6.8: Complexity Happens

From what we have discussed so far, we see that even for the simplest multi-factor studies (i.e. those involving only two factors), there are many possibilities of treatment designs generated by either factor being fixed or random, and factors being crossed or nested.

For any of these possibilities, we can carry out the hypothesis tests using the EMS expressions to identify the correct denominator for the relevant F-statistics.

Crossed				
Source	d.f.	A fixed, B fixed	A fixed, B random	A random, B random
A	$a-1$	$\sigma^2+nb\frac{\sum\alpha_i^2}{a-1}$	$\sigma^2+nb\frac{\sum\alpha_i^2}{a-1}+n\sigma^2_{\alpha\beta}$	$\sigma^2+nb\sigma^2_\alpha+n\sigma^2_{\alpha\beta}$
B	$b-1$	$\sigma^2+na\frac{\sum\beta_i^2}{b-1}$	$\sigma^2+na\sigma^2_\beta$	$\sigma^2+na\sigma^2_\beta+n\sigma^2_{\alpha\beta}$
A×B	$(a-1)(b-1)$	$\sigma^2+n\frac{\sum\sum(\alpha\beta)^2_{ij}}{(a-1)(b-1)}$	$\sigma^2+n\sigma^2_{\alpha\beta}$	$\sigma^2+n\sigma^2_{\alpha\beta}$
		σ^2	σ^2	σ^2

Nested				
Source	d.f.	A fixed, B fixed	A fixed, B random	A random, B random
A	$a-1$	$\sigma^2+bn\frac{\sum\alpha_i^2}{a-1}$	$\sigma^2+bn\frac{\sum\alpha_i^2}{a-1}+n\sigma^2_{\beta(\alpha)}$	$\sigma^2+bn\sigma^2_\alpha+n\sigma^2_{\beta(\alpha)}$
B(A)	$a(b-1)$	$\sigma^2+n\frac{\sum\sum\beta^2_{j(i)}}{a(b-1)}$	$\sigma^2+n\sigma^2_{\beta(\alpha)}$	$\sigma^2+n\sigma^2_{\beta(\alpha)}$
Error	$ab(n-1)$	σ^2	σ^2	σ^2