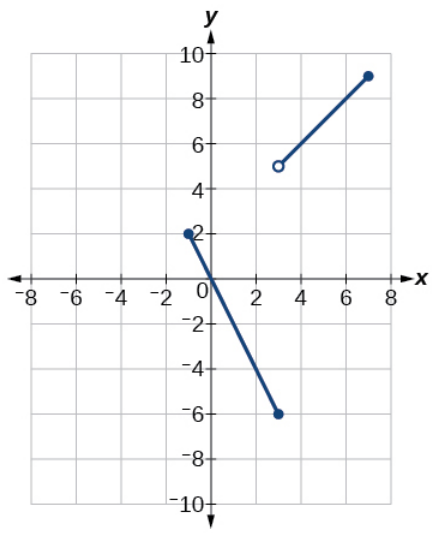
5.R: Introduction to Calculus (Review)

5.1: Finding Limits - Numerical and Graphical Approaches

For the exercises 1-6, use the Figure below.



1) $\lim\limits_{x \to -1^+} f(x)$

Answer

2

2) $\lim\limits_{x \to -1^-} f(x)$

3) $\lim\limits_{x \to -1} f(x)$

Answer

does not exist

4) $\lim\limits_{x \to 3} f(x)$

5) At what values of x is the function discontinuous? What condition of continuity is violated?

Answer

Discontinuous at $x=-1$ $\left(\lim\limits_{x \to a} f(x) \text{ does not exist}\right)$, $x=3$ (jump discontinuity) , and $x=7$ $\left(\lim\limits_{x \to a} f(x) \text{ does not exist}\right)$.

6) Using the Table below, estimate $\lim\limits_{x \to 0} f(x)$.

x	$F(x)$
−0.1	2.875
−0.01	2.92
−0.001	2.998
0	Undefined
0.001	2.9987

x	$F(x)$
0.01	2.865
0.1	2.78145
0.15	2.678

Answer

3

For the exercises 7-9, with the use of a graphing utility, use numerical or graphical evidence to determine the left- and right-hand limits of the function given as x approaches a. If the function has limit as x approaches a, state it. If not, discuss why there is no limit.

7) $f(x)=\begin{cases}|x|-1 & \text{if } x\neq 1\\ x^3 & \text{if } x=1\end{cases}\quad a=1$

8) $f(x)=\begin{cases}\dfrac{1}{x+1} & \text{if } x=-2\\ (x+1)^2 & \text{if } x\neq -2\end{cases}\quad a=-2$

Answer

$\lim\limits_{x\to -2} f(x)=1$

9) $f(x)=\begin{cases}\sqrt{x+3} & \text{if } x<1\\ -\sqrt[3]{x} & \text{if } x>1\end{cases}\quad a=1$

5.2: Finding Limits - Properties of Limits

For the exercises 1-6, find the limits if $\lim\limits_{x\to c} f(x)=-3$ and $\lim\limits_{x\to c} g(x)=5$.

1) $\lim\limits_{x\to c}(f(x)+g(x))$

Answer

2

2) $\lim\limits_{x\to c}\dfrac{f(x)}{g(x)}$

3) $\lim\limits_{x\to c}(f(x)\cdot g(x))$

Answer

-15

4) $\lim\limits_{x\to 0^+} f(x),\ f(x)=\begin{cases}3x^2+2x+1 & x>0\\ 5x+3 & x<0\end{cases}$

5) $\lim\limits_{x\to 0^-} f(x),\ f(x)=\begin{cases}3x^2+2x+1 & x>0\\ 5x+3 & x<0\end{cases}$

Answer

3

6) $\lim\limits_{x\to 3^+}(3x-[\![x]\!])$

For the exercises 7-11, evaluate the limits using algebraic techniques.

