5.11: Solutions

1.

Uniform Distribution

3.

Normal Distribution

5.

$P(6 < x < 7)$

7.

one

9.

zero

11.

one

13.

0.625

15.

The probability is equal to the area from x = 3232" role="presentation" style="position: relative;">$\frac{3}{2}$ $\frac{3}{2}$ to x = 4 above the x-axis and up to $f(x)$ = 1313" role="presentation" style="position: relative;">$\frac{1}{3}$ $\frac{1}{3}$.

17.

It means that the value of x is just as likely to be any number between 1.5 and 4.5.

19.

$1.5 \le x \le 4.5$

21.

0.3333

23.

zero

25.

0.6

27.

b is 12, and it represents the highest value of x.

29.

six

31.

This graph shows a uniform distribution. The horizontal axis ranges from 0 to 12. The distribution is modeled by a rectangle extending from x = 0 to x = 12. A region from x = 9 to x = 12 is shaded inside the rectangle.

Figure 5.52

33.

4.8

35.

X = The age (in years) of cars in the staff parking lot

37.

0.5 to 9.5

39.

$f(x)$ = 1919" role="presentation" style="position: relative;">$\frac{1}{9}$ $\frac{1}{9}$ where x is between 0.5 and 9.5, inclusive.

41.

$\mu = 5$

43.

1. Answers may vary.
2. 3.573.57" role="presentation" style="position: relative;">$\frac{3.5}{7}$ $\frac{3.5}{7}$

45.

1. Answers may vary.
2. $k = 7.25$
3. 7.25

47.

No, outcomes are not equally likely. In this distribution, more people require a little bit of time, and fewer people require a lot of time, so it is more likely that someone will require less time.

49.

five

51.

$f(x) = 0.2e^{-0.2x}$

53.

0.5350

55.

6.02

57.

$f(x) = 0.75e^{-0.75x}$

59.

This graph shows an exponential distribution. The graph slopes downward. It begins at the point (0, 0.75) on the y-axis and approaches the x-axis at the right edge of the graph. The decay parameter, m, equals 0.75.

Figure 5.53

61.

0.4756

63.

The mean is larger. The mean is 1m=10.75≈1.331m=10.75≈1.33" role="presentation" style="position: relative;">$\frac{1}{m} = \frac{1}{0.75} \approx 1.33$$\frac{1}{m} = \frac{1}{0.75} \approx 1.33$, which is greater than 0.9242.

65.

continuous

67.

$m = 0.000121$

69.

1. Answers may vary.
2. $P(x < 5{,}730) = 0.5001$

71.

1. Answers may vary.
2. $k = 2947.73$

73.

Age is a measurement, regardless of the accuracy used.

75.

1. $X \sim U(1, 9)$
2. Answers may vary.
3. f(x)=18f(x)=18" role="presentation" style="position: relative;">$f(x) = \frac{1}{8}$$f(x) = \frac{1}{8}$ where 1≤x≤91≤x≤9" role="presentation" style="position: relative;">$1 \le x \le 9$$1 \le x \le 9$
4. five
5. 2.3
6. 15321532" role="presentation" style="position: relative;">$\frac{15}{32}$ $\frac{15}{32}$
7. 333800333800" role="presentation" style="position: relative;">$\frac{333}{800}$ $\frac{333}{800}$
8. 2323" role="presentation" style="position: relative;">$\frac{2}{3}$ $\frac{2}{3}$
9. 8.2

77.

1. X represents the length of time a commuter must wait for a train to arrive on the Red Line.
2. $X \sim U(0, 8)$
3. Graph the probability distribution.
4. f(x)=18 f (x) = 1 8 " role="presentation" style="position: relative;">$f(x) = \frac{1}{8}$$f(x) = \frac{1}{8}$ where 0≤x≤80 ≤ x ≤ 8" role="presentation" style="position: relative;">$0 \le x \le 8$
   $0 \le x \le 8$
5. four
6. 2.31
7. 18 1 8 " role="presentation" style="position: relative;">$\frac{1}{8}$ $\frac{1}{8}$
8. 18 1 8 " role="presentation" style="position: relative;">$\frac{1}{8}$ $\frac{1}{8}$
9. 3.2

79.

d

81.

b

83.

