# 3.12: Homework

## 3.1 Terminology

72.

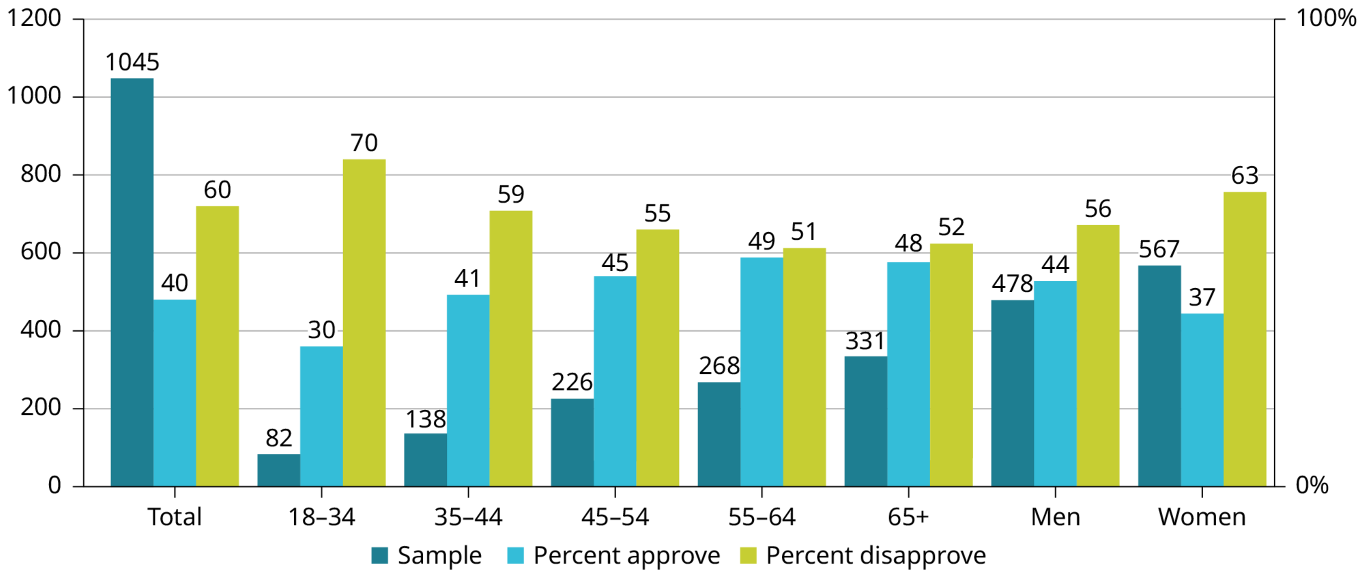



Figure 3.12.1: Copy and Paste Caption here. (Copyright; author via source)

The graph in Figure 3.12.1 displays the sample sizes and percentages of people in different age and gender groups who were polled concerning their approval of Mayor Ford's actions in office. The total number in the sample of all the age groups is 1,045.

1. Define three events in the graph.
2. Describe in words what the entry 40 means.
3. Describe in words the complement of the entry in question 2.
4. Describe in words what the entry 30 means.
5. Out of the men and women, what percent are men?
6. Out of the women, what percent disapprove of Mayor Ford?
7. Out of all the age groups, what percent approve of Mayor Ford?
8. Find P(Approve|Men).
9. Out of the age groups, what percent are more than 44 years old?
10. Find P(Approve|Age < 35).

73. Explain what is wrong with the following statements. Use complete sentences.

1. If there is a 60% chance of rain on Saturday and a 70% chance of rain on Sunday, then there is a 130% chance of rain over the weekend.
2. The probability that a baseball player hits a home run is greater than the probability that he gets a successful hit.

3.2 Independent and Mutually Exclusive Events

Use the following information to answer the next 12 exercises. The graph shown is based on more than 170,000 interviews done by Gallup that took place from January through December in a certain year. The sample consists of employed Americans 18 years of age or older. The Emotional Health Index Scores are the sample space. We randomly sample one Emotional Health Index Score.

