8.1 A Single Population Mean using the Normal Distribution

Section 8.1 A Single Population Mean using the Normal Distribution

Learning Objective:

In this section, you will:

- Apply and interpret point estimates and confidence intervals
- Determine adequate sample sizes needed to estimate population parameters
- Construct and interpret confidence intervals for population means and proportions

We use sample data to make generalizations about an unknown population. This part of statistics is called **inferential statistics**. The sample data help us to make an estimate of a population parameter.

Point Estimate – a single number computed from a sample and used to estimate a population parameter. $\boldsymbol{\mu} = \boldsymbol{mean}, \hat{\boldsymbol{p}} = \boldsymbol{proportion}$

A **confidence interval** is another type of estimate but, instead of being just one number, it is an interval of numbers. The interval of numbers is a range of values calculated from a given set of sample data. The confidence interval is likely to include an unknown population parameter.

$1.8 < \mu < 2.2$ $0.37 < p < 0.43$

Margin of Error (E) **–** maximum difference between the sample mean and the true value of the population mean.

Confidence Level (CL) - the probability that the confidence interval contains the true population parameter; for example, if the CL=90%, then in 90 out of 100 samples the interval estimate will enclose the true population parameter.

α (alpha) - is related to the confidence level. α is the probability that the interval does not contain the unknown population parameter.

- $\boldsymbol{CL = 1 - \alpha \cdot \alpha = 1 - CL}$
- $\boldsymbol{CL + \alpha = 1}$

Confidence intervals for some parameters have the form:

(point estimate – margin of error, point estimate + margin of error)

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Confidence interval for population mean ()

Point estimate for population mean: $\bar{x}$ =

Normal requirement: Either n > 30 **or** population is roughly normally distributed

Confidence Interval: $\boldsymbol{\bar{x} \pm E}$ **or** $\boldsymbol{\bar{x} - E} < \mu < \boldsymbol{\bar{x} + E}$ **or** $(\boldsymbol{\bar{x} - E}, \boldsymbol{\bar{x} + E})$

Example 1: Suppose we have collected data from a sample. We know the sample mean but we do not know the mean for the entire population. The sample mean is seven, and the error bound for the mean is 2.5.

If the confidence level (CL) is 95%, then we say that, "We estimate with 95% confidence that the true value of the population mean is between 4.5 and 9.5."

Example 2: Express the interval 230 < μ < 452 in the $\bar{x}$ ± E

Example 3: Express the interval 34 ± 108 in the form $\bar{x}$ – E < < $\bar{x}$ + E

Example 4: Given the confidence interval (1819, 3457), find $\bar{x}$ and E

Calculating the confidence interval for population mean () when is known **Using the Graphing calculator TI-84**: STAT, TESTS, 7:ZInterval (Data or Stats)

ZInterval (Stats, σ, $\bar{x}$, n, CL) or Enter data L1, ZInterval (Data, σ, L1, 1, CL)

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Example 5: Suppose scores on exams in statistics are normally distributed with an unknown population mean and a population standard deviation of three points. A random sample of 36 scores is taken and gives a sample mean of 68. Find a 90% confidence interval for the true (population) mean of statistics exam scores. Write an interpretation.

Example 6: Randomly selected statistics students participated in an experiment to test their ability to determine when 1 minute (60 seconds) has passed. Forty students yielded a sample mean of 58.3 seconds. Assume that = 9.5 seconds. Construct a 95% confidence interval estimate of the population mean of all statistics students. Based on the results, is it likely that their estimates have a mean that is reasonably close to 60 seconds?

Example 7: When medical students all measured the blood pressure of the same person, they obtained the following results: 138, 130, 135, 140, 120, 125, 120, 130, 130, 144, 143, 140, 130, 150. Assuming that the population standard deviation is 10 mmHg, construct a 99% confidence interval estimate of the population mean. Write an interpretation.

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Critical Values – a z-score on the borderline separating sample statistics that are likely to occur by chance from those that are unlikely. Z/2 is the z-score that separates an area of /2 in the right tail of the standard normal distribution.

Example 8: Find the critical value, Z/2 corresponding to a 90% confidence level.

Calculating the Sample Size n for the Mean

If we know our desired margin of error, we must have a large enough sample to guarantee the

$z\alpha/2\cdot\sigma$ 2 desired error. n = [] . Always round up to next whole number when determining

E

sample size.

Example 9: The population standard deviation for the age of Foothill College students is 15 years. If we want to be 95% confident that the sample mean age is within two years of the true population mean age of Foothill College students, how many randomly selected Foothill College students must be surveyed? Write an interpretation.

Example 10: How many daily rainfall amounts in Boston must be randomly selected to estimate the mean daily rainfall amount? We want 90% confidence that the sample mean is within 0.01 in. of the population mean, and the population standard deviation is known to be 0.212 in. Write an interpretation.

For more information and examples see online textbook OpenStax Introductory Statistics pages 443-456.