2.2: Solving and Graphing Inequalities, and Writing Answers in Interval Notation

2.2.1: Interval Notation

Definition: How to write inequalities in Interval Notation

- **Symbol to not include a value are (or).**
- **Symbol to include a value are [or].**
- **Use the Symbol (or) when using infinity (-∞, ∞).**

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1. **x < -4** The region is from negative infinity (-∞) up to and not including -4. Therefore, the **Interval Notation is (-∞, -4)**.
2. **x ≤ -4** The region is from negative infinity (-∞) up to and including -4. There for , the **Interval Notation is (-∞, -4].**
3. **-3 < x < 2** The region is from -3 up to and not including 2. Therefore, the **Interval Notation is (-3, 2).**
4. **-3 ≤ x ≤ 2** The region is including -3 and up to and including 2. Therefore, the **Interval Notation is [-3, 2].**
5. **x > 5** The region is greater than and not including 5. Therefore, the **Interval Notation is (5, ∞)**.
6. **x ≥ 5** The region is greater than and including 5. Therefore, the **Interval Notation is [5, ∞).**
7. **-3 < x ≤ 10** The region is greater than and not including 3 but less than or equal to 10. Therefore, the **Interval Notation is (-3, 10].**
8. **4 ≤ x < 7** The region is greater than and including 4 and less than not including 7. Therefore, the **Interval Notation is [4, 7).**

2.2.2: To solve and graph inequalities:

1. Solve the inequality using the Properties of Inequalities from the previous section.
2. Graph the solution set on a number line.
3. Write the solution set in interval notation.

Example 2.2.1

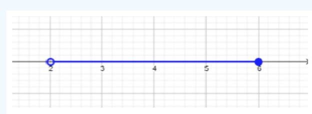
Solve the inequality, graph the solution set on a number line and show the solution set in interval notation:

a. $-1 \leq 2x-5 < 7$
b. $x^2+7x+10 < 0$
c. $-6 < x-2 < 4$

Solution

a.

$-1 \leq 2x-5 < 7$	Example problem
$-1+5 \leq 2x-5+5 < 7+5$	The goal is to isolate the variable x, so start by adding 5 to all three regions in the inequality.
$4 \leq 2x < 12$	Simplify.
$\frac{4}{2} \leq 2x < \frac{4}{2}$	Divide all by 2 to isolate the variable x.
$2 \leq x < 6$	Final answer written in inequality/solution set form.
$[2,6)$	Final answer written in interval notation (see section on Interval Notation for more details)



b.

$x^2+7x+10 < 0$	Example problem
$(x+5)(x+2) < 0$	Factor the polynomial.
$(x+5)(x+2) < 0$	Set each factor equal to 0 and solve for x. x = -5 and x = -2.

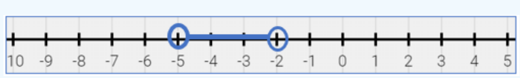
Since the inequality is a strict inequality($<$ or $>$), -5 and -2 is not included in the solution set.

Pick a point less than -5, x = -6. Evaluate the expression (-6+5)(-6+2) = (-1)(-4) = 4. It is positive, x < -5 is not in the solution set.

Pick a point between -5 and x = -2. Evaluate the expression (-4 + 5)(-4+2) = (1)(-2) = -2. It is negative, -5 < x < -2 is in the solution set.

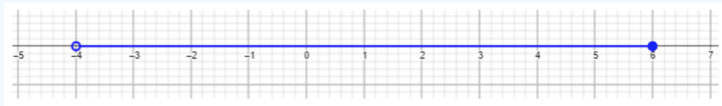
Finally, pick a point greater than -2, x = 0. Evaluate the expression (0 + 5)(0 + 2) = 5(2) = 10. It is positive, x > -2 is not the solution set.

Interval notation: (-5, -2)



-5 < x < -2

c.	$-6 < x - 2 \leq 4$	Example problem
	$-6+2 < x-2+2 \leq 4+2$	The goal is to isolate the variable x, so start by adding 2 to all three regions in the inequality.
	$-4 < x \leq 6$	Final answer written in inequality/solution set form.
	$(-4, 6]$	Final answer written in interval notation (see section on Interval Notation for more details).



Exercise Problems 2.2.1

Solve the inequalities, graph the solution sets on a number line and show the solution sets in interval notation:

1. $0 \leq x+1 \leq 4$
2. $0 < 2(x-1) \leq 4$
3. $6 < 2(x-1) < 12$
4. $x^2 - 6x - 16 < 0$
5. $2x^2 - x - 15 > 0$

Detail answers below

Exercise Answers 2.2.1

1. $0 \leq x+1 \leq 4$

Answer

$0 \leq x+1 \leq 4$

subtract 1 from all three sides

$0-1 \leq x+1-1 \leq 4-1$

$-1 \leq x+0 \leq 3$

$-1 \leq x \leq 3$

[-1, 3] Interval Notation

Exercise Answers 2.2.2

2. $0 < 2(x-1) \leq 4$

Answer

Multiply the 2 through the parenthesis in the middle section.