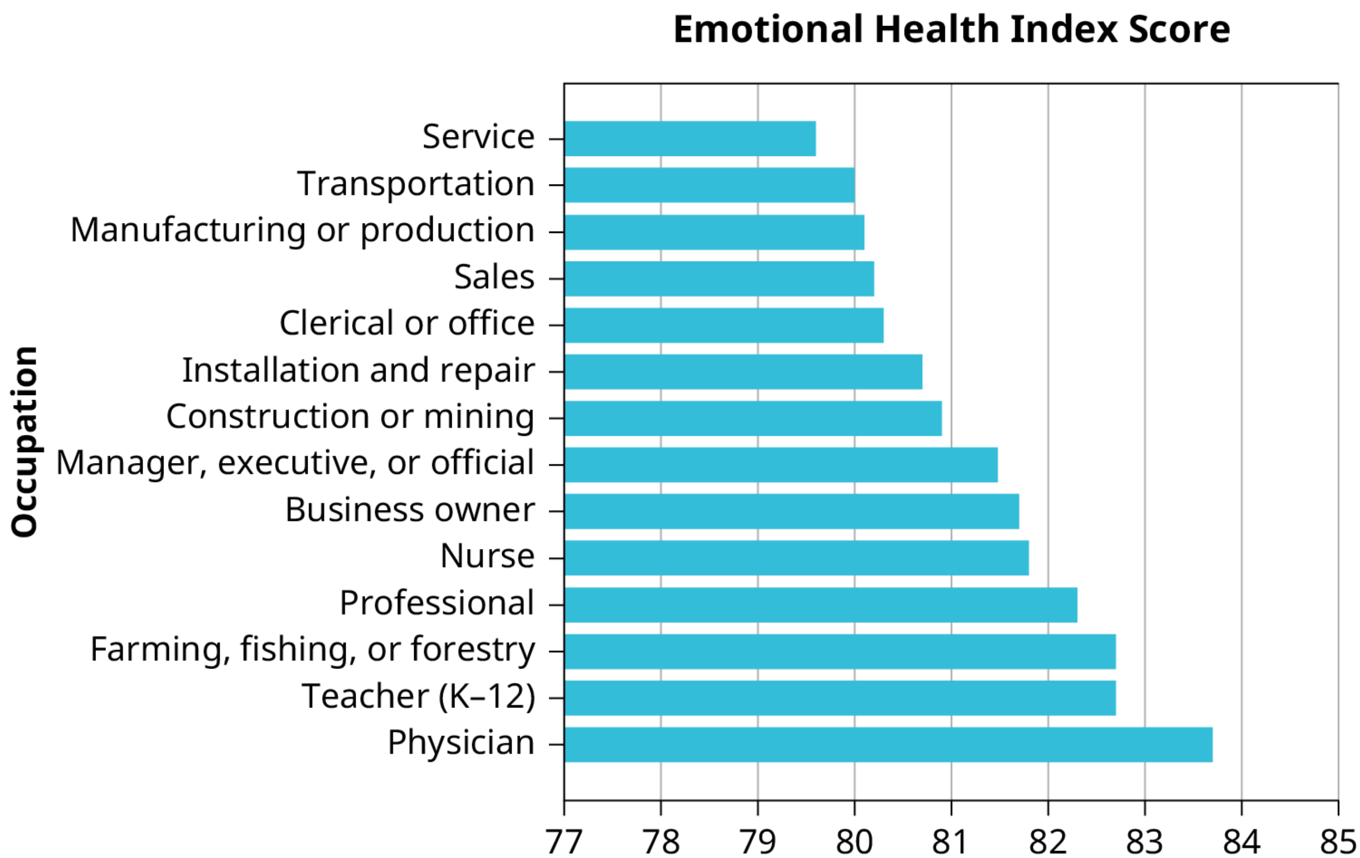


Figure 3.12.2: Copy and Paste Caption here. (Copyright; author via source)

74. Find the probability that the selected occupation has an Emotional Health Index Score of 82.7.

75. Find the probability that the selected occupation has an Emotional Health Index Score of 81.0.

76. Find the probability that the selected occupation has an Emotional Health Index Score more than 81.

77. Find the probability that the selected occupation has an Emotional Health Index Score between 80.5 and 82.

78. If we know an occupation has an Emotional Health Index Score of 81.5 or more, what is the probability that it is 82.7?