1. The probability density function of X is 125−16=19125−16=19" role="presentation" style="position: relative;">$\frac{1}{25-16} = \frac{1}{9}$ $\frac{1}{25-16} = \frac{1}{9}$.
   $P(X > 19) = (25 – 19)$ (19)(19)" role="presentation" style="position: relative;">$\left(\frac{1}{9}\right)$ $\left(\frac{1}{9}\right)$ = 6969" role="presentation" style="position: relative;">$\frac{6}{9}$ $\frac{6}{9}$ =
   2323" role="presentation" style="position: relative;">$\frac{2}{3}$ $\frac{2}{3}$.
   19) = 2/3.">
   This shows the graph of the function f(x) = 1/9, the pdf for a uniform distribution. A horizontal line ranges from the point (16, 1/9) to the point (25, 1.9). Vertical lines extend from the x-axis to the graph at x = 16 and x = 25 creating a rectangle. A region is shaded inside the rectangle from x = 19 to x = 25. Text notes that the shaded area represents P(x
   19) = 2/3." width="968" height="456" data-lazy-src="/apps/archive/20240130.204924/resources/8e6879719f6f0b2c01f4a67c4c3e282fbdcfbc27">
   Figure 5.54
2. $P(19 < X < 22) = (22 – 19)$ (19)(19)" role="presentation" style="position: relative;">$\left(\frac{1}{9}\right)$ $\left(\frac{1}{9}\right)$ = 3939" role="presentation" style="position: relative;">$\frac{3}{9}$ $\frac{3}{9}$ =
   1313" role="presentation" style="position: relative;">$\frac{1}{3}$ $\frac{1}{3}$.
   This shows the graph of the function f(x) = 1/9, the pdf for a uniform distribution. A horizontal line ranges from the point (16, 1/9) to the point (25, 1.9). Vertical lines extend from the x-axis to the graph at x = 16 and x = 25 creating a rectangle. A region is shaded inside the rectangle from x = 19 to x = 22. Text notes that the shaded area represents P(19< x < 22) = 1/3.
   Figure 5.55
3. The area must be 0.25, and 0.25 = (width)(19)(19)" role="presentation" style="position: relative;">$\left(\frac{1}{9}\right)$ $\left(\frac{1}{9}\right)$, so width = (0.25)(9) = 2.25. Thus, the value is 25 – 2.25 = 22.75.
4. This is a conditional probability question. P(x > 21| x > 18). You can do this two ways:
   - Draw the graph where a is now 18 and b is still 25. The height is 1(25−18)1(25−18)" role="presentation" style="position: relative;">$\frac{1}{(25-18)}$ $\frac{1}{(25-18)}$ =
     1717" role="presentation" style="position: relative;">$\frac{1}{7}$ $\frac{1}{7}$
     So, $P(x > 21|x > 18) = (25 – 21)$(17)(17)" role="presentation" style="position: relative;">$\left(\frac{1}{7}\right)$ $\left(\frac{1}{7}\right)$ = 4/7.

- Use the formula: $P(x > 21|x > 18)$ = P(x>21 AND x>18)P(x>18)P(x>21 AND x>18)P(x>18)" role="presentation" style="position: relative;">$\frac{P(x>21 \text{ AND } x>18)}{P(x>18)}$ $\frac{P(x>21 \text{AND} x>18)}{P(x>18)}$ = P(x>21)P(x>18)P(x>21)P(x>18)" role="presentation" style="position: relative;">$\frac{P(x>21)}{P(x>18)}$ $\frac{P(x>21)}{P(x>18)}$ = (25−21)(25−18)(25−21)(25−18)" role="presentation" style="position: relative;">$\frac{(25-21)}{(25-18)}$ $\frac{(25-21)}{(25-18)}$ = 4747" role="presentation" style="position: relative;">$\frac{4}{7}$ $\frac{4}{7}$.

