21.4.2: The Complement of a Set

Learning Outcomes

1. Determine the complement of a set.
2. Write the complement of a set using set notation.

We saw in the section "Represent an Inequality as an Interval on a Number Line" how to graph the complement for a set defined by an inequality. Complements come up very often in statistics, so it is worth revisiting this, but instead of graphically we will focus on set notation. Recall that the complement of a set is everything that is not in that set. Sometimes it is much easier to find the probability of a complement than of the original set, and there is an easy relationship between the probability of an event happening and the probability of the complement of that event happening.

$$P(A) = 1 - P(not\ A)$$

Example 21.4.2.1

Find the complement of the set:

$$A = \{x \mid x < 4\}$$

Solution

The complement of the set of all numbers that are less than 4 is the set of all numbers that are at least as big as 4. Notice that the number 4 is not in the set A, since the inequality is strict (does not have an "="). Therefore the number 4 is in the complement of the set A. In set notation:

$$A^c = \{x \mid x \geq 4\}$$

Example 21.4.2.2

When computing probabilities the complement is sometimes much easier than the original set. For example suppose you roll a die 6 times and want to find the probability that the number 3 comes up at least once. Find the complement of this event.

Solution

First note that the event of at least once means that there could be one 3, two 3's, three 3's, four 3's, five 3's, or six 3's. It turns out that this would be a burden to deal with each of these possibilities. However the complement is quite easy. The complement of getting at least one 3 is that you go no 3's.

Example 21.4.2.3

Suppose that we want to find the probability that at least 20 people in the class have done their homework. Find the complement of this event.

Solution

Sometimes it is easiest to list nearby outcomes and then determine the outcomes that satisfy the event. Finally, to find the complement, you select the rest. First list numbers near 20:

$$\ldots, 17, 18, 19, 20, 21, 22, \ldots$$

Now, the ones that are at least 20 are all the ones including 20 and to the right of 20:

$$20, 21, 22, \ldots$$

These are the large numbers. The complement includes all the small numbers.

$$\ldots, 17, 18, 19$$

We can write this in set notation as:

$$\{x \mid x \leq 19\}$$

or equivalently

$$\{x \mid x < 20\}$$

Example 21.4.2.4

Suppose a number is picked at random from the whole numbers from 1 to 10. Let A be the event that a number is both even and less than 8. Find the complement of A.

Solution

First, the set of numbers that are both even and less than 8 is:

$$A = \{2, 4, 6\}$$

The complement of this set is all the numbers from 1 to 10 that are not in A:

$$A^c = \{1, 3, 5, 7, 8, 9, 10\}$$

Exercise

Suppose that two six sided dice are rolled. Let the A be the event that either the first die is even or the sum of the dice is greater than 5 or both have occurred. Find the complement of A.

- Ex: Find the Intersection of a Set and A Complement Using a Venn Diagram
- https://youtu.be/ek3QwY2gw4w
