

## 11.3: Test of a Single Variance

Thus far our interest has been exclusively on the population parameter  $\mu$  or its counterpart in the binomial,  $p$ . Surely the mean of a population is the most critical piece of information to have, but in some cases we are interested in the variability of the outcomes of some distribution. In almost all production processes quality is measured not only by how closely the machine matches the target, but also the variability of the process. If one were filling bags with potato chips not only would there be interest in the average weight of the bag, but also how much variation there was in the weights. No one wants to be assured that the average weight is accurate when their bag has no chips. Electricity voltage may meet some average level, but great variability, spikes, can cause serious damage to electrical machines, especially computers. I would not only like to have a high mean grade in my classes, but also low variation about this mean. In short, statistical tests concerning the variance of a distribution have great value and many applications.

A **test of a single variance** assumes that the underlying distribution is **normal**. The null and alternative hypotheses are stated in terms of the **population variance**. The test statistic is:

$$\chi_c^2 = \frac{(n-1)s^2}{\sigma_0^2} \quad (11.3.1)$$

where:

- $n$  = the total number of observations in the sample data
- $s^2$  = sample variance
- $\sigma_0^2$  = hypothesized value of the population variance
- $H_0 : \sigma^2 = \sigma_0^2$
- $H_a : \sigma^2 \neq \sigma_0^2$

You may think of  $s$  as the random variable in this test. The number of degrees of freedom is  $df = n - 1$ . A test of a single variance may be right-tailed, left-tailed, or two-tailed. Example 11.1 will show you how to set up the null and alternative hypotheses. The null and alternative hypotheses contain statements about the population variance.

### ? Exercise 11.3.1

Math instructors are not only interested in how their students do on exams, on average, but how the exam scores vary. To many instructors, the variance (or standard deviation) may be more important than the average.

Suppose a math instructor believes that the standard deviation for his final exam is five points. One of his best students thinks otherwise. The student claims that the standard deviation is more than five points. If the student were to conduct a hypothesis test, what would the null and alternative hypotheses be?

### Answer

Even though we are given the population standard deviation, we can set up the test using the population variance as follows.

- $H_0 : \sigma^2 \leq 5^2$
- $H_a : \sigma^2 > 5^2$

### Try It 11.3.1

A SCUBA instructor wants to record the collective depths each of his students' dives during their checkout. He is interested in how the depths vary, even though everyone should have been at the same depth. He believes the standard deviation is three feet. His assistant thinks the standard deviation is less than three feet. If the instructor were to conduct a test, what would the null and alternative hypotheses be?

### ? Exercise 11.3.2

With individual lines at its various windows, a post office finds that the standard deviation for waiting times for customers on Friday afternoon is 7.2 minutes. The post office experiments with a single, main waiting line and finds that for a random sample of 25 customers, the waiting times for customers have a standard deviation of 3.5 minutes on a Friday afternoon.

With a significance level of 5%, test the claim that **a single line causes lower variation among waiting times for customers.**

### Answer

Since the claim is that a single line causes less variation, this is a test of a single variance. The parameter is the population variance,  $\sigma^2$ .

**Random Variable:** The sample standard deviation,  $s$ , is the random variable. Let  $s$  = standard deviation for the waiting times.

- $H_0 : \sigma^2 \geq 7.2^2$
- $H_a : \sigma^2 < 7.2^2$

The word "less" tells you this is a left-tailed test.

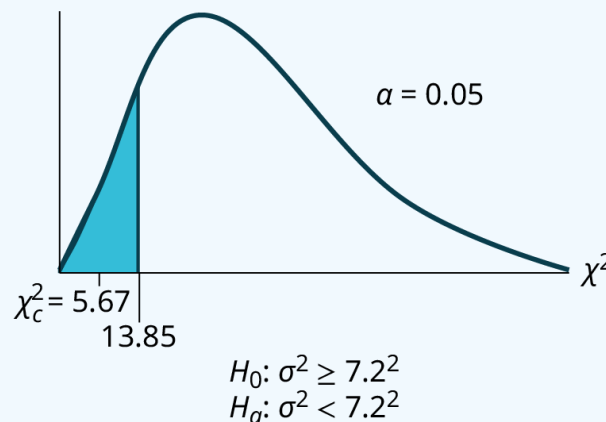
**Distribution for the test:**  $\chi^2_{24}$ , where:

- $n$  = the number of customers sampled
- $df = n - 1 = 25 - 1 = 24$

**Calculate the test statistic:**

$$\chi^2_c = \frac{(n-1)s^2}{\sigma^2} = \frac{(25-1)(3.5)^2}{7.2^2} = 5.67$$

where  $n = 25$ ,  $s = 3.5$ , and  $\sigma = 7.2$ .



