5.3: Repeated Measures ANOVA

Let's take a look at the things we need to find out to make the ANOVA table. What we need to do is calculate all the SSes that we did before for the between-subjects ANOVA. That means the next three steps are identical to the ones you did before. In fact, I will just basically copy the next three steps to find SS_{TOTAL}, SS_{Effect}, and $SS_{\text{Error (within-conditions)}}$. After that we will talk about splitting up $SS_{\text{Error (within-conditions)}}$ into two parts, this is the new thing for this chapter. Here we go!

SS Total

The total sums of squares, or $SS\text{Total}$ measures the total variation in a set of data. All we do is find the difference between each score and the grand mean, then we square the differences and add them all up.

subjects	conditions	scores	diff	diff_squared
1	A	20	13	169
2	A	11	4	16
3	A	2	-5	25
1	B	6	-1	1
2	B	2	-5	25
3	B	7	0	0
1	C	2	-5	25
2	C	11	4	16
3	C	2	-5	25
Sums		63	0	302
Means		7	0	33.5555555555556

The mean of all of the scores is called the **Grand Mean**. It's calculated in the table, the Grand Mean = 7.

We also calculated all of the difference scores **from the Grand Mean**. The difference scores are in the column titled `diff`. Next, we squared the difference scores, and those are in the next column called `diff_squared`.

When you add up all of the individual squared deviations (difference scores) you get the sums of squares. That's why it's called the sums of squares (SS).

Now, we have the first part of our answer:

$$SS_{\text{total}} = SS_{\text{Effect}} + SS_{\text{Error}}$$

$$SS_{\text{total}} = 302$$

and

$$302 = SS_{\text{Effect}} + SS_{\text{Error}}$$

SS Effect

SS_{Total} gave us a number representing all of the change in our data, how they all are different from the grand mean.

What we want to do next is estimate how much of the total change in the data might be due to the experimental intervention/treatment. For example, if we ran an experiment that causes change in the measurement, then the means for each group will be different from other, and the scores in each group will be different from the other groups. As a result, the intervention/treatment forces change onto the numbers, and this will naturally mean that some part of the total variation in the numbers is caused by the intervention.

The way to isolate the variation due to the treatment conditions (also called effect) is to look at the means in each group, and the calculate the difference scores between each group mean and the grand mean, and then the squared deviations to find the sum for SS_{Effect}.

Consider this table, showing the calculations for SS_{Effect}.

subjects	conditions	scores	means	diff	diff_squared
1	A	20	11	4	16
2	A	11	11	4	16
3	A	2	11	4	16
1	B	6	5	-2	4
2	B	2	5	-2	4
3	B	7	5	-2	4
1	C	2	5	-2	4
2	C	11	5	-2	4
3	C	2	5	-2	4
Sums		63	63	0	72
Means		7	7	0	8

Notice we created a new column called `means`, these are the means for each condition, A, B, and C.

SS_{Effect} represents the amount of variation that is caused by differences between the means. The `diff` column is the difference between each condition mean and the grand mean, so for the first row, we have 11-7 = 4, and so on.

We found that $SS_{\text{Effect}} = 72$.

SS Error (within-conditions)

Great, we made it to SS Error. We already found SS Total, and SS Effect, so now we can solve for SS Error just like this:

$$SS_{\text{total}} = SS_{\text{Effect}} + SS_{\text{Error (within-conditions)}}$$

switching around:

$$SS_{\text{Error}} = SS_{\text{total}} - SS_{\text{Effect}}$$

$$SS_{\text{Error (within conditions)}} = 302 - 72 = 230$$

Or, we could compute $SS_{\text{Error (within conditions)}}$ directly from the data:

subjects	conditions	scores	means	diff	diff_squared
1	A	20	11	-9	81
2	A	11	11	0	0
3	A	2	11	9	81
1	B	6	5	-1	1
2	B	2	5	3	9
3	B	7	5	-2	4
1	C	2	5	3	9
2	C	11	5	-6	36

subjects	conditions	scores	means	diff	diff_squared
3	C	2	5	3	9
Sums		63	63	0	230
Means		7	7	0	25.5555555555556

When we compute $SS_{\text{Error (within conditions)}}$ directly, we find the difference between each score and the condition mean for that score. This gives us the remaining error variation around the condition mean, that the condition mean does not explain.

SS Subjects

Now we are ready to calculate new partition, called SS_{Subjects}. We first find the means for each subject. For subject 1, this is the mean of their scores across Conditions A, B, and C. The mean for subject 1 is 9.33 (repeating). Notice there is going to be some rounding error here, that's OK for now.

