

6.6: Standard Scores

Suppose my friend is putting together a new questionnaire intended to measure “grumpiness”. The survey has 50 questions, which you can answer in a grumpy way or not. Across a big sample (hypothetically, let’s imagine a million people or so!) the data are fairly normally distributed, with the mean grumpiness score being 17 out of 50 questions answered in a grumpy way, and the standard deviation is 5. In contrast, when I take the questionnaire, I answer 35 out of 50 questions in a grumpy way. So, how grumpy am I? One way to think about would be to say that I have grumpiness of 35/50, so you might say that I’m 70% grumpy. But that’s a bit weird, when you think about it. If my friend had phrased her questions a bit differently, people might have answered them in a different way, so the overall distribution of answers could easily move up or down depending on the precise way in which the questions were asked. So, I’m only 70% grumpy *with respect to this set of survey questions*. Even if it’s a very good questionnaire, this isn’t very a informative statement.

A simpler way around this is to describe my grumpiness by comparing me to other people. Shockingly, out of my friend’s sample of 1,000,000 people, only 159 people were as grumpy as me (that’s not at all unrealistic, frankly), suggesting that I’m in the top 0.016% of people for grumpiness. This makes much more sense than trying to interpret the raw data. This idea – that we should describe my grumpiness in terms of the overall distribution of the grumpiness of humans – is the qualitative idea that standardisation attempts to get at. One way to do this is to do exactly what I just did, and describe everything in terms of percentiles. However, the problem with doing this is that “it’s lonely at the top”. Suppose that my friend had only collected a sample of 1000 people (still a pretty big sample for the purposes of testing a new questionnaire, I’d like to add), and this time gotten a mean of 16 out of 50 with a standard deviation of 5, let’s say. The problem is that almost certainly, not a single person in that sample would be as grumpy as me.

However, all is not lost. A different approach is to convert my grumpiness score into a **standard score**, also referred to as a z-score. The standard score is defined as the number of standard deviations above the mean that my grumpiness score lies. To phrase it in “pseudo-maths” the standard score is calculated like this:

$$\text{standard score} = \frac{\text{raw score} - \text{mean}}{\text{standard deviation}}$$

In actual maths, the equation for the z-score is

$$z_i = \frac{X_i - \bar{X}}{\hat{\sigma}}$$

So, going back to the grumpiness data, we can now transform Dan’s raw grumpiness into a standardised grumpiness score.⁷⁶ If the mean is 17 and the standard deviation is 5 then my standardised grumpiness score would be⁷⁷

$$z = \frac{35 - 17}{5} = 3.6$$

To interpret this value, recall the rough heuristic that I provided in Section 5.2.5, in which I noted that 99.7% of values are expected to lie within 3 standard deviations of the mean. So the fact that my grumpiness corresponds to a z score of 3.6 indicates that I’m very grumpy indeed. Later on, in Section 9.5, I’ll introduce a function called `pnorm()` that allows us to be a bit more precise than this. Specifically, it allows us to calculate a theoretical percentile rank for my grumpiness, as follows:

```
pnorm( 3.6 )
```

```
## [1] 0.9998409
```

At this stage, this command doesn’t make too much sense, but don’t worry too much about it. It’s not important for now. But the output is fairly straightforward: it suggests that I’m grumpier than 99.98% of people. Sounds about right.

In addition to allowing you to interpret a raw score in relation to a larger population (and thereby allowing you to make sense of variables that lie on arbitrary scales), standard scores serve a second useful function. Standard scores can be compared to one another in situations where the raw scores can’t. Suppose, for instance, my friend also had another questionnaire that measured extraversion using a 24 items questionnaire. The overall mean for this measure turns out to be 13 with standard deviation 4; and I scored a 2. As you can imagine, it doesn’t make a lot of sense to try to compare my raw score of 2 on the extraversion

questionnaire to my raw score of 35 on the grumpiness questionnaire. The raw scores for the two variables are “about” fundamentally different things, so this would be like comparing apples to oranges.

What about the standard scores? Well, this is a little different. If we calculate the standard scores, we get $z=(35-17)/5=3.6$ for grumpiness and $z=(2-13)/4=-2.75$ for extraversion. These two numbers *can* be compared to each other.⁷⁸ I’m much less extraverted than most people ($z=-2.75$) and much grumpier than most people ($z=3.6$); but the extent of my unusualness is much more extreme for grumpiness (since 3.6 is a bigger number than 2.75). Because each standardised score is a statement about where an observation falls *relative to its own population*, it is possible to compare standardised scores across completely different variables.

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