

5.3: Crossed-Nested Designs

Multi-factor studies can involve factor combinations in which factors are crossed and/or nested. These treatment designs are based on the extensions of the concepts discussed so far.

Consider an example (from Canavos and Koutrouvelis, 2009) where machines in an assembly process are evaluated for assembly times. There were three factors of interest: Machine ID (1, 2, or 3), Configuration (1 or 2), and Power level (1, 2, or 3).

3-factor table		Machine (A)					
		1		2		3	
Configuration (B)		1	2	1	2	1	2
	1	10.2	4.2	12.0	4.1	13.1	4.1
		13.1	5.2	13.5	6.1	12.9	6.1
Power (C)	2	16.2	8.0	12.6	4.0	12.9	2.2
		16.9	9.1	14.6	6.1	13.7	3.8
	3	13.8	2.5	12.9	3.7	11.8	2.7
		14.9	4.4	15.0	5.0	13.5	4.1

It turns out that each machine can be operated at each power level, and so these factors can be crossed. Also, each configuration can be operated at each power level and so these factors also are crossed. But the configurations (1 or 2) are unique to each machine. As a result, the configuration is nested within the machine.

The statistical model contains both crossed and nested effects and is:

$$Y_{ijkl} = \mu + \alpha_i + \beta_{j(i)} + \gamma_k + (\alpha\gamma)_{ik} + (\beta\gamma)_{j(i)k} + \epsilon_{ijkl} \quad (5.3.1)$$

with the ANOVA table as follows:

Source	df
Factor A	$a - 1$
Factor B(A)	$a(b - 1)$
Factor C	$c - 1$
AC	$(a - 1)(c - 1)$
CB(A)	$a(b - 1)(c - 1)$
Error	$abc(n - 1)$
Total	$N - 1 = (nabc) - 1$

Notice that the two main effects, *Machine* and *Power*, are included in the model along with their interaction effect. The nested relationship of *Configuration* within *Machine* is represented by the *Configuration(Machine)* term and the crossed relationship between *Configuration* and *Power* is represented by their interaction effect.

Notice that the main effect *Configuration* and the crossed effect *Configuration* \times *Machine* are not included in the model. This is consistent with the facts that a nested effect cannot be represented as the main effect and also that a nested effect cannot interact with its nesting effect.

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