3.5: Hypothesis Test about a Variance

Hypothesis Test about a Variance

When people think of statistical inference, they usually think of inferences involving population means or proportions. However, the particular population parameter needed to answer an experimenter's practical questions varies from one situation to another, and sometimes a population's variability is more important than its mean. Thus, product quality is often defined in terms of low variability.

Sample variance s^2 can be used for inferences concerning a population variance σ^2. For a random sample of n measurements drawn from a normal population with mean **μ** and variance σ^2, the value s^2 provides a **point estimate for σ^2**. In addition, the quantity $\frac{(n-1)s^2}{\sigma^2}$ follows a **Chi-square**(χ^2) **distribution,** with $df = n - 1$.

The properties of **Chi-square (χ^2) distribution** are:

- Unlike Z and t distributions, the values in a chi-square distribution are all positive.
- The chi-square distribution is asymmetric, unlike the Z and t distributions.
- There are many chi-square distributions. We obtain a particular one by specifying the degrees of freedom $(df = n - 1)$ associated with the sample variances s^2.

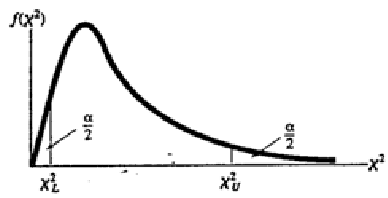


Figure 3.5.1: The chi-square distribution.

One-sample (χ^2) test for testing the hypotheses:

Null hypothesis: $H_0 : \sigma^2 = \sigma_0^2$ (constant)

Alternative hypothesis:

- $H_a : \sigma^2 > \sigma_0^2$ (one-tailed), reject H_0 if the observed $\chi^2 > \chi^2_U$ (upper-tail value at α).
- $H_a : \sigma^2 < \sigma_0^2$ (one-tailed), reject H_0 if the observed $\chi^2 < \chi^2_L$ (lower-tail value at α).
- $H_a : \sigma^2 \neq \sigma_0^2$ (two-tailed), reject H_0 if the observed $\chi^2 > \chi^2_U$ or $\chi^2 < \chi^2_L$ at α/2.

where the χ^2 critical value in the rejection region is based on degrees of freedom $df = n - 1$ and a specified significance level of **α.**

Test statistic:

$$\chi^2 = \frac{(n-1)S^2}{\sigma_0^2} \tag{3.5.1}$$

As with previous sections, if the test statistic falls in the rejection zone set by the critical value, you will reject the null hypothesis.

Example 3.5.1:

A forester wants to control a dense understory of striped maple that is interfering with desirable hardwood regeneration using a mist blower to apply an herbicide treatment. She wants to make sure that treatment has a consistent application rate, in other words, low variability not exceeding 0.25 gal./acre (0.06 gal.2). She collects sample data (n = 11) on this type of mist blower and gets a sample variance of 0.064 gal.2 Using a 5% level of significance, test the claim that the variance is significantly greater than 0.06 gal.2

$H_0 : \sigma^2 = 0.06$

$H_1 : \sigma^2 > 0.06$

The critical value is 18.307. Any test statistic greater than this value will cause you to reject the null hypothesis.

The test statistic is

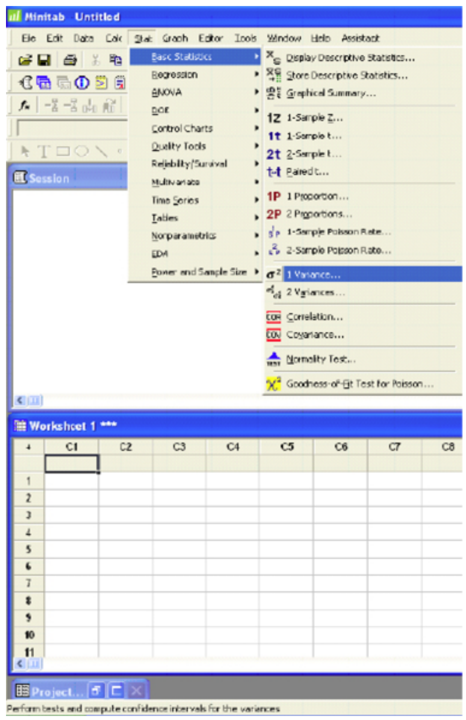
$$\chi^2 = \frac{(n-1)S^2}{\sigma_0^2} = \frac{(11-1)0.064}{0.06} = 10.667$$

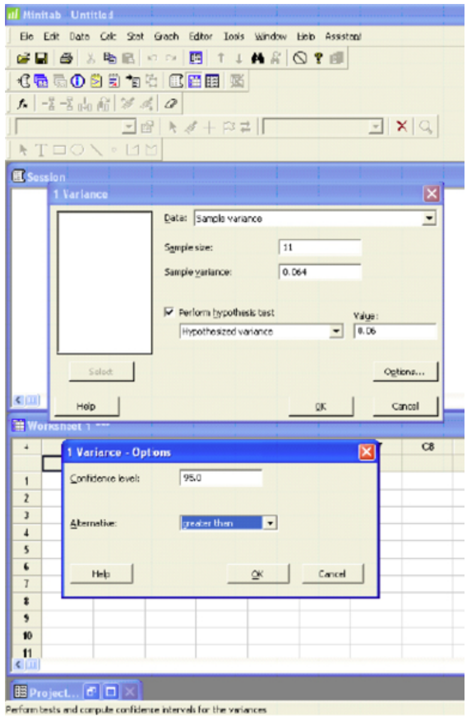
We fail to reject the null hypothesis. The forester does NOT have enough evidence to support the claim that the variance is greater than 0.06 gal.2 You can also estimate the p-value using the same method as for the student t-table. Go across the row for degrees of freedom until you find the two values that your test statistic falls between. In this case going across the row 10, the two table values are 4.865 and 15.987. Now go up those two columns to the top row to estimate the p-value (0.1-0.9). The p-value is greater than 0.1 and less than 0.9. Both are greater than the level of significance (0.05) causing us to fail to reject the null hypothesis.

Software Solutions

Minitab

(referring to Ex. $3.5.1$)





Test and CI for One Variance

Method		
Null hypothesis	Sigma-squared	= 0.06
Alternative hypothesis	Sigma-squared	> 0.06

The chi-square method is only for the normal distribution.

Tests

Test			
Method	Statistic	DF	P-Value
Chi-Square	10.67	10	0.384

Excel

Excel does not offer 1-sample χ^2 testing.
