9.8: Hypothesis Testing with One Sample (Exercises)

These are homework exercises to accompany the Textmap created for "Introductory Statistics" by OpenStax.

9.1: Introduction

9.2: Null and Alternative Hypotheses

Q 9.2.1

Some of the following statements refer to the null hypothesis, some to the alternate hypothesis.

State the null hypothesis, H_0, and the alternative hypothesis. H_a, in terms of the appropriate parameter $(\mu \text{or} p)$.

a. The mean number of years Americans work before retiring is 34.
b. At most 60% of Americans vote in presidential elections.
c. The mean starting salary for San Jose State University graduates is at least $100,000 per year.
d. Twenty-nine percent of high school seniors get drunk each month.
e. Fewer than 5% of adults ride the bus to work in Los Angeles.
f. The mean number of cars a person owns in her lifetime is not more than ten.
g. About half of Americans prefer to live away from cities, given the choice.
h. Europeans have a mean paid vacation each year of six weeks.
i. The chance of developing breast cancer is under 11% for women.
j. Private universities' mean tuition cost is more than $20,000 per year.

S 9.2.1

a. $H_0 : \mu = 34; H_a : \mu \neq 34$
b. $H_0 : p \leq 0.60; H_a : p > 0.60$
c. $H_0 : \mu \geq 100,000; H_a : \mu < 100,000$
d. $H_0 : p = 0.29; H_a : p \neq 0.29$
e. $H_0 : p = 0.05; H_a : p < 0.05$
f. $H_0 : \mu \leq 10; H_a : \mu > 10$
g. $H_0 : p = 0.50; H_a : p \neq 0.50$
h. $H_0 : \mu = 6; H_a : \mu \neq 6$
i. $H_0 : p \geq 0.11; H_a : p < 0.11$
j. $H_0 : \mu \leq 20,000; H_a : \mu > 20,000$

Q 9.2.2

Over the past few decades, public health officials have examined the link between weight concerns and teen girls' smoking. Researchers surveyed a group of 273 randomly selected teen girls living in Massachusetts (between 12 and 15 years old). After four years the girls were surveyed again. Sixty-three said they smoked to stay thin. Is there good evidence that more than thirty percent of the teen girls smoke to stay thin? The alternative hypothesis is:

a. $p < 0.30$
b. $p \leq 0.30$
c. $p \geq 0.30$
d. $p > 0.30$

Q 9.2.3

A statistics instructor believes that fewer than 20% of Evergreen Valley College (EVC) students attended the opening night midnight showing of the latest Harry Potter movie. She surveys 84 of her students and finds that 11 attended the midnight showing. An appropriate alternative hypothesis is:

a. $p = 0.20$
b. $p > 0.20$
c. $p < 0.20$
d. $p \leq 0.20$

S 9.2.3

c

Q 9.2.4

Previously, an organization reported that teenagers spent 4.5 hours per week, on average, on the phone. The organization thinks that, currently, the mean is higher. Fifteen randomly chosen teenagers were asked how many hours per week they spend on the phone. The sample mean was 4.75 hours with a sample standard deviation of 2.0. Conduct a hypothesis test. The null and alternative hypotheses are:

a. $H_0 : \bar{x} = 4.5, H_a : \bar{x} > 4.5$
b. $H_0 : \mu \geq 4.5, H_a : \mu < 4.5$
c. $H_0 : \mu = 4.75, H_a : \mu > 4.75$
d. $H_0 : \mu = 4.5, H_a : \mu > 4.5$

9.3: Outcomes and the Type I and Type II Errors

Q 9.3.1

State the Type I and Type II errors in complete sentences given the following statements.

a. The mean number of years Americans work before retiring is 34.
b. At most 60% of Americans vote in presidential elections.
c. The mean starting salary for San Jose State University graduates is at least $100,000 per year.
d. Twenty-nine percent of high school seniors get drunk each month.
e. Fewer than 5% of adults ride the bus to work in Los Angeles.
f. The mean number of cars a person owns in his or her lifetime is not more than ten.
g. About half of Americans prefer to live away from cities, given the choice.
h. Europeans have a mean paid vacation each year of six weeks.
i. The chance of developing breast cancer is under 11% for women.
j. Private universities mean tuition cost is more than $20,000 per year.

S 9.3.1

a. Type I error: We conclude that the mean is not 34 years, when it really is 34 years. Type II error: We conclude that the mean is 34 years, when in fact it really is not 34 years.
b. Type I error: We conclude that more than 60% of Americans vote in presidential elections, when the actual percentage is at most 60%.Type II error: We conclude that at most 60% of Americans vote in presidential elections when, in fact, more than 60% do.
c. Type I error: We conclude that the mean starting salary is less than $100,000, when it really is at least $100,000. Type II error: We conclude that the mean starting salary is at least $100,000 when, in fact, it is less than $100,000.
d. Type I error: We conclude that the proportion of high school seniors who get drunk each month is not 29%, when it really is 29%. Type II error: We conclude that the proportion of high school seniors who get drunk each month is 29% when, in fact, it is not 29%.
e. Type I error: We conclude that fewer than 5% of adults ride the bus to work in Los Angeles, when the percentage that do is really 5% or more. Type II error: We conclude that 5% or more adults ride the bus to work in Los Angeles when, in fact, fewer that 5% do.
f. Type I error: We conclude that the mean number of cars a person owns in his or her lifetime is more than 10, when in reality it is not more than 10. Type II error: We conclude that the mean number of cars a person owns in his or her lifetime is not more than 10 when, in fact, it is more than 10.
g. Type I error: We conclude that the proportion of Americans who prefer to live away from cities is not about half, though the actual proportion is about half. Type II error: We conclude that the proportion of Americans who prefer to live away from cities is half when, in fact, it is not half.
h. Type I error: We conclude that the duration of paid vacations each year for Europeans is not six weeks, when in fact it is six weeks. Type II error: We conclude that the duration of paid vacations each year for Europeans is six weeks when, in fact, it is not.

i. Type I error: We conclude that the proportion is less than 11%, when it is really at least 11%. Type II error: We conclude that the proportion of women who develop breast cancer is at least 11%, when in fact it is less than 11%.
j. Type I error: We conclude that the average tuition cost at private universities is more than $20,000, though in reality it is at most $20,000. Type II error: We conclude that the average tuition cost at private universities is at most $20,000 when, in fact, it is more than $20,000.

