7.6.1: Applications – Leontief Models (Exercises)

SECTION 7.6 PROBLEM SET: APPLICATIONS - LEONTIEF MODELS

1) Solve the following homogeneous system.

$$\begin{aligned} x+y+z &= 0 \\ 3x+2y+z &= 0 \\ 4x+3y+2z &= 0 \end{aligned}$$

2) Solve the following homogeneous system.

$$\begin{aligned} x-y-z &= 0 \\ x-3y+2z &= 0 \\ 2x-4y+z &= 0 \end{aligned}$$

3) Chris and Ed decide to help each other by doing repairs on each others houses. Chris is a carpenter, and Ed is an electrician. Chris does carpentry work on his house as well as on Ed's house. Similarly, Ed does electrical repairs on his house and on Chris' house. When they are all finished they realize that Chris spent 60% of his time on his own house, and 40% of his time on Ed's house. On the other hand Ed spent half of his time on his house and half on Chris's house. If they originally agreed that each should get about a $1000 for their work, how much money should each get for their work?

4) Chris, Ed, and Paul decide to help each other by doing repairs on each others houses. Chris is a carpenter, Ed is an electrician, and Paul is a plumber. Each does work on his own house as well as on the others houses. When they are all finished they realize that Chris spent 30% of his time on his own house, 40% of his time on Ed's house, and 30% on Paul's house. Ed spent half of his time on his own house, 30% on Chris' house, and remaining on Paul's house. Paul spent 40% of the time on his own house, 40% on Chris' house, and 20% on Ed's house. If they originally agreed that each should get about a $1000 for their work, how much money should each get for their work?

SECTION 7.6 PROBLEM SET: APPLICATIONS - LEONTIEF MODELS

5) Given the internal consumption matrix A, and the external demand matrix D as follows.

$$A = \begin{bmatrix} .30 & .20 & .10 \\ .20 & .10 & .30 \\ .10 & .20 & .30 \end{bmatrix} \quad D = \begin{bmatrix} 100 \\ 150 \\ 200 \end{bmatrix}$$

Solve the system using the open model: $X = AX + D$ or $X = (I - A)^{-1} D$

6) Given the internal consumption matrix A, and the external demand matrix D as follows.

$$A = \begin{bmatrix} .05 & .10 & .10 \\ .10 & .15 & .05 \\ .05 & .20 & .20 \end{bmatrix} \quad D = \begin{bmatrix} 50 \\ 100 \\ 80 \end{bmatrix}$$

Solve the system using the open model: $X = AX + D$ or $X = (I - A)^{-1} D$

7) An economy has two industries, farming and building. For every $1 of food produced, the farmer uses $.20 and the builder uses $.15. For every $1 worth of building, the builder uses $.25 and the farmer uses $.20. If the external demand for food is $100,000, and for building $200,000, what should be the total production for each industry in dollars?

SECTION 7.6 PROBLEM SET: APPLICATIONS - LEONTIEF MODELS

8) An economy has three industries, farming, building, and clothing. For every $1 of food produced, the farmer uses $.20, the builder uses $.15, and the tailor $.05. For every $1 worth of building, the builder uses $.25, the farmer uses $.20, and the tailor $.10. For every $1 worth of clothing, the tailor uses $.10, the builder uses $.20, the farmer uses $.15. If the external demand for food is $100 million, for building $200 million, and for clothing $300 million, what should be the total production for each in dollars?

9) Suppose an economy consists of three industries F, C, and T. The following table gives information about the internal use of each industry's production and external demand in dollars.

	F	C	T	Demand	Total
F	30	10	20	40	100
C	20	30	20	50	120
T	10	10	30	60	110

Find the proportion of the amounts consumed by each of the industries; that is, find the matrix A.

10) If in problem 9, the consumer demand for F, C, and T becomes 60, 80, and 100, respectively, find the total output and the internal use by each industry to meet that demand.