7) $\lim_{h\to 0}\left(\frac{(h+6)^2-36}{h}\right)$

Answer

12

8) $\lim_{x\to 25}\left(\frac{x^2-625}{\sqrt{x}-5}\right)$

9) $\lim_{x\to 1}\left(\frac{-x^2-9x}{x}\right)$

Answer

-10

10) $\lim_{x\to 4}\left(\frac{7-\sqrt{12x+1}}{x-4}\right)$

11) $\lim_{x\to 3}\left(\frac{\frac{1}{3}+\frac{1}{x}}{3+x}\right)$

Answer

$-\frac{1}{9}$

5.3: Continuity

For the exercises 1-5, use numerical evidence to determine whether the limit exists at $x=a$. If not, describe the behavior of the graph of the function at $x=a$.

1) $f(x)=\frac{-2}{x-4};\ a=4$

2) $f(x)=\frac{-2}{(x-4)^2};\ a=4$

Answer

At $x=4$, the function has a vertical asymptote.

3) $f(x)=\frac{-x}{x^2-x-6};\ a=3$

4) $f(x)=\frac{6x^2+23x+20}{4x^2-25};\ a=-\frac{5}{2}$

Answer

removable discontinuity at $a=-\frac{5}{2}$

5) $f(x)=\frac{\sqrt{x}-3}{9-x};\ a=9$

For the exercises 6-12, determine where the given function $f(x)$ is continuous. Where it is not continuous, state which conditions fail, and classify any discontinuities.

6) $f(x)=x^2-2x-15$

Answer

continuous on $(-\infty,\infty)$

7) $f(x)=\dfrac{x^2-2x-15}{x-5}$

8) $f(x)=\dfrac{x^2-2x}{x^2-4x+4}$

Answer

removable discontinuity at $x=2$. $f(2)$ is not defined, but limits exist.

9) $f(x)=\dfrac{x^3-125}{2x^2-12x+10}$

10) $f(x)=\dfrac{x^2-\frac{1}{x}}{2-x}$

Answer

discontinuity at $x=0$ and $x=2$. Both $f(0)$ and $f(2)$ are not defined.

11) $f(x)=\dfrac{x+2}{x^2-3x-10}$

12) $f(x)=\dfrac{x+2}{x^3+8}$

Answer

removable discontinuity at $x=-2$. $f(-2)$ is not defined.

5.4: Derivatives

For the exercises 1-5, find the average rate of change $f(x)=\dfrac{f(x+h)-f(x)}{h}$.

1) $f(x)=3x+2$

2) $f(x)=5$

Answer

0

3) $f(x)=\dfrac{1}{x+1}$

4) $f(x)=\ln(x)$

Answer

$f(x)=\dfrac{\ln(x+h)-\ln(x)}{h}$

5) $f(x)=e^{2x}$

For the exercises 6-7, find the derivative of the function.

6) $f(x)=4x-6$

Answer

4

7) $f(x)=5x^2-3x$

8) Find the equation of the tangent line to the graph of $f(x)$ at the indicated x value.

$$f(x)=-x^3+4x;\ x=2$$

Answer

$y = -8x + 16$

9) For the following exercise, with the aid of a graphing utility, explain why the function is not differentiable everywhere on its domain. Specify the points where the function is not differentiable.

$$f(x) = \frac{x}{|x|}$$

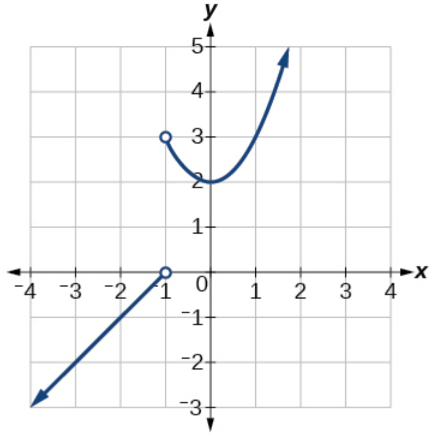
10) Given that the volume of a right circular cone is $V = \frac{1}{3}\pi r^2 h$ and that a given cone has a fixed height of 9 cm and variable radius length, find the instantaneous rate of change of volume with respect to radius length when the radius is 2 cm. Give an exact answer in terms of π.

