2.4: Rational Inequalities

Solving rational inequalities involves finding the zeroes of the numerator and denominator, then using these values to investigate solution set regions on the number line.

Example 2.4.1

Solve the inequalities and write the solution sets in interval notation:

a. $\dfrac{x-1}{x+1} \geq 0$

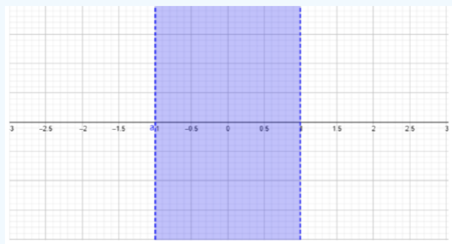
b. $\dfrac{2x-3}{x+1} \leq 0$

c. $\dfrac{x+2}{x-2} \geq 0$

Solution

a.

$\dfrac{x-1}{x+1} \geq 0$	Example problem
$\dfrac{x-1}{x+1} \geq 0$	The quotient must be greater than or equal to 0.
$x-1=0,\ x=1$	Find the zeroes of the numerator
$x+1=0, x=\ -1$	Find the zeroes of the denominator



The zeroes divide the number line into 3 regions, $x<-1$, $-1<x<1$, $x>1$

For $x<-1,$ choose $x=-2.$ $\dfrac{-2-1}{-2+1}=\dfrac{-3}{-1}=3 \geq 0$

Replacing -2 for x results in the answer 3, which is greater than or equal to 0. This region $x<-1$ is included in the solution set.

For $-1<x<1,$ choose $x=0.$ $\dfrac{0-1}{0+1}=\dfrac{-1}{1}=-1<0$

Replacing 0 for x results in the answer -1, which is less than 0, not fulfilling the given inequality in the problem. This region $-1<x<1$ is excluded from the solution set.

For $x>1,$ choose $x=2.$ $\dfrac{2-1}{2+1}=\dfrac{1}{3} \geq 0$

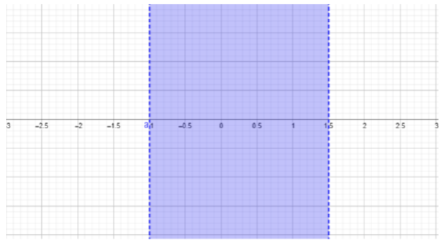
Replacing 2 for x results in the answer $\dfrac{1}{3}$, which is greater than or equal to 0. This region $x>1$ is included in the solution set.

$(-\infty,-1) \cup (1,\infty)$

Final answer written in interval notation (see section on Interval Notation for more details).

b.

$\dfrac{2x-3}{x+1} \leq 0$	Example problem
$\dfrac{2x-3}{x+1} \leq 0$	The quotient must be less than or equal to 0.
$2x-3=0,\ x=1.5$	Find the zeroes of the numerator
$x+1=0,\ x=-1$	Find the zeroes of the denominator



The zeroes divide the number line into 3 regions, $x < -1$, $-1 < x < 1.5$, $x > 1.5$

For $-1 < x$, choose $x = -2$. $\dfrac{2(-2)-3}{-2+1} = \dfrac{-7}{-1} = 7 \geq 0$

Replacing -2 for x results in the answer 7, which is greater than or equal to 0. This region $-1 < x$ is excluded in the solution set.

For $-1 < x < 1.5$, choose $x = 0$. $\dfrac{2(0)-3}{0+1} = \dfrac{-3}{1} = -3 \leq 0$

Replacing 0 for x results in the answer -3, which is less than or equal to 0. This region $-1 < x < 1.5$ is included in the solution set.

For $x > 1.5$, choose $x = 2$. $\dfrac{2(2)-3}{2+1} = \dfrac{1}{3} \geq 0$

Replacing 2 for x results in the answer $\dfrac{1}{3}$, which is greater than or equal to 0. This region $x > 1.5$ is excluded in the solution set.

(-1, 1.5)

Final answer written in interval notation (see section on Interval Notation for more details).

c.

$\dfrac{x+2}{x-2} \geq 0$	Example problem
$\dfrac{x+2}{x-2} \geq 0$	The quotient must be greater than or equal to 0.
$x+2=0, \quad x=-2$	Find the zeroes of the numerator
$x-2=0, \quad x=2$	Find the zeroes of the denominator

The zeroes divide the number line into 3 regions, $x < -2$, $-2 < x < 2$, $x > 2$

For $x<-2,$ choose $x=-3.\dfrac{-3+2}{-3-2}=\dfrac{-1}{-5}=\dfrac{1}{5}\geq 0$

Replacing -3 for x results in the answer $\frac{1}{5}$, which is greater than or equal to 0. This region $x<-2$ is included in the solution set.