Q 9.3.2

For statements a-j in Exercise 9.109, answer the following in complete sentences.

a. State a consequence of committing a Type I error.
b. State a consequence of committing a Type II error.

Q 9.3.3

When a new drug is created, the pharmaceutical company must subject it to testing before receiving the necessary permission from the Food and Drug Administration (FDA) to market the drug. Suppose the null hypothesis is "the drug is unsafe." What is the Type II Error?

a. To conclude the drug is safe when in, fact, it is unsafe.
b. Not to conclude the drug is safe when, in fact, it is safe.
c. To conclude the drug is safe when, in fact, it is safe.
d. Not to conclude the drug is unsafe when, in fact, it is unsafe.

S 9.3.3

b

Q 9.3.4

A statistics instructor believes that fewer than 20% of Evergreen Valley College (EVC) students attended the opening midnight showing of the latest Harry Potter movie. She surveys 84 of her students and finds that 11 of them attended the midnight showing. The Type I error is to conclude that the percent of EVC students who attended is ________.

a. at least 20%, when in fact, it is less than 20%.
b. 20%, when in fact, it is 20%.
c. less than 20%, when in fact, it is at least 20%.
d. less than 20%, when in fact, it is less than 20%.

Q 9.3.4

It is believed that Lake Tahoe Community College (LTCC) Intermediate Algebra students get less than seven hours of sleep per night, on average. A survey of 22 LTCC Intermediate Algebra students generated a mean of 7.24 hours with a standard deviation of 1.93 hours. At a level of significance of 5%, do LTCC Intermediate Algebra students get less than seven hours of sleep per night, on average?

The Type II error is not to reject that the mean number of hours of sleep LTCC students get per night is at least seven when, in fact, the mean number of hours

a. is more than seven hours.
b. is at most seven hours.
c. is at least seven hours.
d. is less than seven hours.

S 9.3.4

d

Q 9.3.5

Previously, an organization reported that teenagers spent 4.5 hours per week, on average, on the phone. The organization thinks that, currently, the mean is higher. Fifteen randomly chosen teenagers were asked how many hours per week they spend on the phone. The sample mean was 4.75 hours with a sample standard deviation of 2.0. Conduct a hypothesis test, the Type I error is:

a. to conclude that the current mean hours per week is higher than 4.5, when in fact, it is higher
b. to conclude that the current mean hours per week is higher than 4.5, when in fact, it is the same
c. to conclude that the mean hours per week currently is 4.5, when in fact, it is higher
d. to conclude that the mean hours per week currently is no higher than 4.5, when in fact, it is not higher

9.4: Distribution Needed for Hypothesis Testing

Q 9.4.1

It is believed that Lake Tahoe Community College (LTCC) Intermediate Algebra students get less than seven hours of sleep per night, on average. A survey of 22 LTCC Intermediate Algebra students generated a mean of 7.24 hours with a standard deviation of 1.93 hours. At a level of significance of 5%, do LTCC Intermediate Algebra students get less than seven hours of sleep per night, on average? The distribution to be used for this test is $\bar{X} \sim$ ________________

a. $N\left(7.24, \frac{1.93}{\sqrt{22}}\right)$
b. $N(7.24, 1.93)$
c. t_{22}
d. t_{21}

S 9.4.1

d

9.5: Rare Events, the Sample, Decision and Conclusion

Q 9.5.1

The National Institute of Mental Health published an article stating that in any one-year period, approximately 9.5 percent of American adults suffer from depression or a depressive illness. Suppose that in a survey of 100 people in a certain town, seven of them suffered from depression or a depressive illness. Conduct a hypothesis test to determine if the true proportion of people in that town suffering from depression or a depressive illness is lower than the percent in the general adult American population.

a. Is this a test of one mean or proportion?
b. State the null and alternative hypotheses.
   H_0 : ____________________ H_a : ____________________
c. Is this a right-tailed, left-tailed, or two-tailed test?
d. What symbol represents the random variable for this test?
e. In words, define the random variable for this test.
f. Calculate the following:
   i. $x =$ ________________
   ii. $n =$ ________________
   iii. $p' =$ _____________
g. Calculate $\sigma_x =$ __________. Show the formula set-up.
h. State the distribution to use for the hypothesis test.
i. Find the p-value.
j. At a pre-conceived $\alpha = 0.05$, what is your:
   i. Decision:
   ii. Reason for the decision:
   iii. Conclusion (write out in a complete sentence):

9.6: Additional Information and Full Hypothesis Test Examples

For each of the word problems, use a solution sheet to do the hypothesis test. The solution sheet is found in [link]. Please feel free to make copies of the solution sheets. For the online version of the book, it is suggested that you copy the .doc or the .pdf files.

Note

If you are using a Student's t-distribution for one of the following homework problems, you may assume that the underlying population is normally distributed. (In general, you must first prove that assumption, however.)

Q 9.6.1.

A particular brand of tires claims that its deluxe tire averages at least 50,000 miles before it needs to be replaced. From past studies of this tire, the standard deviation is known to be 8,000. A survey of owners of that tire design is conducted. From the 28 tires surveyed, the mean lifespan was 46,500 miles with a standard deviation of 9,800 miles. Using $\alpha = 0.05$, is the data highly inconsistent with the claim?

S 9.6.1

a. $H_0 : \mu \geq 50,000$
b. $H_a : \mu < 50,000$
c. Let $\bar{X} =$ the average lifespan of a brand of tires.
d. normal distribution
e. $z = -2.315$
f. $p\text{-value} = 0.0103$
g. Check student’s solution.
h. i. alpha: 0.05
   ii. Decision: Reject the null hypothesis.
   iii. Reason for decision: The p-value is less than 0.05.
   iv. Conclusion: There is sufficient evidence to conclude that the mean lifespan of the tires is less than 50,000 miles.
i. $(43,537,49,463)$

Q 9.6.2

From generation to generation, the mean age when smokers first start to smoke varies. However, the standard deviation of that age remains constant of around 2.1 years. A survey of 40 smokers of this generation was done to see if the mean starting age is at least 19. The sample mean was 18.1 with a sample standard deviation of 1.3. Do the data support the claim at the 5% level?