$0 < 2x-2 \leq 4$

Add 2 to all three sides

$0+2 < 2x-2+2 \leq 4+2$

$2 < 2x+0 \leq 6$

$2 < 2x \leq 6$

Divide all three sides by 2

$2/2 < 2x/2 \leq 6/2$

$1 < x \leq 3$

(1, 3] Interval Notation

Exercise 2.2.3

3. $6 < 2(x-1) < 12$

Answer

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Multiply the 2 through the parenthesis in the middle section.

$6 < 2x-2 < 12$

Add 2 to all three sides

$6+2<2x-2+2<12+2$

$8<2x+0<14$

$8<2x<14$

Divide all sides by 2

$8/2<2x/2<14/2$

$4<x<7$

(4, 7) Interval Notation

Exercise 2.2.4

4. $x^2-6x-16<0$

Answer

Factor $x^2-6x-16$ to (x - 8)(x + 2).

Set x - 8 = 0 and solve for x

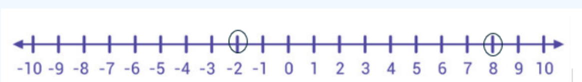
x - 8 = 0   Add 8 to both sides

x - 8 + 8 = 0 + 8
x + 0 = 8
x = 8

Set x + 2 = 0 and solve for x

x + 2 = 0   Subtract 2 from both sides
x + 2 - 2 = 0 - 2
x + 0 = -2
x = -2

Create a number line with the two points selected. Since it is a strict inequality, <, the points will not be included.



Select a point to the left of x = -2, -3. Substitute it in the factored form of the quadratic equation.

(-3 - 8)(-3 +2) = (-11)(-1) = 11 > 0. For values to the left of x = -2 the function is positive.

Select a point between x = -2 and x = 8, 0. Substitute it in the factored form of the quadratic equation.

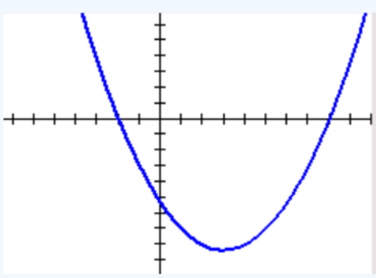
(0 -8)(0 + 2) = (-8)(2) = -16 < 0.   For values between -2 and 8, the function is negative.

Select a point to the right of x = 8, 9. Substitute the value in the factored form of the quadratic equation.

(9 - 8)(9 + 2) = (1)(11) = 11 > 0. For values to the right of x = 8 the function is positive.

The solution is in the interval -2 < x < 8, or (-2, 8) Interval notation.

2.2.3: Graph of quadratic equation



Note the equation is negative between -2 and 8. Therefore, **the solution interval is -2 < x < 8 or (-2, 8) in Interval notation.**

Exercise 2.2.5

5. $2x^2 - x - 15 > 0$

Answer

Using the box method we can factor the $2x^2 - x - 15 > 0$.

	2x	+ 5
x	$2x^2$	5x
- 3	-6x	-15

$2x^2 - x - 15$ = (2x + 5)(x - 3)

Set each factor equal to 0 and solve for x

2x + 5 = 0 subtract 5 from both sides

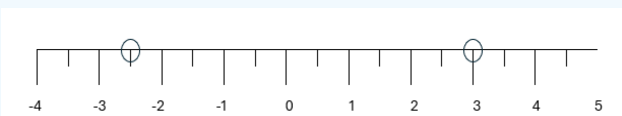
2x + 5 - 5 = 0 - 5
2x + 0 = -5
2x = -5 divide both sides by 2

2x/2 = -5/2

x = -5/2 or -2.5

x - 3 = 0 add 3 to both sides
x - 3 + 3 = 0 + 3
x + 0 = 3
x = 3

Divide the number line into three regions



Select a number to the left of -2.5, -3. Substitute the value in the factored form of the quadratic equation.

(2(-3) + 5)(-3 - 3) = (-6 + 5)(-3 - 3) = (-1)(-6) = 6 > 0. The region to the left of -2.5 is positive.

Select a number between -2.5 and 3, 0. Substitute the value in the factored form of the quadratic equation.

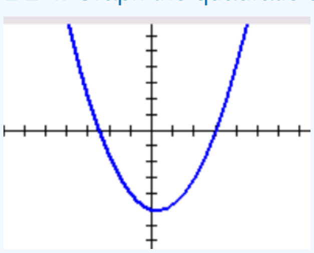
(2(0) + 5)(0 - 3) = (0 + 5)(-3) = (5)(-3) = -15 < 0. The region between -2.5 and 3 is negative.

Select a number to the right of 3, 4. Substitute the value in the factored form of the quadratic equation.

(2(4) + 5)(5 - 3) = (8 + 5)(2) = (13)(2) = 26 > 0. The region to the right of 3 is positive.

The solution is $(-\infty, -2.5) \cup (3, \infty)$.

2.2.4: Graph the quadratic equation



You can see the graph is positive to the left of -2.5 and to the right of 3. The solution in interval notation is $(-\infty, -2.5) \cup (3, \infty)$.