6.5: Classification of Finance Problems

Learning Objectives

In this section, you will review the concepts of chapter 6 to:

1. Re-examine the types of financial problems and classify them.
2. Re-examine the vocabulary words used in describing financial calculations

We'd like to remind the reader that the hardest part of solving a finance problem is determining the category it falls into. So in this section, we will emphasize the classification of problems rather than finding the actual solution.

We suggest that the student read each problem carefully and look for the word or words that may give clues to the kind of problem that is presented. For instance, students often fail to distinguish a lump-sum problem from an annuity. Since the payments are made each period, an annuity problem contains words such as each, every, per etc.. One should also be aware that in the case of a lump-sum, only a single deposit is made, while in an annuity numerous deposits are made at equal spaced time intervals. To help interpret the vocabulary used in the problems, we include a glossary at the end of this section.

Students often confuse the present value with the future value. For example, if a car costs $15,000, then this is its present value. Surely, you cannot convince the dealer to accept $15,000 in some future time, say, in five years. Recall how we found the installment payment for that car. We assumed that two people, Mr. Cash and Mr. Credit, were buying two identical cars both costing $15, 000 each. To settle the argument that both people should pay exactly the same amount, we put Mr. Cash's cash of $15,000 in the bank as a lump-sum and Mr. Credit's monthly payments of x dollars each as an annuity. Then we make sure that the future values of these two accounts are equal. As you remember, at an interest rate of 9%

the future value of Mr. Cash's lump-sum was $\$15,000(1+.09/12)^{60}$, and

the future value of Mr. Credit's annuity was $\frac{x\left[(1+.09/12)^{60}-1\right]}{.09/12}$.

To solve the problem, we set the two expressions equal and solve for m.

The present value of an annuity is found in exactly the same way. For example, suppose Mr. Credit is told that he can buy a particular car for $311.38 a month for five years, and Mr. Cash wants to know how much he needs to pay. We are finding the present value of the annuity of $311.38 per month, which is the same as finding the price of the car. This time our unknown quantity is the price of the car. Now suppose the price of the car is P, then

the future value of Mr. Cash's lump-sum is $\mathrm{P}(1+.09/12)^{60}$, and

the future value of Mr. Credit's annuity is $\frac{\$311.38\left[(1+.09/12)^{60}-1\right]}{.09/12}$.

Setting them equal we get,

$$\begin{aligned}
P(1+.09/12)^{60} &= \frac{\$311.38\left[(1+.09/12)^{60}-1\right]}{.09/12} \\
P(1.5657) &= (\$311.38)(75.4241) \\
P(1.5657) &= \$23,485.57 \\
P &= \$15,000.04
\end{aligned}$$

CLASSIFICATION OF PROBLEMS AND EQUATIONS FOR SolutionS

We now list six problems that form a basis for all finance problems.
Further, we classify these problems and give an equation for the solution.

Example 6.5.1

If $2,000 is invested at 7% compounded quarterly, what will the final amount be in 5 years?

Classification: **Future (accumulated) Value of a Lump-sum** or FV of a lump-sum.

Equation:

$$\mathrm{FV}=\mathrm{A}=\$2000(1+.07/4)^{20}$$

Example 6.5.2

How much should be invested at 8% compounded yearly, for the final amount to be $5,000 in five years?

Classification: **Present Value of a Lump-sum** or PV of a lump-sum.

Equation:

$$\mathrm{PV}(1+.08)^{5}=\$5,000$$

Example 6.5.3

If $200 is invested *each* month at 8.5% compounded monthly, what will the final amount be in 4 years?

Classification: **Future (accumulated) Value of an Annuity** or FV of an annuity.

Equation:

$$\mathrm{FV}=\mathrm{A}=\frac{\$200\left[(1+.085/12)^{48}-1\right]}{.085/12}$$

Example 6.5.4

How much should be invested *each* month at 9% for it to accumulate to $8,000 in three years?

Classification: **Sinking Fund Payment**

Equation:

$$\frac{m\left[(1+.09/12)^{36}-1\right]}{.09/12}=\$8,000$$

Example 6.5.5

Keith has won a lottery paying him $2,000 *per* month for the next 10 years. He'd rather have the entire sum now. If the interest rate is 7.6%, how much should he receive?

Classification: **Present Value of an Annuity** or PV of an annuity.

Equation:

$$\mathrm{PV}(1+.076/12)^{120}=\frac{\$2000\left[(1+.076/12)^{120}-1\right]}{.076/12}$$

Example 6.5.6

Mr. A has just donated $25,000 to his alma mater. Mr. B would like to donate an equivalent amount, but would like to pay by monthly payments over a five year period. If the interest rate is 8.2%, determine the size of the monthly payment?

