9.5: Using Technology - Unequal Slopes Model

SAS Example

If the data collected in the example study were instead as follows:

Females		**Males**	
Salary	Years	Salary	Years
80	5	42	1
50	3	112	4
30	2	92	3
20	1	62	2
60	4	142	5

We would see in **Step 2** of the ANCOVA that we do have a significant treatment × covariate interaction.

Steps for ANCOVA

Using this SAS program with the new data shown below.

```
data unequal_slopes;
input gender $ salary years;
datalines;
m 42 1
m 112 4
m 92 3
m 62 2
m 142 5
f 80 5
f 50 3
f 30 2
f 20 1
f 60 4
;
proc mixed data=unequal_slopes;
class gender;
model salary=gender years gender*years;
title 'Covariance Test for Equal Slopes';
/*Note that we found a significant years*gender interaction*/
/*so we add the lsmeans for comparisons*/
/*With 2 treatments levels we omitted the Tukey adjustment*/
lsmeans gender/pdiff at years=1;
lsmeans gender/pdiff at years=3;
lsmeans gender/pdiff at years=5;
run;
```

We get the following output:

Type 3 Test of Fixed Effects				
Effect	Num DF	De DF	F Value	Pr > F
years	1	6	800.00	F"><.0001
gender	1	6	6.55	F">0.0430
years*gender	1	6	50.00	F">0.0004

Generating Covariate Regression Slopes and Intercepts

```
data unequal_slopes;
input gender $ salary years;
datalines;
m 42 1
m 112 4
m 92 3
m 62 2
m 142 5
f 80 5
f 50 3
f 30 2
f 20 1
f 60 4
;
proc mixed data=unequal_slopes;
class gender;
model salary=gender years gender*years / noint solution;
ods select SolutionF;
title 'Reparmeterized Model';
run;
```

Output:

Solution for Fixed Effects						
Effect	gender	Estimate	Standard Error	DF	t Value	Pr > |t|
gender	f	3.0000	3.3166	6	0.90	|t"">0.4006
gender	m	15.0000	3.3166	6	4.52	|t"">0.0040
years		25.0000	1.0000	6	25.00	|t"">.0001
years*gender	f	-10.0000	1.4142	6	-7.07	|t"">0.0004
years*gender	m	0	.	.	.	|t"">.

Here the intercepts are the Estimates for effects labeled "gender" and the slopes are the Estimates for the effect labeled "years*gender". Thus, the regression equations for this unequal slopes model are:

$$\text{Females} \quad \hat{y} = 3.0 + 15(\text{Years}) \tag{9.5.1}$$

$$\text{Males} \quad \hat{y} = 15 + 25(\text{Years}) \tag{9.5.2}$$

The slopes of the regression lines differ significantly and are not parallel:

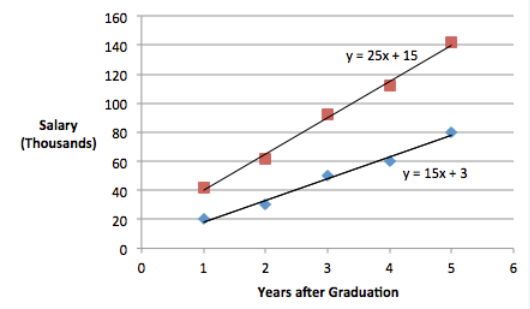

Figure 9.5.a1: Non-parallel regression lines of salary vs years since graduation

And here is the output:

Differences of Least Squares Means								
Effect	gender	_gender	years	Estimate	Standard Error	DF	t Value	Pr > |t|
gender	f	m	1.00	-22.000	3.4641	6	-6.35	|t|">0.0007
gender	f	m	3.00	-42.000	2.0000	6	-21.00	|t|">< .0001
gender	f	m	5.00	-62.000	3.4641	6	-17.90	|t|">< .0001

In this case, we see a significant difference at each level of the covariate specified in the `lsmeans` statement. The magnitude of the difference between males and females differs (giving rise to the interaction significance). In more realistic situations, a significant treatment × covariate interaction often results in significant treatment level differences at certain points along the covariate axis.

