2.6: Try It!

Exercise 2.6.1: Teaching Effectiveness

To compare the teaching effectiveness of 3 teaching methods, the semester average based on 4 midterm exams from five randomly selected students enrolled in each teaching method were used.

1. What is the response in this study?
2. How many replicates are there?
3. Write the appropriate null and alternative hypotheses.
4. Complete the partially filled ANOVA table given below. Round your answers to 4 decimal places.

Source	df	SS	MS	F	p-value
teach_mtd		245			
error					
total		345.1			

5. Find the critical value at $\alpha = .01$
6. Make your conclusion.
7. From the ANOVA analysis, you performed, can you detect the teaching method which yields the highest semester average? If not, suggest a technique that will.

Solution

1. Average of 4 mid-terms
2. 5
3. $H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$, where μ_1, μ_2, μ_3 are the actual semester average of a student enrolled in teaching method 1, method 2, and method 3 respectively. Ha: Not all semester averages are equal. (This means that there are at least two teaching methods that differ in their actual semester averages)
4.

Source	df	SS	MS	F	p-value
teach_mtd	**2**	245	**122.5000**	**14.6853**	**0.0006**
error	**12**	**100.1**	**8.3417**		
total	**14**	345.1			

5. 6.925
6. As the calculated F-statistic value = 14.6853 is more than the critical value of 6.925, H_0 should be rejected. Therefore, we can conclude that all 3 teaching methods do not have the same semester average, indicating that at least 2 teaching methods differ in their actual semester average.
7. The ANOVA conclusion indicated that not all 3 teaching methods are equally effective, but did not indicate which one yields the highest mean score. The Tukey comparison method is one procedure that shows the teaching method that yields the significantly highest average semester score.

Exercise 2.6.2: Commuter Times

In a local commuter bus service, the number of daily passengers for 50 weeks was recorded. The purpose was to determine if the passenger volume is significantly less during weekends compared to workdays. Below are summary statistics for each day of the week. The partially filled ANOVA table, along with a Tukey plot, is shown below.

Statistics

Day	N	Mean	SE Mean	Std Dev

Day	N	Mean	SE Mean	Std Dev
Sun	50	486.500	9.003	63.661
Mon	50	514.600	6.891	48.724
Tue	50	501.340	7.922	56.018
Wed	50	520.640	7.055	49.886
Thu	50	512.880	10.258	72.532
Fri	50	512.600	8.086	57.174
Sat	50	469.860	8.988	63.555

a) State the appropriate null and alternative hypotheses for this test.

Solution

$H_0: \mu_{Sun} = \mu_{Mon} = \mu_{Tues} = \mu_{Wed} = \mu_{Thurs} = \mu_{Fri} = \mu_{Sat}$

\(H_{A}: \ \text{At least one } \mu_{day \ i} \neq \mu_{day \ j}, \text{ for some\) i, j = 1, 2, \ldots, 7 \text{ OR not all means are equal}\)

b) Complete the partially filled ANOVA table given below. Use two decimal places in the F statistic.

Source	df	SS	MS	F	p-value
Groups		100391			
Error					
Total		1306887			

Solution

Source	df	SS	MS	F	p-value
Day	**6**	100391	**16731.8**	**4.76**	**0.0001**
Error	**343**	**1206496**	**3517.5**		
Total	**349**	1306887			

c) Use the appropriate F-distribution cumulative probabilities to verify that the p-value for the test is approximately zero.

Solution

p-value ≈ 0 (from the F-distribution with 6 and 343 degrees of freedom)

d) Use $\alpha = 0.05$ to test if the mean passenger volume differs significantly by day of the week.

Solution

Since the p-value $\leq \alpha = 0.05$, we reject H_0. There is strong evidence to indicate that the mean passenger volume differs significantly by day of the week (i.e., for some days of the week, the average number of commuters is more than others, but this test does not indicate which days have a higher passenger volume).

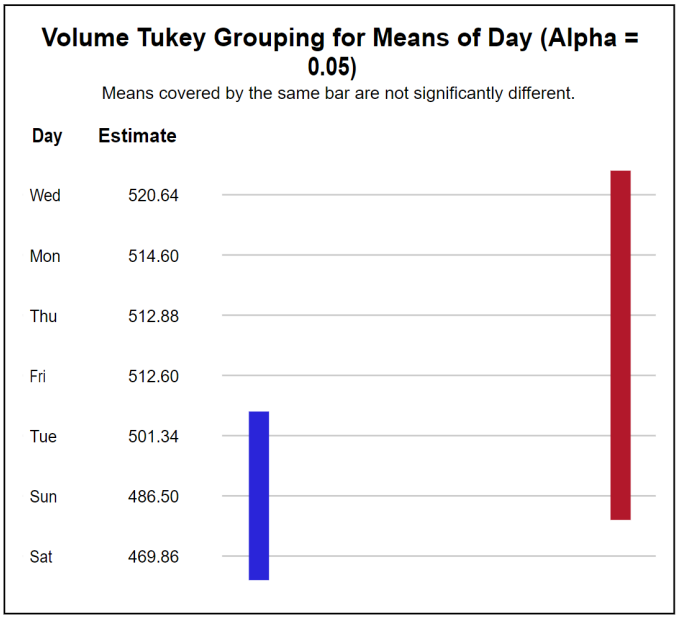

Figure 2.6.1: Grouping information using the tukey method and 95% confidence.

e) Use the output to make a statement about how the mean daily passenger volume differs significantly by day of the week.

Solution

The passenger volume on Sundays is not statistically different from Saturdays and also from Tuesdays. The mean passenger volume on Saturdays is significantly lower than on workdays other than Tuesdays.

f) The management would like to know if the overall number of commuters is significantly more during workdays than during weekends. An appropriate comparison to respond to their query would be to compare the average number of commuters between workdays (Monday through Friday) and the weekend. Write the weight (coefficients) for a linear contrast to make this comparison. Test the hypothesis that the average commuter volume during the weekends is less.

Solution

The weights (coefficients) for the appropriate contrast are given below.

Day	Mon	Tue	Wed	Thu	Fri	Sat	Sun
weight	1	1	1	1	1	-2.5	-2.5

$$t = \frac{\sum_{i=1}^{T} a_i \bar{y}_i}{\sqrt{MSE \sum_{i=1}^{T} \frac{a_i^2}{n_i}}} = \frac{171.16}{\sqrt{3517.5 * \frac{17.5}{50}}} = 4.878$$

Under the null hypothesis, this test statistic has a t-distribution with 343 degrees of freedom. You can obtain the p-value using statistical software. Recall this is a one-tailed test.

Student's t distribution with 343 DF

x	$P(X \leq x)$
4.878	$8.216815 \cdot 10^{-7} \approx 0$

This p-value indicates that the difference in the average number of passengers is statistically significant between workdays and weekends.

See the table below for computations:

Factor	N	Mean	weights	product	weight^2
Mon	50	514.6	1.0	514.6	1.00
Tue	50	501.34	1.0	501.34	1.00
Wed	50	520.64	1.0	520.64	1.00
Thu	50	512.88	1.0	512.88	1.00
Fri	50	512.6	1.0	512.6	1.00
Sat	50	469.86	-2.5	-1174.65	6.25
Sun	50	486.5	-2.5	-1216.25	6.25

Recall that the MSE (error mean squares) is 3517.5 with $df_{error} = 343$.