10.2.1.1: Introduction to Fractions and Mixed Numbers

Learning Objectives

- Identify the numerator and denominator of a fraction.
- Represent a fraction as part of a whole or part of a set.

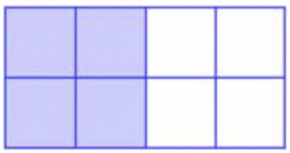
Introduction

Many problems in mathematics deal with whole numbers, which are used to count whole units of things. For example, you can count students in a classroom and the number of dollar bills. You need other kinds of numbers to describe units that are not whole. For example, an aquarium might be partly full. A group may have a meeting, but only some of the members are present.

Fractions are numbers used to refer to a part of a whole. This includes measurements that cannot be written as whole numbers. For example, the width of a piece of notebook paper is more than 8 inches but less than 9 inches. The part longer than 8 inches is written as a fraction. Here, you will investigate how fractions can be written and used to represent quantities that are parts of the whole.

Identifying Numerators and Denominators

A whole can be divided into parts of equal size. In the example below, a rectangle has been divided into eight equal squares. Four of these eight squares are shaded.



The shaded area can be represented by a fraction. A fraction is written vertically as two numbers with a line between them.

The **denominator** (the bottom number) represents the number of equal parts that make up the whole. The **numerator** (the top number) describes the number of parts that you are describing. So returning to the example above, the rectangle has been divided into 8 equal parts, and 4 of them have been shaded. You can use the fraction $\frac{4}{8}$ to describe the shaded part of the whole.

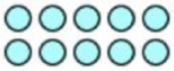
In $\frac{4}{8}$, the 4 is the numerator and tells how many parts are shaded. The 8 is the denominator and tells how many parts are required to make the whole.

Parts of a Set

The rectangle model above provides a good, basic introduction to fractions. However, what do you do with situations that cannot be as easily modeled by shading part of a figure? For example, think about the following situation:

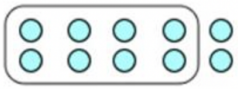
Marc works as a Quality Assurance Manager at an automotive plant. Every hour, he inspects 10 cars; $\frac{4}{5}$ of those pass inspection.

In this case, 10 cars make up the whole group. Each car can be represented as a circle, as shown below.

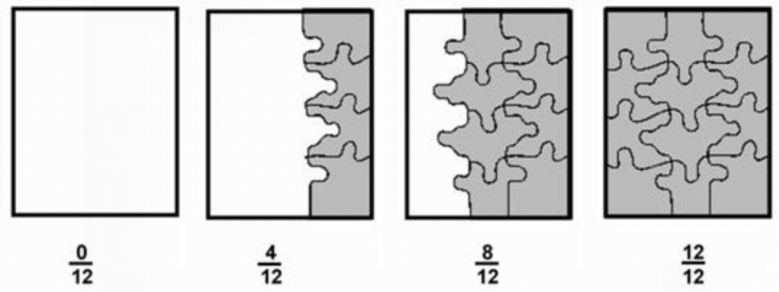


To show $\frac{4}{5}$ of the whole group, you first need to divide the whole group into 5 equal parts. (You know this because the fraction has a denominator of 5.)

To show $\frac{4}{5}$, circle 4 of the equal parts.



Here is another example. Imagine that Aneesh is putting together a puzzle made of 12 pieces. At the beginning, none of the pieces has been put into the puzzle. This means that $\frac{0}{12}$ of the puzzle is complete. Aneesh then puts four pieces together. The puzzle is $\frac{4}{12}$ complete. Soon, he adds four more pieces; 8 out of 12 pieces are now connected. This fraction can be written as $\frac{8}{12}$. Finally, Aneesh adds four more pieces. The puzzle is whole, using all 12 pieces. The fraction can be written as $\frac{12}{12}$.



Note that the number in the denominator cannot be zero. The denominator tells how many parts make up the whole. So if this number is 0, then there are no parts and therefore there can be no whole.

The numerator can be zero, as it tells how many parts you are describing. Notice that in the puzzle example above, you can use the fraction $\frac{0}{12}$ to represent the state of the puzzle when 0 pieces have been placed.

Fractions can also be used to analyze data. In the data table below, 3 out of 5 tosses of a coin came up heads, and 2 out of 5 tosses came up tails. Out of the total number of coin tosses, the portion that was heads can be written as $\frac{3}{5}$. The portion that was tails can be written as $\frac{2}{5}$.

Coin Toss	Result
1	Heads
2	Tails
3	Heads
4	Heads
5	Tails

? Exercise

Sophia, Daphne, and Charlie are all participating in a relay race to raise money for charity. First, Sophia will run 2 miles. Then, Daphne will run 5 miles. Finally, Charlie will end the race by running 3 miles. What fraction of the race will Daphne run?

