3.7: Solving for Time

Often we are interested in how long it will take to accumulate money or how long we'd need to extend a loan to bring payments down to a reasonable level.

Note: This section assumes you've covered solving exponential equations using logarithms in prior classes.

Example 3.7.1

If you invest \$2000 at 6% compounded monthly, how long will it take the account to double in value?

Solution

This is a compound interest problem, since we are depositing money once and allowing it to grow. In this problem,

$$\begin{array}{ll} P=\$2000 & \text{the initial deposit} \\ r=0.06 & 6\% \text{ annual rate} \\ n=12 & 12 \text{ months in } 1 \text{ year} \end{array}$$

So our general equation is $A=2000\left(1+\frac{0.06}{12}\right)^{12t}$. We also know that we want our ending amount to be double of $\$2000$, which is $\$4000,$ so we're looking for t so that $A=4000$. To solve this, we set our equation for A equal to 4000.

$$\begin{array}{ll} 4000=2000\left(1+\frac{0.06}{12}\right)^{12t} & \text{Divide both sides by } 2000 \\ 2=(1.005)^{12t} & \text{To solve for the exponent, take the log of both sides} \\ \log(2)=\log\left((1.005)^{12t}\right) & \text{Use the exponent property of logs on the right side} \\ \log(2)=12t\log(1.005) & \text{Now we can divide both sides by } 12\log 1.005 \\ \frac{\log(2)}{12\log(1.005)}=t & \text{Approximating this to a decimal} \\ t=11.581 & \end{array}$$

It will take about 11.581 years for the account to double in value. Note that your answer may come out slightly differently if you had evaluated the logs to decimals and rounded during your calculations, but your answer should be close. For example if you rounded log(2) to 0.301 and log(1.005) to 0.00217, then your final answer would have been about 11.577 years.

This answer is close to the answer you would get from the rule of 70 ($t=\frac{70}{6}\approx 11.67$), which is just a quick approximation of doubling time. The rule of 70 is based on this method of solving for time using logarithms.

Example 3.7.2

If you invest \$100 each month into an account earning 3% compounded monthly, how long will it take the account to grow to \$10,000?

Solution

This is a savings annuity problem since we are making regular deposits into the account.

$$\begin{array}{ll} d=\$1000 & \text{the monthly deposit} \\ r=0.03 & 3\% \text{ annual rate} \\ n=12 & \text{since we're doing monthly deposits, we'll compound monthly} \end{array}$$

We don't know t, but we want A to be $\$10,000$

Putting this into the equation:

$$10,000=\frac{100\left[\left(1+\frac{0.03}{12}\right)^{12t}-1\right]}{\left(\frac{0.03}{12}\right)} \quad \text{Simplifying the fractions a bit}$$

$$10,000=\frac{100\left[(1.0025)^{12t}-1\right]}{0.0025}$$

We want to isolate the exponential term, 1.0025^{12t}, so multiply both sides by 0.0025

$$\begin{array}{ll} 25=100\left[(1.0025)^{12t}-1\right] & \text{Divide both sides by 100} \\ 0.25=(1.0025)^{12t}-1 & \text{Add 1 to both sides} \\ 1.25=(1.0025)^{12t} & \text{Now take the log of both sides} \\ \log(1.25)=\log\left((1.0025)^{12t}\right) & \text{Use the exponent property of logs} \\ \log(1.25)=12t\log(1.0025) & \text{Divide by } 12\log(1.0025) \\ \dfrac{\log(1.25)}{12\log(1.0025)}=t & \text{Approximating to a decimal} \\ t=7.447 \text{ years} & \end{array}$$

It will take about 7.447 years to grow the account to $\$10,000$.

Try It 3.7.1

Joel is considering putting a $1,000 laptop purchase on his credit card, which has an interest rate of 12% compounded monthly. How long will it take him to pay off the purchase if he makes payments of $30 a month?

Answer

$$\begin{array}{ll} d=\$30 & \text{The monthly payments} \\ r=0.12 & 12\% \text{ annual rate} \\ n=12 & \text{since we're doing monthly deposits} \\ A=\$1000 & \text{we're starting with a \$1,000 loan} \end{array}$$

We are solving for t, the time to pay off the loan.

$$1000=\frac{30\left[1-\left(1+\frac{0.12}{12}\right)^{-12t}\right]}{\frac{0.12}{12}}$$

Solving for t gives 3.396. It will take about 3.4 years to pay off the purchase.