Q 9.6.3

The cost of a daily newspaper varies from city to city. However, the variation among prices remains steady with a standard deviation of 20¢. A study was done to test the claim that the mean cost of a daily newspaper is $1.00. Twelve costs yield a mean cost of 95¢ with a standard deviation of 18¢. Do the data support the claim at the 1% level?

S 9.6.3

a. $H_0 : \mu = \$1.00$
b. $H_a : \mu \neq \$1.00$
c. Let $\bar{X} =$ the average cost of a daily newspaper.
d. normal distribution
e. $z =-0.866$
f. $p\text{-value} = 0.3865$
g. Check student’s solution.
h. i. $\alpha : 0.01$
   ii. Decision: Do not reject the null hypothesis.
   iii. Reason for decision: The p-value is greater than 0.01.
   iv. Conclusion: There is sufficient evidence to support the claim that the mean cost of daily papers is $1. The mean cost could be $1.
i. $(\$0.84, \$1.06)$

Q 9.6.4

An article in the *San Jose Mercury News* stated that students in the California state university system take 4.5 years, on average, to finish their undergraduate degrees. Suppose you believe that the mean time is longer. You conduct a survey of 49 students and obtain a sample mean of 5.1 with a sample standard deviation of 1.2. Do the data support your claim at the 1% level?

Q 9.6.5

The mean number of sick days an employee takes per year is believed to be about ten. Members of a personnel department do not believe this figure. They randomly survey eight employees. The number of sick days they took for the past year are as follows: 12; 4; 15; 3; 11; 8; 6; 8. Let $x =$ the number of sick days they took for the past year. Should the personnel team believe that the mean number is ten?

S 9.6.5

a. $H_0 : \mu = 10$
b. $H_a : \mu \neq 10$
c. Let $\bar{X}$ the mean number of sick days an employee takes per year.
d. Student's *t*-distribution
e. $t = -1.12$
f. $p\text{-value} = 0.300$
g. Check student's solution.
h. i. $\alpha : 0.05$
   ii. Decision: Do not reject the null hypothesis.
   iii. Reason for decision: The $p\text{-value}$ is greater than 0.05.
   iv. Conclusion: At the 5% significance level, there is insufficient evidence to conclude that the mean number of sick days is not ten.
i. $(4.9443, 11.806)$

Q 9.6.6

In 1955, *Life Magazine* reported that the 25 year-old mother of three worked, on average, an 80 hour week. Recently, many groups have been studying whether or not the women's movement has, in fact, resulted in an increase in the average work week for women (combining employment and at-home work). Suppose a study was done to determine if the mean work week has increased. 81 women were surveyed with the following results. The sample mean was 83; the sample standard deviation was ten. Does it appear that the mean work week has increased for women at the 5% level?

Q 9.6.7

Your statistics instructor claims that 60 percent of the students who take her Elementary Statistics class go through life feeling more enriched. For some reason that she can't quite figure out, most people don't believe her. You decide to check this out on your own. You randomly survey 64 of her past Elementary Statistics students and find that 34 feel more enriched as a result of her class. Now, what do you think?

S 9.6.7

a. $H_0 : p \geq 0.6$
b. $H_a : p < 0.6$
c. Let $P' =$ the proportion of students who feel more enriched as a result of taking Elementary Statistics.
d. normal for a single proportion
e. 1.12
f. $p\text{-value} = 0.1308$
g. Check student's solution.
h. i. $\alpha : 0.05$
   ii. Decision: Do not reject the null hypothesis.
   iii. Reason for decision: The $p\text{-value}$ is greater than 0.05.
   iv. Conclusion: There is insufficient evidence to conclude that less than 60 percent of her students feel more enriched.
i. Confidence Interval: $(0.409, 0.654)$

   The "plus-4s" confidence interval is $(0.411, 0.648)$

Q 9.6.8

A Nissan Motor Corporation advertisement read, "The average man's I.Q. is 107. The average brown trout's I.Q. is 4. So why can't man catch brown trout?" Suppose you believe that the brown trout's mean I.Q. is greater than four. You catch 12 brown trout. A fish psychologist determines the I.Q.s as follows: 5; 4; 7; 3; 6; 4; 5; 3; 6; 3; 8; 5. Conduct a hypothesis test of your belief.

Q 9.6.9

Refer to Exercise 9.119. Conduct a hypothesis test to see if your decision and conclusion would change if your belief were that the brown trout's mean I.Q. is **not** four.

S 9.6.9

a. $H_0 : \mu = 4$
b. $H_a : \mu \neq 4$
c. Let $\bar{X}$ the average I.Q. of a set of brown trout.
d. two-tailed Student's t-test
e. $t = 1.95$
f. $p\text{-value} = 0.076$
g. Check student's solution.
h.
   i. $\alpha : 0.05$
   ii. Decision: Reject the null hypothesis.
   iii. Reason for decision: The $p\text{-value}$ is greater than 0.05
   iv. Conclusion: There is insufficient evidence to conclude that the average IQ of brown trout is not four.
i. $(3.8865, 5.9468)$

Q 9.6.10

According to an article in *Newsweek*, the natural ratio of girls to boys is 100:105. In China, the birth ratio is 100: 114 (46.7% girls). Suppose you don't believe the reported figures of the percent of girls born in China. You conduct a study. In this study, you count the number of girls and boys born in 150 randomly chosen recent births. There are 60 girls and 90 boys born of the 150. Based on your study, do you believe that the percent of girls born in China is 46.7?

Q 9.6.11

A poll done for *Newsweek* found that 13% of Americans have seen or sensed the presence of an angel. A contingent doubts that the percent is really that high. It conducts its own survey. Out of 76 Americans surveyed, only two had seen or sensed the presence of an angel. As a result of the contingent's survey, would you agree with the *Newsweek* poll? In complete sentences, also give three reasons why the two polls might give different results.