Answer

12π

Practice Test

For the exercises 1-6, use the graph of f in the Figure below.



1) $f(1)$

Answer

3

2) $\lim\limits_{x \to -1^+} f(x)$

3) $\lim\limits_{x \to -1^-} f(x)$

Answer

0

4) $\lim\limits_{x \to -1} f(x)$

5) $\lim\limits_{x \to -2} f(x)$

Answer

-1

6) At what values of x is f discontinuous? What property of continuity is violated?

7) $f(x)=\begin{cases}\dfrac{1}{3}-3 & \text{if } x\leq 2\\ x^3+1 & \text{if } x>2\end{cases}$ $a=2$

Answer

$$\lim_{x\to 2^-} f(x)=-\frac{5}{2}a \text{ and } \lim_{x\to 2^+} f(x)=9$$

Thus, the limit of the function as x approaches 2 does not exist.

8) $f(x)=\begin{cases}x^3+1 & \text{if } x<1\\ 3x^2-1 & \text{if } x=1\\ -\sqrt{x+3}+4 & \text{if } x>1\end{cases}$ $a=1$

For the exercises 9-11, evaluate each limit using algebraic techniques.

9) $\lim\limits_{x\to -5}\left(\dfrac{\frac{1}{5}+\frac{1}{x}}{10+2x}\right)$

Answer

$-\dfrac{1}{50}$

10) $\lim\limits_{h\to 0}\left(\dfrac{\sqrt{h^2+25}-5}{h^2}\right)$

11) $\lim\limits_{h\to 0}\left(\dfrac{1}{h}-\dfrac{1}{h^2+h}\right)$

Answer

1

For the exercises 12-13, determine whether or not the given function f is continuous. If it is continuous, show why. If it is not continuous, state which conditions fail.

12) $f(x)=\sqrt{x^2-4}$

13) $f(x)=\dfrac{x^3-4x^2-9x+36}{x^3-3x^2+2x-6}$

Answer

removable discontinuity at $x=3$

For the exercises 14-16, use the definition of a derivative to find the derivative of the given function at $x=a$.

14) $f(x)=\dfrac{3}{5+2x}$

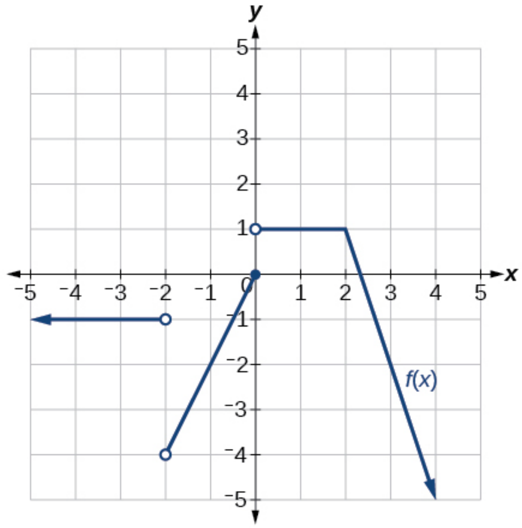
15) $f(x)=\dfrac{3}{\sqrt{x}}$

Answer

$$f'(x)=-\frac{3}{2a^{\frac{3}{2}}}$$

16) $f(x)=2x^2+9x$

17) For the graph in the Figure below, determine where the function is continuous/discontinuous and differentiable/not differentiable.



Answer

discontinuous at $-2, 0$, not differentiable at $-2, 0, 2$.

For the exercises 18-19, with the aid of a graphing utility, explain why the function is not differentiable everywhere on its domain. Specify the points where the function is not differentiable.