85.

1. $P(X > 650)$ = 700−650700−300=50400=18700−650700−300=50400=18" role="presentation" style="position: relative;">$\frac{700-650}{700-300} = \frac{50}{400} = \frac{1}{8}$ $\frac{700-650}{700-300} = \frac{50}{400} = \frac{1}{8}$ = 0.125
2. $P(400 < X < 650)$ = 650−400700−300=250400650−400700−300=250400" role="presentation" style="position: relative;">$\frac{650-400}{700-300} = \frac{250}{400}$ $\frac{650-400}{700-300} = \frac{250}{400}$ = 0.625
3. 0.10 = width700−300width700−300" role="presentation" style="position: relative;">$\frac{\text{width}}{700-300}$ $\frac{\text{width}}{700-300}$, so width = 400(0.10) = 40. Since 700 – 40 = 660, the drivers travel at least 660 miles on the furthest 10% of days.

87.

1. X = the useful life of a particular car battery, measured in months.
2. X is continuous.
3. $X \sim Exp(0.025)$
4. 40 months
5. 360 months
6. 0.4066
7. 14.27

89.

1. X = the time (in years) after reaching age 60 that it takes an individual to retire
2. X is continuous.
3. $X \sim Exp$(15)(15)" role="presentation" style="position: relative;">$\left(\frac{1}{5}\right)$ $\left(\frac{1}{5}\right)$
4. five
5. five
6. Answers may vary.
7. 0.1353
8. before
9. 18.3

91.

a

93.

c

95.

Let T = the life time of a light bulb.

The decay parameter is m = 1/8, and $T \sim$ Exp(1/8). The cumulative distribution function is P(T<t)=1−e−t8P(T<t)=1−e−t8" role="presentation" style="position: relative;">$P(T < t) = 1 - e^{-\frac{t}{8}}$ $P(T < t) = 1 - e^{-\frac{t}{8}}$