S 9.6.11

a. $H_0 : p \geq 0.13$
b. $H_a : p < 0.13$
c. Let $P' =$ the proportion of Americans who have seen or sensed angels
d. normal for a single proportion
e. –2.688
f. $p\text{-value} = 0.0036$
g. Check student's solution.
h.
   i. alpha: 0.05
   ii. Decision: Reject the null hypothesis.
   iii. Reason for decision: The p-valuee is less than 0.05.
   iv. Conclusion: There is sufficient evidence to conclude that the percentage of Americans who have seen or sensed an angel is less than 13%.
i. $(0, 0.0623)$

   The"plus-4s" confidence interval is (0.0022, 0.0978)

Q 9.6.12

The mean work week for engineers in a start-up company is believed to be about 60 hours. A newly hired engineer hopes that it's shorter. She asks ten engineering friends in start-ups for the lengths of their mean work weeks. Based on the results that follow, should she count on the mean work week to be shorter than 60 hours?

Data (length of mean work week): 70; 45; 55; 60; 65; 55; 55; 60; 50; 55.

Q 9.6.13

Use the “Lap time” data for Lap 4 (see [link]) to test the claim that Terri finishes Lap 4, on average, in less than 129 seconds. Use all twenty races given.

S 9.6.13

a. $H_0 : \mu \geq 129$
b. $H_a : \mu < 129$
c. Let $\bar{X} =$ the average time in seconds that Terri finishes Lap 4.
d. Student's t-distribution
e. $t = 1.209$
f. 0.8792
g. Check student’s solution.
h.
   i. $\alpha : 0.05$
   ii. Decision: Do not reject the null hypothesis.
   iii. Reason for decision: The $p\text{-value}$ is greater than 0.05.
   iv. Conclusion: There is insufficient evidence to conclude that Terri’s mean lap time is less than 129 seconds.
i. $(128.63, 130.37)$

Q 9.6.14

Use the “Initial Public Offering” data (see [link]) to test the claim that the mean offer price was $18 per share. Do not use all the data. Use your random number generator to randomly survey 15 prices.

Note

The following questions were written by past students. They are excellent problems!

Q 9.6.15

"Asian Family Reunion," by Chau Nguyen

Every two years it comes around.

We all get together from different towns.

In my honest opinion,

It's not a typical family reunion.

Not forty, or fifty, or sixty,

But how about seventy companions!

The kids would play, scream, and shout

One minute they're happy, another they'll pout.

The teenagers would look, stare, and compare

From how they look to what they wear.

The men would chat about their business

That they make more, but never less.

Money is always their subject

And there's always talk of more new projects.

The women get tired from all of the chats

They head to the kitchen to set out the mats.

Some would sit and some would stand

Eating and talking with plates in their hands.

Then come the games and the songs

And suddenly, everyone gets along!

With all that laughter, it's sad to say

That it always ends in the same old way.

They hug and kiss and say "good-bye"

And then they all begin to cry!

I say that 60 percent shed their tears

But my mom counted 35 people this year.

She said that boys and men will always have their pride,

So we won't ever see them cry.

I myself don't think she's correct,

So could you please try this problem to see if you object?

S 9.6.15

a. $H_0 : p = 0.60$
b. $H_a : p < 0.60$
c. Let P' = the proportion of family members who shed tears at a reunion.
d. normal for a single proportion
e. –1.71
f. 0.0438
g. Check student’s solution.
h. i. $\alpha : 0.05$
   ii. Decision: Reject the null hypothesis.
   iii. Reason for decision: p-value $< \alpha$
   iv. Conclusion: At the 5% significance level, there is sufficient evidence to conclude that the proportion of family members who shed tears at a reunion is less than 0.60. However, the test is weak because the p-value and alpha are quite close, so other tests should be done.
i. We are 95% confident that between 38.29% and 61.71% of family members will shed tears at a family reunion. $(0.3829, 0.6171)$ The“plus-4s” confidence interval (see chapter 8) is $(0.3861, 0.6139)$

Note that here the “large-sample” $1 - \text{PropZTest}$ provides the approximate p-value of 0.0438. Whenever a p-value based on a normal approximation is close to the level of significance, the exact p-value based on binomial probabilities should be calculated whenever possible. This is beyond the scope of this course.

Q 9.6.16

"The Problem with Angels," by Cyndy Dowling

Although this problem is wholly mine,

The catalyst came from the magazine, Time.

On the magazine cover I did find

The realm of angels tickling my mind.

Inside, 69% I found to be

In angels, Americans do believe.

Then, it was time to rise to the task,

Ninety-five high school and college students I did ask.

Viewing all as one group,

Random sampling to get the scoop.

So, I asked each to be true,

"Do you believe in angels?" Tell me, do!

Hypothesizing at the start,

Totally believing in my heart

That the proportion who said yes

Would be equal on this test.

Lo and behold, seventy-three did arrive,

Out of the sample of ninety-five.

Now your job has just begun,

Solve this problem and have some fun.

Q 9.6.17

"Blowing Bubbles," by Sondra Prull

Studying stats just made me tense,

I had to find some sane defense.

Some light and lifting simple play

To float my math anxiety away.

Blowing bubbles lifts me high

Takes my troubles to the sky.

POIK! They're gone, with all my stress

Bubble therapy is the best.

The label said each time I blew

The average number of bubbles would be at least 22.

I blew and blew and this I found

From 64 blows, they all are round!

But the number of bubbles in 64 blows

Varied widely, this I know.

20 per blow became the mean

They deviated by 6, and not 16.

From counting bubbles, I sure did relax

But now I give to you your task.

Was 22 a reasonable guess?

Find the answer and pass this test!

S 9.6.17

a. $H_0 : \mu \geq 22$
b. $H_a : \mu < 22$
c. Let $\bar{X} =$ the mean number of bubbles per blow.
d. Student's t-distribution

e. –2.667
f. $p\text{-value} = 0.00486$
g. Check student’s solution.
h. i. $\alpha : 0.05$
   ii. Decision: Reject the null hypothesis.
   iii. Reason for decision: The $p\text{-value}$ is less than 0.05.
   iv. Conclusion: There is sufficient evidence to conclude that the mean number of bubbles per blow is less than 22.
i. $(18.501, 21.499)$

Q 9.6.18

"Dalmatian Darnation," by Kathy Sparling

A greedy dog breeder named Spreckles

Bred puppies with numerous freckles

The Dalmatians he sought

Possessed spot upon spot

The more spots, he thought, the more shekels.

His competitors did not agree

That freckles would increase the fee.