Cannot accept  $H_0$

Figure 11.3.1

The graph of the Chi-square shows the distribution and marks the critical value with 24 degrees of freedom at 95% level of confidence,  $\alpha = 0.05$ , 13.85. The critical value of 13.85 came from the Chi squared table which is read very much like the students  $t$  table. The difference is that the students  $t$  distribution is symmetrical and the Chi squared distribution is not. At the top of the Chi squared table we see not only the familiar 0.05, 0.10 etc. but also 0.95, 0.975 etc. These are the columns used to find the left hand critical value. The graph also marks the calculated  $\chi^2$  test statistic of 5.67.

Comparing the test statistic with the critical value, as we have done with all other hypothesis tests, we reach the conclusion.

**Make a decision:** Because the calculated test statistic is in the tail we cannot accept  $H_0$ . This means that you reject  $\sigma^2 \geq 7.2^2$ . In other words, you do not think the variation in waiting times is 7.2 minutes or more; you think the variation in waiting times is less.

**Conclusion:** At a 5% level of significance, from the data, there is sufficient evidence to conclude that a single line causes a lower variation among the waiting times or with a single line, the customer waiting times vary less than 7.2 minutes.

### Try It 11.3.2

The FCC conducts broadband speed tests to measure how much data per second passes between a consumer's computer and the internet. At a certain point in time, the standard deviation of Internet speeds across Internet Service Providers (ISPs) was 12.2 percent. Suppose a sample of 15 ISPs is taken, and the standard deviation is 13.2. An analyst claims that the standard deviation of speeds is more than what was reported. State the null and alternative hypotheses, compute the degrees of freedom, the test statistic, sketch the graph of the  $p$ -value, and draw a conclusion. Test at the 1% significance level.

### ? Exercise 11.3.3

Professor Holmes has a weakness for cream filled donuts, but they believe that some bakeries are not properly filling the donuts. A sample of 24 donuts reveals a mean amount of filling equal to 0.04 cups, and the sample standard deviation is 0.11 cups. Professor Holmes has an interest in the average quantity of filling, of course, but he is particularly distressed if one donut is radically different from another. Professor Holmes does not like surprises.

#### Problem

Test at 95% the null hypothesis that the population variance of donut filling is significantly different from the average amount of filling.

#### Answer

This is clearly a problem dealing with variances. In this case we are testing a single sample rather than comparing two samples from different populations. The null and alternative hypotheses are thus:

$$\begin{aligned} H_0 : \sigma^2 &= 0.04 \\ H_a : \sigma^2 &\neq 0.04 \end{aligned} \quad (11.3.2)$$

The test is set up as a two-tailed test because Professor Holmes has shown concern with too much variation in filling as well as too little: their dislike of a surprise is any level of filling outside the expected average of 0.04 cups. The test statistic is calculated to be:

$$\chi^2_c = \frac{(n-1)s^2}{\sigma_o^2} = \frac{(24-1)0.11^2}{0.04^2} = 6.9575 \quad (11.3.3)$$

The calculated  $\chi^2$  test statistic, 6.96, is in the tail therefore at a 0.05 level of significance, we cannot accept the null hypothesis that the variance in the donut filling is equal to 0.04 cups. It seems that Professor Holmes is destined to meet disappointment with each bit.

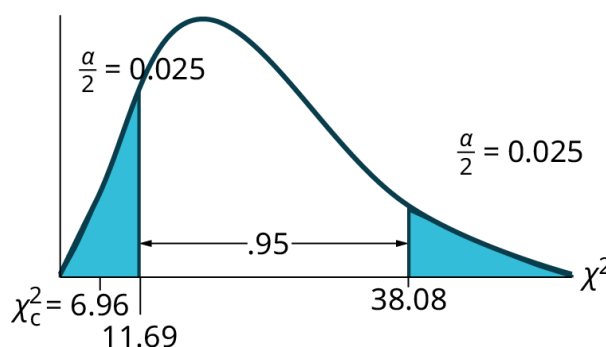


Figure 11.3.2

### Try It 11.3.3

The FCC conducts broadband speed tests to measure how much data per second passes between a consumer's computer and the internet. At a certain point in time, the standard deviation of Internet speeds across Internet Service Providers (ISPs) was 12.2 percent. Suppose a sample of 15 ISPs is taken, and the standard deviation is 13.2. An analyst claims that the standard deviation of speeds is more than what was reported. State the null and alternative hypotheses, compute the degrees of freedom,

the test statistic, sketch the graph of the distribution and mark the area associated with the level of confidence, and draw a conclusion. Test at the 1% significance level.

---

This page titled [11.3: Test of a Single Variance](#) is shared under a [CC BY 4.0](#) license and was authored, remixed, and/or curated by [OpenStax](#) via [source content](#) that was edited to the style and standards of the LibreTexts platform.