4.5: F-Test for Comparing Two Population Variances

F-Test for Comparing Two Population Variances

One major application of a test for the equality of two population variances is for **checking** the validity of the equal variance assumption $(\sigma_1^2 = \sigma_2^2)$ for a two-sample t-test. First we hypothesize two populations of measurements that are normally distributed. We label these populations as 1 and 2, respectively. We are interested in comparing the variance of population 1 (σ_1^2) to the variance of population 2 (σ_2^2).

When independent random samples have been drawn from the respective populations, the ratio

$$\frac{S_1^2/S_2^2}{\sigma_1^2/\sigma_2^2} \tag{4.5.1}$$

possesses a probability distribution in repeated sampling that is referred to as an **F** distribution and its properties are:

- Unlike Z and t, but like χ^2, F can assume only positive values.
- The **F** distribution, unlike the Z and t distributions, but like the (\chi^2\) distribution, is non-symmetrical.
- There are many **F** distributions, and each one has a different shape. We specify a particular one by designating the degrees of freedom associated with S_1^2 and S_2^2. We denote these quantities by df_1 and df_2, respectively.

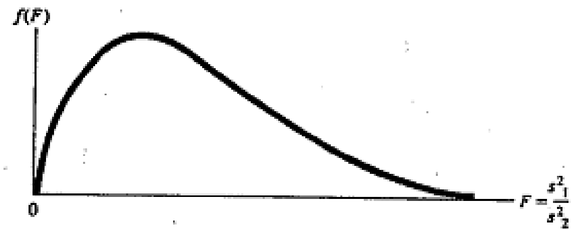


Figure 4.5.1: The F-distribution.

Note: A statistical test of the null hypothesis $\sigma_1^2 = \sigma_2^2$ utilizes the test statistic S_1^2/S_2^2. It may require either upper tail or lower tail rejection region, depending on which sample variance is larger. To alleviate this situation, we are at liberty to designate the population with the larger sample variance as **population 1** (i.e., used as the numerator of the ratio S_1^2/S_2^2). By this convention, the rejection region is only located in the **upper tail of the F distribution**.

Null hypothesis: $H_0 : \sigma_1^2 = \sigma_2^2$

Alternative hypothesis:

- $H_a : \sigma_1^2 > \sigma_2^2$ (one-tailed), reject H_0 if the observed F > Fα
- $H_a : \sigma_1^2 \neq \sigma_2^2$ (two-tailed), reject H_0 if the observed F > Fα/2.

Test statistic: $F = \frac{S_1^2}{S_2^2}$ **assuming** $S_1^2 > S_2^2$,

where the **F** critical value in the rejection region is based on 2 degrees of freedom $df_1 = n_1 - 1$ (associated with numerator S_1^2) and $df_2 = n_2 - 1$ (associated with denominator S_2^2).

Example 4.5.1:

A forester wants to compare two different mist blowers for consistent application. She wants to use the mist blower with the smaller variance, which means more consistent application. She wants to test that the variance of Type A (0.087 gal.2) is significantly greater than the variance of Type B (0.073 gal.2) using α = 0.05.

Type A	Type B
S_1^2 = 0.087	S_2^2=0.073
n_1= 16	n_2 = 21

Solution

$H_0 : \sigma_1^2 = \sigma_2^2$

$H_1 : \sigma_1^2 > \sigma_2^2$

The critical value $(df_1 = 15 \text{ and } df_2 = 20)$ is 2.20.

