3.15: Solutions

1. a. $P(L)=P(S)$
b. $P(M \cup S)$
c. $P(F \cap L)$
d. $P(M \mid L)$
e. $P(L \mid M)$
f. $P(S \mid F)$
g. $P(F \mid L)$
h. $P(F \cup L)$
i. $P(M \cap S)$
j. $P(F)$

3. $P(N)=\frac{15}{42}=\frac{5}{14}=0.36$
5. $P(C)=\frac{5}{42}=0.12$
7. $P(G)=\frac{20}{150}=\frac{2}{15}=0.13$
9. $P(R)=\frac{22}{150}=\frac{11}{75}=0.15$
11. $P(O)=\frac{150-22-38-20-28-26}{150}=\frac{16}{150}=\frac{8}{75}=0.11$
13. $P(E)=\frac{47}{194}=0.24$
15. $P(N)=\frac{23}{194}=0.12$
17. $P(S)=\frac{12}{194}=\frac{6}{97}=0.06$
19. $\frac{13}{52}=\frac{1}{4}=0.25$
21. $\frac{3}{6}=\frac{1}{2}=0.5$
23. $P(R)=\frac{4}{8}=0.5$
25. $P(O \cup H)$
27. $P(H \mid I)$
29. $P(N \mid O)$
31. $P(I \cup N)$
33. $P(I)$
35. The likelihood that an event will occur given that another event has already occurred.
37. 1
39. the probability of landing on an even number or a multiple of three
41. $P(J)=0.3$
43. $P(Q \cap R)=P(Q)P(R)$
$0.1=(0.4)P(R)$
$P(R)=0.25$
44. Find $P(Z)$: $P(Z)=0.87$
45. Find $P(S)$: $P(S)=0.32$
46. Find $P(S \mid Z)$: $P(S \mid Z)=0.55$
47. In words, what is $S \mid Z$? : $S \mid Z$ means, given that the customer has ordered pizza, the person also orders a salad.
48. Find $P(ZANDS)$: $P(ZANDS)=0.4785$

49. In words, what is $ZANDS$? : $ZANDS$ represents the event that a customer orders a pizza and salad.

50. Are P and S independent events? Show why or why not. No, because $P(S)$ does not equal $P(S \mid Z)$.

51. Find $P(ZORS)$: $P(ZORS)=0.7115$

52. In words, what is $ZORS$? $ZORS$ represents the event that a customer orders a pizza or salad.

53. Are Z and S mutually exclusive events? Show why or why not. No, because $P(S$ and Z) does not equal 0 .

55. P (musician is a male $\cap$ had private instruction $)= \frac{15}{130} = \frac{3}{26} = 0.12$

57. The events are not mutually exclusive. It is possible to be a female musician who learned music in school.

58.

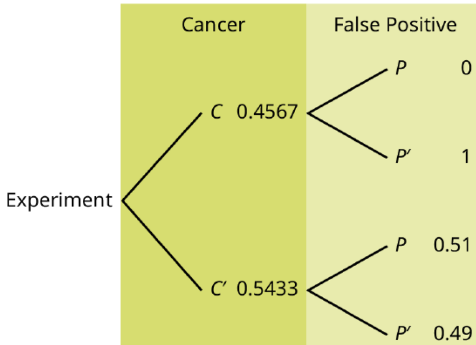

Figure 3.15.1:

60. $\frac{35,065}{100,450}$

62. To pick one person from the study who is Japanese American AND smokes 21 to 30 cigarettes per day means that the person has to meet both criteria: both Japanese American and smokes 21 to 30 cigarettes. The sample space should include everyone in the study. The probability is $\frac{4,715}{100,450}$.

64. To pick one person from the study who is Japanese American given that person smokes 21-30 cigarettes per day, means that the person must fulfill both criteria and the sample space is reduced to those who smoke 2130 cigarettes per day. The probability is $\frac{4715}{15,273}$.

66. a.