1. Therefore, $P(T < 1)$ = 1 – e–18–18" role="presentation" style="position: relative;">$-\frac{1}{8}$ $-\frac{1}{8}$ ≈ 0.1175.
2. We want to find $P(6 < t < 10)$.
   To do this, $P(6 < t < 10) – P(t < 6)$
   =(1–e–18*10)–(1–e–18*6)=(1–e–18*10)–(1–e–18*6)" role="presentation" style="position: relative;">$= \left(1 - e^{-\frac{1}{8}*10}\right) - \left(1 - e^{-\frac{1}{8}*6}\right)$
   $= \left(1 - e^{-\frac{1}{8}*10}\right) - \left(1 - e^{-\frac{1}{8}*6}\right)$ ≈ 0.7135 – 0.5276 = 0.1859
   This graph shows an exponential distribution. The graph slopes downward. It begins at the point (0, 1.2) and approaches the horizontal t-axis at the right edge of the graph. The region under the graph from x = 6 to x = 10 is shaded. Text notes that the shaded area represents P(6 < t < 10) = 0.1859.
   Figure 5.56
3. We want to find 0.70
   =P(T>t)=1–(1–e−t8)=e−t8.=P(T>t)=1–(1–e−t8)=e−t8." role="presentation" style="position: relative;">$= P(T > t) = 1 - \left(1 - e^{-\frac{t}{8}}\right) = e^{-\frac{t}{8}}$.
   $= P(T > t) = 1 - \left(1 - e^{-\frac{t}{8}}\right) = e^{-\frac{t}{8}}$.
   Solving for t, e–t8–t8" role="presentation" style="position: relative;">$-\frac{t}{8}$ $-\frac{t}{8}$ = 0.70, so –t8–t8" role="presentation" style="position: relative;">$-\frac{t}{8}$ $-\frac{t}{8}$ = $ln(0.70)$, and $t = –8ln(0.70)$ ≈ 2.85 years.
   Or use t =
   ln(area_to_the_right)(–m)=ln(0.70)–18≈2.85 yearsln(area_to_the_right)(–m)=ln(0.70)–18≈2.85 years" role="presentation" style="position: relative;">$\frac{ln(\text{area}$
   $\frac{ln(\text{area\_to\_the\_right})}{(-m)} = \frac{ln(0.70)}{-\frac{1}{8}} \approx 2.85 \text{ years}$.
   2.85) = 0.70."> This graph shows an exponential distribution. The graph slopes downward. It begins at the point (0, 1.2) and approaches the horizontal t-axis at the right edge of the graph. The region under the graph from x = 2.85 to the edge of the graph is shaded. Text notes that the shaded area represents P(t
   2.85) = 0.70." width="981" height="564" src="/apps/archive/20240130.204924/resources/443d2d74b21ce16f1b3033b0eaee2d8aa8fe7a3d">
   Figure 5.57
4. We want to find 0.02 = $P(T < t)$ = 1 – e–t8–t8" role="presentation" style="position: relative;">$-\frac{t}{8}$ $-\frac{t}{8}$.
   Solving for t, e–t8–t8" role="presentation" style="position: relative;">$-\frac{t}{8}$ $-\frac{t}{8}$ = 0.98, so –t8–t8" role="presentation" style="position: relative;">$-\frac{t}{8}$ $-\frac{t}{8}$ = $ln(0.98)$, and t = $–8ln(0.98)$ ≈ 0.1616 years, or roughly two months.
   The warranty should cover light bulbs that last less than 2 months.
   Or use
   ln(area_to_the_right)(–m)=ln(1–0.2)–18ln(area_to_the_right)(–m)=ln(1–0.2)–18" role="presentation" style="position: relative;">$\frac{\ln(\text{area\_to\_the\_right})}{(-\text{m})} = \frac{\ln(1-0.2)}{-\frac{1}{8}}$
   $\frac{\ln(\text{area\_to\_the\_right})}{(-\text{m})} = \frac{\ln(1-0.2)}{-\frac{1}{8}}$ = 0.1616.
5. We must find $P(T < 8|T > 7)$.
   Notice that by the rule of complement events, $P(T < 8|T > 7) = 1 – P(T > 8|T > 7)$.
   By the memoryless property ($P(X > r + t|X > r) = P(X > t)$).
   So $P(T > 8|T > 7) = P(T > 1)$ =
   1–(1–e–18)=e–18≈0.88251–(1–e–18)=e–18≈0.8825" role="presentation" style="position: relative;">$1 - \left(1 - e^{-\frac{1}{8}}\right) = e^{-\frac{1}{8}} \approx 0.8825$

$1-\left(1-e^{-\frac{1}{8}}\right)=e^{-\frac{1}{8}}\approx 0.8825$

Therefore, $P(T < 8|T > 7) = 1 - 0.8825 = 0.1175$.

97.

Let X = the number of no-hitters throughout a season. Since the duration of time between no-hitters is exponential, the number of no-hitters per season is Poisson with mean $\lambda = 3$.

Therefore, $(X = 0)$ = 30e–30!30e–30!" role="presentation" style="position: relative;">$\frac{3^0e^{-3}}{0!}$ $\frac{3^0e^{-3}}{0!}$ = $e^{-3} \approx 0.0498$

NOTE

You could let T = duration of time between no-hitters. Since the time is exponential and there are 3 no-hitters per season, then the time between no-hitters is 1313" role="presentation" style="position: relative;">$\frac{1}{3}$ $\frac{1}{3}$ season. For the exponential, μ = 1313" role="presentation" style="position: relative;">$\frac{1}{3}$ $\frac{1}{3}$.

Therefore, m = 1μ1μ" role="presentation" style="position: relative;">$\frac{1}{\mu}$ $\frac{1}{\mu}$ = 3 and $T \sim Exp(3)$.