They said, “Spots are quite nice

But they don't affect price;

One should breed for improved pedigree.”

The breeders decided to prove

This strategy was a wrong move.

Breeding only for spots

Would wreak havoc, they thought.

His theory they want to disprove.

They proposed a contest to Spreckles

Comparing dog prices to freckles.

In records they looked up

One hundred one pups:

Dalmatians that fetched the most shekels.

They asked Mr. Spreckles to name

An average spot count he'd claim

To bring in big bucks.

Said Spreckles, “Well, shucks,

It's for one hundred one that I aim.”

Said an amateur statistician

Who wanted to help with this mission.

“Twenty-one for the sample

Standard deviation's ample:

They examined one hundred and one

Dalmatians that fetched a good sum.

They counted each spot,

Mark, freckle and dot

And tallied up every one.

Instead of one hundred one spots

They averaged ninety six dots

Can they muzzle Spreckles'

Obsession with freckles

Based on all the dog data they've got?

Q 9.6.19

"Macaroni and Cheese, please!!" by Nedda Misherghi and Rachelle Hall

As a poor starving student I don't have much money to spend for even the bare necessities. So my favorite and main staple food is macaroni and cheese. It's high in taste and low in cost and nutritional value.

One day, as I sat down to determine the meaning of life, I got a serious craving for this, oh, so important, food of my life. So I went down the street to Greatway to get a box of macaroni and cheese, but it was SO expensive! $2.02 !!! Can you believe it? It made me stop and think. The world is changing fast. I had thought that the mean cost of a box (the normal size, not some super-gigantic-family-value-pack) was at most $1, but now I wasn't so sure. However, I was determined to find out. I went to 53 of the closest grocery stores and surveyed the prices of macaroni and cheese. Here are the data I wrote in my notebook:

Price per box of Mac and Cheese:

- 5 stores @ $2.02
- 15 stores @ $0.25
- 3 stores @ $1.29
- 6 stores @ $0.35
- 4 stores @ $2.27
- 7 stores @ $1.50
- 5 stores @ $1.89
- 8 stores @ 0.75.

I could see that the cost varied but I had to sit down to figure out whether or not I was right. If it does turn out that this mouth-watering dish is at most $1, then I'll throw a big cheesy party in our next statistics lab, with enough macaroni and cheese for just me. (After all, as a poor starving student I can't be expected to feed our class of animals!)

S 9.6.19

a. $H_0: \mu \leq 1$
b. $H_a: \mu > 1$
c. Let $\bar{X} =$ the mean cost in dollars of macaroni and cheese in a certain town.
d. Student's t-distribution
e. $t = 0.340$
f. $p\text{-value} = 0.36756$
g. Check student's solution.
h.
   i. $\alpha: 0.05$
   ii. Decision: Do not reject the null hypothesis.
   iii. Reason for decision: The p-value is greater than 0.05
   iv. Conclusion: The mean cost could be $1, or less. At the 5% significance level, there is insufficient evidence to conclude that the mean price of a box of macaroni and cheese is more than $1.

i. $(0.8291, 1.241)$

Q 9.6.20

"William Shakespeare: The Tragedy of Hamlet, Prince of Denmark," by Jacqueline Ghodsi

THE CHARACTERS (in order of appearance):

- HAMLET, Prince of Denmark and student of Statistics
- POLONIUS, Hamlet's tutor
- HOROTIO, friend to Hamlet and fellow student

Scene: The great library of the castle, in which Hamlet does his lessons

Act I

(The day is fair, but the face of Hamlet is clouded. He paces the large room. His tutor, Polonius, is reprimanding Hamlet regarding the latter's recent experience. Horatio is seated at the large table at right stage.)

POLONIUS: My Lord, how cans't thou admit that thou hast seen a ghost! It is but a figment of your imagination!

HAMLET: I beg to differ; I know of a certainty that five-and-seventy in one hundred of us, condemned to the whips and scorns of time as we are, have gazed upon a spirit of health, or goblin damn'd, be their intents wicked or charitable.

POLONIUS If thou doest insist upon thy wretched vision then let me invest your time; be true to thy work and speak to me through the reason of the null and alternate hypotheses. (He turns to Horatio.) Did not Hamlet himself say, "What piece of work is man, how noble in reason, how infinite in faculties? Then let not this foolishness persist. Go, Horatio, make a survey of three-and-sixty and discover what the true proportion be. For my part, I will never succumb to this fantasy, but deem man to be devoid of all reason should thy proposal of at least five-and-seventy in one hundred hold true.

HORATIO (to Hamlet): What should we do, my Lord?

HAMLET: Go to thy purpose, Horatio.

HORATIO: To what end, my Lord?

HAMLET: That you must teach me. But let me conjure you by the rights of our fellowship, by the consonance of our youth, but the obligation of our ever-preserved love, be even and direct with me, whether I am right or no.

(Horatio exits, followed by Polonius, leaving Hamlet to ponder alone.)

Act II

(The next day, Hamlet awaits anxiously the presence of his friend, Horatio. Polonius enters and places some books upon the table just a moment before Horatio enters.)

POLONIUS: So, Horatio, what is it thou didst reveal through thy deliberations?

HORATIO: In a random survey, for which purpose thou thyself sent me forth, I did discover that one-and-forty believe fervently that the spirits of the dead walk with us. Before my God, I might not this believe, without the sensible and true avouch of mine own eyes.

POLONIUS: Give thine own thoughts no tongue, Horatio. (Polonius turns to Hamlet.) But look to't I charge you, my Lord. Come Horatio, let us go together, for this is not our test. (Horatio and Polonius leave together.)

HAMLET: To reject, or not reject, that is the question: whether 'tis nobler in the mind to suffer the slings and arrows of outrageous statistics, or to take arms against a sea of data, and, by opposing, end them. (Hamlet resignedly attends to his task.)

(Curtain falls)

Q 9.6.21

"Untitled," by Stephen Chen

I've often wondered how software is released and sold to the public. Ironically, I work for a company that sells products with known problems. Unfortunately, most of the problems are difficult to create, which makes them difficult to fix. I usually use the test program X, which tests the product, to try to create a specific problem. When the test program is run to make an error occur, the likelihood of generating an error is 1%.

