11.1: Sample Spaces and Probability

Learning Objectives

In this section, you will learn to:

1. Write sample spaces.
2. Calculate probabilities by examining simple events in sample spaces.

If two coins are tossed, what is the probability that both coins will fall heads? The problem seems simple enough, but it is not uncommon to hear the incorrect answer 1/3. A student may incorrectly reason that if two coins are tossed there are three possibilities, one head, two heads, or no heads. Therefore, the probability of two heads is one out of three. The answer is wrong because if we toss two coins there are four possibilities and not three. For clarity, assume that one coin is a penny and the other a nickel. Then we have the following four possibilities.

HH HT TH TT

The possibility HT, for example, indicates a head on the penny and a tail on the nickel, while TH represents a tail on the penny and a head on the nickel. It is for this reason, we emphasize the need for understanding sample spaces.

Sample Spaces

An act of flipping coins, rolling dice, drawing cards, or surveying people are referred to as a probability **experiment**. A **sample space** of an experiment is the set of all possible outcomes.

Example 11.1.1

If a die is rolled, write a sample space.

Solution

A die has six faces each having an equally likely chance of appearing. Therefore, the set of all possible outcomes S is

$$\{ 1, 2, 3, 4, 5, 6 \}.$$

Example 11.1.2

A family has three children. Write a sample space.

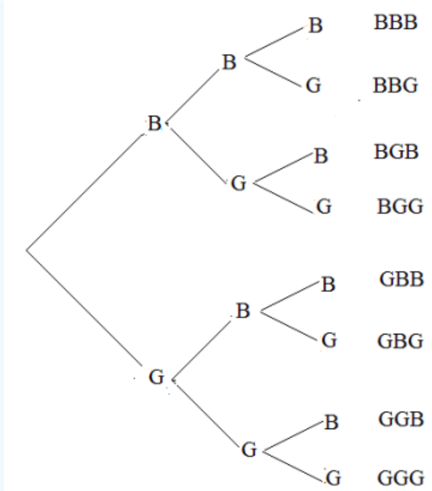
Solution

The sample space consists of eight possibilities.

$$\{ BBB, BBG, BGB, BGG, GBB, GBG, GGB, GGG \}$$

The possibility BGB, for example, indicates that the first born is a boy, the second born a girl, and the third a boy.

We illustrate these possibilities with a tree diagram.



Example 11.1.3

Two dice are rolled. Write the sample space.

Solution

We assume one of the dice is red, and the other green. We have the following 36 possibilities.

Green						
Red	**1**	**2**	**3**	**4**	**5**	**6**
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

The entry (2, 5), for example, indicates that the red die shows a 2, and the green a 5.

Probability

Now that we understand the concept of a sample space, we will define probability.

Definition: Probability

For a sample space S, and an outcome A of S, the following two properties are satisfied.

1. If A is an outcome of a sample space, then the probability of A, denoted by $P(A)$, is between 0 and 1, inclusive.

$$0 \leq P(A) \leq 1$$

2. The sum of the probabilities of all the outcomes in S equals 1.

The probability $P(A)$ of an event A describes the chance or likelihood of that event occurring.

- If $P(A)=0$, event A is certain not to occur. If $P(A)=1$, event A is certain to occur.
- If $P(A)=0.5$, then event A is equally likely to occur or not occur.
- If we toss a fair coin that is equally likely to land on heads or tails, then P(Head) = 0.50.

- If the weather forecast says there is a 70% chance of rain today, then P(Rain) = 0.70, indicating is it more likely to rain than to not rain.

Example 11.1.4

If two dice, one red and one green, are rolled, find the probability that the red die shows a 3 and the green shows a six.

Solution

Since two dice are rolled, there are 36 possibilities. The probability of each outcome, listed in Example $11.1.3$, is equally likely.

Since (3, 6) is one such outcome, the probability of obtaining (3, 6) is 1/36.

The example we just considered consisted of only one outcome of the sample space. We are often interested in finding probabilities of several outcomes represented by an event.

An **event** is a subset of a sample space. If an event consists of only one outcome, it is called a **simple event.**

Example 11.1.5

If two dice are rolled, find the probability that the sum of the faces of the dice is 7.

Solution

Let E represent the event that the sum of the faces of two dice is 7.

The possible cases for the sum to be equal to 7 are: (1, 6), (2,5), (3, 4), (4, 3), (5, 2), and (6, 1), so event E is

E = {(1, 6), (2,5), (3, 4), (4, 3), (5, 2), (6, 1)}

The probability of the event E is

P(E) = 6/36 or 1/6.

Example 11.1.6

A jar contains 3 red, 4 white, and 3 blue marbles. If a marble is chosen at random, what is the probability that the marble is a red marble or a blue marble?