79. What is the probability that an occupation has an Emotional Health Index Score of 80.7 or 82.7?

80. What is the probability that an occupation has an Emotional Health Index Score less than 80.2 given that it is already less than 81?

81. What occupation has the highest Emotional Health Index Score?

82. What occupation has the lowest Emotional Health Index Score?

83. What is the range of the data?

84. Compute the average EHIS.

85. If all occupations are equally likely for a certain individual, what is the probability that they will have an occupation with lower than average EHIS?

3.3 Two Basic Rules of Probability

86. Prior to the 2015 Supreme Court decision legalizing same-sex marriage nationwide, a survey reported that 61% of California registered voters approved of allowing two people of the same gender to marry and have regular marriage laws apply to them. Among 18 to 39 year olds (California registered voters), the approval rating was 78%. Six in ten California registered voters said that the upcoming Supreme Court's ruling about the constitutionality of California's Proposition 8 was either very or somewhat important to them. Out of those CA registered voters who support same-sex marriage, 75% say the ruling is important to them.

In this problem, let:

- C = California registered voters who support same-sex marriage.
- B = California registered voters who say the Supreme Court's ruling about the constitutionality of California's Proposition 8 is very or somewhat important to them
- A = California registered voters who are 18 to 39 years old.

1. Find P(C).
2. Find P(B).
3. Find P(C|A).
4. Find P(B|C).
5. In words, what is C|A?
6. In words, what is B|C?
7. Find P(C AND B).
8. In words, what is C AND B?
9. Find P(C OR B).
10. Are C and B mutually exclusive events? Show why or why not.

87. A survey was conducted in a large city to measure the popularity of that city's mayor. The survey was repeated every year for three years. The survey polled 1,046 people to measure the mayor's popularity. Everyone polled expressed either approval or disapproval. These are the results the poll produced:

- In Year 1, 60% of the population approved of the mayor's actions in office.
- In Year 2, 57% of the population approved of his actions.
- In Year 3, the percentage of popular approval was measured at 42%.

1. What is the sample size for this study?
2. What proportion in the poll disapproved of the mayor, according to the results from Year 3?
3. How many people polled responded that they approved the mayor based on results from Year 3?
4. What is the probability that a person supported the mayor, based on the data collected in Year 2?
5. What is the probability that a person supported the mayor, based on the data collected in Year 1?

Use the following information to answer the next three exercises. The casino game, roulette, allows the gambler to bet on the probability of a ball, which spins in the roulette wheel, landing on a particular color, number, or range of numbers. The table used to place bets contains of 38 numbers, and each number is assigned to a color and a range.

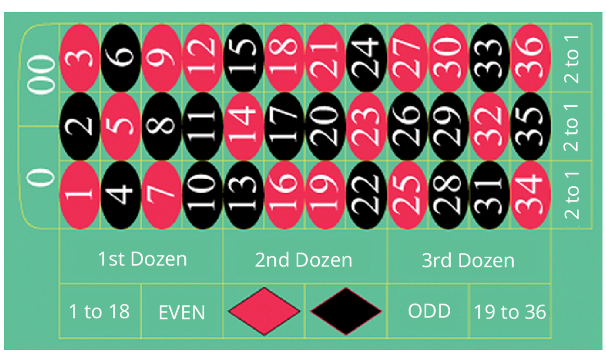


Figure 3.12.3: (credit: film8ker/wikibooks)

88. List the sample space of the 38 possible outcomes in roulette.

a. You bet on red. Find *P*(red).

b. You bet on -1st 12- (1st Dozen). Find $P$(-1st 12-).
c. You bet on an even number. Find $P$(even number).
d. Is getting an odd number the complement of getting an even number? Why?
e. Find two mutually exclusive events.
f. Are the events Even and 1st Dozen independent?