So, armed with this knowledge, I wrote a new test program Y that will generate the same error that test program X creates, but more often. To find out if my test program is better than the original, so that I can convince the management that I'm right, I ran my test program to find out how often I can generate the same error. When I ran my test program 50 times, I generated the error twice. While this may not seem much better, I think that I can convince the management to use my test program instead of the original test program. Am I right?

S 9.6.21

a. $H_0 : p = 0.01$
b. $H_a : p > 0.01$
c. Let $P' =$ the proportion of errors generated
d. Normal for a single proportion
e. 2.13
f. 0.0165
g. Check student's solution.
h. i. $\alpha : 0.05$
   ii. Decision: Reject the null hypothesis
   iii. Reason for decision: The $p\text{-value}$ is less than 0.05.
   iv. Conclusion: At the 5% significance level, there is sufficient evidence to conclude that the proportion of errors generated is more than 0.01.
i. Confidence interval: $(0, 0.094)$.

   The"plus-4s" confidence interval is $(0.004, 0.144)$

Q 9.6.22

"Japanese Girls' Names"

by Kumi Furuichi

It used to be very typical for Japanese girls' names to end with "ko." (The trend might have started around my grandmothers' generation and its peak might have been around my mother's generation.) "Ko" means "child" in Chinese characters. Parents would name their daughters with "ko" attaching to other Chinese characters which have meanings that they want their daughters to become, such as Sachiko—happy child, Yoshiko—a good child, Yasuko—a healthy child, and so on.

However, I noticed recently that only two out of nine of my Japanese girlfriends at this school have names which end with "ko." More and more, parents seem to have become creative, modernized, and, sometimes, westernized in naming their children.

I have a feeling that, while 70 percent or more of my mother's generation would have names with "ko" at the end, the proportion has dropped among my peers. I wrote down all my Japanese friends', ex-classmates', co-workers, and acquaintances' names that I could remember. Following are the names. (Some are repeats.) Test to see if the proportion has dropped for this generation.

Ai, Akemi, Akiko, Ayumi, Chiaki, Chie, Eiko, Eri, Eriko, Fumiko, Harumi, Hitomi, Hiroko, Hiroko, Hidemi, Hisako, Hinako, Izumi, Izumi, Junko, Junko, Kana, Kanako, Kanayo, Kayo, Kayoko, Kazumi, Keiko, Keiko, Kei, Kumi, Kumiko, Kyoko, Kyoko, Madoka, Maho, Mai, Maiko, Maki, Miki, Miki, Mikiko, Mina, Minako, Miyako, Momoko, Nana, Naoko, Naoko, Naoko, Noriko, Rieko, Rika, Rika, Rumiko, Rei, Reiko, Reiko, Sachiko, Sachiko, Sachiyo, Saki, Sayaka, Sayoko, Sayuri, Seiko, Shiho, Shizuka, Sumiko, Takako, Takako, Tomoe, Tomoe, Tomoko, Touko, Yasuko, Yasuko, Yasuyo, Yoko, Yoko, Yoko, Yoshiko, Yoshiko, Yoshiko, Yuka, Yuki, Yuki, Yukiko, Yuko, Yuko.

Q 9.6.23

"Phillip's Wish," by Suzanne Osorio

My nephew likes to play

Chasing the girls makes his day.

He asked his mother

If it is okay

To get his ear pierced.

She said, “No way!”

To poke a hole through your ear,

Is not what I want for you, dear.

He argued his point quite well,

Says even my macho pal, Mel,

Has gotten this done.

It’s all just for fun.

C’mon please, mom, please, what the hell.

Again Phillip complained to his mother,

Saying half his friends (including their brothers)

Are piercing their ears

And they have no fears

He wants to be like the others.

She said, “I think it’s much less.

We must do a hypothesis test.

And if you are right,

I won’t put up a fight.

But, if not, then my case will rest.”

We proceeded to call fifty guys

To see whose prediction would fly.

Nineteen of the fifty

Said piercing was nifty

And earrings they’d occasionally buy.

Then there’s the other thirty-one,

Who said they’d never have this done.

So now this poem’s finished.

Will his hopes be diminished,

Or will my nephew have his fun?

S 9.6.23

a. $H_0: p = 0.50$
b. $H_a: p < 0.50$
c. Let $P' =$ the proportion of friends that has a pierced ear.
d. normal for a single proportion
e. –1.70
f. $p\text{-value} = 0.0448$
g. Check student’s solution.
h. i. $\alpha: 0.05$
   ii. Decision: Reject the null hypothesis
   iii. Reason for decision: The $p\text{-value}$ is less than 0.05. (However, they are very close.)
   iv. Conclusion: There is sufficient evidence to support the claim that less than 50% of his friends have pierced ears.
i. Confidence Interval: $(0.245, 0.515)$ The “plus-4s” confidence interval is $(0.259, 0.519)$

Q 9.6.24

"The Craven," by Mark Salangsang

Once upon a morning dreary
In stats class I was weak and weary.
Pondering over last night’s homework
Whose answers were now on the board
This I did and nothing more.
While I nodded nearly napping
Suddenly, there came a tapping.
As someone gently rapping,
Rapping my head as I snore.
Quoth the teacher, “Sleep no more.”
“In every class you fall asleep,”
The teacher said, his voice was deep.
“So a tally I’ve begun to keep
Of every class you nap and snore.
The percentage being forty-four.”
“My dear teacher I must confess,
While sleeping is what I do best.
The percentage, I think, must be less,
A percentage less than forty-four.”
This I said and nothing more.
“We’ll see,” he said and walked away,
And fifty classes from that day
He counted till the month of May
The classes in which I napped and snored.
The number he found was twenty-four.
At a significance level of 0.05,
Please tell me am I still alive?
Or did my grade just take a dive
Plunging down beneath the floor?
Upon thee I hereby implore.

Q 9.6.25

Toastmasters International cites a report by Gallop Poll that 40% of Americans fear public speaking. A student believes that less than 40% of students at her school fear public speaking. She randomly surveys 361 schoolmates and finds that 135 report they fear public speaking. Conduct a hypothesis test to determine if the percent at her school is less than 40%.

