4.4: Exponential Relationships (2 of 6)

Learning Objectives

- Use an exponential model (when appropriate) to describe the relationship between two quantitative variables. Interpret the model in context.

Now we investigate what the numbers in the exponential model tell us.

Example

Understanding the Numbers in the Exponential Model

Our goal in this example is to understand the meaning of the numbers 121 and 1.083 in the exponential model for predicting the number of eagle mating pairs.

$$\text{Predicted eagle pairs} = 121\,(1.083)^t$$

The 121 is the *initial value for the exponential model*. It is the predicted value for y when $t = 0$. It is also the ***y*-intercept**, where the exponential model crosses the y-axis.

- To see this, plug $t = 0$ into the exponential model: Predicted eagle pairs = $121\,(1.083)^0 = 121\,(1) = 121$.
- Interpretation in context: When $t = 0$, the year is 1950. In 1950, the number of eagle mating pairs is predicted to be 121.

Note: A number with an exponent of 0 is equal to 1. For example, $2^0 = 1$ and $1.5120^0 = 1$. (This is not true for 0: 0^0 is not defined.)

Now let's investigate the meaning of 1.083 in the context of eagle mating pairs.

For 1951, when $t = 1$, the model predicts $\hat{y}=121(1.083)^{1}\approx 131$ pairs of eagles that are mating.

For 1952, when $t = 2$, there are $\hat{y}=121(1.083)^{2}\approx 142$ pairs.

We can also view the calculation for $t = 2$ as repeated multiplication by 1.083:

$$\begin{array}{l}\hat{y}=121(1.083)^{2}\approx 142\\ \hat{y}=\underbrace{121\cdot(1.083)}_{131\ \mathrm{pairs\ when}\ t=1}\cdot(1.083)\approx \underbrace{142}_{142\ \mathrm{pairs\ when}\ t=2}\end{array}$$

Note: From this viewpoint, we find the estimated number of mating pairs for 1952 by multiplying the estimated 131 pairs from the previous year by 1.083.

Here is another example: For 1953, when $t = 3$, we can rewrite $(1.083)^3$ as repeated multiplication: (1.083)(1.083)(1.083). The exponent 3 tells us to multiply the initial value 121 by 1.083 three times.

$$\begin{array}{l}\hat{y}=121(1.083)^{3}\approx 154\\ \hat{y}=\underbrace{121\cdot(1.083)\cdot(1.083)}_{142\ \mathrm{pairs\ when}\ t=2}\cdot(1.083)\approx \underbrace{154}_{154\ \mathrm{pairs\ when}\ t=3}\end{array}$$

Note: We can also view this process as multiplying the estimated 142 eagle pairs from the previous year by 1.083.

In general, to find the number of mating pairs for the next year, we multiply the previous year's estimate by 1.083. We call this the **growth factor**.

We view the growth factor as containing information about the **percentage increase** in the population over the previous year. To see how this works, let's start with a hypothetical situation in which there is no change in the number of eagle mating pairs from one year to the next. Then we look at different percentages of growth for the first year to build to an understanding of the meaning of 1.083:

No change in the number of eagle pairs: If there is no change in a year, we have 100% of the mating pairs from the previous year. The growth factor is 1.00, which is 100% written in decimal form. This is important to understand. A growth factor of 1.00 means no growth. This makes sense because 121(1.00) = 121; there is no change when we multiply by 1.00.

5% growth in the first year:

- 100% of the mating pairs + **5% increase** in mating pairs = 105%.
- Convert to decimal form to find the growth factor: 105% = 1.05.
- Now multiply the growth factor by 121 to find the number of mating pairs for the next year: 121**(1.05)** = 127.

If we multiply by 1.05, this is a 5% increase.

6.8% growth in the first year:

- 100% of the mating pairs + **6.8% increase** in mating pairs = 106.8%.
- Convert to decimal form to find the growth factor: 106.8% = 1.068.
- Now multiply the growth factor by 121 to find the number of mating pairs for the next year: 121**(1.068)** = 129. **If we multiply by 1.068, this is a 6.8% increase.**

What is the meaning of 1.083 in the model Predicted eagle pairs = 121 $(1.083)^t$?

Answer: The 1.083 is the growth factor; as a percentage, it is 108.3%. We view 108.3% as 100% + 8.3%. There is an *estimated* 8.3% increase in the number of eagle pairs each year. (Remember, the 100% represents no change in the population.)

Spotlight on Converting Percentages

Percent means "per 100." So a percent means "divide by 100."

To convert from a percent to the decimal form, divide by 100.

- For example, 105% means 105 ÷ 100 = 1.05, so 105% = 1.05
- 106.8% means 106.8 ÷ 100 = 1.068, so 106.8% = 1.068
- 96.8% means 96.8 ÷ 100 = .968, so 98.6% = 0.968
- Notice that dividing by 100 moves the decimal two places to the left.

To convert from a decimal form to a percent, we are converting in the opposite direction, so multiply by 100.

- To convert 1.03 to a percent: 1.03 x 100 = 103, so 1.03 = 103%
- To convert 1.083 to a percent: 1.083 x 100 = 108.3, so 1.083 = 108.3%
- To convert 0.834 to a percent: 0.834 x 100 = 83.4, so 0.834 = 83.4%
- Notice that multiplying by 100 moves the decimal two places to the right.

Note: A number is in "decimal form" when it is not a percentage. A percentage like 108.3% is not in "decimal form" even though it has a decimal in the number.

Learn By Doing

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