

6.4: Special Case - Fully Nested Random Effects Design

Here, we will consider a special case of random effects models where each factor is nested within the levels of the next "order" of a hierarchy. This Fully Nested Random Effects model is similar to Russian Matryoshka dolls, where the smaller dolls are nested within the next larger one.

Consider 3 random factors A, B, and C that are hierarchically nested. That is, C is nested in (B, A) combinations and B is nested within levels of A. Suppose there are n observations made at the lowest level.

The statistical model for this case is:

$$Y_{ijkl} = \mu + \alpha_i + \beta_{i(j)} + \gamma_{k(ij)} + \epsilon_{ijkl} \quad (6.4.1)$$

where $i = 1, 2, \dots, a$, $j = 1, 2, \dots, b$, $k = 1, 2, \dots, c$ and $l = 1, 2, \dots, n$.

We will also have $\epsilon_{ijkl} \stackrel{iid}{\sim} \mathcal{N}(0, \sigma_\epsilon^2)$, $\gamma_{k(ij)} \stackrel{iid}{\sim} \mathcal{N}(0, \sigma_\gamma^2)$, $\beta_{i(j)} \stackrel{iid}{\sim} \mathcal{N}(0, \sigma_\beta^2)$, and $\alpha_i \stackrel{iid}{\sim} \mathcal{N}(0, \sigma_\alpha^2)$.

The DFs and expected mean squares for this design would be as follows:

Source	DF	EMS	F
A	$(a - 1)$	$\sigma_\epsilon^2 + n\sigma_\gamma^2 + nc\sigma_\beta^2 + ncb\sigma_\alpha^2$	MSA / MSB(A)
B(A)	$a(b - 1)$	$\sigma_\epsilon^2 + n\sigma_\gamma^2 + nc\sigma_\beta^2$	MSB(A) / MSC(AB)
C(A,B)	$ab(c - 1)$	$\sigma_\epsilon^2 + n\sigma_\gamma^2$	MSC(AB) / MSE
Error	$abc(n - 1)$	σ_ϵ^2	
Total	$abcn - 1$		

In this case, each F -test we construct for the sources will be based on different denominators.

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