S 9.6.25

a. $H_0 : p = 0.40$
b. $H_a : p < 0.40$

c. Let $P'=$ the proportion of schoolmates who fear public speaking.
d. normal for a single proportion
e. –1.01
f. $p\text{-value} = 0.1563$
g. Check student's solution.
h. i. $\alpha : 0.05$
 ii. Decision: Do not reject the null hypothesis.
 iii. Reason for decision: The $p\text{-value}$ is greater than 0.05.
 iv. Conclusion: There is insufficient evidence to support the claim that less than 40% of students at the school fear public speaking.
i. Confidence Interval: $(0.3241, 0.4240)$: The "plus-4s" confidence interval is $(0.3257, 0.4250)$

Q 9.6.26

Sixty-eight percent of online courses taught at community colleges nationwide were taught by full-time faculty. To test if 68% also represents California's percent for full-time faculty teaching the online classes, Long Beach City College (LBCC) in California, was randomly selected for comparison. In the same year, 34 of the 44 online courses LBCC offered were taught by full-time faculty. Conduct a hypothesis test to determine if 68% represents California. NOTE: For more accurate results, use more California community colleges and this past year's data.

Q 9.6.27

According to an article in *Bloomberg Businessweek*, New York City's most recent adult smoking rate is 14%. Suppose that a survey is conducted to determine this year's rate. Nine out of 70 randomly chosen N.Y. City residents reply that they smoke. Conduct a hypothesis test to determine if the rate is still 14% or if it has decreased.

S 9.6.27

a. $H_0 : p = 0.14$
b. $H_a : p < 0.14$
c. Let $P'=$ the proportion of NYC residents that smoke.
d. normal for a single proportion
e. –0.2756
f. $p\text{-value} = 0.3914$
g. Check student's solution.
h. i. $\alpha : 0.05$
 ii. Decision: Do not reject the null hypothesis.
 iii. Reason for decision: The $p\text{-value}$ is greater than 0.05.
 iv. At the 5% significance level, there is insufficient evidence to conclude that the proportion of NYC residents who smoke is less than 0.14.
i. Confidence Interval: $(0.0502, 0.2070)$: The "plus-4s" confidence interval (see chapter 8) is $(0.0676, 0.2297)$

Q 9.6.28

The mean age of De Anza College students in a previous term was 26.6 years old. An instructor thinks the mean age for online students is older than 26.6. She randomly surveys 56 online students and finds that the sample mean is 29.4 with a standard deviation of 2.1. Conduct a hypothesis test.

Q 9.6.29

Registered nurses earned an average annual salary of $69,110. For that same year, a survey was conducted of 41 California registered nurses to determine if the annual salary is higher than $69,110 for California nurses. The sample average was $71,121 with a sample standard deviation of $7,489. Conduct a hypothesis test.

S 9.6.29

a. $H_0 : \mu = 69,110$
b. $H_0 : \mu > 69,110$
c. Let $\bar{X} =$ the mean salary in dollars for California registered nurses.

d. Student's t-distribution
e. $t = 1.719$
f. p-value : 0.0466
g. Check student’s solution.
h. i. $\alpha : 0.05$
   ii. Decision: Reject the null hypothesis.
   iii. Reason for decision: The p-value is less than 0.05.
   iv. Conclusion: At the 5% significance level, there is sufficient evidence to conclude that the mean salary of California registered nurses exceeds $69,110.
i. $(\$68,757,\$73,485)$

Q 9.6.30

La Leche League International reports that the mean age of weaning a child from breastfeeding is age four to five worldwide. In America, most nursing mothers wean their children much earlier. Suppose a random survey is conducted of 21 U.S. mothers who recently weaned their children. The mean weaning age was nine months (3/4 year) with a standard deviation of 4 months. Conduct a hypothesis test to determine if the mean weaning age in the U.S. is less than four years old.

Q 9.6.31

Over the past few decades, public health officials have examined the link between weight concerns and teen girls' smoking. Researchers surveyed a group of 273 randomly selected teen girls living in Massachusetts (between 12 and 15 years old). After four years the girls were surveyed again. Sixty-three said they smoked to stay thin. Is there good evidence that more than thirty percent of the teen girls smoke to stay thin?

After conducting the test, your decision and conclusion are

a. Reject H_0: There is sufficient evidence to conclude that more than 30% of teen girls smoke to stay thin.
b. Do not reject H_0: There is not sufficient evidence to conclude that less than 30% of teen girls smoke to stay thin.
c. Do not reject H_0: There is not sufficient evidence to conclude that more than 30% of teen girls smoke to stay thin.
d. Reject H_0: There is sufficient evidence to conclude that less than 30% of teen girls smoke to stay thin.

S 9.6.31

c

Q 9.6.32

A statistics instructor believes that fewer than 20% of Evergreen Valley College (EVC) students attended the opening night midnight showing of the latest Harry Potter movie. She surveys 84 of her students and finds that 11 of them attended the midnight showing.

At a 1% level of significance, an appropriate conclusion is:

a. There is insufficient evidence to conclude that the percent of EVC students who attended the midnight showing of Harry Potter is less than 20%.
b. There is sufficient evidence to conclude that the percent of EVC students who attended the midnight showing of Harry Potter is more than 20%.
c. There is sufficient evidence to conclude that the percent of EVC students who attended the midnight showing of Harry Potter is less than 20%.
d. There is insufficient evidence to conclude that the percent of EVC students who attended the midnight showing of Harry Potter is at least 20%.

Q 9.6.33

Previously, an organization reported that teenagers spent 4.5 hours per week, on average, on the phone. The organization thinks that, currently, the mean is higher. Fifteen randomly chosen teenagers were asked how many hours per week they spend on the phone. The sample mean was 4.75 hours with a sample standard deviation of 2.0. Conduct a hypothesis test.

At a significance level of $a = 0.05$, what is the correct conclusion?

a. There is enough evidence to conclude that the mean number of hours is more than 4.75
b. There is enough evidence to conclude that the mean number of hours is more than 4.5
c. There is not enough evidence to conclude that the mean number of hours is more than 4.5
d. There is not enough evidence to conclude that the mean number of hours is more than 4.75

S 9.6.33

c

Instructions: For the following ten exercises,

Hypothesis testing: For the following ten exercises, answer each question.

State the null and alternate hypothesis.

State the p-value.

