

10.2.1.4: Simplifying Fractions

Learning Objectives

- Find an equivalent fraction with a given denominator.
- Simplify a fraction to lowest terms.

Introduction

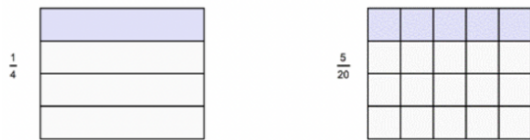
Fractions are used to represent a part of a whole. Fractions that represent the same part of a whole are called **equivalent fractions**. Factoring, multiplication, and division are all helpful tools for working with equivalent fractions.

Equivalent Fractions

We use equivalent fractions every day. Fifty cents can be 2 quarters, and we have $\frac{2}{4}$ of a dollar, because there are 4 quarters in a dollar. Fifty cents is also 50 pennies out of 100 pennies, or $\frac{50}{100}$ of a dollar. Both of these fractions are the same amount of money, but written with a different numerator and denominator.

Think about a box of crackers that contains 3 packets of crackers. Two of these packets are $\frac{2}{3}$ of the box. Suppose each packet has 30 crackers in it. Two packets are also $60(30 \cdot 2)$ crackers out of $90(30 \cdot 3)$ crackers. This is $\frac{60}{90}$ of the box. The fractions $\frac{2}{3}$ and $\frac{60}{90}$ both represent two packets of crackers, so they are equivalent fractions.

Equivalent fractions represent the same part of a whole, even if the numerator and denominator are different. For example, $\frac{1}{4} = \frac{5}{20}$. In these diagrams, both fractions represent one of four rows in the rectangle.



Since $\frac{1}{4}$ and $\frac{5}{20}$ are naming the same part of a whole, they are equivalent.

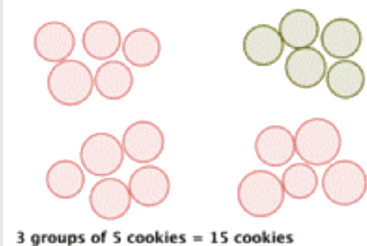
There are many ways to name the same part of a whole using equivalent fractions.

Let's look at an example where you need to find an equivalent fraction.

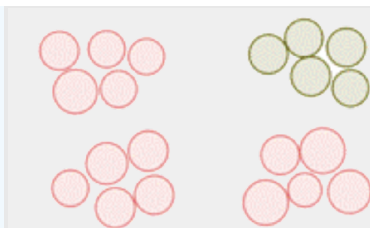
Example

John is making cookies for a bake sale. He made 20 large cookies, but he wants to give away only $\frac{3}{4}$ of them for the bake sale. What fraction of the cookies does he give away, using 20 as the denominator?

Solution

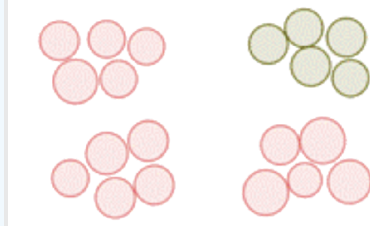


Start with 20 cookies.



3 groups of 5 cookies = 15 cookies

Because the denominator of $\frac{3}{4}$ is 4, make 4 groups of cookies, 5 in each group.



3 groups of 5 cookies = 15 cookies

$$\frac{3}{4} = \frac{3 \cdot 5}{4 \cdot 5} \text{ because there are 5 cookies in each group.}$$

$$\frac{3}{4} = \frac{3 \cdot 5}{4 \cdot 5} = \frac{15}{20}$$

He gives away $\frac{15}{20}$ of the cookies.

When you regroup and reconsider the parts and whole, you are multiplying the numerator and denominator by the same number. In the above example, you multiply 4 by 5 to get the needed denominator of 20, so you also need to multiply the numerator 3 by 5, giving the new numerator of 15.

Finding Equivalent Fractions

To find equivalent fractions, multiply or divide *both* the numerator and the denominator by the same number.

Examples:

$$\frac{20}{25} = \frac{20 \div 5}{25 \div 5} = \frac{4}{5}$$

$$\frac{2}{7} = \frac{2 \cdot 6}{7 \cdot 6} = \frac{12}{42}$$

Exercise

Write an equivalent fraction to $\frac{2}{3}$ that has a denominator of 27.

- A. $\frac{26}{27}$
- B. $\frac{11}{27}$
- C. $\frac{18}{27}$
- D. $\frac{12}{18}$

Answer

- A. Incorrect. You may have added the difference between the two denominators to the numerator ($27-3=24$, $24+2=26$). Instead, you need to use a multiplying factor of $9(27 \div 3)$. The correct answer is $\frac{18}{27}$.
- B. Incorrect. You may have added the multiplying factor of 9 to the numerator. Instead, multiply the numerator by this factor. The correct answer is $\frac{18}{27}$.
- C. Correct. The multiplying factor is 9, so the denominator is $3 \cdot 9 = 27$ and the numerator is $2 \cdot 9 = 18$.
- D. Incorrect. Although this is an equivalent fraction to $\frac{2}{3}$, the denominator is not 27. The correct answer is $\frac{18}{27}$.

Simplifying Fractions

A fraction is in its **simplest form**, or **lowest terms**, when it has the least numerator and the least denominator possible for naming this part of a whole. The numerator and denominator have no common factor other than 1.

Here are 10 blocks, 4 of which are green. So, the fraction that is green is $\frac{4}{10}$. To simplify, you find a common factor and then regroup the blocks by that factor.

