5.3: The Uniform Distribution

The uniform distribution is a continuous probability distribution and is concerned with events that are equally likely to occur. When working out problems that have a uniform distribution, be careful to note if the data is inclusive or exclusive of endpoints.

The mathematical statement of the uniform distribution is

$$f(x)=\frac{1}{b-a} \text{ for } a \leq x \leq b \tag{5.3.1}$$

where $a=$ the lowest value of x and $b=$ the highest value of x.
Formulas for the theoretical mean and standard deviation are

$$\mu=\frac{a+b}{2} \text{ and } \sigma=\sqrt{\frac{(b-a)^2}{12}} \tag{5.3.2}$$

Exercise 5.3.1

The amount of time, in minutes, that a person must wait for a bus is uniformly distributed between zero and 15 minutes, inclusive.

a. What is the probability that a person waits fewer than 12.5 minutes?

Answer

a. Let $X=$ the number of minutes a person must wait for a bus. $a=0$ and $b=15$. $X \sim U(0,15)$. Write the probability density function. $f(x)=\frac{1}{15-0}=\frac{1}{15}$ for $0 \leq x \leq 15$.

Find $P(x<12.5)$. Draw a graph.

$$P(x<k)=(\text{ base })(\text{ height })=(12.5-0)\left(\frac{1}{15}\right)=0.8333 \tag{5.3.3}$$

The probability a person waits less than 12.5 minutes is 0.8333 .

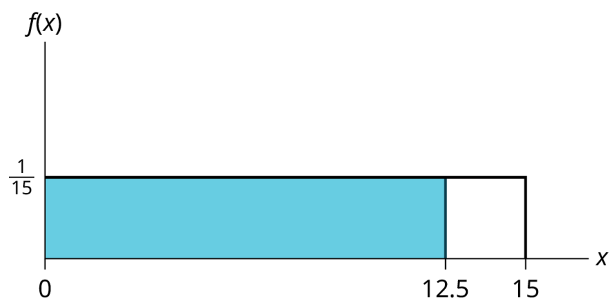

Figure 5.3.1:

Exercise 5.3.2

b. On the average, how long must a person wait? Find the mean, μ, and the standard deviation, σ.

Answer

b. $\mu=\frac{a+b}{2}=\frac{15+0}{2}=7.5$. On the average, a person must wait 7.5 minutes. $\sigma=\sqrt{\frac{(b-a)^2}{12}}=\sqrt{\frac{(15-0)^2}{12}}=$ 4.3. The Standard deviation is 4.3 minutes.

Exercise 5.3.3

c. Ninety percent of the time, the time a person must wait falls below what value?

NOTE

This asks for the 90^{th} percentile.

Answer

c. Find the 90^{th} percentile. Draw a graph. Let $k =$ the 90^{th} percentile.

$$\begin{aligned} &P(x<k)=(\text{ base })(\text{ height })=(k-0)\left(\frac{1}{15}\right) \\ &0.90=(k)\left(\frac{1}{15}\right) \\ &k=(0.90)(15)=13.5 \end{aligned} \tag{5.3.4}$$

The 90^{th} percentile is 13.5 minutes. Ninety percent of the time, a person must wait at most 13.5 minutes.

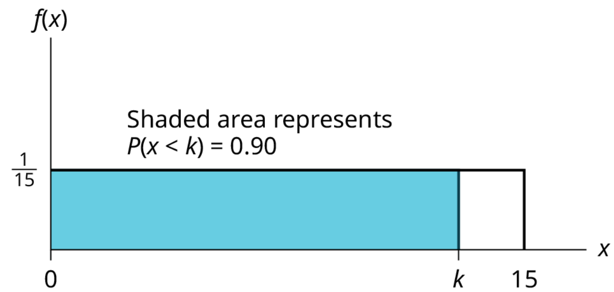

Figure 5.3.2:

Try It 5.3.1

The total duration of baseball games in the major league in a typical season is uniformly distributed between 447 hours and 521 hours inclusive.

a. Find a and b and describe what they represent.
b. Write the distribution.
c. Find the mean and the standard deviation.
d. What is the probability that the duration of games for a team in a single season is between 480 and 500 hours?