For $\ -2<x<2,$ choose $x=0.\ \dfrac{0+2}{0-2}=\dfrac{2}{-2}=-1<0$

Replacing 0 for x results in the answer -1, which is less than 0, not fulfilling the given inequality in the problem. This region $-2<x<2$ is not included in the solution set.

For $x>2,$ choose $x=3.\ \dfrac{3+2}{3-2}=\dfrac{5}{1}=5\geq 0$

Replacing 3 for x results in the answer 5, which is greater than or equal to 0. This region $x>2$ is included in the solution set.

$(-\infty,-2)\cup(2,\infty)$

Final answer written in interval notation (see section on Interval Notation for more details).

Exercise 2.4.1

1. $\dfrac{x+3}{x-2}\geq 0$
2. $\dfrac{x-2}{x-1}\leq 0$
3. $\dfrac{8}{x+2}\leq\dfrac{1}{x+2}$
4. $\dfrac{2x-3}{x+1}\geq 0$

Exercise Solutions 2.4.1

$\dfrac{x+3}{x-2}\geq 0$

Solution

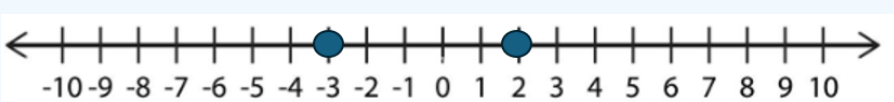
Set x + 3 = 0 and solve for x.

x + 3 = 0
 -3 -3
x = -3

Set x - 2 = 0 and solve for x.
x - 2= 0
 2 2
X + 0 = 2
x = 2

Because we have the line under the inequality, ≥, we will include -3 and 2 in the solution.

The number line is divided into three regions. Left of -3, between -3 and 2, to the right of 2. Pick a point in each region.



x = -4

$\dfrac{-4+3}{-4-2}=\dfrac{-1}{-6}=\dfrac{1}{6}\geq 0$. The area to the left of -3 is a solution. The interval notation is $(-\infty,-3]$

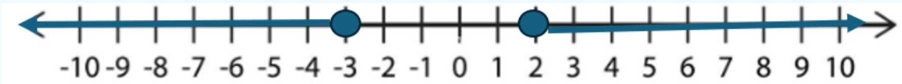
x = 0

$\dfrac{0+3}{0-2}=\dfrac{3}{-2}<0$. The area between -3 and 2 is not a solution.

x = 3

$\frac{3+3}{3-2}$ = $\frac{6}{1}$ = 6 ≥ 0. The area to the right of 2 is a solution. The interval notation is $[2, \infty)$

The solution is $(-\infty, -3] \cup [2, \infty)$ in Interval Notation.



Exercise 2.4.1

$\frac{8}{x+2} \leq \frac{1}{x+2}$

Answer

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Subtract $\frac{1}{x-2}$ from both sides

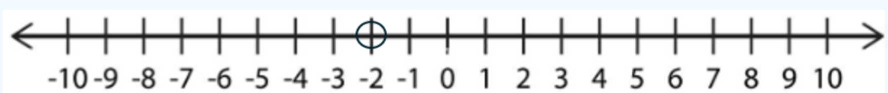
$\frac{8}{x+2}$ - $\frac{1}{x+2}$ ≤ 0

$\frac{7}{x+2}$ ≤ 0

There is only a variable in the denominator so we set x + 2 = 0 and solve for x.

x + 2 = 0
  -2   -2
x + 0 = -2
x = -2

Since x = -2 results in 0 in the denominator x = -2 is not a part of the solution. -2 breaks the number line into two parts: to the left of -2 and to the right of -2.



Pick a number both regions.

x = -3 and substitute it in the rational expression.

$\frac{8}{-3+2}$ = $\frac{8}{-1}$ = -8

$\frac{1}{-3+2}$ = $\frac{1}{-1}$ = -1

-8 ≤ -1 The region to the left of -2 is part of the solution. Interval Notation $(-\infty, -2)$.

x = 0

$\frac{8}{0+2}$ = $\frac{8}{2}$ = 4

$\frac{1}{0+2}$ = $\frac{1}{2}$

4 ≥ $\frac{1}{2}$ Therefore, this region is not a part of the solution.

The solution is $(-\infty, -2)$.