✓ Example

Simplify $\frac{4}{10}$.

Solution



$$\frac{\text{green blocks}}{\text{blocks}} = \frac{2(2)}{5(2)} = \frac{2}{5}$$



We start with 4 green blocks out of 10 total blocks.



$$\frac{\text{green blocks}}{\text{blocks}} = \frac{2(2)}{5(2)} = \frac{2}{5}$$



Group the blocks in twos, since 2 is a common factor. You have 2 groups of green blocks and a total of 5 groups, each group containing 2 blocks.



$$\frac{\text{green blocks}}{\text{blocks}} = \frac{2(2)}{5(2)} = \frac{2}{5}$$



Now, consider the groups as the part and you have 2 green groups out of 5 total groups.

$$\frac{4}{10} = \frac{2}{5} \quad \text{The simplified fraction is } \frac{2}{5}.$$

Once you have determined a common factor, you can divide the blocks into the groups by dividing both the numerator and denominator to determine the number of groups that you have.

For example, to simplify $\frac{6}{9}$ you find a common factor of 3, which will divide evenly into both 6 and 9. So, you divide 6 and 9 into groups of 3 to determine how many groups of 3 they contain. This gives $\frac{6 \div 3}{9 \div 3} = \frac{2}{3}$, which means 2 out of 3 groups, and $\frac{2}{3}$ is equivalent to $\frac{6}{9}$.

It may be necessary to group more than one time. Each time, determine a common factor for the numerator and denominator using the tests of divisibility, when possible. If both numbers are even numbers, start with 2. For example:

✓ Example

Simplify $\frac{32}{48}$.

Solution

$$\frac{32}{48} = \frac{32 \div 2}{48 \div 2} = \frac{16}{24}$$

32 and 48 have a common factor of 2. Divide each by 2.

$$\frac{16}{24} = \frac{16 \div 2}{24 \div 2} = \frac{8}{12}$$

16 and 24 have a common factor of 2. Divide each by 2.

$$\frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}$$

8 and 12 have a common factor of 4. Divide each by 4.

$$\frac{32}{48} = \frac{2}{3}$$

$\frac{2}{3}$ is the simplified fraction equivalent to $\frac{32}{48}$.

In the example above, 16 is a factor of both 32 and 48, so you could have shortened the solution.

$$\frac{32}{48} = \frac{2 \cdot 16}{3 \cdot 16} = \frac{2}{3}$$

You can also use **prime factorization** to help regroup the numerator and denominator.

✓ Example

Simplify $\frac{54}{72}$.

Solution

$$\frac{54}{72} = \frac{2 \cdot 3 \cdot 3 \cdot 3}{2 \cdot 2 \cdot 2 \cdot 3 \cdot 3}$$

The prime factorization of 54 is $2 \cdot 3 \cdot 3 \cdot 3$.

The prime factorization of 72 is $2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$.

$$\frac{3 \cdot (2 \cdot 3 \cdot 3)}{2 \cdot 2 \cdot (2 \cdot 3 \cdot 3)}$$

Rewrite, finding common factors.

$$\frac{3}{2 \cdot 2} \cdot 1$$

$$\frac{2 \cdot 3 \cdot 3}{2 \cdot 3 \cdot 3} = 1$$

$$\frac{3}{4}$$

Multiply: $2 \cdot 2$.

$$\frac{54}{72} = \frac{3}{4}$$

$\frac{3}{4}$ is the simplified fraction equivalent to $\frac{54}{72}$.

Notice that when you *simplify* a fraction, you *divide* the numerator and denominator by the same number, in the same way you *multiply* by the same number to find an *equivalent* fraction with a greater denominator. In the example above, you could have divided the numerator and denominator by 9, a common factor of 54 and 72.

$$\frac{54 \div 9}{72 \div 9} = \frac{6}{8}$$

Since the numerator (6) and the denominator (8) still have a common factor, the fraction is not yet in lowest terms. So, again divide by the common factor 2.

$$\frac{6 \div 2}{8 \div 2} = \frac{3}{4}$$

Repeat this process of dividing by a common factor until the only common factor is 1.

📌 Simplifying Fractions to Lowest Terms

To simplify a fraction to lowest terms, divide both the numerator and the denominator by their common factors. Repeat as needed until the only common factor is 1.

? Exercise

Simplify $\frac{36}{72}$.

- A. $\frac{3}{6}$
- B. $\frac{9}{18}$
- C. $\frac{18}{38}$
- D. $\frac{1}{2}$

Answer

- A. Incorrect. Although $\frac{3}{6}$ is an equivalent fraction to $\frac{36}{72}$, it is not in lowest terms. There is a common factor of 3 in the numerator and denominator. The correct answer is $\frac{1}{2}$.
- B. Incorrect. Although $\frac{9}{18}$ is an equivalent fraction to $\frac{36}{72}$, it is not in lowest terms. There are other common factors (9 and 3). The correct answer is $\frac{1}{2}$.
- C. Incorrect. You may have divided 72 by 2 and got 38 rather than 36. $\frac{18}{36}$ is an equivalent fraction to $\frac{36}{72}$, but it is not in lowest terms. The correct answer is $\frac{1}{2}$.
- D. Correct. $\frac{1}{2}$ is in lowest terms, as 1 is the only common factor of 1 and 2.

Summary

Equivalent fractions do not always have the same numerator and denominator, but they have the same value. A fraction is in lowest terms when the numerator and denominator of the fraction share no common factors other than 1. A fraction written in lowest terms is called a simplified fraction.

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