

## 4.2: Probability Distribution Function (PDF) for a Discrete Random Variable

A discrete probability distribution function has two characteristics:

- Each probability is between zero and one, inclusive.
- The sum of the probabilities is one.

### ✓ Example 4.2.1

A child psychologist is interested in the number of times a newborn baby's crying wakes its mother after midnight. For a random sample of 50 mothers, the following information was obtained. Let  $X$  = the number of times per week a newborn baby's crying wakes its mother after midnight. For this example,  $x = 0, 1, 2, 3, 4, 5$

$P(x)$  = probability that  $X$  takes on a value  $x$ .

$x$	$P(x)$
0	$P(x = 0) = \frac{2}{50}$
1	$P(x = 1) = \frac{11}{50}$
2	$P(x = 2) = \frac{23}{50}$
3	$P(x = 3) = \frac{9}{50}$
4	$P(x = 4) = \frac{4}{50}$
5	$P(x = 5) = \frac{1}{50}$

$X$  takes on the values 0, 1, 2, 3, 4, 5. This is a discrete PDF because:

- Each  $P(x)$  is between zero and one, inclusive.
- The sum of the probabilities is one, that is,

$$\frac{2}{50} + \frac{11}{50} + \frac{23}{50} + \frac{9}{50} + \frac{4}{50} + \frac{1}{50} = 1 \quad (4.2.1)$$

### ? Exercise 4.2.1

A hospital researcher is interested in the number of times the average post-op patient will ring the nurse during a 12-hour shift. For a random sample of 50 patients, the following information was obtained. Let  $X$  = the number of times a patient rings the nurse during a 12-hour shift. For this exercise,  $x = 0, 1, 2, 3, 4, 5$   $P(x)$  = the probability that  $X$  takes on value  $x$ . Why is this a discrete probability distribution function (two reasons)?

$X$	$P(x)$
0	$P(x = 0) = \frac{4}{50}$
1	$P(x = 1) = \frac{8}{50}$
2	$P(x = 2) = \frac{16}{50}$
3	$P(x = 3) = \frac{14}{50}$
4	$P(x = 4) = \frac{6}{50}$

$X$	$P(x)$
5	$P(x = 5) = \frac{2}{50}$

### Answer

Each  $P(x)$  is between 0 and 1, inclusive, and the sum of the probabilities is 1, that is:

$$\frac{4}{50} + \frac{8}{50} + \frac{16}{50} + \frac{14}{50} + \frac{6}{50} + \frac{2}{50} = 1 \quad (4.2.2)$$

### ✓ Example 4.2.2

Suppose Nancy has classes three days a week. She attends classes three days a week 80% of the time, two days 15% of the time, one day 4% of the time, and no days 1% of the time. Suppose one week is randomly selected.

- Let  $X$  = the number of days Nancy \_\_\_\_\_.
- $X$  takes on what values?
- Suppose one week is randomly chosen. Construct a probability distribution table (called a PDF table) like the one in [Example](#). The table should have two columns labeled  $x$  and  $P(x)$ . What does the  $P(x)$  column sum to?

### Solutions

a. Let  $X$  = the number of days Nancy attends class per week.

b. 0, 1, 2, and 3

c

$x$	$P(x)$
0	0.01
1	0.04
2	0.15
3	0.80

### ? Exercise 4.2.2

Jeremiah has basketball practice two days a week. Ninety percent of the time, he attends both practices. Eight percent of the time, he attends one practice. Two percent of the time, he does not attend either practice. What is  $X$  and what values does it take on?

### Answer

$X$  is the number of days Jeremiah attends basketball practice per week.  $X$  takes on the values 0, 1, and 2.

## Review

The characteristics of a probability distribution function (PDF) for a discrete random variable are as follows:

- Each probability is between zero and one, inclusive (*inclusive* means to include zero and one).
- The sum of the probabilities is one.

