3.4: Two Basic Rules of Probability

When calculating probability, there are two rules to consider when determining if two events are independent or dependent and if they are mutually exclusive or not.

The Multiplication Rule

If A and B are two events defined on a **sample space**, then: $P(A \cap B) = P(B)P(A|B)$. We can think of the intersection symbol as substituting for the word "and".

This rule may also be written as: $P(A|B) = \frac{P(A \cap B)}{P(B)}$

This equation is read as the probability of A given B equals the probability of A and B divided by the probability of B.

If A and B are **independent**, then $P(A|B) = P(A)$. Then $P(A \cap B) = P(A|B)P(B)$ becomes $P(A \cap B) = P(A)(B)$ because the $P(A|B) = P(A)$ if *A* and *B* are independent.

One easy way to remember the multiplication rule is that the word "and" means that the event has to satisfy two conditions. For example the name drawn from the class roster is to be both a female and a Part 2 student. It is harder to satisfy two conditions than only one and of course when we multiply fractions the result is always smaller. This reflects the increasing difficulty of satisfying two conditions.

The Addition Rule

If A and B are defined on a sample space, then: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. We can think of the union symbol substituting for the word "or". The reason we subtract the intersection of A and B is to keep from double counting elements that are in both A and B.

If A and B are **mutually exclusive**, then $P(A \cap B) = 0$. Then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ becomes $P(A \cup B) = P(A) + P(B)$.

Exercise 3.4.1

A student goes to the library. Let events B = the student checks out a book and D = the student checks out a DVD. Suppose that $P(B) = 0.40$, $P(D) = 0.30$ and $P(D|B) = 0.5$.

1. Find $P(B^C)$.
2. Find $P(D \cap B)$.
3. Find $P(B|D)$.
4. Find $P(D \cap B^C)$.
5. Find $P(D|B^C)$.