3.1: Percents

Learning Objectives

- Convert between percent, decimal, and fraction form.
- Solve problems involving percents.
- Find percent increase or decrease.

In the 2004 vice-presidential debates, Edwards's claimed that US forces have suffered "90% of the coalition casualties" in Iraq. Cheney disputed this, saying that in fact Iraqi security forces and coalition allies "have taken almost 50 percent" of the casualties [1]. Who is correct? How can we make sense of these numbers?

Percent literally means “per 100,” or “parts per hundred.” When we write 40%, this is equivalent to the fraction $\frac{40}{100}$ or the decimal 0.40. Notice that 80 out of 200 and 10 out of 25 are also 40%, since $\frac{80}{200}=\frac{10}{25}=\frac{40}{100}$.

Example 3.1.1

243 people out of 400 state that they like dogs. What percent is this?

Solution

$\frac{243}{400}=0.6075=\frac{60.75}{100}$. This is 60.75%.

Notice that the percent can be found from the equivalent decimal by moving the decimal point two places to the right.

Converting between Decimals and Percents

- To change a decimal to a percent, move the decimal point two places to the right and add the percent sign
- To change a percent to a decimal, drop the percent sign and move the decimal point two places to the left

Converting between Fractions and Percents

- To change a fraction to a percent, first change the fraction to a decimal by dividing, then change the decimal to a percent
- To change a percent to a fraction, write a fraction with the percent number (without the percent sign) in the numerator and 100 in the denominator, then simplify if necessary.

Example 3.1.2

Write each as a percent:

a. $\frac{1}{4}$
b. 0.02
c. 2.35

Solution

a. $\frac{1}{4}=0.25=25\%$
b. $0.02=2\%$
c. $2.35=235\%$

Percents

If we have a *part* that is some *percent* of a *whole*, then

$\text{percent} = \dfrac{\text{part}}{\text{whole}}$, or equivalently, $\text{part} = \text{percent} \cdot \text{whole}$

To do the calculations, we write the percent as a decimal.

When writing a percentage statement in symbols, the word "of" becomes multiplication, and the word "is" becomes an equal sign. Also, we must change the percent into decimal or fractional form. For example, the statement \$2 is 5% of \$40 would read as $2 = 0.05 * 40$ as a mathematical equation.

Example 3.1.3

The sales tax in a town is 9.4%. How much tax will you pay on a \$140 purchase?

Solution

Here, \$140 is the whole, and we want to find 9.4% *of* \$140. We start by writing the percent as a decimal by moving the decimal point two places to the left (which is equivalent to dividing by 100). We can then compute:

$\text{tax} = 0.094(140) = \$13.16$in tax.

Example 3.1.4

In the news, you hear "tuition is expected to increase by 7% next year." If tuition this year was \$1200 per semester, what will it be next year?

Solution

The tuition next year will be the current tuition plus an additional 7%, so it will be 107% of this year's tuition:

$\$1200(1.07) = \$1284$

Alternatively, we could have first calculated 7% of \$1200: $\$1200(0.07) = \$84$

Notice this is *not* the expected tuition for next year (we could only wish). Instead, this is the expected *increase*, so to calculate the expected tuition, we'll need to add this change to the previous year's tuition:

$\$1200 + \$84 = \$1284$

Try It 3.1.1

A TV originally priced at \$799 is on sale for 30% off. There is a 9.2% sales tax. Find the final price including the discount and sales tax.

Answer

The sale price is $\$799(0.70) = \$559.30$After tax, the price is $\$559.30(1.092) = \$610.76$

Calculating a Percent Increase or Decrease

Procedure to find **percent increase (decrease):**

- Find the amount of increase (decrease)
- Calculate the percent increase (decrease) by using a fraction $\dfrac{\text{amount of increase (decrease)}}{\text{original amount}}$
- Convert the fraction to a percent

Example 3.1.5

The value of a car dropped from \$7400 to \$6800 over the last year. What percent decrease is this?

Solution

To compute the percent change, we first need to find the dollar value change: $\$6800 - \$7400 = -\$600$ Often we will take the absolute value of this amount, which is called the **absolute change**: $|-600| = 600$.

Since we are computing the decrease relative to the starting value, we compute this percent out of $\$7400$:

$\frac{600}{7400} = 0.081 = 8.1\%$ decrease. This is called a **relative change**.

Absolute and Relative Change

Given two quantities,

Absolute change = $|\text{ending quantity } - \text{ starting quantity}|$

Relative change: $\frac{\text{absolute change}}{\text{starting quantity}}$

Absolute change has the same units as the original quantity.

Relative change gives a percent change.

The starting quantity is called the **base** of the percent change.

The base of a percent is very important. For example, while Nixon was president, it was argued that marijuana was a "gateway" drug, claiming that 80% of marijuana smokers went on to use harder drugs like cocaine. The problem is, this isn't true. The true claim is that 80% of harder drug users first smoked marijuana. The difference is one of base: 80% of marijuana smokers using hard drugs, vs. 80% of hard drug users having smoked marijuana. These numbers are not equivalent. As it turns out, only one in 2,400 marijuana users actually go on to use harder drugs [2].

Example 3.1.6

There are about 75 QFC supermarkets in the U.S. Albertsons has about 215 stores. Compare the size of the two companies.

Solution

When we make comparisons, we must ask first whether an absolute or relative comparison. The absolute difference is $215 - 75 = 140$. From this, we could say "Albertsons has 140 more stores than QFC." However, if you wrote this in an article or paper, that number does not mean much. The relative difference may be more meaningful. There are two different relative changes we could calculate, depending on which store we use as the base:

Using QFC as the base, $\frac{140}{75} = 1.867$.

