5.1: Logic Statements

Learning Objectives

- Identify a logical statement
- Construct the negation of a statement, including the use of quantifiers
- Construct a compound statement using conjunctions and disjunctions

Logic is the study of the methods and principles of reasoning. In logic, a **statement** is a declarative sentence that is either true or false, but not both. The key to constructing a good logical statement is that there must be no ambiguity. To be a statement, a sentence must be true or false. It cannot be both. In logic, the truth of a statement is established beyond ANY doubt by a well-reasoned argument.

So, a sentence such as "The house is beautiful" is not a statement, since whether the sentence is true or not is a matter of opinion.

A question such as "Is it snowing?" is not a statement, because it is a question and is not declaring that something is true.

Some sentences that are mathematical in nature often are not logical statements because we may not know precisely what a variable represents. For example, the equation $3x+5=10$ is not a logical statement, since we do not know what x represents. If we substitute a specific value for x (such as $x=4$), then the resulting equation, $3x+5=10$ is a logical statement (which is a false statement).

Statement

A **statement** is a sentence that is either true or false.

In logic, lower case letters are often used to represent statements such as p, q or r.

Example 5.1.1

The following are statements:

a. Zero times any real number is zero.
b. $1+1=2$.
c. All birds can fly. (This is a false statement. How can you establish that?)

The following are not statements:

a. Come here.
b. Who are you?
c. I am lying right now. (This is a paradox. If I'm lying I'm telling the truth and if I'm telling the truth I'm lying.)

Try It 5.1.1

Which of the following are statements?

a. I like sports cars.
b. $2+3=6$.
c. Where are you?

Answer

Only b is correct since you cannot determine if a is true, and c is a question that is neither true nor false.

Negation

The **negation** of a statement is a statement that has the opposite truth value of the original statement.

Notation: $\sim p$ (read: "the negation of p" or "not p")

The negation of a true statement must be a false statement and vice-versa. A simple statement can often be negated by adding or removing the word "not." A statement can also be negated by adding "It is not true that ... (statement)" or "It is not the case that ... (statement)."

Example 5.1.2

Find the negation of the following statements:

a. p: The sky is blue.
b. q: Homework is not due today.

Solution

a. $\sim p$: The sky is not blue. Note that p is true and $\sim p$ is false.
b. $\sim q$: Homework is due today. Note that q and $\sim q$ have opposite truth values. If q is true, then $\sim q$ is false, or if q is false, then $\sim q$ is true.

Logical statements are related to sets and set operations. Words that describe an entire set, such as “all”, “every”, or “none”, are called **universal quantifiers** because that set could be considered a universal set. In contrast, words or phrases such as “some”, “one”, or “at least one” are called **existential quantifiers** because they describe the existence of at least one element in a set.

Quantifiers

A **universal quantifier** states that an entire set of things share a characteristic.

An **existential quantifier** states that a set contains at least one element.

Something interesting happens when we negate a quantified statement. When we negate a statement with a universal quantifier, we get a statement with an existential quantifier, and vice-versa.

Negating a quantified statement

Statement	Negation
All A are B.	At least one A is not B. Some A are not B.
No A are B.	At least one A is B. Some A are B.
At least one A is B. Some A are B.	No A are B.
At least one A is not B. Some A are not B.	All A are B.

In logic, when you have a statement and a negation, one must be negative, meaning it contains "no" or "not", and the other must be positive. For example, for the statement "All students love math," the negation cannot be "Some students love math" since neither statement is negative, even though they have opposite truth values. The correct negation is "Some students do not love math."

Example 5.1.3

Write the negation of “Somebody brought a flashlight.”

Solution

Since the statement is of the form "Some A are B," the negation will be of the form "No A are B." The negation is “Nobody brought a flashlight.”

Example 5.1.4

Write the negation of “There are no prime numbers that are even.”

Solution

Since the statement is of the form "No *A* are *B*," the negation will be of the form "At least one *A* is *B*." The negation is “At least one prime number is even.”

Try It 5.1.2

Write the negation of “All Icelandic children learn English in school.”

Answer

Some Icelandic children do not learn English in school.

We can make a new statement from other statements; we call these **compound statements**. Compound statements are formed by connecting 2 or more simple statements with operators such as *and* and *or*.

Symbols

The symbol $\wedge$ is a conjunction and is used for "and": p and q is notated $p \wedge q$

The symbol $\vee$ is a disjunction and is used for "or" (where "or" is not exclusive): p or q is notated $p \vee q$

You can remember the first two symbols by relating them to the shapes for the union and intersection. $p \wedge q$ would be the elements that exist in both sets, in $p \cap q$. Likewise, $p \vee q$ would be the elements that exist in either set, in $p \cup q$. When we are working with sets, we use the rounded version of the symbols; when we are working with statements, we use the pointy version.

Example 5.1.5

Translate each statement into symbolic notation. Let p represent "I like Pepsi" and let c represent "I like Coke".

a. I like Pepsi or I like Coke.
b. I like Pepsi and I like Coke.
c. I do not like Pepsi.
d. It is not the case that I like Pepsi or Coke.
e. I like Pepsi but I do not like Coke.

Solution

a. $p \vee c$
b. $p \wedge c$
c. $\sim p$
d. $\sim (p \vee c)$
e. $p \wedge \sim c$

As you can see, we can use parentheses to organize more complicated statements.

Try It 5.1.3

Translate “We have carrots or we will not make soup” into symbols. Let c represent “we have carrots” and let s represent “we will make soup”.

Answer

$c \vee \sim s$

This page titled 5.1: Logic Statements is shared under a CC BY-SA 3.0 license and was authored, remixed, and/or curated by David Lippman (The OpenTextBookStore) via source content that was edited to the style and standards of the LibreTexts platform.

- **1.0 : Introduction to the Basic Language of Mathematics** by Pamini Thangarajah is licensed CC BY-NC-SA 4.0.
- **17.4: Quantified Statements** by David Lippman is licensed CC BY-SA 3.0. Original source: http://www.opentextbookstore.com/mathinsociety.
- **17.5: Truth Tables: Conjunction (and), Disjunction (or), Negation (not)** by David Lippman is licensed CC BY-SA 3.0. Original source: http://www.opentextbookstore.com/mathinsociety.