3.6: Venn Diagrams

A **Venn diagram** is a picture that represents the outcomes of an experiment. It generally consists of a box that represents the sample space S together with circles or ovals. The circles or ovals represent events. Venn diagrams also help us to convert common English words into mathematical terms that help add precision.

Venn diagrams are named for their inventor, John Venn, a mathematics professor at Cambridge and an Anglican minister. His main work was conducted during the late 1870's and gave rise to a whole branch of mathematics and a new way to approach issues of logic. We will develop the probability rules just covered using this powerful way to demonstrate the probability postulates including the Addition Rule, Multiplication Rule, Complement Rule, Independence, and Conditional Probability.

Exercise 3.6.1

Suppose an experiment has the outcomes $1, 2, 3, \ldots, 12$where each outcome has an equal chance of occurring. Let event $A = \{1, 2, 3, 4, 5, 6\}$and event $B = \{6, 7, 8, 9\}$. Then A intersect $B = A \cap B = \{6\}$ and A union $B = A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. The Venn diagram is as follows:

Answer

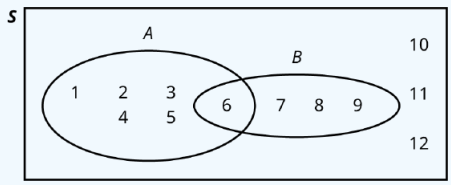


Figure 3.6.1

Figure $3.6.1$ shows the most basic relationship among these numbers. First, the numbers are in groups called sets; set A and set B. Some number are in both sets; we say in set $A\cap$ in set B. The English word "and" means inclusive, meaning having the characteristics of both A and B, or in this case, being a part of both A and B. This condition is called the INTERSECTION of the two sets. All members that are part of both sets constitute the intersection of the two sets. The intersection is written as $A \cap B$ where $\cap$ is the mathematical symbol for intersection. The statement $A \cap B$ is read as "A intersect B." You can remember this by thinking of the intersection of two streets.

There are also those numbers that form a group that, for membership, the number must be in either one or the other group. The number does not have to be in BOTH groups, but instead only in either one of the two. These numbers are called the UNION of the two sets and in this case they are the numbers 1-5 (from A exclusively), 7-9 (from set B exclusively) and also 6 , which is in both sets A and B . The symbol for the UNION is $\cup$, thus $A \cup B =$ numbers 1-9, but excludes number 10, 11, and 12. The values 10,11 , and 12 are part of the universe, but are not in either of the two sets.

Translating the English word "AND" into the mathematical logic symbol $\cap$, intersection, and the word "OR" into the mathematical symbol $\cup$, union, provides a very precise way to discuss the issues of probability and logic. The general terminology for the three areas of the Venn diagram in Figure $3.6.1$is shown in Figure $3.6.2$.

▼

Solution

a. 0.51; b. 0.075; c. 0.04; d. 0.545; e. The area represents the African Americans that have type O blood and the Rh- factor. f. The area represents the African Americans that have neither type O blood nor the Rh- factor.

Try It 3.6.1

Suppose an experiment has outcomes black, white, red, orange, yellow, green, blue, and purple, where each outcome has an equal chance of occurring. Let event $C = \{$ green, blue, purple $\}$ and event $P = \{$ red, yellow, blue $\}$. Then $C \cap P = \{$ blue $\}$ and $C \cup P = \{$ green, blue, purple, red, yellow $\}$. Draw a Venn diagram representing this situation.

Exercise 3.6.2

Flip two fair coins. Let $A =$ tails on the first coin. Let $B =$ tails on the second coin. Then $A = \{TT, TH\}$ and $B = \{TT, HT\}$. Therefore, $A \cap B = \{\mathrm{TT}\}$. $A \cup B = \{\mathrm{TH}, \mathrm{TT}, \mathrm{HT}\}$.

The sample space when you flip two fair coins is $X = \{HH, HT, TH, TT\}$. The outcome HH is in NEITHER A NOR B. The Venn diagram is as follows:

Answer

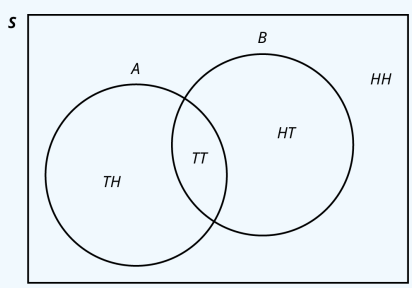


Figure 3.6.2

▼

Solution

a. 0.51; b. 0.075; c. 0.04; d. 0.545; e. The area represents the African Americans that have type O blood and the Rh- factor. f. The area represents the African Americans that have neither type O blood nor the Rh- factor.