? Minitab Example

Steps in Minitab

When we re-run the program with the new dataset Salary-new Data, we find a significant interaction between gender and years.

To do this, open the Minitab dataset Salary-new Data.

Go to **`Stat`** > **`ANOVA`** > **`General Linear model`** > **`Fit General Linear Model`** and follow the same sequence of steps as in the previous section. In Step 2, Minitab will display the following output.

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
years	1	8000.0	8000.0	800.00	0.000
gender	1	65.5	65.45	6.55	0.043
years*gender	1	500.0	500.0	50.00	0.000
Error	6	60.0	10.00		
Total	9	12970.0			

It is clear the interaction term is significant and should not be removed. This suggests the slopes are not equal. Thus, the magnitude of the difference between males and females differs (giving rise to the interaction significance).

R Example

Steps:

- Fit an unequal slopes model.
- Plot the regression lines.

Steps in R

1. Fit an unequal slopes model by using the following commands:

```
setwd("~/path-to-folder/")
unequal_slopes_data <- read.table("unequal_slopes.txt",header=T)
attach(unequal_slopes_data)
unequal_slopes_model<-lm(salary ~ gender + years + gender:years,unequal_slopes_d
anova(unequal_slopes_model)
#Analysis of Variance Table
#Response: salary
#             Df Sum Sq Mean Sq F value     Pr(>F)
#gender        1   4410    4410     441  7.596e-07 ***
#years         1   8000    8000     800  1.293e-07 ***
#gender:years  1    500     500      50  0.0004009 ***
#Residuals     6     60      10
#---
#Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

With a p-value of 0.0004009 in the interaction term (`gender*years`), we can conclude that the slopes are unequal. To estimate the two regression lines, we need the following output:

```
#summary(unequal_slopes_model)$coefficients
#             Estimate  Std. Error    t value      Pr(>|t|)
#(Intercept)         3    3.316625   0.904534  4.005719e-01
#genderm            12    4.690416   2.558409  4.300074e-02
#years              15    1.000000  15.000000  5.530240e-06
#genderm:years      10    1.414214  7.071068   4.008775e-04
```

Here the intercept for females is the estimate for `intercept` and the intercept for males is the summation of the estimates `intercept` + `genderm` (note the letter *m* after *gender*). The slope for females is the estimate for `years` and the slope for males is the summation of the estimates `years` + `genderm: years` (note the letter *m* after *gender*). Thus, the regression equations for the unequal slopes model are: $y = 3 + 15x$ for females and $y = 15 + 25x$ for males.

2. Plot the regression lines by using the following commands:

```
males_regression <- lm(salary~years,data=subset(unequal_slopes_data,gender=="m")
females_regression <- lm(salary~years,data=subset(unequal_slopes_data,gender=="f
plot(years,salary, xlab="Years after graduation", ylab="Salary(Thousands)",pch=2
abline(males_regression)
abline(females_regression)
text(locator(1),"y=25x+15",col="red")
```

```
text(locator(1),"y=15x+3",col="blue")
detach(unequal_slopes_data)
```

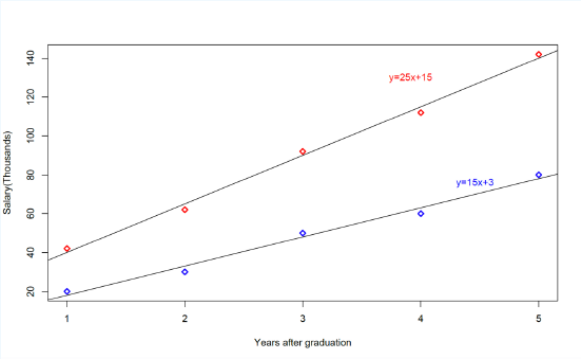

Figure 9.5.c1: Regression line plot in R