2.2: Properties of Angles

Learning Objectives

- Identify parallel and perpendicular lines.
- Find measures of angles.
- Identify complementary and supplementary angles.

Introduction

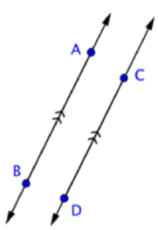
Imagine two separate and distinct lines on a plane. There are two possibilities for these lines: they will either intersect at one point, or they will never intersect. When two lines intersect, four angles are formed. Understanding how these angles relate to each other can help you figure out how to measure them, even if you only have information about the size of one angle.

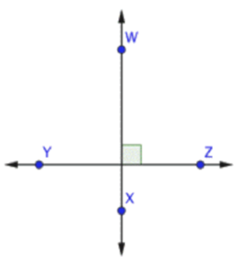
Parallel and Perpendicular

Parallel lines are two or more lines that never intersect. Likewise, parallel line segments are two line segments that never intersect even if the line segments were turned into lines that continued forever. Examples of parallel line segments are all around you, in the two sides of this page and in the shelves of a bookcase. When you see lines or structures that seem to run in the same direction, never cross one another, and are always the same distance apart, there's a good chance that they are parallel.

Perpendicular lines are two lines that intersect at a 90° (right) angle. And perpendicular line segments also intersect at a 90° (right) angle. You can see examples of perpendicular lines everywhere as well: on graph paper, in the crossing pattern of roads at an intersection, to the colored lines of a plaid shirt. In our daily lives, you may be happy to call two lines perpendicular if they merely seem to be at right angles to one another. When studying geometry, however, you need to make sure that two lines intersect at a 90° angle before declaring them to be perpendicular.

The image below shows some parallel and perpendicular lines. The geometric symbol for parallel is $\|$, so you can show that $\overleftrightarrow{AB} \| \overleftrightarrow{CD}$. Parallel lines are also often indicated by the marking >> on each line (or just a single > on each line). Perpendicular lines are indicated by the symbol $\perp$, so you can write $\overleftrightarrow{WX} \perp \overleftrightarrow{YZ}$.

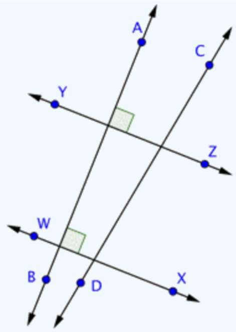




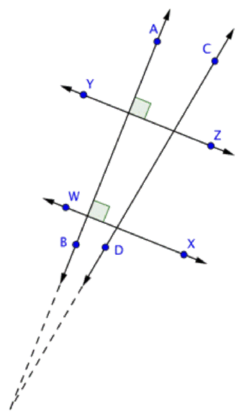
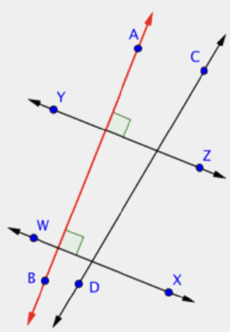
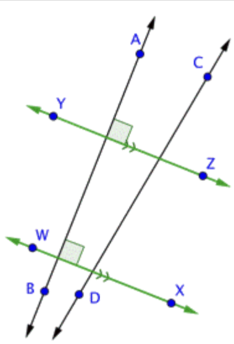
If two lines are parallel, then any line that is perpendicular to one line will also be perpendicular to the other line. Similarly, if two lines are both perpendicular to the same line, then those two lines are parallel to each other. Let's take a look at one example and identify some of these types of lines.

Example 2.2.1

Identify a set of parallel lines and a set of perpendicular lines in the image below.



Solution

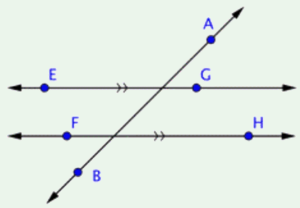
	Parallel lines never meet, and perpendicular lines intersect at a right angle. $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ do not intersect in this image, but if you imagine extending both lines, they will intersect soon. So, they are neither parallel nor perpendicular.
	$\overleftrightarrow{AB}$ is perpendicular to both $\overleftrightarrow{WX}$ and $\overleftrightarrow{YZ}$, as indicated by the right-angle marks at the intersection of those lines.
	Since $\overleftrightarrow{AB}$ is perpendicular to both lines, then $\overleftrightarrow{WX}$ and $\overleftrightarrow{YZ}$ are parallel.

