

## 6.2: The Standard Normal Distribution

The **standard normal distribution** is a normal distribution of **standardized values called z-scores**. A z-score is measured in **units of the standard deviation**.

The mean for the standard normal distribution is 0, and the standard deviation is 1. What this does is dramatically simplify the mathematical calculation of probabilities. Take a moment and substitute 0 and 1 in the appropriate places in the above formula and you can see that the equation collapses into one that can be much more easily solved using integral calculus. The transformation  $z = \frac{x - \mu}{\sigma}$  produces the distribution  $Z \sim N(0, 1)$ . The value  $x$  in the given equation comes from a known normal distribution with known mean  $\mu$  and known standard deviation  $\sigma$ . The z-score tells how many standard deviations a particular  $x$  is away from the mean.

### Z-Scores

If  $X$  is a normally distributed random variable and  $X \sim N(\mu, \sigma)$ , then the z-score for a particular value  $x$  is:

$$z = \frac{x - \mu}{\sigma}$$

The z-score tells you how many standard deviations the value  $x$  is above (to the right of) or below (to the left of) the mean,  $\mu$ . Values of  $x$  that are larger than the mean have positive z-scores, and values of  $x$  that are smaller than the mean have negative z-scores. If  $x$  equals the mean, then  $x$  has a z-score of zero.

#### ? Example 6.2.1

Suppose  $X \sim N(5, 6)$ . This says that  $X$  is a normally distributed random variable with mean  $\mu = 5$  and standard deviation  $\sigma = 6$ . Suppose  $x = 17$ . Then:

$$z = \frac{x - \mu}{\sigma} = \frac{17 - 5}{6} = 2$$

This means that  $x = 17$  is **two standard deviations** ( $2\sigma$ ) above or to the right of the mean  $\mu = 5$ .

Now suppose  $x = 1$ . Then:  $z = \frac{x - \mu}{\sigma} = \frac{1 - 5}{6} = -0.67$  (rounded to two decimal places)

**This means that  $x = 1$  is 0.67 standard deviations ( $-0.67\sigma$ ) below or to the left of the mean  $\mu = 5$ .**

### The Empirical Rule

If  $X$  is a random variable and has a normal distribution with mean  $\mu$  and standard deviation  $\sigma$ , then **the Empirical Rule** states the following:

- About 68% of the  $x$  values lie between  $-1\sigma$  and  $+1\sigma$  of the mean  $\mu$  (within one standard deviation of the mean).
- About 95% of the  $x$  values lie between  $-2\sigma$  and  $+2\sigma$  of the mean  $\mu$  (within two standard deviations of the mean).
- About 99.7% of the  $x$  values lie between  $-3\sigma$  and  $+3\sigma$  of the mean  $\mu$  (within three standard deviations of the mean). Notice that almost all the  $x$  values lie within three standard deviations of the mean.
- The z-scores for  $+1\sigma$  and  $-1\sigma$  are  $+1$  and  $-1$ , respectively.
- The z-scores for  $+2\sigma$  and  $-2\sigma$  are  $+2$  and  $-2$ , respectively.
- The z-scores for  $+3\sigma$  and  $-3\sigma$  are  $+3$  and  $-3$  respectively.

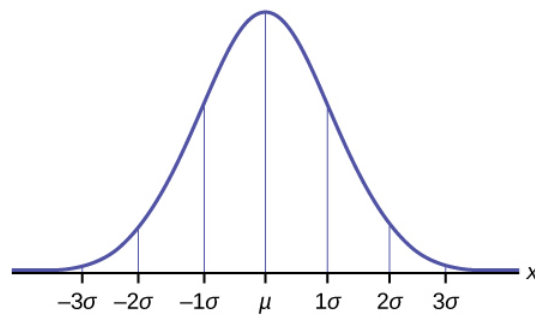


Figure 6.2.1

### ? Example 6.2.2

Suppose  $X$  has a normal distribution with mean 50 and standard deviation 6.

- About 68% of the  $X$  values lie within one standard deviation of the mean. Therefore, about 68% of the  $X$  values lie between  $-1\sigma = (-1)(6) = -6$  and  $1\sigma = (1)(6) = 6$  of the mean 50. The values  $50 - 6 = 44$  and  $50 + 6 = 56$  are within one standard deviation from the mean 50. The z-scores are  $-1$  and  $+1$  for 44 and 56, respectively.
- About 95% of the  $X$  values lie within two standard deviations of the mean. Therefore, about 95% of the  $X$  values lie between  $-2\sigma = (-2)(6) = -12$  and  $2\sigma = (2)(6) = 12$ . The values  $50 - 12 = 38$  and  $50 + 12 = 62$  are within two standard deviations from the mean 50. The z-scores are  $-2$  and  $+2$  for 38 and 62, respectively.
- About 99.7% of the  $X$  values lie within three standard deviations of the mean. Therefore, about 99.7% of the  $X$  values lie between  $-3\sigma = (-3)(6) = -18$  and  $3\sigma = (3)(6) = 18$  of the mean 50. The values  $50 - 18 = 32$  and  $50 + 18 = 68$  are within three standard deviations from the mean 50. The z-scores are  $-3$  and  $+3$  for 32 and 68, respectively.

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