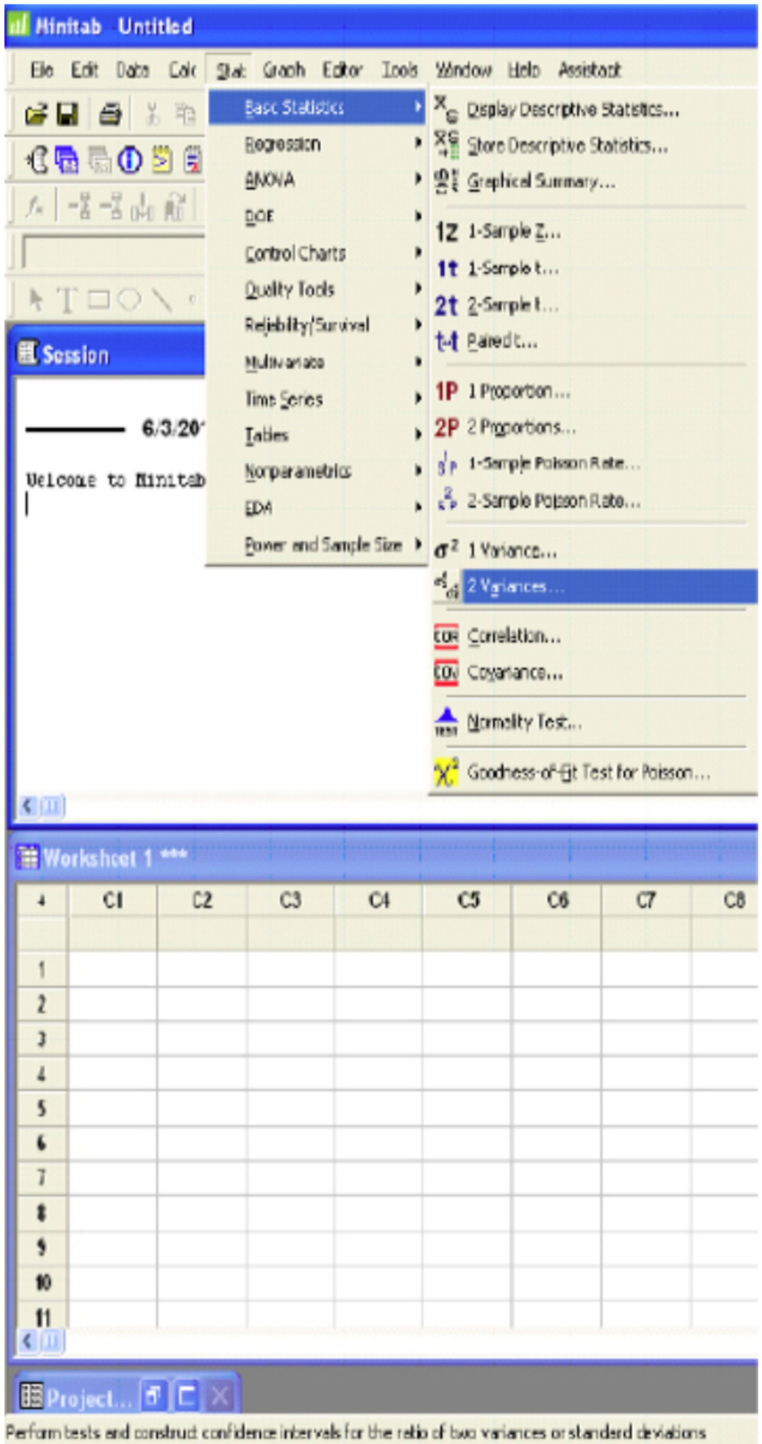
The test statistic is:

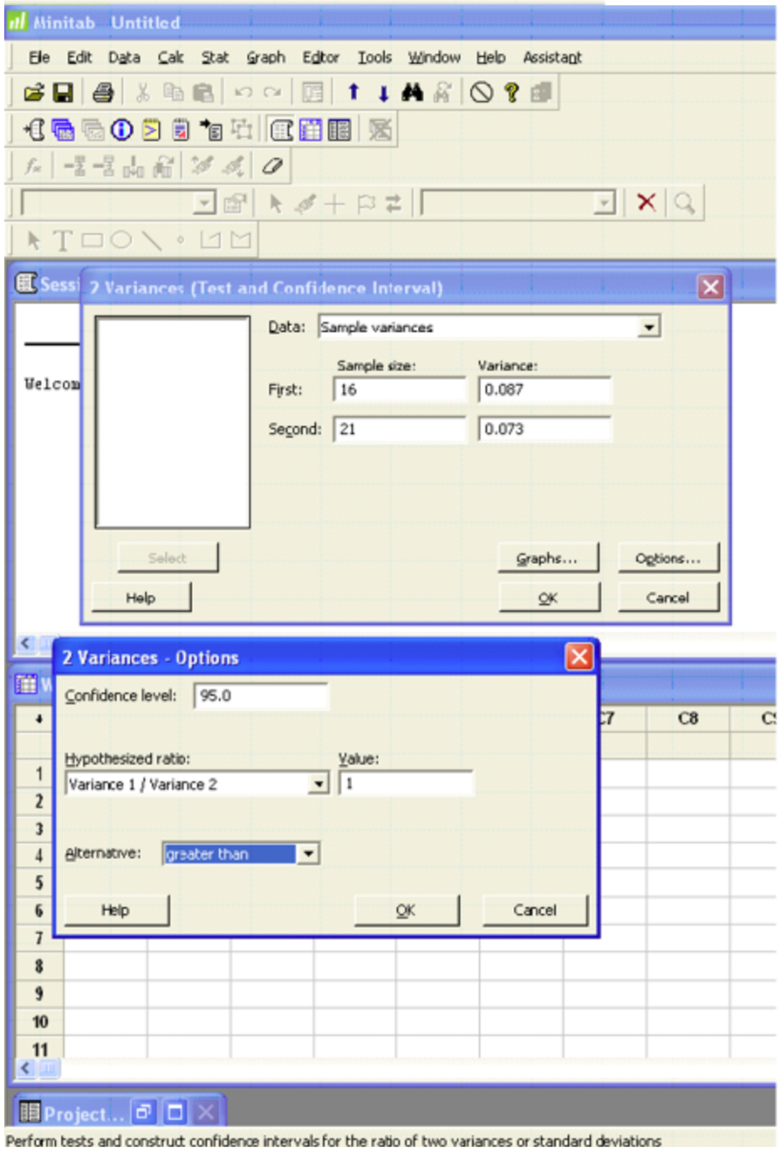
$$F = \frac{S_1^2}{S_2^2} = \frac{0.087}{0.073} = 1.192 \tag{4.5.2}$$