Solution

We assume the marbles are r_1, r_2, r_3, w_1, w_2, w_3, w_4, b_1, b_2, b_3. Let the event C represent that the marble is red or blue.

The sample space $\mathrm{S}=\{\mathrm{r}_1, \mathrm{r}_2, \mathrm{r}_3, \mathrm{w}_1, \mathrm{w}_2, \mathrm{w}_3, \mathrm{w}_4, \mathrm{b}_1, \mathrm{b}_2, \mathrm{b}_3\}$.

And the event $\mathrm{C}=\{\mathrm{r}_1, \mathrm{r}_2, \mathrm{r}_3, \mathrm{b}_1, \mathrm{b}_2, \mathrm{b}_3\}$

Therefore, the probability of C,

$$\mathrm{P}(\mathrm{C})=6/10 \text{ or } 3/5$$

Example 11.1.7

A jar contains three marbles numbered 1, 2, and 3. If two marbles are drawn **without replacement**, what is the probability that the sum of the numbers is 5?

Note: The two marbles in this example are drawn consecutively **without replacement**. That means that after a marble is drawn it is not replaced in the jar, and therefore is no longer available to select on the second draw.

Solution

Since two marbles are drawn without replacement, the sample space consists of the following six possibilities.

$$\mathrm{S}=\{(1,2),(1,3),(2,1),(2,3),(3,1),(3,2)\}$$

Note that (1,1), (2,2) and (3,3) are not listed in the sample space. These outcomes are not possible when drawing without replacement, because once the first marble is drawn but not replaced into the jar, that marble is not available in the jar to be selected again on the second draw.

Let the event E represent that the sum of the numbers is five. Then

$$\mathrm{E}=\{(2,3),(3,2)\}$$

Therefore, the probability of E is

$$\mathrm{P}(\mathrm{E})=2/6 \text{ or } 1/3.$$

✓ Example 11.1.8

A jar contains three marbles numbered 1, 2, and 3. If two marbles are drawn **without replacement**, what is the probability that the sum of the numbers is *at least* 4?

Solution

The sample space, as in Example 11.1.7, consists of the following six possibilities.

$$\mathrm{S}=\{(1,2),(1,3),(2,1),(2,3),(3,1),(3,2)\}$$

Let the event F represent that the sum of the numbers is at least four. Then

$$\mathrm{F}=\{(1,3),(3,1),(2,3),(3,2)\}$$

Therefore, the probability of F is

$$\mathrm{P}(\mathrm{F})=4/6 \text{ or } 2/3$$

✓ Example 11.1.9

A jar contains three marbles numbered 1, 2, and 3. If two marbles are drawn **with replacement**, what is the probability that the sum of the numbers is 5?

Note: The two marbles in this example are drawn consecutively **with replacement**. That means that after a marble is drawn it IS replaced in the jar, and therefore is available to select again on the second draw.

Solution

When two marbles are drawn with replacement, the sample space consists of the following nine possibilities.

$$\mathrm{S}=\{(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3)\}$$

Note that (1,1), (2,2) and (3,3) are listed in the sample space. These outcomes are possible when drawing with replacement, because once the first marble is drawn and replaced, that marble is not available in the jar to be drawn again.

Let the event E represent that the sum of the numbers is four. Then

$$\mathrm{E}=(2,3),(3,2)$$

Therefore, the probability of F is $\mathrm{P}(\mathrm{E})=2/9$

Note that in Example 11.1.9 when we selected marbles with replacement, the probability has changed from Example 11.1.7 where we selected marbles without replacement.

✓ Example 11.1.10

A jar contains three marbles numbered 1, 2, and 3. If two marbles are drawn **with replacement**, what is the probability that the sum of the numbers is *at least* 4?

Solution

The sample space when drawing with replacement consists of the following nine possibilities.

$$S = (1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)$$

Let the event F represent that the sum of the numbers is at least four. Then

$$F = (1,3), (3,1), (2,3), (3,2), (2,2), (3,3)$$

Therefore, the probability of F is

$$P(F) = 6/9 \text{ or } 2/3.$$

Note that in Example $11.1.10$ when we selected marbles with replacement, the probability is the same as in Example $11.1.8$ where we selected marbles without replacement.

Thus sampling with or without replacement MAY change the probabilities, but may not, depending on the situation in the particular problem under consideration. We'll re-examine the concepts of sampling with and without replacement in Section 8.3.

✓ Example 11.1.11

One 6 sided die is rolled once. Find the probability that the result is greater than 4.

Solution

The sample space consists of the following six possibilities in set S: $S = 1,2,3,4,5,6$

Let E be the event that the number rolled is greater than four: $E = 5,6$

Therefore, the probability of E is: $P(E) = 2/6$ or $1/3$.

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