Try It 3.6.2

Roll a fair, six-sided die. Let $A =$ a prime number of dots is rolled. Let $B =$ an odd number of dots is rolled. Then $A = \{2, 3, 5\}$ and $B = \{1, 3, 5\}$. Therefore, $A \cap B = \{3, 5\}$. $A \cup B = \{1, 2, 3, 5\}$. The sample space for rolling a fair die is $S = \{1, 2, 3, 4, 5, 6\}$. Draw a Venn diagram representing this situation.

Exercise 3.6.3

A person with type O blood and a negative Rh factor (Rh-) can donate blood to any person with any blood type. Four percent of African Americans have type O blood and a negative RH factor, 5−10% of African Americans have the Rh- factor, and 51% have type O blood.

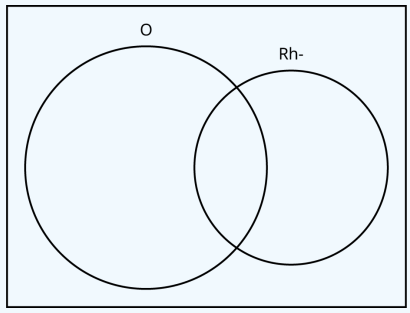


Figure 3.6.3

The "O" circle represents the African Americans with type O blood. The "Rh-" oval represents the African Americans with the Rh- factor.

We will take the average of 5% and 10% and use 7.5% as the percent of African Americans who have the Rh- factor. Let *O* = African American with Type O blood and *R* = African American with Rh- factor.

a. $P(O) =$
b. $P(R) =$
c. $P(O \cap R) =$
d. $P(O \cup R) =$
e. In the Venn Diagram, describe the overlapping area using a complete sentence.
f. In the Venn Diagram, describe the area in the rectangle but outside both the circle and the oval using a complete sentence.

Answer

a. 0.51; b. 0.075; c. 0.04; d. 0.545; e. The area represents the African Americans that have type O blood and the Rh- factor. f. The area represents the African Americans that have neither type O blood nor the Rh- factor.

▼

Solution

a. 0.51; b. 0.075; c. 0.04; d. 0.545; e. The area represents the African Americans that have type O blood and the Rh- factor. f. The area represents the African Americans that have neither type O blood nor the Rh- factor.

Try It 3.6.3

Roll a fair, six-sided die. Let *A* = a prime number of dots is rolled. Let *B* = an odd number of dots is rolled. Then *A* = {2, 3, 5} and *B* = {1, 3, 5}. Therefore, *A* AND *B* = {3, 5}. *A* OR *B* = {1, 2, 3, 5}. The sample space for rolling a fair die is *S* = {1, 2, 3, 4, 5, 6}. Draw a Venn diagram representing this situation.

Exercise 3.6.4

Forty percent of the students at a local college belong to a club and **50%** work part time. **Five percent** of the students work part time and belong to a club. Draw a Venn diagram showing the relationships. Let *C* = student belongs to a club and *PT* = student works part time.

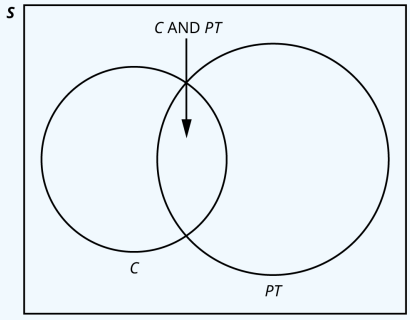


Figure 3.6.4

If a student is selected at random, find

- the probability that the student belongs to a club. $P(C) = 0.40$
- the probability that the student works part time. $P(PT) = 0.50$
- the probability that the student belongs to a club AND works part time. $P(C \cap \mathrm{PT}) = 0.05$
- the probability that the student belongs to a club given that the student works part time.

$$P(C \mid PT) = \frac{P(C \cap PT)}{P(PT)} = \frac{0.05}{0.50} = 0.1$$

- the probability that the student belongs to a club OR works part time.

$$P(C \cup \mathrm{PT}) = P(C) + P(\mathrm{PT}) - P(C \cap \mathrm{PT}) = 0.40 + 0.50 - 0.05 = 0.85$$

In order to solve Example 3.6.4 we had to draw upon the concept of conditional probability from the previous section. There we used tree diagrams to track the changes in the probabilities, because the sample space changed as we drew without replacement. In short, conditional probability is the chance that something will happen given that some other event has already happened. Put another way, the probability that something will happen conditioned upon the situation that something else is also true. In Example 3.6.4 the probability $P(C||PT)$ is the conditional probability that the randomly drawn student is a member of the club, conditioned upon the fact that the student also is working part time. This allows us to see the relationship between Venn diagrams and the probability postulates.