89. Compute the probability of winning the following types of bets:

a. Betting on two lines that touch each other on the table as in 1-2-3-4-5-6
b. Betting on three numbers in a line, as in 1-2-3
c. Betting on one number
d. Betting on four numbers that touch each other to form a square, as in 10-11-13-14
e. Betting on two numbers that touch each other on the table, as in 10-11 or 10-13
f. Betting on 0-00-1-2-3
g. Betting on 0-1-2; or 0-00-2; or 00-2-3

90. Compute the probability of winning the following types of bets:

a. Betting on a color
b. Betting on one of the dozen groups
c. Betting on the range of numbers from 1 to 18
d. Betting on the range of numbers 19–36
e. Betting on one of the columns
f. Betting on an even or odd number (excluding zero)

91. Suppose that you have eight cards. Five are green and three are yellow. The five green cards are numbered 1, 2, 3, 4, and 5 . The three yellow cards are numbered 1 , 2 , and 3 . The cards are well shuffled. You randomly draw one card.

$G =$ card drawn is green
$E =$ card drawn is even-numbered
a. List the sample space.
b. $P(G) =$
c. $P(G \mid E) =$
d. $P(G \cap E) =$
e. $P(G \cup E) =$
f. Are $G$ and $E$ mutually exclusive? Justify your answer numerically.

92. Roll two fair dice separately. Each die has six faces.

a. List the sample space.
b. Let $A$ be the event that either a three or four is rolled first, followed by an even number. Find $P(A)$.
c. Let $B$ be the event that the sum of the two rolls is at most seven. Find $P(B)$.
d. In words, explain what " $P(A \mid B)$ " represents. Find $P(A \mid B)$.
e. Are $A$ and $B$ mutually exclusive events? Explain your answer in one to three complete sentences, including numerical justification.
f. Are $A$ and $B$ independent events? Explain your answer in one to three complete sentences, including numerical justification.

93. A special deck of cards has ten cards. Four are green, three are blue, and three are red. When a card is picked, its color of it is recorded. An experiment consists of first picking a card and then tossing a coin.

a. List the sample space.
b. Let $A$ be the event that a blue card is picked first, followed by landing a head on the coin toss. Find $P(A)$.
c. Let $B$ be the event that a red or green is picked, followed by landing a head on the coin toss. Are the events $A$ and $B$ mutually exclusive? Explain your answer in one to three complete sentences, including numerical justification.
d. Let $C$ be the event that a red or blue is picked, followed by landing a head on the coin toss. Are the events $A$ and $C$ mutually exclusive? Explain your answer in one to three complete sentences, including numerical justification.

94. An experiment consists of first rolling a die and then tossing a coin.

a. List the sample space.
b. Let $A$ be the event that either a three or a four is rolled first, followed by landing a head on the coin toss. Find $P(A)$.

c. Let $B$ be the event that the first and second tosses land on heads. Are the events $A$ and $B$ mutually exclusive? Explain your answer in one to three complete sentences, including numerical justification.

95. An experiment consists of tossing a nickel, a dime, and a quarter. Of interest is the side the coin lands on.

a. List the sample space.
b. Let $A$ be the event that there are at least two tails. Find $P(A)$.
c. Let $B$ be the event that the first and second tosses land on heads. Are the events $A$ and $B$ mutually exclusive? Explain your answer in one to three complete sentences, including justification.

96. Consider the following scenario:

Let $P(C) = 0.4$.
Let $P(D) = 0.5$.
Let $P(C \mid D) = 0.6$.

a. Find $P(C \cap D)$.
b. Are $C$ and $D$ mutually exclusive? Why or why not?
c. Are $C$ and $D$ independent events? Why or why not?
d. Find $P(C \cup D)$.
e. Find $P(D \mid C)$.