1. The desired probability is $P(T > 1) = 1 - P(T < 1) = 1 - (1 - e^{-3}) = e^{-3} \approx 0.0498$.
2. Let T = duration of time between no-hitters. We find $P(T > 2|T > 1)$, and by the **memoryless property** this is simply $P(T > 1)$, which we found to be 0.0498 in part a.
3. Let X = the number of no-hitters is a season. Assume that X is Poisson with mean $\lambda = 3$. Then $P(X > 3) = 1 - P(X \le 3) = 0.3528$.

99.

1. 10091009" role="presentation" style="position: relative;">$\frac{100}{9}$ $\frac{100}{9}$ = 11.11
2. $P(X > 10) = 1 - P(X \le 10) = 1 -$ Poissoncdf(11.11, 10) ≈ 0.5532.
3. The number of people with Type B positive blood encountered roughly follows the Poisson distribution, so the number of people X who arrive between successive Type B positive arrivals is roughly exponential with mean $\mu = 9$ and m = 1919" role="presentation" style="position: relative;">$\frac{1}{9}$ $\frac{1}{9}$. The cumulative distribution function of X is P(X<x)=1−e−x9P(X<x)=1−e−x9" role="presentation" style="position: relative;">$P\left(X<x\right)=1-e^{-\frac{x}{9}}$ $P\left(X<x\right)=1-e^{-\frac{x}{9}}$. Thus, $P(X > 20) = 1 - P(X \le 20)$ = 1−(1−e−209)≈0.1084.1−(1−e−209)≈0.1084." role="presentation" style="position: relative;">$1-\left(1-e^{-\frac{20}{9}}\right)\approx 0.1084.$$1-\left(1-e^{-\frac{20}{9}}\right)\approx 0.1084.$

NOTE

We could also deduce that each person arriving has a 8/9 chance of not having Type B positive blood. So the probability that none of the first 20 people arrive have Type B positive blood is (89)20≈0.0948(89)20≈0.0948" role="presentation" style="position: relative;">$\left(\frac{8}{9}\right)^{20}\approx 0.0948$$\left(\frac{8}{9}\right)^{20}\approx 0.0948$. (The geometric distribution is more appropriate than the exponential because the number of people between Type B positive people is discrete instead of continuous.)

101.

Let T = duration (in minutes) between successive visits. Since patients arrive at a rate of one patient every seven minutes, $\mu = 7$ and the decay constant is m = 1717" role="presentation" style="position: relative;">$\frac{1}{7}$ $\frac{1}{7}$. The cdf is $P(T < t)$ = 1−et71−et7" role="presentation" style="position: relative;">$1-e^{\frac{t}{7}}$ $1-e^{\frac{t}{7}}$

1. $P(T < 2)$ = 1 - 1−e−271−e−27" role="presentation" style="position: relative;">$1-e^{-\frac{2}{7}}$ $1-e^{-\frac{2}{7}}$ ≈ 0.2485.
2. $P(T > 15)$ = 1−P(T<15)=1−(1−e−157)≈e−157≈0.11731−P(T<15)=1−(1−e−157)≈e−157≈0.1173" role="presentation" style="position: relative;"> $1-P\left(T<15\right)=1-\left(1-e^{-\frac{15}{7}}\right)\approx e^{-\frac{15}{7}}\approx 0.1173$.
3. $P(T > 15|T > 10) = P(T > 5)$ = 1−(1−e−57)=e−57≈0.48951−(1−e−57)=e−57≈0.4895" role="presentation" style="position: relative;">$1-\left(1-e^{-\frac{5}{7}}\right)=e^{-\frac{5}{7}}\approx 0.4895$ $1-\left(1-e^{-\frac{5}{7}}\right)=e^{-\frac{5}{7}}\approx 0.4895$.
4. Let X = # of patients arriving during a half-hour period. Then X has the Poisson distribution with a mean of 307307" role="presentation" style="position: relative;">$\frac{30}{7}$ $\frac{30}{7}$, $X \sim$ Poisson(307)(307)" role="presentation" style="position: relative;">$\left(\frac{30}{7}\right)$ $\left(\frac{30}{7}\right)$. Find $P(X > 8) = 1 - P(X \le 8) \approx 0.0311$.