6.1 The Standard Normal Distribution

Section 6.1 The Standard Normal Distribution

Learning Objective:

In this section, you will:

- Solve and explain real-world applications of probability

The **standard normal distribution** is a normal distribution of **standardized values called *z*-scores**. **A *z*-score is measured in units of the standard deviation** with . = 0 and . = 1. The total area under its density curve is equal to 1.

Z-Score

x−μ

If *X* is a normally distributed random variable and *X* ~ *N(μ, σ)*, then the *z*-score is: z=

σ

- The *z*-score tells you how many standard deviations the value *x* is above (to the right of) or below (to the left of) the mean, *μ*.
- Values of *x* that are larger than the mean have positive *z*-scores, and values of *x* that are smaller than the mean have negative *z*-scores. If *x* equals the mean, then *x* has a *z*-score of zero.
- The *z*-score allows us to compare data that are scaled differently.

Example 1: Suppose X ~ N(5, 6). This says that X is a normally distributed random variable with mean μ = 5 and standard deviation σ= 6. Suppose x= 17. Find the z-score of x.

Example 2: Suppose X ~ N(8, 1). What value of x has a z-score of -2.25?

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Example 3: Heights of women have a normal distribution with a mean of 161 cm and a standard deviation of 7 cm.

1. What is the z-score for a woman who is 170 cm tall?
2. How many standard deviations away from the mean is a woman who is 170cm tall?

Example 4: Some doctors believe that a person can lose five pounds, on the average, in a month by reducing his or her fat in take and by exercising consistently. Suppose weight loss has a normal distribution. Let X = the amount of weight lost (in pounds) by a person in a month. Use a standard deviation of two pounds. X~N(5, 2).

1. Suppose a person lost ten pounds in a month. Find the z-score.
2. Fill in the blanks: The z-score when x = 10 pounds is z = 2.5. This z-score tells you that x = 10 is

________ standard deviations to the ________ (right or left) of the mean _____ (What is the mean?).

3. Fill in the blanks: Suppose a person gained three pounds (a negative weight loss). Then z =

__________. This z-score tells you that x = –3 is ________ standard deviations to the __________ (right or left) of the mean.

4. Suppose the random variables X and Y have the following normal distributions: X ~ N(5, 6) and Y ~ N(2, 1). If x= 17, then z= 2. (This was previously shown.) If y= 4, what is z?

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Example 5: Compare data that are scaled differently. Which is a better score? A score of 76 on a quiz where the mean score was 64 and the standard deviation was 6.7, or a score of 12 on a quiz with a mean of 10 and a standard deviation of 1.1?

Empirical Rule

If *X* is a random variable and has a normal distribution with mean *μ* and standard deviation *σ*, then the Empirical Rule says the following:

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- About 99.7% of the *x* values lie between -3σ and $+3\sigma$ of the mean μ Notice that almost all the *x* values lie within three standard deviations of the mean. The empirical rule is also known as the 68-95-99.7 rule.

Example 6: Heights of women have a normal distribution with a mean of 161 cm and a standard deviation of 7 cm.

1. Approximately what is the percentage of women between 147 cm and 175 cm?

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2. About 99.7% of women are between what two heights?
3. What percentage of women are between 154 and 182 cm

Example 7: The mean height of 15 to 18-year-old males from Chile from 2009 to 2010 was 170 cm with a standard deviation of 6.28 cm. Male heights are known to follow a normal distribution. Let X = the height of a 15 to 18-year-old male from Chile in 2009 to 2010. Then X~N(170, 6.28).

1. Suppose a 15 to 18-year-old male from Chile was 168cm tall from 2009 to 2010. The z-score when x = 168cm is z =_______. This z-score tells you that x = 168 is ________ standard deviations to the ________ (right or left) of the mean _____ (What is the mean?).
2. Suppose that the height of a 15 to 18-year-old male from Chile from 2009 to 2010 has a z-score of z = 1.27. What is the male's height? The z-score (z=1.27) tells you that the male's height is ________ standard deviations to the __________ (right or left) of the mean.

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For more information and examples see online textbook OpenStax Introductory Statistics pages 366371.