97. $Y$ and $Z$ are independent events.

a. Rewrite the basic Addition Rule $P(Y \cup Z) = P(Y) + P(Z) - P(Y \cap Z)$ using the information that $Y$ and $Z$ are independent events.
b. Use the rewritten rule to find $P(Z)$ if $P(Y \cup Z) = 0.71$ and $P(Y) = 0.42$.

98. $G$ and $H$ are mutually exclusive events. $P(G) = 0.5 P(H) = 0.3$

a. Explain why the following statement MUST be false: $P(H \mid G) = 0.4$.
b. Find $P(H \cup G)$.
c. Are $G$ and $H$ independent or dependent events? Explain in a complete sentence.

99. According to the 2019 U.S. Census, approximately 331,449,281 people live in the United States. Of these people, $67,800,000$ speak a language other than English at home. Of those who speak another language at home, $61.6\%$ speak Spanish.

Let: $E =$ speaks English at home; $E' =$ speaks another language at home; $S =$ speaks Spanish.
Finish each probability statement by matching the correct answer.

| Probability Statements | Answers |
|---|---|
| a. $P(E') =$ | i. 0.7954 |
| b. $P(E) =$ | ii. 0.616 |
| c. $P(S \cap E') =$ | iii. 0.2046 |
| d. $P(S \mid E') =$ | iv. 0.1260 |

100. 1994, the U.S. government held a lottery to issue 55,000 Green Cards (permits for non-citizens to work legally in the U.S.). Renate Deutsch, from Germany, was one of approximately 6.5 million people who entered this lottery. Let $G$ = won green card.

a. What was Renate's chance of winning a Green Card? Write your answer as a probability statement.
b. In the summer of 1994, Renate received a letter stating she was one of 110,000 finalists chosen. Once the finalists were chosen, assuming that each finalist had an equal chance to win, what was Renate's chance of winning a Green Card? Write your answer as a conditional probability statement. Let $F$ = was a finalist.
c. Are $G$ and $F$ independent or dependent events? Justify your answer numerically and also explain why.
d. Are $G$ and $F$ mutually exclusive events? Justify your answer numerically and explain why.

101. Three professors at George Washington University did an experiment to determine if economists are more selfish than other people. They dropped 64 stamped, addressed envelopes with $10 cash in different classrooms on the George Washington campus.

44% were returned overall. From the economics classes 56% of the envelopes were returned. From the business, psychology, and history classes 31% were returned.

Let: $R$ = money returned; $E$ = economics classes; $O$ = other classes

a. Write a probability statement for the overall percent of money returned.
b. Write a probability statement for the percent of money returned out of the economics classes.
c. Write a probability statement for the percent of money returned out of the other classes.
d. Is money being returned independent of the class? Justify your answer numerically and explain it.
e. Based upon this study, do you think that economists are more selfish than other people? Explain why or why not. Include numbers to justify your answer.

102. The following table of data obtained from www.baseball-almanac.com shows hit information for four players. Suppose that one hit from the table is randomly selected.

| Name | Single | Double | Triple | Home Run | Total Hits |
|---|---|---|---|---|---|
| Babe Ruth | 1,517 | 506 | 136 | 714 | 2,873 |
| Jackie Robinson | 1,054 | 273 | 54 | 137 | 1,518 |
| Ty Cobb | 3,053 | 724 | 295 | 117 | 4,189 |
| Hank Aaron | 2,294 | 624 | 98 | 755 | 3,771 |
| Total | 7,918 | 2,127 | 583 | 1,723 | 12,351 |

Are "the hit being made by Hank Aaron" and "the hit being a double" independent events?

a. Yes, because $P$ (hit by Hank Aaron|hit is a double) $=$ $P$ (hit by Hank Aaron)
b. No, because $P$ (hit by Hank Aaron|hit is a double) $\neq$ $P$ (hit is a double)
c. No, because $P$ (hit is by Hank Aaron|hit is a double) $\neq$ $P$ (hit by Hank Aaron)
d. Yes, because $P$ (hit is by Hank Aaron|hit is a double) $=$ $P$ (hit is a double)

103. According to the American Red Cross, a person with type O blood and a negative Rh factor (Rh-) can donate blood to any person with any blood type. Their data show that 45% of people have type O blood and 7% of people have Rh- factor; 38% of people have type O or Rh- factor.

a. Find the probability that a person has both type O blood and the Rh- factor.
b. Find the probability that a person does NOT have both type O blood and the Rh- factor.

