8.3: Split-Split-Plot Design

The idea of split-plots can easily be extended to multiple splits. In a 3-factor factorial, for example, it is possible to assign Factor A to whole plots, then Factor B to subplots within the applications of Factor A, and then split the experimental units used for Factor B into sub-subplots to receive the levels of Factor C.

For a fixed effect factorial treatment design in an RCBD (with blocks, levels of Factor A, levels of Factor B, and levels of Factor C), the split-split-plot would produce the following table:

Source	d.f.
(Whole plots)	
Block	$r-1$
Factor A	$a-1$
Whole plot error	$(r-1)(a-1)$
(Subplots)	
Factor B	$b-1$
A × B	$(a-1)(b-1)$
Subplot error	$a(r-1)(b-1)$
(Sub-subplots)	
Factor C	$c-1$
A × C	$(a-1)(c-1)$
B × C	$(b-1)(c-1)$
A × B × C	$(a-1)(b-1)(c-1)$
Sub-subplot error	\(ab(r - 1)(c - 1)\)
Total	$(rabc)-1$

The model is specified as we did earlier for the split-plot in RCBD, retaining only the interactions involving replication where they form denominators for F-tests for factor effects. For the model above, we would need to include the block, block × A, and block × A × B terms in the random statement in SAS. In SAS, Block × A × B would automatically include the Block × B effect SS and *df*. All other interactions involving replications and factor C would be included in the residual error term. The block × A term is often referred to as "Error a" ("Whole plot error" in the table), the Block × A × B term as "Error b" ("Subplot error" in the table), and the residual error as "Error c" ("Sub-subplot error" in the table) because of their roles as the denominator in the F-tests.