Classification: **Installment Payment**.

Equation:

$$\frac{m\left[(1+.082/12)^{60}-1\right]}{.082/12}=\$25,000(1+.082/12)^{60}$$

GLOSSARY: VOCABULARY AND SYMBOLS USED IN FINANCIAL CALCULATIONS

As we've seen in these examples, it's important to read the problems carefully to correctly identify the situation. It is essential to understand to vocabulary for financial problems. Many of the vocabulary words used are listed in the glossary below for easy reference.

t	Term	Time period for a loan or investment. In this book t is represented in years and should be converted into years when it is stated in months or other units.
P	Principal	Principal is the amount of money borrowed in a loan. If a sum of money is invested for a period of time, the sum invested at the start is the Principal.
P	Present Value	Value of money at the beginning of the time period.
A	Accumulated Value Future Value	Value of money at the end of the time period
D	Discount	In loans involving simple interest, a discount occurs if the interest is deducted from the loan amount at the beginning of the loan period, rather than being repaid at the end of the loan period.
m	Periodic Payment	The amount of a constant periodic payment that occurs at regular intervals during the time period under consideration (examples: periodic payments made to repay a loan, regular periodic payments into a bank account as savings, regular periodic payment to a retired person as an annuity,)
n	Number of payment periods and compounding periods per year	In this book, when we consider periodic payments, we will always have the compounding period be the same as the payment period. In general the compounding and payment periods do not have to be the same, but the calculations are more complicated if they are different. If the periods differ, formulas for the calculations can be found in finance textbooks or various online resources. Calculations can easily be done using technology such as an online financial calculator, or financial functions in a spreadsheet, or a financial pocket calculator.

nt	Number of periods	nt = (number of periods per year) $\times$(number of years) nt gives the total number of payment and compounding periods In some situations we will calculate nt as the multiplication shown above. In other situations the problem may state nt, such as a problem describing an investment of 18 months duration compounded monthly. In this example: nt = 18 months and n = 12; then t = 1.5 years but t is not stated explicitly in the problem. The TI-84+ calculators built in TVM solver uses $N = nt$.
r	Annual interest rate Nominal rate	The stated annual interest rate. This is stated as a percent but converted to decimal form when using financial calculation formulas. If a bank account pays 3% interest compounded quarterly, then 3% is the nominal rate, and it is included in the financial formulas as r = 0.03
r/n	Interest rate per compounding period	If a bank account pays 3% interest compounded quarterly, then r/n = 0.03/4 = 0. 075, corresponding to a rate of 0.75% per quarter. Some Finite Math books use the symbol i to represent r/n
r_{EFF}	Effective Rate Effective Annual Interest Rate APY Annual Percentage Yield APR Annual Percentage Rate	The effective rate is the interest rate compounded annually that would give the same interest rate as the compounded rate stated for the investment. The effective rate provides a uniform way for investors or borrowers to compare different interest rates with different compounding periods.
I	Interest	Money paid by a borrower for the use of money borrowed as a loan. Money earned over time when depositing money into a savings account, certificate of deposit, or money market account. When a person deposits money in a bank account, the person depositing the funds is essentially temporarily lending the money to the bank and the bank pays interest to the depositor.

	Sinking Fund	A fund set up by making payments over a period of time into a savings or investment account in order to save to fund a future purchase. Businesses use sinking funds to save for a future purchase of equipment at the end of the savings period by making periodic installment payments into a sinking fund.
	Annuity	An annuity is a stream of periodic payments. In this book it refers to a stream of constant periodic payments made at the end of each compounding period for a specific amount of time. In common use the term annuity generally refers to a constant stream of periodic payments received by a person as retirement income, such as from a pension. Annuity payments in general may be made at the end of each payment period (ordinary annuity) or at the start of each period (annuity due). The compounding periods and payment periods do not need to be equal, but in this textbook we only consider situations when these periods are equal.
	Lump Sum	A single sum of money paid or deposited at one time, rather than being spread out over time. An example is lottery winnings if the recipient chooses to receive a single “lump sum” one-time payment, instead of periodic payments over a period of time or as. Use of the word lump sum indicates that this is a one time transaction and is not a stream of periodic payments.
	Loan	An amount of money that is borrowed with the understanding that the borrower needs to repay the loan to the lender in the future by the end of a period of time that is called the term of the loan. The repayment is most often accomplished through periodic payments until the loan has completely repaid over the term of the loan. However there are also loans that can be repaid as a single sum at the end of the term of the loan, with interest paid either periodically over the term or in a lump sum at the end of the loan or as a discount at the start of the loan.