6.7: Probability Rules (2 of 3)

Learning Objectives

- Reason from probability distributions, using probability rules, to answer probability questions.

Here we continue to use probability distributions to answer probability questions. We look for some patterns that suggest general rules for determining probabilities.

Example

When Can We Add Probabilities?

Compare these two questions. What do the solutions have in common?

Question 1: A person with blood type A can receive blood from individuals with type A or O blood. *What is the probability that a randomly selected person from the United States can donate blood to someone with type A blood?*

Blood Type	O	A	B	AB
Probability	0.45	0.41	0.10	0.04

Answer P(donate to A) = P(blood type A or blood type O) = 0.45 + 0.41 = 0.86. There is an 86% chance that a randomly selected person in the United States can donate blood to someone with type A blood.

Question 2: *What is the probability that a randomly chosen boreal owl nest will either be empty or contain only 1 egg?*

Number of Eggs	0	1	2	3	4	5	6
Probability	0.2	0.1	0.1	0.25	0.25	0.05	0.05

Answer P(no eggs or 1 egg) = P(no egg) + P(1 egg) = 0.2 + 0.1 = 0.3. There is a 30% chance that a randomly selected boreal owl nest will be empty or contain only one egg.

What do these solutions have in common?

In each case, we have two events and we want to find the probability that either event A *or* event B occurs. In each case, we added the probabilities. This works because the events have no outcomes in common. When two events have no outcomes in common, they are **disjoint**.

The events “type A blood” and “type O blood” are disjoint. These events cannot both happen at the same time for a single person. A person cannot have both type A blood and type O blood.

The events “no eggs” and “1 egg” are disjoint. These outcomes cannot both happen at the same time for a single nest. A nest cannot contain no eggs and at the same time contain 1 egg.

If two events are disjoint, then we can add their individual probabilities. We write this fact as a rule:

$P(A \text{ or } B) = P(A) + P(B)$

Comment

We stated the addition rule as a formal rule. A rule is a concise way to summarize a general principle from specific examples. This is one advantage of a rule. One disadvantage of a rule is that sometimes it discourages us from just thinking through a problem. Students often have the experience that they misremember a rule or forget the conditions required for the rule to work. This leads to mistakes that we can avoid if we just think through the problem without worrying about rules. We encourage you to think through probability problems whenever possible without resorting to rules. If you use a rule, be careful to check that the situation meets the conditions required for using the rule.

This addition rule for probabilities only works when the events are disjoint. If the events are not disjoint, the rule does not work. Here is an example of when the rule does not work because the events are not disjoint.

Example

When Can We NOT Add Probabilities?

	Arts-Sci	Bus-Econ	Info Tech	Health Science	Graphics Design	Culinary Arts	Row Totals
Female	4,660	435	494	421	105	83	6,198
Male	4,334	490	564	223	97	94	5,802
Column Totals	8,994	925	1,058	644	202	177	12,000

Question: *What is the probability that a randomly selected student is either a Health Science major or a female?*

Answer There are 644 Health Science majors and 6,198 females, but 421 students are counted twice because they are both Health Science majors and female. We must subtract these students before calculating the relative frequency:

$$P(\mathrm{Health\; Science\; or\; female})=\frac{644+6,198-421}{12,000}=\frac{6,421}{12,000}\approx 0.54$$

Now let’s calculate the individual probabilities and see if the rule works:

$$\text{P}(\mathrm{Health\; Science})+\text{P}(\mathrm{female})=\frac{\text{644}}{\text{12,000}}+\frac{\text{6,198}}{\text{12,000}}\approx \text{0.57}$$

Main point: P(Health Science or female) ≠ P(Health Science) + P(female). In other words, the addition rule does not work here. Why not? The two events “Health Science” and “female” are not disjoint. The data set contains people who are both in the Health Science program and female.

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Example

Do We Ever Subtract Probabilities?

Compare these two questions. What do the solutions have in common?

Question 1: People with blood type O can donate blood to people with any other blood type. For this reason, people with blood type O are called universal donors. *What is the probability that a randomly selected person from the United States is* ***not*** *a universal donor?*

Blood Type	O	A	B	AB
Probability	0.45	0.41	0.10	0.04

Answer P(NOT a universal donor) = P(blood type is not type O) = P(blood type A, B, or AB) = 0.41 + 0.10 + 0.04 = 0.55. There is a 55% chance that a randomly selected person in the United States is not a universal donor.

Here is another way we can solve this problem. We can use the idea that all of the probabilities together make up 100% of the possibilities. If we add up all the probabilities in the table, we get 1. We can subtract the probability that someone is type O from 1 to find the probability that the person is not type O:

P(NOT a universal donor) = P(blood type is not type O) = 1 – P(type O) = 1 – 0.45 = 0.55

Question 2: *What is the probability that a randomly selected boreal owl nest is* ***not*** *empty?*

Number of Eggs	0	1	2	3	4	5	6

Probability	0.2	0.1	0.1	0.25	0.25	0.05	0.05

Answer *P*(nest is not empty) = *P*(at least one egg) = *P*(1, 2, 3, 4, 5, or 6 eggs) = 0.1 + 0.1 + 0.25 + 0.25 + 0.05 + 0.05 = 0.80. There is an 80% chance that the nest you observe has at least one egg.

Here is another approach:

P(nest is not empty) = *P*(at least one egg) = 1 – *P*(0 eggs) = 1 – 0.2 = 0.8.

What do these solutions have in common?

In each case, we have an event that can be interpreted as a "not" statement. The probability that a person is not a universal donor means the person is *not type O*. The probability that a boreal owl nest is empty means the nest *does not contain 0 eggs*. In each case, the easy way to compute the probability is to use the **complement** event. The complement of event A is the event composed of outcomes that are "not *A*." In our examples, the complement of "type O blood" is the event composed of "blood types A, B, or AB." The complement of "0 eggs" is the event composed of "1, 2, 3, 4, 5, or 6 eggs." When two sets of events are complements, their probabilities add to 1.

When one event is the complement of another, then we can use the complement rule:

$$P(\text{not } A) = 1 - P(A)$$

We can use this rule to find probabilities only when the two events are complements. Two events are complements when their probabilities add to 1.

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