

## 4.7: Try It!

### ? Exercise 4.7.1: Design Matrix

Below is a design matrix for a data set of a recent study.

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & -1 & -1 & -1 \\ 1 & -1 & -1 & -1 \\ 1 & -1 & -1 & -1 \end{bmatrix}$$

a) Identify the number of treatment levels and replicates.

#### Solution

4 treatment levels and 3 replicates

b) Name the model and write its equation.

#### Solution

This design matrix corresponds to the effects model, and the model equation is  $Y_{ij} = \mu + \tau_i + \epsilon_{ij}$ , where  $i = 1, 2, 3, 4$ ,  $j = 1, 2, 3$ , and  $\sum_{i=1}^4 \tau_i = 0$ .

c) Write the equation and the design matrix that corresponds to the cell means model.

#### Solution

The equation for the cell means model is:  $Y_{ij} = \mu + \epsilon_{ij}$ , where  $i = 1, 2, 3, 4$  and  $j = 1, 2, 3$ . The design matrix corresponding to the cell means model is:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

d) Write the equation and the design matrix that corresponds to the dummy variable regressions model.

#### Solution

The equation for the 'dummy variable regression' model is:  $Y_{ij} = \mu + \mu_i + \epsilon_{ij}$  for  $i = 1, 2, 3$  and  $j = 1, 2, 3$ .  
 $Y_{4j} = \mu + \epsilon_{4j}$

The design matrix is given below. Note that the last 3 rows correspond to the 4th treatment level which is the reference category and its effect is estimated by the model intercept.

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

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