Parallel lines: $\overleftrightarrow{WX} \| \overleftrightarrow{YZ}$

Perpendicular lines: $\overleftrightarrow{AB} \perp \overleftrightarrow{WX}, \overleftrightarrow{AB} \perp \overleftrightarrow{YZ}$

Try It 2.2.1

Which statement most accurately represents the image below?



A. $\overleftrightarrow{EF} \| \overleftrightarrow{GH}$
B. $\overleftrightarrow{AB} \perp \overleftrightarrow{EG}$
C. $\overleftrightarrow{FH} \| \overleftrightarrow{EG}$
D. $\overleftrightarrow{AB} \| \overleftrightarrow{FH}$

Answer

A. Incorrect. This image shows the lines $\overleftrightarrow{EG}$ and $\overleftrightarrow{FH}$, not $\overleftrightarrow{EF}$ and $\overleftrightarrow{GH}$.

B. Incorrect. $\overleftrightarrow{AB}$ does intersect $\overleftrightarrow{EG}$, but the intersection does not form a right angle. This means that they cannot be perpendicular.

C. Correct. Both $\overleftrightarrow{EG}$ and $\overleftrightarrow{FH}$ are marked with >> on each line, and those markings mean they are parallel.

D. Incorrect. $\overleftrightarrow{AB}$ and $\overleftrightarrow{FH}$ intersect, so they cannot be parallel.

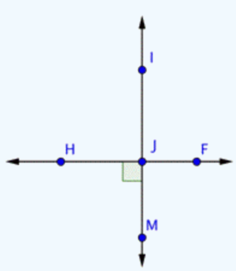
Finding Angle Measurements

Understanding how parallel and perpendicular lines relate can help you figure out the measurements of some unknown angles. To start, all you need to remember is that perpendicular lines intersect at a 90° angle, and that a straight angle measures 180°.

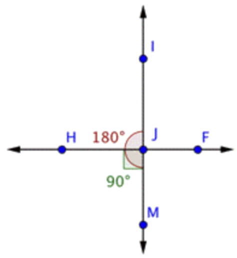
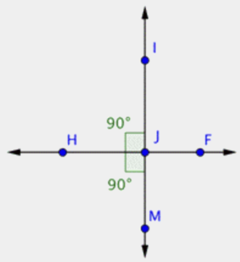
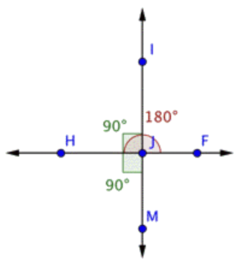
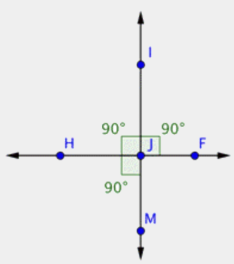
The measure of an angle such as $\angle A$ is written as $m\angle A$. Look at the example below. How can you find the measurements of the unmarked angles?

Example 2.2.2

Find the measurement of $\angle IJF$.



Solution

	Only one angle, $\angle HJM$, is marked in the image. Notice that it is a right angle, so it measures 90°. $\angle HJM$ is formed by the intersection of lines $\overleftrightarrow{IM}$ and $\overleftrightarrow{HF}$. Since $\overleftrightarrow{IM}$ is a line, $\angle IJM$ is a straight angle measuring 180°.
	You can use this information to find the measurement of $\angle HJI$: $m\angle HJM + m\angle HJI = m\angle IJM$ $90^\circ + m\angle HJI = 180^\circ$ $m\angle HJI = 90^\circ$
	Now use the same logic to find the measurement of $\angle IJF$. $\angle IJF$ is formed by the intersection of lines $\overleftrightarrow{IM}$ and $\overleftrightarrow{HF}$. Since $\overleftrightarrow{HF}$ is a line, $\angle HJF$ will be a straight angle measuring 180º.
	You know that $\angle HJI$ measures 90°. Use this information to find the measurement of $\angle IJF$: $m\angle HJI + m\angle IJF = m\angle HJF$ $90^\circ + m\angle IJF = 180^\circ$ $m\angle IJF = 90^\circ$

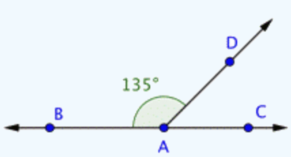
Thus, $m\angle IJF = 90^\circ$.