This tells us Albertsons is 186.7% larger than QFC.

Using Albertsons as the base, $\frac{140}{215} = 0.651$.

This tells us QFC is 65.1% smaller than Albertsons.

Notice both of these are showing percent *differences*. We could also calculate the size of Albertsons relative to QFC: , which tells us Albertsons is 2.867 times the size of QFC. Likewise, we could calculate the size of QFC relative to Albertsons: , which tells us that QFC is 34.9% of the size of Albertsons.

Example 3.1.7

Suppose a stock drops in value by 60% one week, then increases in value the next week by 75%. Is the value higher or lower than where it started?

Solution

To answer this question, suppose the value started at $100. After one week, the value dropped by 60%:

$\$100 - \$100(0.60) = \$100 - \$60 = \$40.$

In the next week, notice that base of the percent has changed to the new value, $40. Computing the 75% increase:

$\$40 + \$40(0.75) = \$40 + \$30 = \$70.$

In the end, the stock is still $30 lower, or $\frac{\$30}{\$100} = 30\%$ lower, valued than it started.

Try It 3.1.2

The U.S. federal debt at the end of 2001 was $5.77 trillion, and grew to $6.20 trillion by the end of 2002. At the end of 2005 it was $7.91 trillion, and grew to $8.45 trillion by the end of 2006 [3]. Calculate the absolute and relative increase for 2001-2002 and 2005-2006. Which year saw a larger increase in federal debt?

Answer

2001-2002: Absolute change: $0.43 trillion. Relative change: 7.45%

2005-2006: Absolute change: $0.54 trillion. Relative change: 6.83%

2005-2006 saw a larger absolute increase, but a smaller relative increase.

Example 3.1.8

A Seattle Times article on high school graduation rates reported “The number of schools graduating 60 percent or fewer students in four years – sometimes referred to as “dropout factories” – decreased by 17 during that time period. The number of kids attending schools with such low graduation rates was cut in half.”

a. Is the “decrease by 17” number a useful comparison?
b. Considering the last sentence, can we conclude that the number of “dropout factories” was originally 34?

Solution

a. This number is hard to evaluate, since we have no basis for judging whether this is a larger or small change. If the number of “dropout factories” dropped from 20 to 3, that’d be a very significant change, but if the number dropped from 217 to 200, that’d be less of an improvement.
b. The last sentence provides relative change which helps put the first sentence in perspective. We can estimate that the number of “dropout factories” was probably previously around 34. However, it’s possible that students simply moved schools rather than the school improving, so that estimate might not be fully accurate.

Example 3.1.9

In the 2004 vice-presidential debates, Edwards's claimed that US forces have suffered "90% of the coalition casualties" in Iraq. Cheney disputed this, saying that in fact Iraqi security forces and coalition allies "have taken almost 50 percent" of the casualties. Who is correct?

Solution

Without more information, it is hard for us to judge who is correct, but we can easily conclude that these two percents are talking about different things, so one does not necessarily contradict the other. Edward’s claim was a percent with coalition forces as the base of the percent, while Cheney’s claim was a percent with both coalition and Iraqi security forces as the base of the percent. It turns out both statistics are in fact fairly accurate.

Try It 3.1.3

In the 2012 presidential elections, one candidate argued that "the president's plan will cut $716 billion from Medicare, leading to fewer services for seniors," while the other candidate rebuts that "our plan does not cut current spending and actually expands benefits for seniors, while implementing cost saving measures." Are these claims in conflict, in agreement, or not comparable because they're talking about different things?

Answer

Without more information, it is hard to judge these arguments. This is compounded by the complexity of Medicare. As it turns out, the $716 billion is not a cut in current spending, but a cut in future increases in spending, largely reducing future growth in health care payments. In this case, at least the numerical claims in both statements could be considered at least partially true. Here is one source of more information if you're interested: http://factcheck.org/2012/08/a-campaign-full-of-mediscare/

We'll wrap up our review of percents with a couple cautions. First, when talking about a change of quantities that are already measured in percents, we have to be careful in how we describe the change.

Example 3.1.10

A politician's support increases from 40% of voters to 50% of voters. Describe the change.

Solution

We could describe this using an absolute change: $|50\% - 40\%| = 10\%$. Notice that since the original quantities were percents, this change also has the units of percent. In this case, it is best to describe this as an increase of 10 **percentage points**.

In contrast, we could compute the percent change: $\frac{10\%}{40\%} = 0.25 = 25\%$ increase. This is the relative change, and we'd say the politician's support has increased by 25%.

Lastly, a caution against averaging percents.

Example 3.1.11

A basketball player scores on 40% of 2-point field goal attempts, and on 30% of 3-point of field goal attempts. Find the player's overall field goal percentage.

Solution

It is very tempting to average these values, and claim the overall average is 35%, but this is likely not correct, since most players make many more 2-point attempts than 3-point attempts. We don't actually have enough information to answer the question. Suppose the player attempted 200 2-point field goals and 100 3-point field goals. Then they made 200(0.40) = 80 2-point shots and $100(0.30) = 30$ 3-point shots. Overall, they made 110 shots out of 300, for a $\frac{110}{300} = 0.367 = 36.7\%$ overall field goal percentage.

[1] www.factcheck.org/cheney_edwards_mangle_facts.html

[2] http://tvtropes.org/pmwiki/pmwiki.php/Main/LiesDamnedLiesAndStatistics

[3] www.whitehouse.gov/sites/defa...s/hist07z1.xls

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