

10.6.5: Problem Solving and Estimating

Finally, we will bring together the mathematical tools we've reviewed, and use them to approach more complex problems. In many problems, it is tempting to take the given information, plug it into whatever formulas you have handy, and hope that the result is what you were supposed to find. Chances are, this approach has served you well in other math classes.

This approach does not work well with real life problems. Instead, problem solving is best approached by first starting at the end: identifying exactly what you are looking for. From there, you then work backwards, asking "what information and procedures will I need to find this?" Very few interesting questions can be answered in one mathematical step; often times you will need to chain together a solution pathway, a series of steps that will allow you to answer the question.

Problem Solving Process

1. Identify the question you're trying to answer.
2. Work backwards, identifying the information you will need and the relationships you will use to answer that question.
3. Continue working backwards, creating a solution pathway.
4. If you are missing necessary information, look it up or estimate it. If you have unnecessary information, ignore it.
5. Solve the problem, following your solution pathway.

In most problems we work, we will be approximating a solution, because we will not have perfect information. We will begin with a few examples where we will be able to approximate the solution using basic knowledge from our lives.

✓ Example 24

How many times does your heart beat in a year?

Solution

This question is asking for the rate of heart beats per year. Since a year is a long time to measure heart beats for, if we knew the rate of heart beats per minute, we could scale that quantity up to a year. So the information we need to answer this question is heart beats per minute. This is something you can easily measure by counting your pulse while watching a clock for a minute.

Suppose you count 80 beats in a minute. To convert this beats per year:

$$\frac{80 \text{ beats}}{1 \text{ minute}} \cdot \frac{60 \text{ minutes}}{1 \text{ hour}} \cdot \frac{24 \text{ hours}}{1 \text{ day}} \cdot \frac{365 \text{ days}}{1 \text{ year}} = 42,048,000 \text{ beats per year}$$

✓ Example 25

How thick is a single sheet of paper? How much does it weigh?

Solution

While you might have a sheet of paper handy, trying to measure it would be tricky. Instead we might imagine a stack of paper, and then scale the thickness and weight to a single sheet. If you've ever bought paper for a printer or copier, you probably bought a ream, which contains 500 sheets. We could estimate that a ream of paper is about 2 inches thick and weighs about 5 pounds. Scaling these down,

$$\frac{2 \text{ inches}}{\text{ream}} \cdot \frac{1 \text{ ream}}{500 \text{ pages}} = 0.004 \text{ inches per sheet}$$

$$\frac{5 \text{ pounds}}{\text{ream}} \cdot \frac{1 \text{ ream}}{500 \text{ pages}} = 0.01 \text{ pounds per sheet, or } 0.16 \text{ ounces per sheet.}$$

✓ Example 26

A recipe for zucchini muffins states that it yields 12 muffins, with 250 calories per muffin. You instead decide to make mini-muffins, and the recipe yields 20 muffins. If you eat 4, how many calories will you consume?

Solution

There are several possible solution pathways to answer this question. We will explore one.

To answer the question of how many calories 4 mini-muffins will contain, we would want to know the number of calories in each mini-muffin. To find the calories in each mini-muffin, we could first find the total calories for the entire recipe, then divide it by the number of mini-muffins produced. To find the total calories for the recipe, we could multiply the calories per standard muffin by the number per muffin. Notice that this produces a multi-step solution pathway. It is often easier to solve a problem in small steps, rather than trying to find a way to jump directly from the given information to the solution.

We can now execute our plan:

$$12 \text{ muffins} \$ \cdot \frac{250 \text{ calories}}{\text{muffin}} = 3000 \$ \text{ calories for the whole recipe}$$

$$\frac{3000 \text{ calories}}{20 \text{ mini - muffins}} \text{ gives } 150 \text{ calories per mini-muffin}$$

$$4 \text{ mini muffins} \cdot \frac{150 \text{ calories}}{\text{mini - muffin}} \text{ totals } 600 \text{ calories consumed.}$$

✓ Example 27

You need to replace the boards on your deck. About how much will the materials cost?

Solution

There are two approaches we could take to this problem: 1) estimate the number of boards we will need and find the cost per board, or 2) estimate the area of the deck and find the approximate cost per square foot for deck boards. We will take the latter approach.

For this solution pathway, we will be able to answer the question if we know the cost per square foot for decking boards and the square footage of the deck. To find the cost per square foot for decking boards, we could compute the area of a single board, and divide it into the cost for that board. We can compute the square footage of the deck using geometric formulas. So first we need information: the dimensions of the deck, and the cost and dimensions of a single deck board.

Suppose that measuring the deck, it is rectangular, measuring 16 ft by 24 ft, for a total area of 384ft^2 .