18) $f(x) = |x-2| - |x+2|$

19) $f(x) = \dfrac{2}{1+e^{\frac{2}{x}}}$

Answer

not differentiable at $x = 0$ (no limit)

For the exercises 20-24, explain the notation in words when the height of a projectile in feet, s, is a function of time t in seconds after launch and is given by the function $s(t)$.

20) $s(0)$

21) $s(2)$

Answer

the height of the projectile at $t = 2$ seconds

22) $s'(2)$

23) $\dfrac{s(2)-s(1)}{2-1}$

Answer

the average velocity from $t = 1$ to $t = 2$

24) $s(t) = 0$

For the exercises 25-28, use technology to evaluate the limit.

25) $\lim\limits_{x \to 0} \dfrac{\sin(x)}{3x}$

Answer

$\frac{1}{3}$

26) $\lim_{x\to 0} \frac{\tan^2(x)}{2x}$

27) $\lim_{x\to 0} \frac{\sin(x)(1-\cos(x))}{2x^2}$

Answer

0

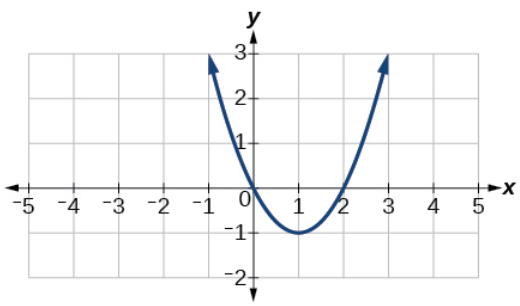
28) Evaluate the limit by hand.

$$\lim_{x\to 1} f(x),\ \text{where } f(x)=\begin{cases} 4x-7 & x\neq 1 \\ x^2-4 & x=1 \end{cases}$$

At what value(s) of x is the function below discontinuous?

$$f(x)=\begin{cases} 4x-7 & x\neq 1 \\ x^2-4 & x=1 \end{cases}$$

For the exercises 29-32, consider the function whose graph appears in Figure.



29) Find the average rate of change of the function from $x=1$ to $x=3$.

Answer

2

30) Find all values of x at which $f'(x)=0$.

Answer

$x=1$

31) Find all values of x at which $f'(x)$ does not exist.

32) Find an equation of the tangent line to the graph of f the indicated point: $f(x)=3x^2-2x-6,\ x=-2$

Answer

$y=-14x-18$

For the exercises 33-34, use the function $f(x)=x(1-x)^{\frac{2}{5}}$

33) Graph the function $f(x)=x(1-x)^{\frac{2}{5}}$ by entering $f(x)=x\left((1-x)^2\right)^{\frac{1}{5}}$ and then by entering $f(x)=x\left((1-x)^{\frac{1}{5}}\right)^2$.

34) Explore the behavior of the graph of $f(x)$ around $x=1$ by graphing the function on the following domains, $[0.9,1.1]$, $[0.99,1.01]$, $[0.999,1.001]$, and $[0.9999,1.0001]$. Use this information to determine whether the function appears to be differentiable at $x=1$.

Answer

The graph is not differentiable at $x = 1$ (cusp).

For the exercises 35-42, find the derivative of each of the functions using the definition: $\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

35) $f(x) = 2x - 8$

36) $f(x) = 4x^2 - 7$

Answer

$f'(x) = 8x$

37) $f(x) = x - \frac{1}{2}x^2$

38) $f(x) = \dfrac{1}{x+2}$

Answer

$$f'(x) = -\frac{1}{(2+x)^2}$$

39) $f(x) = \dfrac{3}{x-1}$

40) $f(x) = -x^3 + 1$

Answer

$f'(x) = -3x^2$

41) $f(x) = x^2 + x^3$

42) $f(x) = \sqrt{x-1}$

Answer

$$f'(x) = -\frac{1}{2\sqrt{x-1}}$$

Contributors and Attributions

- Template:OpenStaxPreCalc