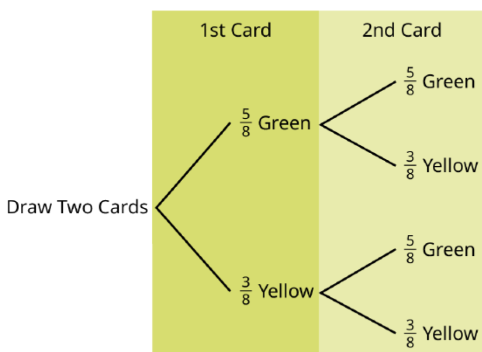

Figure 3.15.2

b. $P(GG) = \left(\frac{5}{8}\right)\left(\frac{5}{8}\right) = \frac{25}{64}$

c. $P($ at least one green $) = P(GG) + P(GY) + P(YG) = \frac{25}{64} + \frac{15}{64} + \frac{15}{64} = \frac{55}{64}$

d. $P(G \mid G) = \frac{5}{8}$

e. Yes, they are independent because the first card is placed back in the bag before the second card is drawn; the composition of cards in the bag remains the same from draw one to draw two.

68.

a.

	<20	20–64	>64	Totals
Female	0.0244	0.3954	0.0661	0.486
Male	0.0259	0.4186	0.0695	0.514

	<20	20–64	>64	Totals
Totals	0.0503	0.8140	0.1356	1

b. $P(F) = 0.486$
c. $P(>64 \mid\mid F) = 0.1361$
d. $P(>64 \text{ and } F) = P(F)\ P(>64|F) = (0.486)(0.1361) = 0.0661$
e. $P(>64 \mid\mid F)$ is the percentage of female drivers who are 65 or older and $P(>64 \cap\cap F)$ is the percentage of drivers who are female and 65 or older.
f. $P(>64) = P(>64 \cap\cap F) + P(>64 \cap\cap M) = 0.1356$
g. No, being female and 65 or older are not mutually exclusive because they can occur at the same time $P(>64 \cap\cap F) = 0.0661$.

70.

a.

	Car, truck or van	Walk	Public transportation	Other	Totals
Alone	0.7318				
Not alone	0.1332				
Totals	0.8650	0.0390	0.0530	0.0430	1

b. If we assume that all walkers are alone and that none from the other two groups travel alone (which is a big assumption) we have: $P(\text{Alone}) = 0.7318 + 0.0390 = 0.7708$.
c. Make the same assumptions as in (b) we have: (0.7708)(1,000) = 771
d. (0.1332)(1,000) = 133'

73.

a. You can't calculate the joint probability knowing the probability of both events occurring, which is not in the information given; the probabilities should be multiplied, not added; and probability is never greater than 100%
b. A home run by definition is a successful hit, so he has to have at least as many successful hits as home runs.

75. 0

77. 0.3571

79. 0.2142

81. Physician (83.7)

83. 83.7 − 79.6 = 4.1

85. $P(\text{Occupation} < 81.3) = 0.5$

87.

a. The Forum Research surveyed 1,046 Torontonians.
b. 58%
c. 42% of 1,046 = 439 (rounding to the nearest integer)
d. 0.57
e. 0.60.

89. a. P (Betting on two line that touch each other on the table $)=\dfrac{6}{38}$

b. $P($ Betting on three numbers in a line $)=\dfrac{3}{38}$

c. $P($ Betting on one number $)=\dfrac{1}{38}$

d. P (Betting on four number that touch each other to form a square $)=\dfrac{4}{38}$

e. P (Betting on two number that touch each other on the table)$=\frac{2}{38}$

f. $P($ Betting on $0-00-1-2-3)=\frac{5}{38}$

g. P (Betting on 0-1-2; or 0-00-2; or 00-2-3) $=\frac{3}{38}$

91. a. $\{G1, G2, G3, G4, G5, Y1, Y2, Y3\}$
b. $\frac{5}{8}$
c. $\frac{2}{3}$
d. $\frac{2}{8}$
e. $\frac{6}{8}$
f. No, because $P(G \cap E)$ does not equal 0 .