From a visit to the local home store, you find that an 8 foot by 4 inch cedar deck board costs about \$7.50. The area of this board, doing the necessary conversion from inches to feet, is:

$$8 \text{ feet} \cdot 4 \text{ inches} \cdot \frac{1 \text{ foot}}{12 \text{ inches}} = 2.667\text{ft}^2. \text{ The cost per square foot is then}$$

$$\frac{\$7.50}{2.667\text{ft}^2} = \$2.8125 \text{ per ft}^2.$$

This will allow us to estimate the material cost for the whole 384ft^2 deck

$$\$384\text{ft}^2 \cdot \frac{\$2.8125}{\text{ft}^2} = \$1080 \text{ total cost.}$$

Of course, this cost estimate assumes that there is no waste, which is rarely the case. It is common to add at least 10% to the cost estimate to account for waste.

✓ Example 28

Is it worth buying a Hyundai Sonata hybrid instead the regular Hyundai Sonata?

Solution

To make this decision, we must first decide what our basis for comparison will be. For the purposes of this example, we'll focus on fuel and purchase costs, but environmental impacts and maintenance costs are other factors a buyer might consider.

It might be interesting to compare the cost of gas to run both cars for a year. To determine this, we will need to know the miles per gallon both cars get, as well as the number of miles we expect to drive in a year. From that information, we can find the number of gallons required from a year. Using the price of gas per gallon, we can find the running cost.

From Hyundai's website, the 2013 Sonata will get 24 miles per gallon (mpg) in the city, and 35 mpg on the highway. The hybrid will get 35 mpg in the city, and 40 mpg on the highway.

An average driver drives about 12,000 miles a year. Suppose that you expect to drive about 75% of that in the city, so 9,000 city miles a year, and 3,000 highway miles a year.

We can then find the number of gallons each car would require for the year.

Sonata:

$$9000 \text{ city miles} \cdot \frac{1 \text{ gallon}}{24 \text{ city miles}} + 3000 \text{ highway miles} \cdot \frac{1 \text{ gallon}}{35 \text{ highway miles}} = 460.7 \text{ gallons}$$

Hybrid:

$$9000 \text{ city miles} \cdot \frac{1 \text{ gallon}}{35 \text{ city miles}} + 3000 \text{ highway miles} \cdot \frac{1 \text{ gallon}}{40 \text{ highway miles}} = 332.1 \text{ gallons}$$

If gas in your area averages about \$3.50 per gallon, we can use that to find the running cost:

$$\text{Sonata: } 460.7 \text{ gallons} \cdot \frac{\$3.50}{\text{gallon}} = \$1612.45$$

$$\text{Hybrid: } 332.1 \text{ gallons} \cdot \frac{\$3.50}{\text{gallon}} = \$1162.35$$

The hybrid will save \$450.10 a year. The gas costs for the hybrid are about $\frac{\$450.10}{\$1612.45} = 0.279 = 27.9\%$ lower than the costs for the standard Sonata.

While both the absolute and relative comparisons are useful here, they still make it hard to answer the original question, since “is it worth it” implies there is some tradeoff for the gas savings. Indeed, the hybrid Sonata costs about \$25,850, compared to the base model for the regular Sonata, at \$20,895.

To better answer the “is it worth it” question, we might explore how long it will take the gas savings to make up for the additional initial cost. The hybrid costs \$4965 more. With gas savings of \$451.10 a year, it will take about 11 years for the gas savings to make up for the higher initial costs.

We can conclude that if you expect to own the car 11 years, the hybrid is indeed worth it. If you plan to own the car for less than 11 years, it may still be worth it, since the resale value of the hybrid may be higher, or for other non-monetary reasons. This is a case where math can help guide your decision, but it can’t make it for you.

? Try it Now 6

If traveling from Seattle, WA to Spokane WA for a three-day conference, does it make more sense to drive or fly?

Answer

There is not enough information provided to answer the question, so we will have to make some assumptions, and look up some values.

Assumptions:

- a) We own a car. Suppose it gets 24 miles to the gallon. We will only consider gas cost.
- b) We will not need to rent a car in Spokane, but will need to get a taxi from the airport to the conference hotel downtown and back.
- c) We can get someone to drop us off at the airport, so we don’t need to consider airport parking.
- d) We will not consider whether we will lose money by having to take time off work to drive.

Values looked up (your values may be different)

- a) Flight cost: \$184
- b) Taxi cost: \$25 each way (estimate, according to hotel website)
- c) Driving distance: 280 miles each way
- d) Gas cost: \$3.79 a gallon

Cost for flying: \$184 flight cost + \$50 in taxi fares = \$234.

Cost for driving: 560 miles round trip will require 23.3 gallons of gas, costing \$88.31

Based on these assumptions, driving is cheaper. However, our assumption that we only include gas cost may not be a good one. Tax law allows you deduct \$0.55 (in 2012) for each mile driven, a value that accounts for gas as well as a portion of

the car cost, insurance, maintenance, etc. Based on this number, the cost of driving would be \$319

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