5.5: Arguments with Truth Tables

Learning Objectives

- Define a logical argument
- Determine if an argument is valid using a truth table

Logic is the study of the methods and principles of reasoning. An **argument** is a set of facts or assumptions, called **premises**, used to support a conclusion. For a logical argument to be **valid**, it is the case that, if the premises are true then the conclusion **must** be true.

Argument

An **argument** is a set of statements called **premises** together with a **conclusion**. An argument consisting of two premises and a conclusion is called **a syllogism**.

There are two general types of arguments: inductive and deductive arguments.

Argument types

An **inductive** argument uses a collection of specific examples as its premises and uses them to propose a general conclusion.

A **deductive** argument uses a collection of general statements as its premises and uses them to propose a specific situation as the conclusion.

Example 5.5.1

The argument "When I went to the store last week I forgot my purse, and when I went today I forgot my purse. I always forget my purse when I go the store" is an inductive argument.

The premises are:

I forgot my purse last week

I forgot my purse today

The conclusion is:

I always forget my purse

Notice that the premises are specific situations, while the conclusion is a general statement. In this case, this is a fairly weak argument, since it is based on only two instances.

Example 5.5.2

The argument "Every day for the past year, a plane flies over my house at 2:00 P.M. A plane will fly over my house every day at 2:00 P.M." is a stronger inductive argument, since it is based on a larger set of evidence. While it is not necessarily true—the airline may have cancelled its afternoon flight—it is probably a safe bet.

Evaluating inductive arguments

An inductive argument is never able to prove the conclusion true, but it can provide either weak or strong evidence to suggest that it may be true.

Many scientific theories, such as the big bang theory, can never be proven. Instead, they are inductive arguments supported by a wide variety of evidence. Usually in science, an idea is considered a hypothesis until it has been well tested, at which point it graduates to being considered a theory. Common scientific theories, like Newton's theory of gravity, have all stood up to years of

testing and evidence, though sometimes they need to be adjusted based on new evidence, such as when Einstein proposed the theory of general relativity.

A deductive argument is more clearly valid or not, which makes it easier to evaluate.

Evaluating deductive arguments

A deductive argument is considered valid if, assuming that all the premises are true, the conclusion follows logically from those premises. In other words, when the premises are all true, the conclusion *must* be true.

One way to determine if an argument is valid is by using truth tables.

Analyzing arguments using truth tables

To analyze an argument with a truth table:

1. Represent each of the premises symbolically.
2. Create a conditional statement, joining all the premises with a conjunction to form the antecedent, and using the conclusion as the consequent.
3. Create a truth table for the conditional statement. If it is always true, then the argument is valid.

Example 5.5.3

Consider the argument

Premise:	If you bought bread, then you went to the store.
Premise:	You bought bread.
Conclusion:	You went to the store.

Solution

While this example is fairly obviously a valid argument, we can analyze it using a truth table by representing each of the premises symbolically. We can then form a conditional statement showing that the premises together imply the conclusion. If the truth table is a tautology (always true), then the argument is valid.

We'll let b represent "you bought bread" and s represent "you went to the store". Then the argument becomes:

Premise:	$b \to s$
Premise:	b
Conclusion:	s

To test the validity, we look at whether the combination of both premises implies the conclusion; is it true that $[(b \to s) \wedge b] \to s$?

b	s	$b \to s$
T	T	T
T	F	F
F	T	T
F	F	T

b	s	$b \to s$	$(b \to s) \wedge b$
T	T	T	T
T	F	F	F
F	T	T	F
F	F	T	F

b	s	$b \to s$	$(b \to s) \wedge b$	$[(b \to s) \wedge b] \to s$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Since the truth table for $[(b \to s) \wedge b] \to s$ is always true, this is a valid argument.

Try It 5.5.1

Determine whether the argument is valid:

Premise: If I have a shovel, I can dig a hole.
Premise: I dug a hole.
Conclusion: Therefore, I had a shovel.

Answer

Let $S =$ have a shovel, $D =$ dig a hole. The first premise is equivalent to $S \to D$. The second premise is D. The conclusion is S. We are testing $[(S \to D) \wedge D] \to S$

S	D	$S \to D$	$(S \to D) \wedge D$	$[(S \to D) \wedge D] \to S$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

This is not a tautology, so this is an invalid argument.

Example 5.5.4

Premise: If I go to the mall, then I'll buy new jeans.
Premise: If I buy new jeans, I'll buy a shirt to go with it.
Conclusion: If I go to the mall, I'll buy a shirt.

Solution

Let $m =$ I go to the mall, $j =$ I buy jeans, and $s =$ I buy a shirt.

The premises and conclusion can be stated as:

Premise: $m \to j$
Premise: $j \to s$
Conclusion: $m \to s$

We can construct a truth table for $[(m \to j) \wedge (j \to s)] \to (m \to s)$. Try to recreate each step and see how the truth table was constructed.

m	j	s	$m \to j$	$j \to s$	$(m \to j) \wedge (j \to s)$	$m \to s$	$[(m \to j) \wedge (j \to s)] \to (m \to s)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

From the final column of the truth table, we can see this is a valid argument.