Use the following information to answer the next five exercises: A company wants to evaluate its attrition rate, in other words, how long new hires stay with the company. Over the years, they have established the following probability distribution.

Let  $X$  = the number of years a new hire will stay with the company.

Let  $P(x)$  = the probability that a new hire will stay with the company  $x$  years.

### ? Exercise 4.2.3

Complete Table using the data provided.

$x$	$P(x)$
0	0.12
1	0.18
2	0.30
3	0.15
4	
5	0.10
6	0.05

### Answer

$x$	$P(x)$
0	0.12
1	0.18
2	0.30
3	0.15
4	0.10
5	0.10
6	0.05

### ? Exercise 4.2.4

$$P(x = 4) = \underline{\hspace{2cm}}$$

### ? Exercise 4.2.5

$$P(x \geq 5) = \underline{\hspace{2cm}}$$

### Answer

$$0.10 + 0.05 = 0.15$$

### ? Exercise 4.2.6

On average, how long would you expect a new hire to stay with the company?

### ? Exercise 4.2.7

What does the column " $P(x)$ " sum to?

### Answer

1

Use the following information to answer the next six exercises: A baker is deciding how many batches of muffins to make to sell in his bakery. He wants to make enough to sell every one and no fewer. Through observation, the baker has established a probability distribution.

$x$	$P(x)$
1	0.15
2	0.35
3	0.40
4	0.10

#### ? Exercise 4.2.8

Define the random variable  $X$ .

#### ? Exercise 4.2.9

What is the probability the baker will sell more than one batch?  $P(x > 1) = \underline{\hspace{2cm}}$

**Answer**

$$0.35 + 0.40 + 0.10 = 0.85$$

#### ? Exercise 4.2.10

What is the probability the baker will sell exactly one batch?  $P(x = 1) = \underline{\hspace{2cm}}$

#### ? Exercise 4.2.11

On average, how many batches should the baker make?

**Answer**

$$1(0.15) + 2(0.35) + 3(0.40) + 4(0.10) = 0.15 + 0.70 + 1.20 + 0.40 = 2.45$$

Use the following information to answer the next four exercises: Ellen has music practice three days a week. She practices for all of the three days 85% of the time, two days 8% of the time, one day 4% of the time, and no days 3% of the time. One week is selected at random.

#### ? Exercise 4.2.12

Define the random variable  $X$ .

#### ? Exercise 4.2.13

Construct a probability distribution table for the data.

**Answer**

$x$	$P(x)$
0	0.03
1	0.04
2	0.08
3	0.85

### ? Exercise 4.2.14

We know that for a probability distribution function to be discrete, it must have two characteristics. One is that the sum of the probabilities is one. What is the other characteristic?

Use the following information to answer the next five exercises: Javier volunteers in community events each month. He does not do more than five events in a month. He attends exactly five events 35% of the time, four events 25% of the time, three events 20% of the time, two events 10% of the time, one event 5% of the time, and no events 5% of the time.

### ? Exercise 4.2.15

Define the random variable  $X$ .

**Answer**

Let  $X$  = the number of events Javier volunteers for each month.

### ? Exercise 4.2.16

What values does  $x$  take on?

### ? Exercise 4.2.17

Construct a PDF table.

**Answer**

$x$	$P(x)$
0	0.05
1	0.05
2	0.10
3	0.20
4	0.25
5	0.35

### ? Exercise 4.2.18

Find the probability that Javier volunteers for less than three events each month.  $P(x < 3) = \underline{\hspace{2cm}}$

### ? Exercise 4.2.19

Find the probability that Javier volunteers for at least one event each month.  $P(x > 0) = \underline{\hspace{2cm}}$

**Answer**

$$1 - 0.05 = 0.95$$

## Glossary

### Probability Distribution Function (PDF)

a mathematical description of a discrete random variable (RV), given either in the form of an equation (formula) or in the form of a table listing all the possible outcomes of an experiment and the probability associated with each outcome.

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