93. NOTE The coin toss is independent of the card picked first.

a. $\{(G, H)(G, T)(B, H)(B, T)(R, H)(R, T)\}$
b. $P(A)=P($ blue $) P($ head $)=\left(\frac{3}{10}\right)\left(\frac{1}{2}\right)=\frac{3}{20}$
c. Yes, A and B are mutually exclusive because they cannot happen at the same time; you cannot pick a card that is both blue and also (red or green). $P(A \cap B)=0$
d. No, A and C are not mutually exclusive because they can occur at the same time. In fact, C includes all of the outcomes of A; if the card chosen is blue it is also (red or blue). $P(A \cap C)=P(A)=\frac{3}{20}$

95. a. $S=\{(HHH),(HHT),(HTH),(HTT),(THH),(THT),(TTH),(TT)\}$

b. $\frac{4}{8}$
c. Yes, because if A has occurred, it is impossible to obtain two tails. In other words, $P(A \cap B)=0$.

97. a. If Y and Z are independent, then $P(Y \cap Z)=P(Y) P(Z)$, so $P(Y \cup Z)=P(Y)+P(Z)-P(Y) P(Z)$.
b. 0.5

99. iii i iv ii

101.

a. $P(R)=0.44$
b. $P(R \mid E)=0.56$
c. $P(R \mid O)=0.31$
d. No, whether the money is returned is not independent of which class the money was placed in. There are several ways to justify this mathematically, but one is that the money placed in economics classes is not returned at the same overall rate; $P(R \mid E) \neq P(R)$.
e. No, this study definitely does not support that notion; in fact, it suggests the opposite. The money placed in the economics classrooms was returned at a higher rate than the money place in all classes collectively; $P(R \mid E)>P(R)$.

103. a. P (type O OR Rh-) $=P($ type O) $+P($ Rh- $)-P($ type O AND Rh- $)$ $0.38=0.45+0.07-P$ (type O AND Rh-); Solve to find P (type O AND Rh-) $=0.14$. 14% of people have type O, Rh- blood.
b. $P($ NOT(type O AND Rh- $))=1-P($ type O AND Rh- $)=1-0.14=0.86$
86% of people do not have type O, Rh- blood.

105. a. Let $C=$ be the event that the cookie contains chocolate. Let $N=$ the event that the cookie contains nuts.
b. $P(C \cup N)=P(C)+P(N)-P(C \cap N)=0.36+0.12-0.08=0.40$
c. $P($ NEITHER chocolate NOR nuts $)=1-P(C \cup N)=1-0.40=0.60$

107. 0

109. $\frac{10}{67}$

111. $\frac{10}{34}$

113. d

115.

a.

Race and sex	1–14	15–24	25–64	Over 64	TOTALS
White, male	1,165	2,036	3,703	1,491	8,395
White, female	1,076	2,242	4,060	1,751	9,129
Black, male	142	194	384	104	824
Black, female	131	290	486	154	1,061
All others				156	
TOTALS	2,792	5,279	9,354	3,656	21,081

b.

Race and sex	1–14	15–24	25–64	Over 64	TOTALS
White, male	1,165	2,036	3,703	1,491	8,395
White, female	1,076	2,242	4,060	1,751	9,129
Black, male	142	194	384	104	824
Black, female	131	290	486	154	1,061
All others	278	517	721	156	1672
TOTALS	2,792	5,279	9,354	3,656	21,081

c. $\frac{8,395}{21,081} \approx 0.3982$
d. $\frac{1,061}{21,081} \approx 0.0503$
e. $\frac{1,885}{21,081} \approx 0.0894$
f. $\frac{9,219}{19,409} \approx 0.475$
g. $\frac{1,595}{3,656} \approx 0.456$

117. b

119.

a. $\frac{26}{106}$
b. $\frac{33}{106}$
c. $\frac{21}{106}$
d. $\left(\frac{26}{106}\right) + \left(\frac{33}{106}\right) - \left(\frac{21}{106}\right) = \left(\frac{38}{106}\right)$
e. $\frac{21}{33}$

121. a

124.

a. $P(C) = 0.4567$
b. not enough information
c. not enough information
d. No, because over half (0.51) of men have at least one false positive text

126. a. $(J \cup K) = P(J) + P(K) - P(J \cap K); 0.45 = 0.18 + 0.37 - P(J \cap K)$; solve to find $P(J \cap K) = 0.10$
b. $P(\text{NOT}(J \cap K)) = 1 - P(J \cap K) = 1 - 0.10 = 0.90$
c. $P(\text{NOT}(J \cup K)) = 1 - P(J \cup K) = 1 - 0.45 = 0.55$