

10.3: Two Population Means with Known Standard Deviations

Even though this situation is not likely (knowing the population standard deviations is not likely), the following example illustrates hypothesis testing for independent means, known population standard deviations. The sampling distribution for the difference between the means is normal and both populations must be normal. The random variable is $\bar{X}_1 - \bar{X}_2$. The normal distribution has the following format:

Normal distribution is:

$$\bar{X}_1 - \bar{X}_2 \sim N \left[\mu_1 - \mu_2, \sqrt{\frac{(\sigma_1)^2}{n_1} + \frac{(\sigma_2)^2}{n_2}} \right] \quad (10.3.1)$$

The standard deviation is:

$$\sqrt{\frac{(\sigma_1)^2}{n_1} + \frac{(\sigma_2)^2}{n_2}} \quad (10.3.2)$$

The test statistic (z-score) is:

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{(\sigma_1)^2}{n_1} + \frac{(\sigma_2)^2}{n_2}}} \quad (10.3.3)$$

✓ Example 10.3.1

Independent groups, population standard deviations known: The mean lasting time of two competing floor waxes is to be compared. Twenty floors are randomly assigned to test each wax. Both populations have a normal distributions. The data are recorded in Table.

Wax	Sample Mean Number of Months Floor Wax Lasts	Population Standard Deviation
1	3	0.33
2	2.9	0.36

Does the data indicate that **wax 1 is more effective than wax 2**? Test at a 5% level of significance.

Answer

This is a test of two independent groups, two population means, population standard deviations known.

Random Variable: $\bar{X}_1 - \bar{X}_2$ = difference in the mean number of months the competing floor waxes last.

- $H_0 : \mu_1 \leq \mu_2$
- $H_a : \mu_1 > \mu_2$

The words "**is more effective**" says that **wax 1 lasts longer than wax 2**, on average. "Longer" is a ">" symbol and goes into H_a . Therefore, this is a right-tailed test.

Distribution for the test: The population standard deviations are known so the distribution is normal. Using Equation 10.3.1, the distribution is:

$$\bar{X}_1 - \bar{X}_2 \sim N \left(0, \sqrt{\frac{0.33^2}{20} + \frac{0.36^2}{20}} \right) \quad (10.3.4)$$

Since $\mu_1 \leq \mu_2$ then $\mu_1 - \mu_2 \leq 0$ and the mean for the normal distribution is zero.

Calculate the p-value using the normal distribution: $p\text{-value} = 0.1799$

Graph:

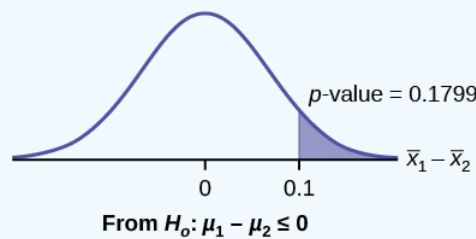


Figure 10.3.1.

$$\bar{X}_1 - \bar{X}_2 = 3 - 2.9 = 0.1$$

Compare α and the p -value: $\alpha = 0.05$ and $p\text{-value} = 0.1799$. Therefore, $\alpha < p\text{-value}$.

Make a decision: Since $\alpha < p\text{-value}$, do not reject H_0 .

Conclusion: At the 5% level of significance, from the sample data, there is not sufficient evidence to conclude that the mean time wax 1 lasts is longer (wax 1 is more effective) than the mean time wax 2 lasts.

Press **STAT**. Arrow over to **TESTS** and press **3:2-SampZTest**. Arrow over to **Stats** and press **ENTER**. Arrow down and enter **.33** for sigma1, **.36** for sigma2, **3** for the first sample mean, **20** for n1, **2.9** for the second sample mean, and **20** for n2. Arrow down to μ_1 : and arrow to $> \mu_2$. Press **ENTER**. Arrow down to **Calculate** and press **ENTER**. The p -value is $p = 0.1799$ and the test statistic is 0.9157. Do the procedure again, but instead of **Calculate** do **Dra**.

? Exercise 10.3.1

The means of the number of revolutions per minute of two competing engines are to be compared. Thirty engines are randomly assigned to be tested. Both populations have normal distributions. Table shows the result. Do the data indicate that Engine 2 has higher RPM than Engine 1? Test at a 5% level of significance.

Engine	Sample Mean Number of RPM	Population Standard Deviation
1	1,500	50
2	1,600	60

Answer

The p -value is almost 0, so we reject the null hypothesis. There is sufficient evidence to conclude that Engine 2 runs at a higher RPM than Engine 1.