In this example, you may have noticed that angles $\angle HJI, \angle IJF,$ and $\angle HJM$ are all right angles. (If you were asked to find the measurement of $\angle FJM$, you would find that angle to be 90°, too.) This is what happens when two lines are perpendicular: the four angles created by the intersection are all right angles.

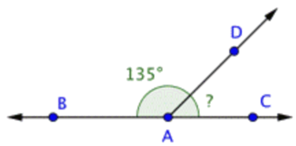
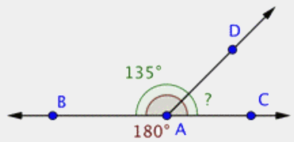
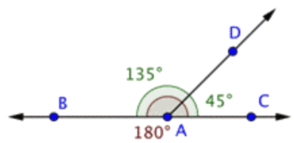
Not all intersections happen at right angles, though. In the example below, notice how you can use the same technique as shown above (using straight angles) to find the measurement of a missing angle.

Example 2.2.3

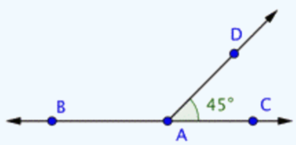
Find the measurement of $\angle DAC$.



Solution

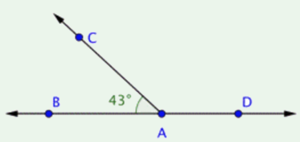
	This image shows the line $\overleftrightarrow{BC}$ and the ray $\overrightarrow{AD}$ intersecting at point A. The measurement of $\angle BAD$ is 135°. You can use straight angles to find the measurement of $\angle DAC$.
	$\angle BAC$ is a straight angle, so it measures 180°.
	Use this information to find the measurement of $\angle DAC$. $m\angle BAD + m\angle DAC = m\angle BAC$ $135^\circ + m\angle DAC = 180^\circ$ $m\angle DAC = 45^\circ$

Thus, $m\angle DAC = 45^\circ$.



Try It 2.2.2

Find the measurement of $\angle CAD$.



Answer

$m\angle CAD$ = 137° since 180° - 43° = 137°.

Supplementary and Complementary

In the example above, $m\angle BAC$ and $m\angle DAC$ add up to 180°. Two angles whose measures add up to 180° are called **supplementary angles**. There's also a term for two angles whose measurements add up to 90°; they are called **complementary angles**.

One way to remember the difference between the two terms is that "corner" and "complementary" each begin with c (a 90° angle looks like a corner), while straight and "supplementary" each begin with s (a straight angle measures 180°).

If you can identify supplementary or complementary angles within a problem, finding missing angle measurements is often simply a matter of adding or subtracting.

Example 2.2.4

Two angles are supplementary. If one of the angles measures 48°, what is the measurement of the other angle?

Solution

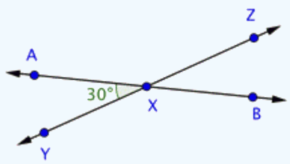
$m\angle A + m\angle B = 180^\circ$	Two supplementary angles make up a straight angle, so the sum of the measurements of the two angles will be 180°.

$48^\circ + m\angle B = 180^\circ$ $m\angle B = 180^\circ - 48^\circ$ $m\angle B = 132^\circ$	You know the measurement of one angle. To find the measurement of the second angle, subtract 48° from 180°.

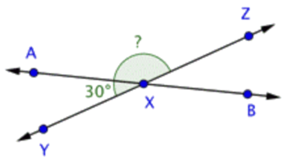
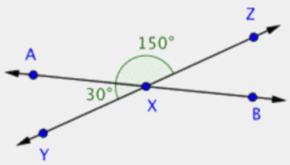
The measurement of the other angle is 132°.

Example 2.2.5

Find the measurement of $\angle AXZ$.



