8.5: Estimating a Population Proportion (3 of 3)

Learning Objectives

- Construct a confidence interval to estimate a population proportion when conditions are met. Interpret the confidence interval in context.
- For a confidence interval, interpret the meaning of a confidence level and relate it to the margin of error.

Introduction

On the previous page, we learned the general formula for a confidence interval for a population proportion:

$$\stackrel{^}{p}\text{ }\pm\text{ }{Z}_{c}\sqrt{\frac{\stackrel{^}{p}(1-\stackrel{^}{p})}{n}}$$

Recall that, for our most common confidence levels, the values of Z_c are:

90% confidence interval: $Z_c \approx 1.645$
95% confidence interval: $Z_c \approx 1.960$
99% confidence interval: $Z_c \approx 2.576$

Learn By Doing

https://assessments.lumenlearning.co...sessments/3594

Confidence Interval Width

The goal of the confidence interval is to estimate the population proportion. If the interval contains the population proportion, a small amount of error means we have a more precise estimate. Narrower confidence intervals give more precise interval estimates for the population proportion, but this is true only if the intervals contain the population proportion.

We saw in the previous activity that a lower confidence level corresponds to a smaller margin of error. In general, 90% confidence intervals are narrower than 95% confidence intervals because there is a smaller margin of error. But we are less confident that 90% confidence intervals contain the population proportion. Recall that in the long run, 10% of these intervals will *not* contain the population proportion at all! We therefore have to choose between precision and confidence.

Of course, ideally, we would like to have a narrow interval *and* a high level of confidence. We can achieve this by increasing the size of the sample.

Example

College Students and Marijuana Legalization

National surveys show that about 43% of American adults support the legalization of marijuana. What proportion of students at Capital Community College support the legalization of marijuana? Suppose students conduct two surveys. For one survey, they randomly select a sample of 100 students. For the other survey, they randomly select a sample of 400 students. Surprisingly, in both surveys, the proportion in favor of legalization is 55%. The students calculate the 95% confidence interval for both surveys.

What is the impact of the size of the sample on the confidence interval?

For the sample of size 100, the confidence interval is

$$0.55\text{ }\pm\text{ }1.96\sqrt{\frac{(0.55)(0.45)}{100}}\text{ }=\text{ }0.55\text{ }\pm\text{ }0.098\text{ }=\text{ }(0.452,0.648)$$

For the sample of size 400, the confidence interval is

$$0.55\text{ }\pm\text{ }1.96\sqrt{\frac{(0.55)(0.45)}{400}}\text{ }=\text{ }0.55\text{ }\pm\text{ }0.049\text{ }=\text{ }(0.501,0.599)$$

Notice that the larger sample gives a smaller margin of error. The margin of error for the sample of 400 is half that of the sample of 100.

This makes sense. Our intuition tells us that larger samples should give more precise estimates of the population proportion. We also saw this in Module 7 where we investigated the impact of sample size on the variability in the sample proportions. We saw that proportions from larger samples vary less. If there is less variability in the sampling distribution, the standard error is smaller. Since we use the standard error to find the margin of error, larger samples will produce a smaller margin of error.

More specifically, we can see that a sample four times larger gives a margin of error half as large because we divide by $\sqrt{n}$, the square root of the sample size, in the formula. Similarly, a sample nine times larger gives a margin of error one-third as large.

In general, to decrease the margin of error, we can increase the sample size or decrease the confidence level. We always prefer to increase the sample size because it allows us to keep a higher level of confidence. We want a higher level of confidence because the confidence level is the proportion of intervals that actually contains the population proportion.

Learn By Doing

https://assessments.lumenlearning.co...sessments/3595

Learn By Doing

https://assessments.lumenlearning.co...sessments/3907

Let’s Summarize

- The sample proportion is a point estimate for the population proportion, but it is almost always wrong. We therefore use an interval estimate, called a *confidence interval*, to give us a range of values for the population proportion.
- We can calculate a confidence interval for a population proportion when we can use a normal distribution to model the long-run behavior of sample proportions. We can use a normal distribution model when there are at least 10 observed successes and 10 observed failures.
- The interpretation of a confidence interval depends on the confidence level. For example, using a 95% confidence level, we are 95% confident that the population proportion falls within the interval.
- A confidence interval is a sample proportion plus or minus a margin of error. The margin of error is related to the confidence level. For a 95% confidence level, the margin of error is approximately two standard errors. The formula is $\stackrel{^}{p}\text{}\pm\text{}{Z}_{c}\sqrt{\frac{\stackrel{^}{p}(1-\stackrel{^}{p})}{n}}$
- Lower confidence levels and higher sample sizes lead to narrower confidence intervals. A narrower confidence interval has a smaller error. Since we want to be confident that an interval accurately estimates the population proportion, high levels of confidence are desirable. So larger sample sizes are the preferred way to decrease the error and create narrower confidence intervals.