The test statistic is not larger than the critical value (it does not fall in the rejection zone) so we fail to reject the null hypothesis. While the variance of Type B is mathematically smaller than the variance of Type A, it is not statistically smaller. There is not enough statistical evidence to support the claim that the variance of Type A is significantly greater than the variance of Type B. Both mist blowers will deliver the chemical with equal consistency.

Software Solutions

Minitab


Minitab - Untitled
File Edit Data Calc Stat Graph Editor Tools Window Help Assistant
Basic Statistics
Regression
ANOVA
DOE
Control Charts
Quality Tools
Reliability/Survival
Multivariate
Time Series
Tables
Nonparametrics
EDA
Power and Sample Size
Display Descriptive Statistics...
Store Descriptive Statistics...
Graphical Summary...
1-Sample Z...
1-Sample t...
2-Sample t...
Paired t...
1 Proportion...
2 Proportions...
1-Sample Poisson Rate...
2-Sample Poisson Rate...
1 Variance...
2 Variances...
Correlation...
Covariance...
Normality Test...
Goodness-of-Fit Test for Poisson...
Session
Welcome to Minitab
Worksheet 1 ***
C1 C2 C3 C4 C5 C6 C7 C8
1 2 3 4 5 6 7 8 9 10 11
Project...
Perform tests and construct confidence intervals for the ratio of two variances or standard deviations