State α.

What is your decision?

Write a conclusion.

Answer any other questions asked in the problem.

Q 9.6.34

According to the Center for Disease Control website, in 2011 at least 18% of high school students have smoked a cigarette. An Introduction to Statistics class in Davies County, KY conducted a hypothesis test at the local high school (a medium sized–approximately 1,200 students–small city demographic) to determine if the local high school's percentage was lower. One hundred fifty students were chosen at random and surveyed. Of the 150 students surveyed, 82 have smoked. Use a significance level of 0.05 and using appropriate statistical evidence, conduct a hypothesis test and state the conclusions.

Q 9.6.35

A recent survey in the *N.Y. Times Almanac* indicated that 48.8% of families own stock. A broker wanted to determine if this survey could be valid. He surveyed a random sample of 250 families and found that 142 owned some type of stock. At the 0.05 significance level, can the survey be considered to be accurate?

S 9.6.35

a. $H_0: p = 0.488$ $H_a: p \neq 0.488$
b. p-value $= 0.0114$
c. $\alpha = 0.05$
d. Reject the null hypothesis.
e. At the 5% level of significance, there is enough evidence to conclude that 48.8% of families own stocks.
f. The survey does not appear to be accurate.

Q 9.6.36

Driver error can be listed as the cause of approximately 54% of all fatal auto accidents, according to the American Automobile Association. Thirty randomly selected fatal accidents are examined, and it is determined that 14 were caused by driver error. Using $\alpha = 0.05$, is the AAA proportion accurate?

Q 9.6.37

The US Department of Energy reported that 51.7% of homes were heated by natural gas. A random sample of 221 homes in Kentucky found that 115 were heated by natural gas. Does the evidence support the claim for Kentucky at the $\alpha = 0.05$ level in Kentucky? Are the results applicable across the country? Why?

S 9.6.37

a. $H_0: p = 0.517$ $H_0: p \neq 0.517$
b. p-value $= 0.9203$.
c. $\alpha = 0.05$.
d. Do not reject the null hypothesis.

e. At the 5% significance level, there is not enough evidence to conclude that the proportion of homes in Kentucky that are heated by natural gas is 0.517.
f. However, we cannot generalize this result to the entire nation. First, the sample's population is only the state of Kentucky. Second, it is reasonable to assume that homes in the extreme north and south will have extreme high usage and low usage, respectively. We would need to expand our sample base to include these possibilities if we wanted to generalize this claim to the entire nation.

Q 9.6.38

For Americans using library services, the American Library Association claims that at most 67% of patrons borrow books. The library director in Owensboro, Kentucky feels this is not true, so she asked a local college statistic class to conduct a survey. The class randomly selected 100 patrons and found that 82 borrowed books. Did the class demonstrate that the percentage was higher in Owensboro, KY? Use $\alpha = 0.01$ level of significance. What is the possible proportion of patrons that do borrow books from the Owensboro Library?

Q 9.6.39

The Weather Underground reported that the mean amount of summer rainfall for the northeastern US is at least 11.52 inches. Ten cities in the northeast are randomly selected and the mean rainfall amount is calculated to be 7.42 inches with a standard deviation of 1.3 inches. At the $\alpha = 0.05 level$, can it be concluded that the mean rainfall was below the reported average? What if $\alpha = 0.01$? Assume the amount of summer rainfall follows a normal distribution.

S 9.6.39

a. $H_0 : \mu \geq 11.52$ $H_a : \mu < 11.52$
b. $p\text{-value} = 0.000002$which is almost 0.
c. $\alpha = 0.05$.
d. Reject the null hypothesis.
e. At the 5% significance level, there is enough evidence to conclude that the mean amount of summer rain in the northeaster US is less than 11.52 inches, on average.
f. We would make the same conclusion if alpha was 1% because the $p\text{-value}$ is almost 0.

Q 9.6.40

A survey in the *N.Y. Times Almanac* finds the mean commute time (one way) is 25.4 minutes for the 15 largest US cities. The Austin, TX chamber of commerce feels that Austin's commute time is less and wants to publicize this fact. The mean for 25 randomly selected commuters is 22.1 minutes with a standard deviation of 5.3 minutes. At the $\alpha = 0.10$ level, is the Austin, TX commute significantly less than the mean commute time for the 15 largest US cities?

Q 9.6.41

A report by the Gallup Poll found that a woman visits her doctor, on average, at most 5.8 times each year. A random sample of 20 women results in these yearly visit totals

3; 2; 1; 3; 7; 2; 9; 4; 6; 6; 8; 0; 5; 6; 4; 2; 1; 3; 4; 1

At the $\alpha = 0.05$ level can it be concluded that the sample mean is higher than 5.8 visits per year?

S 9.6.42

1. $H_0 : \mu \leq 5.8$ $H_a : \mu > 5.8$
2. $p\text{-value} = 0.9987$
3. $\alpha = 0.05$
4. Do not reject the null hypothesis.
5. At the 5% level of significance, there is not enough evidence to conclude that a woman visits her doctor, on average, more than 5.8 times a year.

Q 9.6.42

According to the *N.Y. Times Almanac* the mean family size in the U.S. is 3.18. A sample of a college math class resulted in the following family sizes:

5; 4; 5; 4; 4; 3; 6; 4; 3; 3; 5; 5; 6; 3; 3; 2; 7; 4; 5; 2; 2; 2; 3; 2

At $\alpha = 0.05$ level, is the class' mean family size greater than the national average? Does the Almanac result remain valid? Why?

Q 9.6.43

The student academic group on a college campus claims that freshman students study at least 2.5 hours per day, on average. One Introduction to Statistics class was skeptical. The class took a random sample of 30 freshman students and found a mean study time of 137 minutes with a standard deviation of 45 minutes. At α = 0.01 level, is the student academic group's claim correct?

S 9.6.43

a. $H_0 : \mu \geq 150$ $H_0 : \mu < 150$
b. p-value $= 0.0622$
c. $\alpha = 0.01$
d. Do not reject the null hypothesis.
e. At the 1% significance level, there is not enough evidence to conclude that freshmen students study less than 2.5 hours per day, on average.
f. The student academic group's claim appears to be correct.

9.7: Hypothesis Testing of a Single Mean and Single Proportion