The `means` column now shows all of the subject means. We then find the difference between each subject mean and the grand mean. These deviations are shown in the `diff` column. Then we square the deviations, and sum them up.

subjects	conditions	scores	means	diff	diff_squared
1	A	20	9.33	2.33	5.4289
2	A	11	8	1	1
3	A	2	3.66	-3.34	11.1556
1	B	6	9.33	2.33	5.4289
2	B	2	8	1	1
3	B	7	3.66	-3.34	11.1556
1	C	2	9.33	2.33	5.4289
2	C	11	8	1	1
3	C	2	3.66	-3.34	11.1556
Sums		63	62.97	-0.0299999999999994	52.7535
Means		7	6.99666666666667	-0.00333333333333326	5.8615

We found that the sum of the squared deviations SS_{Subjects} = 52.75. Note again, this has some small rounding error because some of the subject means had repeating decimal places, and did not divide evenly.

We can see the effect of the rounding error if we look at the sum and mean in the `diff` column. We know these should be both zero, because the Grand mean is the balancing point in the data. The sum and mean are both very close to zero, but they are not zero because of rounding error.

SS Error (left-over)

Now we can do the last thing. Remember we wanted to split up the $SS_{\text{Error (within conditions)}}$ into two parts, SS_{Subjects} and $SS_{\text{Error (left-over)}}$. Because we have already calculate $SS_{\text{Error (within conditions)}}$ and SS_{Subjects}, we can solve for $SS_{\text{Error (left-over)}}$:

$$SS_{\text{Error (left-over)}} = SS_{\text{Error (within conditions)}} - SS_{\text{Subjects}}$$

$$SS_{\text{Error (left-over)}} = SS_{\text{Error (within conditions)}} - SS_{\text{Subjects}} = 230 - 52.75 = 177.25$$

We have finished our job of computing the sums of squares that we need in order to do the next steps, which include computing the MS (mean squared) for the effect and the error term. Once we do that, we can find the F-value, which is the ratio of the two MS's.

Compute the MS

Calculating the MS's (mean squares) that we need for the F-value involves the same general steps as last time. We divide each SS by the degrees of freedom for the SS.

The degrees of freedom for SS_{Effect} are the same as before, the number of conditions - 1. We have three conditions, so the df is 2. Now we can compute the MS_{Effect}.

$$MS_{\text{Effect}} = \frac{SS_{\text{Effect}}}{df} = \frac{72}{2} = 36$$

The degrees of freedom for SS_{Subjects} are the number of subjects - 1. We have three participants, so the df is 2. Now we can compute the MS_{Subjects}.

$$MS_{\text{Subjects}} = \frac{SS_{\text{Subjects}}}{df} = \frac{52.75}{2} = 26.375$$

The degrees of freedom for $SS_{\text{Error (left-over)}}$ are different than before, they are the (number of subjects - 1) multiplied by the (number of conditions -1). We have 3 subjects and three conditions, so $(3-1)*(3-1) = 2*2 = 4$. Or you can go the easy way, it is the difference of degrees of freedom between Total, Effect/Treatment, and Subjects.

$$df_{\text{Error (left-over)}} = df_{\text{Total}} - df_{\text{Effect}} - df_{\text{Subjects}} = 8 - 2 - 2 = 4$$

Regardless, now we can compute the $MS_{\text{Error (left-over)}}$.

$$MS_{\text{Error (left-over)}} = \frac{SS_{\text{Error (left-over)}}}{df} = \frac{177.33}{4} = 44.33$$

Compute F

We just found the two MS's that we need to compute F. We went through all of this to compute F for our data, so let’s do it:

$$F = \frac{MS_{\text{Effect}}}{MS_{\text{Error (left-over)}}} = \frac{36}{44.33} = 0.812$$

And, there we have it!

p-value

We already conducted the repeated-measures ANOVA using statistical program and found the p-value associated with our F-value is 0.505. We might write up the results of our experiment and say that the main effect condition was not significant, F(2, 4) = 0.812, p = 0.505.

What does this statement mean? Remember, that the p-value represents the probability of getting the F value we observed or larger under the null (assuming that the samples come from the same distribution, the assumption of no differences). So, we know that an F-value of 0.812 or larger happens fairly often by chance (when there are no real differences), in fact it happens 50.5% of the time. As a result, we do not reject the idea that any differences in the means we have observed could have been produced by chance.