▼

Solution

a. 0.51; b. 0.075; c. 0.04; d. 0.545; e. The area represents the African Americans that have type O blood and the Rh- factor. f. The area represents the African Americans that have neither type O blood nor the Rh- factor.

Try It 3.6.4

1. Fifty percent of the workers at a factory work a second job, 25% have a spouse who also works, 5% work a second job and have a spouse who also works. Draw a Venn diagram showing the relationships. Let W = works a second job and S = spouse also works.
2. In a bookstore, the probability that the customer buys a novel is 0.6, and the probability that the customer buys a non-fiction book is 0.4. Suppose that the probability that the customer buys both is 0.2.
   a. Draw a Venn diagram representing the situation.
   b. Find the probability that the customer buys either a novel or a non-fiction book.
   c. In the Venn diagram, describe the overlapping area using a complete sentence.
   d. Suppose that some customers buy only compact disks. Draw an oval in your Venn diagram representing this event.

Exercise 3.6.5

A set of 20 German Shepherd dogs is observed. 12 are male, 8 are female, 10 have some brown coloring, and 5 have some white sections of fur. Answer the following using Venn Diagrams.

a. Draw a Venn diagram simply showing the sets of male and female dogs.

b. Draw a second Venn diagram illustrating that 10 of the male dogs have brown coloring.

c. Now draw a situation depicting a scenario in which the non-shaded region represents "No white fur and female," or White fur' ∩ Female. the prime above "fur" indicates "not white fur." The prime above a set means not in that set, e.g. A' means not A. Sometimes, the notation used is a line above the letter. For example, $\bar{A} = \mathrm{A}'$.

Answer

a. The Venn diagram below demonstrates the situation of mutually exclusive events where the outcomes are independent events. If a dog cannot be both male and female, then there is no intersection. Being male precludes being female and being female precludes being male: in this case, the characteristic gender is therefore mutually exclusive. A Venn diagram shows this as two sets with no intersection. The intersection is said to be the null set using the mathematical symbol ∅. a.

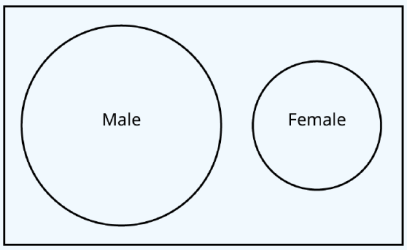


Figure 3.6.5

b. The Venn diagram below shows the overlap between male and brown where the number 10 is placed in it. This represents Male ∩ Brown: both male and brown. This is the intersection of these two characteristics. To get the union of Male and Brown, then it is simply the two circled areas minus the overlap. In proper terms, Male ∪ Brown = Male + Brown - Male ∩ Brown will give us the number of dogs in the union of these two sets. If we did not subtract the intersection, we would have double counted some of the dogs.

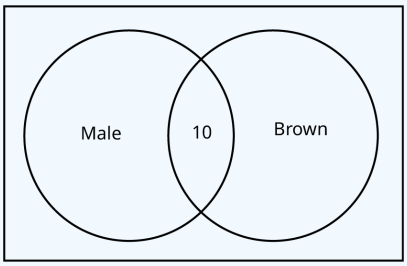


Figure 3.6.6: Copy and Paste Caption here. (Copyright; author via source)

c.