A. 5 miles
B. $\frac{5}{10}$
C. $\frac{2}{5}$
D. $\frac{5}{3}$

Answer

A. Incorrect. Daphne will run 5 miles, but that does not indicate the fractional part of the race that she will run. To find the fraction, first find the whole length of the race by combining the distances the three people will run: 2+5+3=10. Then consider the distance Daphne will run. The correct answer is $\frac{5}{10}$.
B. Correct. The entire race is 10 miles long, and Daphne will run 5 miles. This means she will run $\frac{5}{10}$ of the race.
C. Incorrect. To find the fraction, first find the whole length of the race by combining the distances the three people will run: 2+5+3=10. Then consider the distance Daphne will run. The correct answer is $\frac{5}{10}$.
D. Incorrect. To find the fraction, first find the whole length of the race by combining the distances the three people will run: 2+5+3=10. Then consider the distance Daphne will run. The correct answer is $\frac{5}{10}$.

Parts of a Whole

The “parts of a whole” concept can be modeled with pizzas and pizza slices. For example, imagine a pizza is cut into 4 pieces, and someone takes 1 piece. Now, $\frac{1}{4}$ of the pizza is gone and $\frac{3}{4}$ remains. Note that both of these fractions have a denominator of 4, which refers to the number of slices the whole pizza has been cut into.

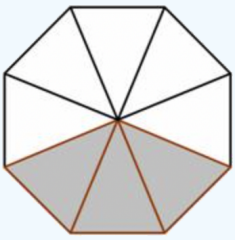
Example

Joaquim bakes a blueberry pie for a potluck dinner. The total pie is cut into 6 equal slices. After everybody eats dessert, only one slice of the pie remains. What fraction of the pie remains?

Solution

$\frac{?}{6}$	The pie was cut into six equal slices, so the denominator of the fraction will be 6.
$\frac{1}{6}$	Only 1 slice remains, so the numerator of the fraction will be 1.
	$\frac{1}{6}$ of the pie remains.

Example



Write a fraction to represent the portion of the octagon that is *not* shaded.

Solution

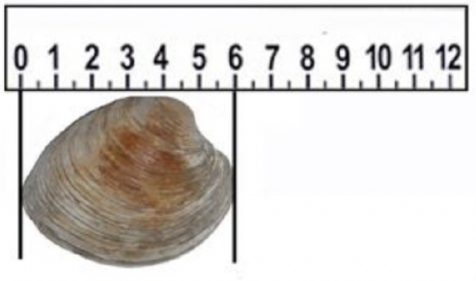
$\frac{?}{8}$	The octagon has eight equal sections, so the denominator of the fraction will be 8.
$\frac{5}{8}$	Five sections are not shaded, so the numerator of the fraction will be 5.
	$\frac{5}{8}$ of the octagon is not shaded.

Measurement Contexts

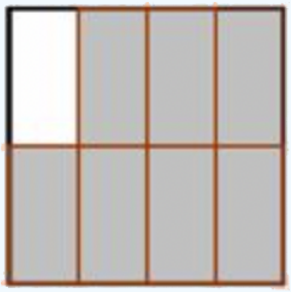
You can use a fraction to represent the quantity in a container. This measuring cup is $\frac{3}{4}$ filled with a liquid. Note that if the cup were $\frac{4}{4}$ full, it would be a whole cup.

You can also use fractions in measuring the length, width, or height of something that is not a full unit. Using a 12-inch ruler, you measure a shell that is 6 inches long. You know that 12 inches equals one foot. So, the length of this shell is $\frac{6}{12}$ of a foot; the 12-

inch ruler is the “whole,” and the length of the shell is the “part.”



? Exercise



Which fraction represents the portion of the shape that is shaded?

A. $\frac{8}{7}$
B. $\frac{7}{8}$

Answer

A. Incorrect. The total number of parts that make up the whole, 8, is the denominator. That is the number *below* the fraction bar. The number of shaded parts, 7, is the numerator. That is the number *above* the fraction bar. The correct answer is $\frac{7}{8}$.
B. Correct. The total number of parts that make up the whole, 8, is the denominator (below the fraction bar). The number of shaded parts, 7, is the numerator (above the fraction bar).

Summary

Fractions are used to represent parts of a whole. You can use fractions when describing substances, quantities, or diagrams that are not complete. You also use fractions to describe numbers of people or objects that do not make up a complete group. Fractions are written with a numerator and denominator. The numerator (above the fraction bar) tells the number of parts being described, and the denominator (below the fraction bar) tells the number of parts that make up the whole.

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