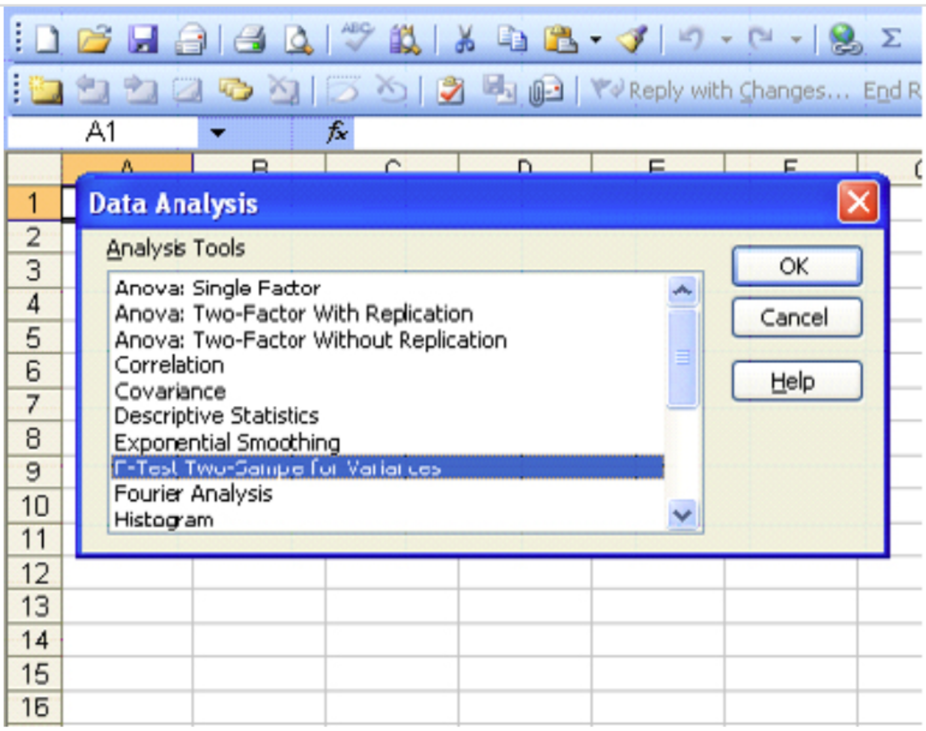


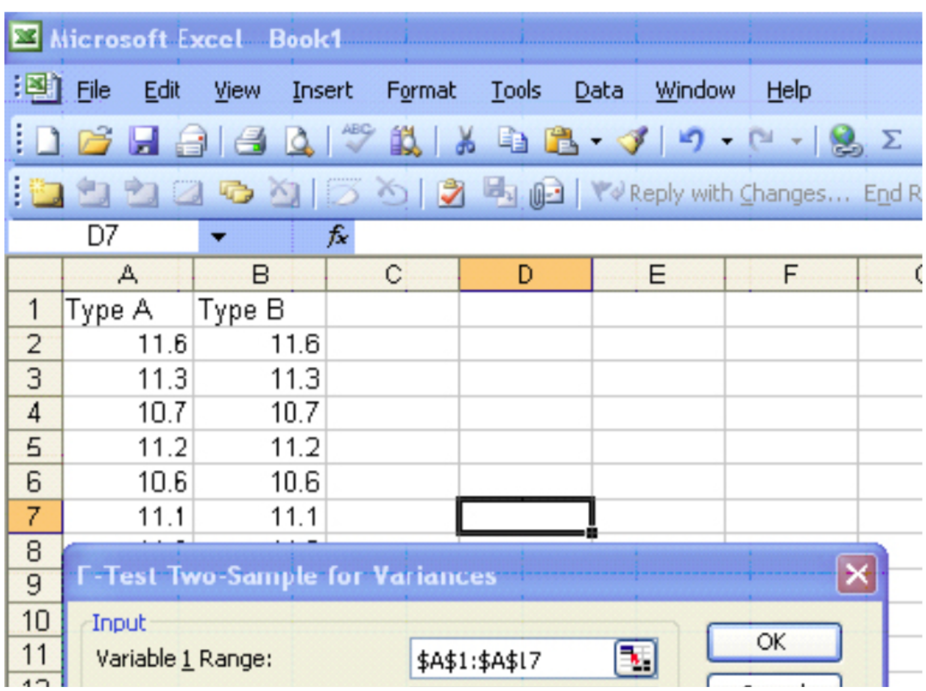
Test and CI for Two Variances - Methods

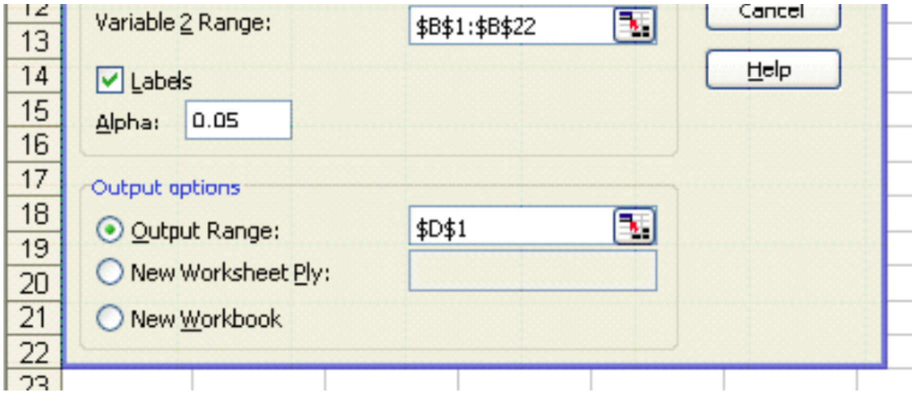
Null hypothesis	Variance(1) / Variance(2) = 1		
Alternative hypothesis	Variance(1) / Variance(2) > 1		
Significance level	Alpha = 0.05		
Statistics			
Sample	N	StDev	Variance
1	16	0.295	0.087
2	21	0.270	0.073
Ratio of standard deviations = 1.092			
Ratio of variances = 1.192			
Tests			
	Test		

Null hypothesis	Variance(1) / Variance(2) = 1			
Method	DF1	DF2	Statistic	p-value
F Test (normal)	15	20	1.19	0.351

Excel







F-Test Two-Sample for Variances

	Type A	Type B
Mean	11.07188	11.10595
Variance	0.08699	0.073379
Observations	16	21
df	15	20
F	1.185483	
$P(F \leq f)$ one-tail	0.355098	
F Critical one-tail	2.203274	