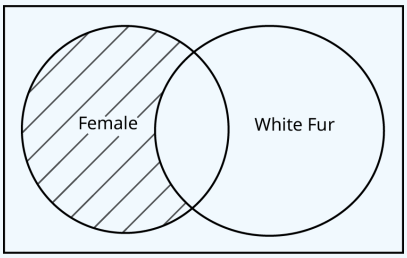


Figure 3.6.7

The Addition Rule of Probability

We met the addition rule earlier but without the help of Venn diagrams. Venn diagrams help visualize the counting process that is inherent in the calculation of probability. To restate the Addition Rule of Probability:

$$P(A \cup B) = P(\text{ A}) + P(B) - P(A \cap B) \tag{3.6.1}$$

Remember that probability is simply the proportion of the objects we are interested in relative to the total number of objects. This is why we can see the usefulness of the Venn diagrams. Example $3.6.5$ shows how we can use Venn diagrams to count the number of dogs in the union of brown and male by reminding us to subtract the intersection of brown and male. We can see the effect of this directly on probabilities in the addition rule.

▼

Solution

a. 0.51; b. 0.075; c. 0.04; d. 0.545; e. The area represents the African Americans that have type O blood and the Rh- factor. f. The area represents the African Americans that have neither type O blood nor the Rh- factor.

Try It $3.6.5$

A set of 40 people are selected randomly. 24 are male, 16 are female, 21 like coffee, and 19 like tea. Answer the following using Venn diagrams.

a. Draw a Venn diagram simply showing the sets of male and female.
b. Draw a second Venn diagram illustrating 14 males liking coffee.
c. Draw a situation with a region depicting female and not liking tea.

Exercise $3.6.6$

Let's sample 50 students who are in a statistics class. 20 are first-year students and 30 are sophomores. 15 students get a "B" in the course, and 5 students both get a "B" and are first-year students.

Find the probability of selecting a student who either earns a "B" OR is a first-year student. We are translating the word OR to the mathematical symbol for the addition rule, which is the union of the two sets.

Answer

We know that there are 50 students in our sample, so we know the denominator of our fraction to give us probability. We need only to find the number of students that meet the characteristics we are interested in, i.e. any first-year students and any student who earned a grade of "B." With the Addition Rule of probability, we can skip directly to probabilities.

Let "A" = the number of first-year students, and let "B" = the grade of "B." Below we can see the process for using Venn diagrams to solve this.

The $P(A) = \frac{20}{50} = 0.40, P(B) = \frac{15}{50} = 0.30$, and $P(A \cap B) = \frac{5}{50} = 0.10$.

Therefore, $P(A \cap B) = 0.40 + 0.30 - 0.10 = 0.60$.

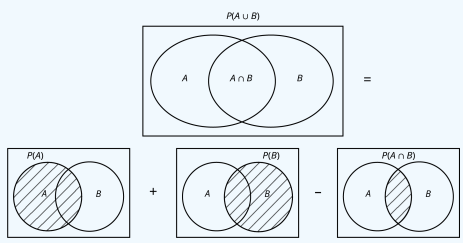


Figure 3.6.8

If two events are mutually exclusive, then, like the example where we diagram the male and female dogs, the addition rule is simplified to just $P(A \cup B) = P(\text{ A}) + P(B) - 0$. This is true because, as we saw earlier, the union of mutually exclusive events is the null set, $\varnothing$. The diagrams below demonstrate this.

