values for slope, aspect, elevation, density, and a measure of site potential are included for each plot. There are 22 geographically specific versions of FVS called variants.

NE-TWIGS (Belcher 1982) is a common variant applicable to fourteen northeastern states. Stand growth projections are based on simulating the growth and mortality for trees in the 5-inch and larger DBH classes. Ingrowth can be manually entered or simulated using an automatic ingrowth function. The growth equation annually estimates a diameter for each sample tree and updates the crown ratio of the tree (Miner et al. 1988).

*Annual diameter growth = potential growth*competition modifier*

Potential growth is defined as the growth of the top 10% of the fastest growing trees and is predicted using the following equation:

$$Potential\ growth = b_1 * SI * [1.0 - e^{(-b_2 * D)}] \tag{9.1.8}$$

where,

potential growth is defined as the potential annual basal area growth of a tree (sq. ft./yr)
$b1$ and $b2$ are species specific coefficients
SI is site index (index age 50 years) and
D is current tree diameter in in.

The competition modifier is an index bounded from 0 to 1, and is found by:

Competition modifier = $e^{-b_3 * BA}$

where $b3$ is a species-specific coefficient and
BA is the current basal area (sq. ft./ac).

Tree mortality is calculated by estimating the probability of death of each tree in a given year:

Survival = 1-[1/(1+en)]

where $n = c_1 + c_2 * (D+1)^{c_3} * e^{c_4 * D - c_5 * BA - c_6 * SI}$
$c1, \ldots, c6$ are species-specific coefficients
D is current tree diameter (inches)
BA is stand basal area (sq. ft./ac) and
SI is site index.

Inventory data and site information are entered into FVS, and a self-calibration process adjusts the growth models to match the rates present in the entered data. Harvests can be simulated with growth and mortality rates based on post-removal stand densities. Growth cycles run for 5-10 years and output includes a summary of current stand conditions, sampling statistics, and calibration results.

Applications of Regression Techniques

Regression models serve many purposes in the management of natural and forest resources. The following examples serve to highlight some of these applications.

Weight Scaling for Sawlogs

In 1962, Bower created the following equation for predicting loblolly pine sawlog volume based on truckload weights and the number of logs per truck:

$$Y = -3.954N + 0.0925W \tag{9.1.9}$$

where Y = total board-foot volume (International1/4- rule) for a truckload of logs
N = number of 16-ft logs on the truck
W = total load weight (lb.)

Notice that there is no y-intercept in the model. When there are no logs on the truck, there is no volume to be estimated.

Rates of Stem Taper

Kozak et al. (1969) developed a technique for estimating the fraction of volume per tree located in logs of any specified length and dib for any system of scaling (board feet, cubic feet, or weight). Their regression model also predicted taper curves and upper stem diameters (dib) for some conifer species.

$$\frac{d^2}{dbh^2} = b_0 + b_1\left(\frac{h}{H}\right) + b_2\left(\frac{h^2}{H^2}\right) \tag{9.1.10}$$

where d = stem diameter at any height h above ground
H = total tree height

This equation resolves to:

$$d = dbh\sqrt{b_0 + b_1\left(\frac{h}{H}\right) + b_2\left(\frac{h^2}{H^2}\right)} \tag{9.1.11}$$

The predictor variables are the ratio, and squared ratio, of any height to total height.

Multiple Entry Volume Table that Allows for Variable Utilization Standards

Foresters commonly want to predict tree volume for various top diameters but many of the available volume equations were created for specific top limits. Burkhart (1977) created a regression model to predict volume (cubic feet) of loblolly pine to any desired merchantable top limit. His approach predicted total stem volume, then converted total volume to merchantable volume by applying predicted ratios of merchantable volume to total volume.

$$V = 0.34864 + 0.00232dbh^2H \tag{9.1.12}$$

$$R = 1 - 0.32354\left(\frac{d_t^{3.1579}}{dbh^{2.7115}}\right) \tag{9.1.13}$$

where dbh = diameter at breast height (in.)
H = total tree height (ft.)
V = total stem cubic-foot volume
R = merchantable cubic-foot volume to top diameter dt divided by total stem cubic-foot volume
dt = top dob (in.)

Weight Tables for Tree Boles

Belanger (1973) utilized a combined-variable approach to develop predictions of green-weight and dry weight of sycamore tree:

$$GBW = -32.35109 + 0.15544 dbh^2 H \tag{9.1.14}$$

$$DBW = -17.67910 + 0.06684 dbh^2 H \tag{9.1.15}$$

where GBW = green bole weight to 3-in.top (lb.)
DBW = dry bole weight to 3-in.top (lb.)
dbh = diameter at breast height (in.)
H = total tree height (ft.)

Biomass Prediction

A common approach to predicting tree biomass weight has been to use a logarithmic combined-variable formula (e.g. Edwards and McNab 1979). The observed relationship between these variables is typically non-linear, therefore a log or natural log transformation is needed to linearize the relationship.

$$log\, Y = b_0 + b_1 log\, dbh^2 H \tag{9.1.16}$$

where Y = total tree weight
dbh = diameter at breast height
H = total tree height

However, past studies (Tritton and Hornbeck 1982 and Wiant et al. 1979) indicated that there was little model improvement when height was added. Many dry-weight biomass models now follow this form:

$$ln\ wt = b_0 + b_1 ln\ dbh \tag{9.1.17}$$

$$wt = e^{b_0} dbh^{b_1} \tag{9.1.18}$$

where wt = total tree weight
dbh = diameter at breast height

Volume Predictions based on Stump Diameter

Bylin (1982) created a regression model to predict tree volume using stump diameter and stump height for species in Louisiana.

$$V = b_0 + b_1 S_{DIB}^2 + b_2 H_s \tag{9.1.19}$$

where V = tree volume (cu. ft.)
SDIB = stump diameters inside bark (in.)
HS = stump height (ft.)

Yield Estimation

MacKinney and Chaiken (1939) were the first to use multiple regression, with stand density as a predictor variable, to predict yield for loblolly pine trees.

$$log\, Y = b_0 + b_1 \frac{1}{A} + b_2 S + b_3 log\, SDI + b_4 C \tag{9.1.20}$$

where

- Y = yield (cu. ft./ac)

- A = stand age
- S = site index
- SDI = stand-density index
- C = composition index (loblolly pine BA/total BA)

Growth and Yield Prediction for Uneven-aged Stands

Moser and Hall (1969) developed a yield equation, expressed as a function of time, initial volume, and basal area, to predict volume in mixed northern hardwoods.

$$Y = [(Y_0)(8.3348BA_0^{-1.3175})]x[0.9348 - (0.9348 - 1.0203BA_0^{-0.0125})e^{-0.0062t}]^{-105.5} \tag{9.1.21}$$

where

- $Y0$ = initial volume (cu. ft./ac)
- $BA0$ = initial basal area (sq. ft./ac)
- t = elapsed time interval (years from initial condition)
- Y = predicted volume (cu. ft./ac) t years after observation of initial conditions $Y0$ and $BA0$ at time $t0$