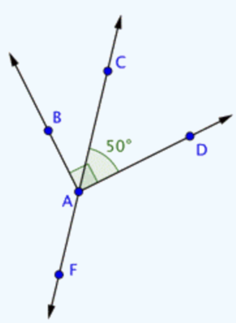
Solution

	This image shows two intersecting lines, $\overleftrightarrow{AB}$ and $\overleftrightarrow{YZ}$. They intersect at point X, forming four angles. Angles $\angle AXY$ and $\angle AXZ$ are supplementary because together they make up the straight angle $\angle YXZ$.
	Use this information to find the measurement of $\angle AXZ$. $m\angle AXY + m\angle AXZ = m\angle YXZ$ $30^\circ + m\angle AXZ = 180^\circ$ $m\angle AXZ = 150^\circ$

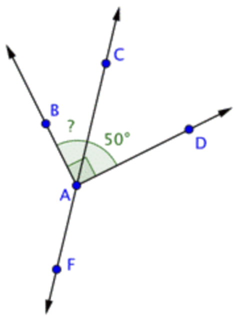
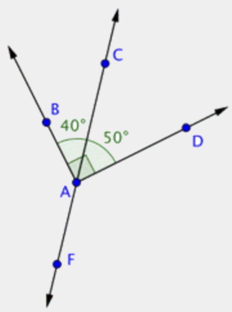
Thus, $m\angle AXZ = 150^\circ$.

Example 2.2.6

Find the measurement of $\angle BAC$.



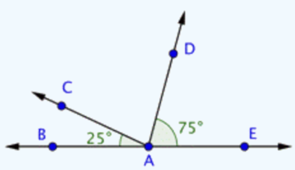
Solution

	This image shows the line $\overleftrightarrow{CF}$ and the rays $\overrightarrow{AB}$ and $\overrightarrow{AD}$, all intersecting at point A. Angle $\angle BAD$ is a right angle. Angles $\angle BAC$ and $\angle CAD$ are complementary, because together they create $\angle BAD$.
	Use this information to find the measurement of $\angle BAC$. $m\angle BAC + m\angle CAD = m\angle BAD$ $m\angle BAC + 50^\circ = 90^\circ$ $m\angle BAC = 40^\circ$

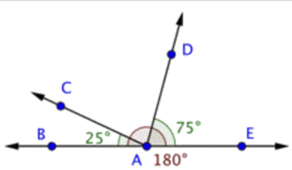
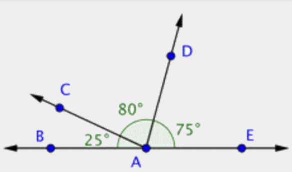
Thus, $m\angle BAC = 40^\circ$.

Example 2.2.7

Find the measurement of $\angle CAD$.



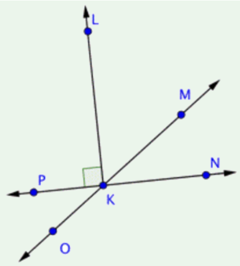
Solution

	You know the measurements of two angles here: $\angle CAB$ and $\angle DAE$. You also know that $m\angle BAE = 180^\circ$.
	Use this information to find the measurement of $\angle CAD$. $m\angle BAC + m\angle CAD + m\angle DAE = m\angle BAE$ $25^\circ + m\angle CAD + 75^\circ = 180^\circ$ $m\angle CAD + 100^\circ = 180^\circ$ $m\angle CAD = 80^\circ$

Thus, $m\angle CAD = 80^\circ$.

Try It 2.2.3

Which pair of angles is complementary?



Answer

$\angle LKM$ and $\angle MKN$. The measurements of two complementary angles will add up to 90°. $\angle LKP$ is a right angle, so $\angle LKN$ must be a right angle as well. $\angle LKM + \angle MKN = \angle LKN$, so $\angle LKM$ and $\angle MKN$ are complementary.

Summary

Parallel lines do not intersect, while perpendicular lines cross at a 90°. angle. Two angles whose measurements add up to 180° are said to be supplementary, and two angles whose measurements add up to 90° are said to be complementary. For most pairs of intersecting lines, all you need is the measurement of one angle to find the measurements of all other angles formed by the intersection.