5.1.1a: The Additive Model (No Interaction)

In a factorial design, we first look at the interactions for significance. In the case where interaction is not significant, then we can drop the interaction term from our model, and we end up with an additive model.

For a two-factor factorial, the model we initially consider (as we have discussed in Section 5.1) is:

$$Y_{ij} = \mu_{..} + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ijk} \tag{5.1.1a.1}$$

Note that the interaction term, $(\alpha\beta)_{ij}$, is a multiplicative term.

If the interaction is found to be non-significant, then the model reduces to:

$$Y_{ij} = \mu_{..} + \alpha_i + \beta_j + \epsilon_{ijk} \tag{5.1.1a.2}$$

Here we can see that the response variable is simply a function of adding the effects of the two factors.

Example $5.1.1a.1$: Glucose in Blood Serum

As an example, (adapted from Kuehl, 2000), let's look at a study designed to evaluate two chemical methods used for assaying the amount of glucose in blood serum. A large volume of blood serum served as a starting point for the experiment. The blood serum was divided into three portions, each of which was 'doped' or augmented by adding an additional amount of glucose. Three doping levels were used. Samples of the doped serum were then assayed for glucose concentration by one of two chemical methods. This type of ‘doping’ experiment is commonly used to compare the sensitivity of assay methods.

The amount of glucose detected in each sample was recorded and is presented in the table below.

	Chemical Assay Method					
	Method 1			Method 2		
Doping Level	1	2	3	1	2	3
	46.5	138.4	180.9	39.8	132.4	176.8
	47.3	144.4	180.5	40.3	132.4	173.6
	46.9	142.7	183	41.2	130.3	174.9

Solution

The model was run as a two-factor factorial and produced the following results:

Type 3 Analysis of Variance							
Source	DF	Sum of Squares	Mean Square	Expected Mean Square	Error Term	Error DF	F Value	Pr > F
method	1	263.733889	263.733889	Var(Residual) + Q(method, method*doping)	MS(Residual)	12	98.35	<.0001
doping	2	57026	28513	Var(Residual) + Q(doping, method*doping)	MS(Residual)	12	10632.5	<.0001

Type 3 Analysis of Variance								
method*doping	2	13.821111	6.910556	Var(Residual) + Q(method*doping)	MS(Residual)	12	2.58	0.1172
Residual	12	32.180000	2.681667	Var(Residual)				

Here we can see that the interaction of *method*doping* was not significant (*p-value* > 0.05) at a 5% level. We drop the interaction effect from the model and run the additive model. The resulting ANOVA table is:

The Mixed Procedure								
Type 3 Analysis of Variance								
Source	DF	Sum of Squares	Mean Square	Expected Mean Square	Error Term	Error DF	F Value	Pr > F
method	1	263.733889	263.733889	Var(Residual)+Q(method, method)	MS(Residual)	14	80.26	<.0001
doping	2	57026	28513	Var(Residual) + Q(doping,doping)	MS(Residual)	14	8677.63	<.0001
1Residual	14	46.001111	3.285794	Var(Residual)				

The Error SS is now 46.001, which is the sum of the interaction SS and the error SS of the model with the interaction. The df values were also added the same way. This example shows that any term not included in the model gets added into the error term, which may erroneously inflate the error especially if the impact of excluded term on the response is not negligible.

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method Least Squares Means								
method	Estimate	Standard Error	DF	t Value	Pr >|t|	Alpha	Lower	Upper
1	123.40	0.6042	14	204.23	<.0001	0.05	122.10	124.70
2	115.74	0.6042	14	191.56	<.0001	0.05	114.45	117.04

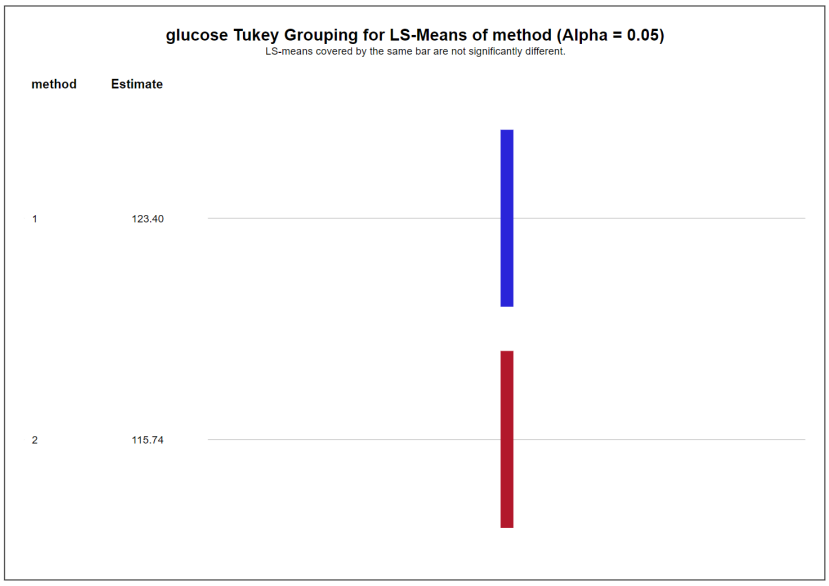

Figure 5.1.1$a.1$: Glucose Tukey grouping for LS-Means of method.

doping Least Squares Means								
Doping	Estimate	Standard Error	DF	t Value	Pr >|t|	Alpha	Lower	Upper
1	43.67	0.7400	14	59.01	<.0001	0.05	42.08	45.25
2	136.77	0.7400	14	184.81	<.0001	0.05	135.18	138.35
3	178.28	0.7400	14	240.92	<.0001	0.05	176.70	179.87

Here, we can see that the response variable, the amount of glucose detected in a sample, is the overall mean **PLUS** the effect of the method used **PLUS** the effect of the glucose amount added to the original sample. (Hence, the additive nature of this model!)