✓ Example 10.3.2: Age of Senators

An interested citizen wanted to know if Democratic U. S. senators are older than Republican U.S. senators, on average. On May 26 2013, the mean age of 30 randomly selected Republican Senators was 61 years 247 days old (61.675 years) with a standard deviation of 10.17 years. The mean age of 30 randomly selected Democratic senators was 61 years 257 days old (61.704 years) with a standard deviation of 9.55 years.

Do the data indicate that Democratic senators are older than Republican senators, on average? Test at a 5% level of significance.

Answer

This is a test of two independent groups, two population means. The population standard deviations are unknown, but the sum of the sample sizes is $30 + 30 = 60$, which is greater than 30, so we can use the normal approximation to the Student's-t distribution. Subscripts: 1: Democratic senators 2: Republican senators

Random variable: $\bar{X}_1 - \bar{X}_2$ = difference in the mean age of Democratic and Republican U.S. senators.

- $H_0 : \mu_1 \leq \mu_2$ $H_0 : \mu_1 - \mu_2 \leq 0$

- $H_a : \mu_1 > \mu_2$ $H_a : \mu_1 - \mu_2 > 0$

The words "older than" translates as a ">" symbol and goes into H_a . Therefore, this is a right-tailed test.

Distribution for the test: The distribution is the normal approximation to the Student's t for means, independent groups. Using the formula, the distribution is:

$$\bar{X}_1 - \bar{X}_2 \sim N \left[0, \sqrt{\frac{(9.55)^2}{30} + \frac{(10.17)^2}{30}} \right] \quad (10.3.5)$$

Since $\mu_1 \leq \mu_2$, $\mu_1 - \mu_2 \leq 0$ and the mean for the normal distribution is zero.

(Calculating the p -value using the normal distribution gives p -value = 0.4040)

Graph:

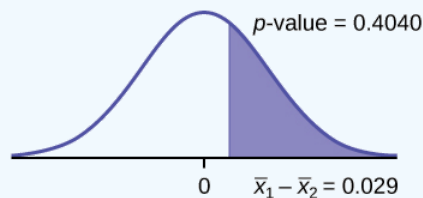


Figure 10.3.2.

Compare α and the p -value: $\alpha = 0.05$ and p -value = 0.4040. Therefore, $\alpha < p$ -value.

Make a decision: Since $\alpha < p$ -value, do not reject H_0 .

Conclusion: At the 5% level of significance, from the sample data, there is not sufficient evidence to conclude that the mean age of Democratic senators is greater than the mean age of the Republican senators.

References

1. Data from the United States Census Bureau. Available online at www.census.gov/prod/cen2010/b...c2010br-02.pdf
2. Hinduja, Sameer. "Sexting Research and Gender Differences." Cyberbullying Research Center, 2013. Available online at cyberbullying.us/blog/sexting...r-differences/ (accessed June 17, 2013).
3. "Smart Phone Users, By the Numbers." Visually, 2013. Available online at <http://visual.ly/smart-phone-users-numbers> (accessed June 17, 2013).
4. Smith, Aaron. "35% of American adults own a Smartphone." Pew Internet, 2013. Available online at www.pewinternet.org/~media/F...martphones.pdf (accessed June 17, 2013).
5. "State-Specific Prevalence of Obesity Among Adults—United States, 2007." MMWR, CDC. Available online at <http://www.cdc.gov/mmwr/preview/mmwrhtml/mm5728a1.htm> (accessed June 17, 2013).
6. "Texas Crime Rates 1960–2012." FBI, Uniform Crime Reports, 2013. Available online at: <http://www.disastercenter.com/crime/txcrime.htm> (accessed June 17, 2013).

Review

A hypothesis test of two population means from independent samples where the population standard deviations are known will have these characteristics:

- Random variable: $\bar{X}_1 - \bar{X}_2 =$ the difference of the means
- Distribution: normal distribution

Formula Review

Normal Distribution:

$$\bar{X}_1 - \bar{X}_2 \sim N \left[\mu_1 - \mu_2, \sqrt{\frac{(\sigma_1)^2}{n_1} + \frac{(\sigma_2)^2}{n_2}} \right] \quad (10.3.6)$$

Generally $\mu_1 - \mu_2 = 0$.

Test Statistic (z-score):

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{(\sigma_1)^2}{n_1} + \frac{(\sigma_2)^2}{n_2}}} \quad (10.3.7)$$

Generally $\mu_1 - \mu_2 = 0$.

where:

σ_1 and σ_2 are the known population standard deviations. n_1 and n_2 are the sample sizes. \bar{x}_1 and \bar{x}_2 are the sample means. μ_1 and μ_2 are the population means

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