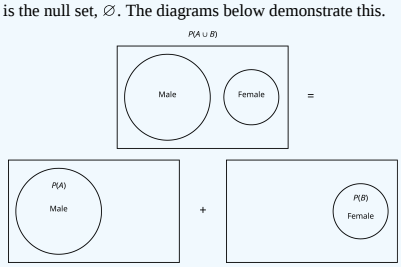
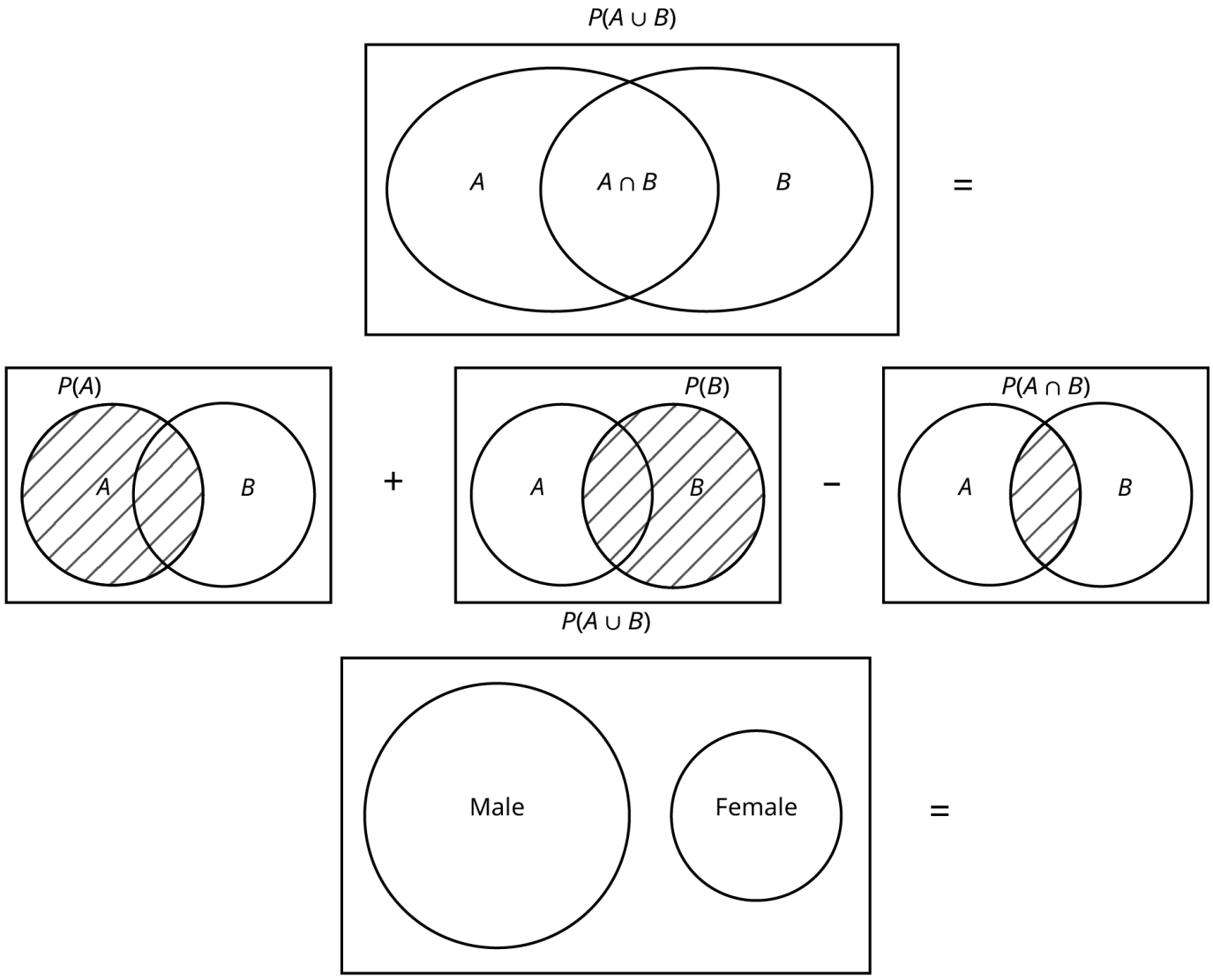


Figure 3.6.9

▼



Solution

a. 0.51; b. 0.075; c. 0.04; d. 0.545; e. The area represents the African Americans that have type O blood and the Rh- factor. f. The area represents the African Americans that have neither type O blood nor the Rh- factor.

Try It 3.6.6

A set of 40 random people is taken. 15 are male, and 25 are female. 18 people like coffee. 10 people are male and like coffee. Find the probability of selecting a person that likes coffee or is male.

The Multiplication Rule of Probability

Restating the Multiplication Rule of Probability using the notation of Venn diagrams, we have:

P(A∩B)=P(A|B)·P(B)P(A∩B)=P(A|B)·P(B)

The multiplication rule can be modified with a bit of algebra into the following conditional rule. Then Venn diagrams can then be used to demonstrate the process.

The conditional rule: P(A|B)=P(A∩B)P(B)P(A|B)=P(A∩B)P(B)

Using the same facts from Example $3.6.6$ above, find the probability that someone will earn a "B" if they are a "first-year students."

P(A|B)=0.100.30=13P(A|B)=0.100.30=13

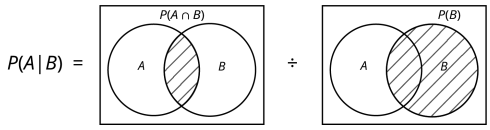


Figure 3.6.10

The multiplication rule must also be altered if the two events are independent. **Independent events** are defined as a situation where the conditional probability is simply the probability of the event of interest. Formally, independence of events is defined as P(A|B)=P(A)P(A|B)=P(A) or P(B|A)=P(B)P(B|A)=P(B). When flipping coins, the outcome of the second flip is independent of the outcome of the first flip; coins do not have memory. The Multiplication Rule of Probability for independent events thus becomes:

$$P(A \cap B)=P(A) \cdot P(B) \tag{3.6.2}$$

One easy way to remember this is to consider what we mean by the word "and." We see that the Multiplication Rule has translated the word "and" to the Venn notation for intersection. Therefore, the outcome must meet the two conditions of first-year and grade of "B" in the above example. It is harder, less probable, to meet two conditions than just one or some other one. We can attempt to see the logic of the Multiplication Rule of probability due to the fact that fractions multiplied times each other become smaller.

