8.3 A Population Proportion

Section 8.3 A Population Proportion

Learning Objective:

In this section, you will:

- Apply and interpret point estimates and confidence intervals
- Determine adequate sample sizes needed to estimate population parameters
- Construct and interpret confidence intervals for population proportions

Proportion = Probability = Percent – Example: If 28% of scores are higher than yours, then the probability of a score being higher than yours is 0.28, and the proportion of scores higher than yours is 0.28

Point Estimate – the best estimate for a population proportion, *p*, is the sample proportion, $\hat{p}$.

$\hat{p} = x$ $\hat{q} = 1 - \hat{p}$

n

Margin of Error (E) – maximum difference between the sample proportion and the true value of the population proportion.

Confidence Interval – a range of values used to estimate the true value of a population parameter.

$\hat{p} \pm E$ **or** $\hat{p} - E < \mu < \hat{p} + E$ **or** $(\hat{p} - E, \hat{p} + E)$

Confidence level- the probability 1 – (usually expressed as a percentage) that the confidence interval actually does contain the population parameter, assuming that the estimation process is repeated a large number of times.

Example 1: The 90% confidence interval for the proportion of all students with a GPA over 3.5 is

$.0997 < p < .2203$.

Interpretation: "We estimate with 90% confident that the true value of the proportion of all students with a GPA over 3.5 is between 0.0997 and .2203." If we construct similar confidence intervals using sample proportions numerous times, we expect that 90% of those intervals would contain the true population proportion.

Calculating Confidence Intervals for population proportions:

Using the Graphing calculator TI-84: STAT, TESTS, A:1-PROPZINT

1-PropZInt(x, n, CL)

Example 2: When Mendel conducted his famous genetics experiments with peas, one sample of offspring consisted of 428 green peas and 152 yellow peas. Find a 95% confidence interval estimate of the percentage of green peas.

Example 2 (continued): Mendel expected that 75% of the peas would be green. Given that the percentage of green peas in our sample is not 75%, do the results contradict Mendel's theory? Why or why not?

Example 3: If 230 out of 600 teenagers plan to see the new Hunger Games movie, find a 90% confidence interval estimate for the percentage of all teenagers planning to see the movie.

Example 3 (continued): The movie theater claims that 25% of teenagers are planning to see the movie. Does their claim appear to be correct?

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Calculating the Minimum Sample Size

If we know our desired margin of error, we must have a large enough sample to guarantee the desired

2 $[z\alpha/2]$ $\hat{p}\hat{q}$ error. $n = E2$, when $\hat{p}$ and $\hat{q}$ are known

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$[z\alpha/2]$ 0.25

If we don't know $\hat{p}$ in advance, we use 0.5 for both $\hat{p}$ and $\hat{q}$. This gives us $n = E2$

Always round up to next whole number when determining sample size.

Example 4: Find the minimum sample size needed if the margin of error must be two percentage points, the confidence level is 99%, and the point estimate for the population proportion is 14%. Write an interpretation.

Example 5: A survey was conducted to determine the percentage of car owners who would pay to put nitrogen in their tires (Nitrogen supposedly leaks out at a slower rate than air, which keeps the tire pressure at the ideal level.) How many randomly selected car owners should be surveyed? Assume that we want to be 95% confident that the sample percentage is within 3% of the true percentage of all car owners who would be willing to pay for nitrogen. Write an interpretation.

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For more information and examples see online textbook OpenStax Introductory Statistics pages 460-467.