10.6: Combinations- Involving Several Sets

Learning Objectives

In this section you will learn to

1. count the number of items selected from more than one set
2. count the number of items selected when there are restrictions on the selections

So far we have solved the basic combination problem of r objects chosen from n different objects. Now we will consider certain variations of this problem.

Example 10.6.1

How many five-people committees consisting of 2 men and 3 women can be chosen from a total of 4 men and 4 women?

Solution

We list 4 men and 4 women as follows:

$$M_1 M_2 M_3 M_4 W_1 W_2 W_3 W_4$$

Since we want 5-people committees consisting of 2 men and 3 women, we'll first form all possible two-man committees and all possible three-woman committees. Clearly there are 4C2 = 6 two-man committees, and 4C3 = 4 three-woman committees, we list them as follows:

2-Man Committees	3-Woman Committees
$\mathrm{M_1M_2}$ $\mathrm{M_1M_3}$ $\mathrm{M_1M_4}$ $\mathrm{M_2M_3}$ $\mathrm{M_2M_4}$ $\mathrm{M_3M_4}$	$\mathrm{W_1W_2W_3}$ $\mathrm{W_1W_2W_4}$ $\mathrm{W_1W_3W_4}$ $\mathrm{W_2W_3W_4}$

For every 2-man committee there are four 3-woman committees that can be chosen to make a 5-person committee. If we choose M_1M_2 as our 2-man committee, then we can choose any of $W_1W_2W_3$, $W_1W_2W_4$, $W_1W_3W_4$, or $W_2W_3W_4$ as our 3-woman committees. As a result, we get

$$\boxed{M_1M_2}W_1W_2W_3, \boxed{M_1M_2}W_1W_2W_4, \boxed{M_1M_2}W_1W_3W_4, \boxed{M_1M_2}W_2W_3W_4$$

Similarly, if we choose M_1M_3 as our 2-man committee, then, again, we can choose any of $W_1W_2W_3$, $W_1W_2W_4$, $W_1W_3W_4$, or $W_2W_3W_4$ as our 3-woman committees.

$$\boxed{M_1M_3}W_1W_2W_3, \boxed{M_1M_3}W_1W_2W_4, \boxed{M_1M_3}W_1W_3W_4, \boxed{M_1M_3}W_2W_3W_4$$

And so on.

Since there are six 2-man committees, and for every 2-man committee there are four 3-woman committees, there are altogether $6 \cdot 4 = 24$ five-people committees.

In essence, we are applying the multiplication axiom to the different combinations.

Example 10.6.2

A high school club consists of 4 freshmen, 5 sophomores, 5 juniors, and 6 seniors. How many ways can a committee of 4 people be chosen that includes

a. One student from each class?
b. All juniors?
c. Two freshmen and 2 seniors?
d. No freshmen?
e. At least three seniors?

Solution

a. Applying the multiplication axiom to the combinations involved, we get

$$(4C1)(5C1)(5C1)(6C1) = 600$$

b. We are choosing all 4 members from the 5 juniors, and none from the others.

$$5C4 = 5$$

c. $4C2 \cdot 6C2 = 90$

d. Since we don't want any freshmen on the committee, we need to choose all members from the remaining 16. That is

$$16C4 = 1820$$

e. Of the 4 people on the committee, we want at least three seniors. This can be done in two ways. We could have three seniors, and one non-senior, or all four seniors.

$$(6C3)(14C1) + 6C4 = 295$$

Example 10.6.3

How many five-letter word sequences consisting of 2 vowels and 3 consonants can be formed from the letters of the word INTRODUCE?

Solution

First we select a group of five letters consisting of 2 vowels and 3 consonants.
Since there are 4 vowels and 5 consonants, we have

$$(4C2)(5C3)$$

Since our next task is to make word sequences out of these letters, we multiply these by 5!.

$$(4C2)(5C3)(5!) = 7200.$$

Example 10.6.4

A standard deck of playing cards has 52 cards consisting of 4 suits each with 13 cards. In how many different ways can a 5-card hand consisting of four cards of one suit and one of another be drawn?

Solution

We will do the problem using the following steps.
Step 1. Select a suit.
Step 2. Select four cards from this suit.
Step 3. Select another suit.
Step 4. Select a card from that suit.

Applying the multiplication axiom, we have

Ways of selecting the first suit	Ways of selecting 4 cards from this suit	Ways of selecting the next suit	Ways of selecting a card from that suit
4C1	13C4	3C1	13C1

$(4C1)(13C4)(3C1)(13C1) = 111,540.$

A STANDARD DECK OF 52 PLAYING CARDS

As in the previous example, many examples and homework problems in this book refer to a standard deck of 52 playing cards. Before we end this section, we take a minute to describe a standard deck of playing cards, as some readers may not be familiar with this.

A standard deck of 52 playing cards has 4 suits with 13 cards in each suit.

♦ diamonds ♥ hearts ♠ spades ♣ clubs

Each suit is associated with a color, either black (spades, clubs) or red (diamonds, hearts)

Each suit contains 13 denominations (or values) for cards:

nine numbers 2, 3, 4, …., 10 and Jack(J), Queen (Q), King (K), Ace (A).

The Jack, Queen and King are called "face cards" because they have pictures on them. Therefore a standard deck has 12 face cards: (3 values JQK) x (4 suits ♦♥♠ ♣)

We can visualize the 52 cards by the following display

Suit	Color	Values (Denominations)
♦ Diamonds	Red	2 3 4 5 6 7 8 9 10 J Q K A
♥ Hearts	Red	2 3 4 5 6 7 8 9 10 J Q K A
♠ Spades	Black	2 3 4 5 6 7 8 9 10 J Q K A
♣ Clubs	Black	2 3 4 5 6 7 8 9 10 J Q K A