The development of the Rules of Probability with the use of Venn diagrams can be shown to help as we wish to calculate probabilities from data arranged in a contingency table.

? Exercise $3.6.7$

Table $3.6.1$ is from a sample of 200 people who were asked how much education they completed. The columns represent the highest education they completed, and the rows separate the individuals by men and women.

	Less than high school grad	High school grad	Some college	College grad	Total
Men	5	15	40	60	120
Women	8	12	30	30	80
Total	13	27	70	90	200

Table $3.6.1$

Now, we can use this table to answer probability questions. The following examples are designed to help understand the format above while connecting the knowledge to both Venn diagrams and the probability rules.

a. What is the probability that a selected person both finished college and is a woman?

b. What is the probability of selecting either a woman or someone who finished college?

c. What is the probability of selecting a high school graduate if we only select from the group of men?

d. Can we conclude that the level of education attained by these 200 people is independent of the gender of the person?

Answer

a. This is a simple task of finding the value where the two characteristics intersect on the table, and then applying the postulate of probability, which states that the probability of an event is the proportion of outcomes that match the event in which we are interested as a proportion of all total possible outcomes.

$P($ College Grad $\cap$ Female $)=\frac{30}{200}=0.15$

b. This task involves the use of the addition rule to solve for this probability.

$P($ College Grad $\cup$ Female $)=P(F)+P(CG)-P(F \cap CG)$

$P($ College Grad $\cup$ Female $)=\frac{80}{200}+\frac{90}{200}-\frac{30}{200}=\frac{140}{200}=0.70$

c. Here we must use the conditional probability rule (the modified multiplication rule) to solve for this probability.

P\left(H S \text { Grad } \left\lvert\, M a n \right)=\dfrac{P(\operatorname{HS~Grad} \cap \operatorname{Man})}{\mathrm{P}(\mathrm{Man})}=\dfrac{\left(\dfrac{15}{200}\right)}{\left(\dfrac{120}{200}\right)}=

d.

There are two ways to approach this test. The first method seeks to test if the intersection of two events equals the product of the events separately remembering that if two events are independent than $P(A)^{*} P(B)=P(A \cap B)$. For simplicity's sake, we can use calculated values from above.

Does $P($ College Grad $\cap$ Woman $)=P(CG) \cdot P(F)$? $\frac{30}{200} \neq \frac{90}{200} \cdot \frac{80}{200}$ because $0.15 \neq 0.18$

Therefore, gender and education here are not independent.

The second method is to test if the conditional probability of A given B is equal to the probability of A. Again for simplicity, we can use an already calculated value from above.

Does $P(HS$ Grad | Man $)=P(HS$ Grad) ?

$\frac{15}{120} \neq \frac{27}{200}$ because $0.125 \neq 0.135$

Therefore, again gender and education here are not independent.

▼
▼

Solution

a. 0.51; b. 0.075; c. 0.04; d. 0.545; e. The area represents the African Americans that have type O blood and the Rh- factor. f. The area represents the African Americans that have neither type O blood nor the Rh- factor.

Try It 3.6.5

The table shows a spread of what 100 random people like to eat. The columns represent the fruit that they like to eat, and the rows separate the individuals by men and women.

	Grape	Watermelon	Apple	Pomegranate	**Total**
Men	12	18	20	10	60
Women	13	7	15	5	40
Total	25	25	35	15	**100**

Table 3.6.2

Answer the following questions:

a. What is the probability that a selected person likes pomegranate and is a man?
b. What is the probability of selecting either a man or someone who likes apple?
c. What is the probability of selecting a person who likes watermelon if we select from the group of women?

This page titled 3.6: Venn Diagrams is shared under a CC BY 4.0 license and was authored, remixed, and/or curated by OpenStax via source content that was edited to the style and standards of the LibreTexts platform.