104. At a college, 72% of courses have final exams and 46% of courses require research papers. Suppose that 32% of courses have a research paper and a final exam. Let $F$ be the event that a course has a final exam. Let $R$ be the event that a course requires a research paper.

a. Find the probability that a course has a final exam or a research project.
b. Find the probability that a course has NEITHER of these two requirements.

105. In a box of assorted cookies, 36% contain chocolate and 12% contain nuts. In the box, 8% contain both chocolate and nuts. Sean is allergic to both chocolate and nuts.

a. Find the probability that a cookie contains chocolate or nuts (he can't eat it).
b. Find the probability that a cookie does not contain chocolate or nuts (he can eat it).

106. A college finds that 10% of students have taken a distance learning class and that 40% of students are part time students. Of the part time students, 20% have taken a distance learning class. Let $D$ = event that a student takes a distance learning class and $E$ = event that a student is a part time student

a. Find $P(D \cap E)$.
b. Find $P(E \mid D)$.

c. Find $P(D \cup E)$.
d. Using an appropriate test, show whether $D$ and $E$ are independent.
e. Using an appropriate test, show whether $D$ and $E$ are mutually exclusive.

## 3.5 Venn Diagrams

*Use the information in the table below to answer the next eight exercises.* The table shows the political party affiliation for various members of the U.S. Senate during two separate years when they are up for reelection.

| Up for reelection: | Democratic Party | Republican Party | Other | Total |
|---|---|---|---|---|
| Year A | 20 | 13 | 0 | |
| Year B | 10 | 24 | 0 | |
| Total | | | | |

107. What is the probability that a randomly selected senator has an “Other” affiliation?

108. What is the probability that a randomly selected senator is up for reelection in Year B?

109. What is the probability that a randomly selected senator is a Democrat and up for reelection in Year B?

110. What is the probability that a randomly selected senator is a Republican or is up for reelection in Year A?

111. Suppose that a member of the U.S. Senate is randomly selected. Given that the randomly selected senator is up for reelection in Year B, what is the probability that this senator is a Democrat?

112. Suppose that a member of the U.S. Senate is randomly selected. What is the probability that the senator is up for reelection in Year A, knowing that this senator is a Republican?

113. The events “Republican” and “Up for reelection in Year B” are ________

a. mutually exclusive.
b. independent.
c. both mutually exclusive and independent.
d. neither mutually exclusive nor independent.

114. The events “Other” and “Up for reelection in Year B” are ________

a. mutually exclusive.
b. independent.
c. both mutually exclusive and independent.
d. neither mutually exclusive nor independent.

115. The table below gives the number of participants in the recent National Health Interview Survey who had been treated for cancer in the previous 12 months. The results are sorted by age, race (Black or White), and sex. We are interested in possible relationships between age, race, and sex.

| Race and sex | 15–24 | 25–40 | 41–65 | Over 65 | TOTALS |
|---|---|---|---|---|---|
| White, male | 1,165 | 2,036 | 3,703 | | 8,395 |
| White, female | 1,076 | 2,242 | 4,060 | | 9,129 |
| Black, male | 142 | 194 | 384 | | 824 |
| Black, female | 131 | 290 | 486 | | 1,061 |

| Race and sex | 15–24 | 25–40 | 41–65 | Over 65 | TOTALS |
|---|---|---|---|---|---|
| All others | | | | | |
| TOTALS | 2,792 | 5,279 | 9,354 | | 21,081 |

Do not include "all others" for parts f and g.

a. Fill in the column for cancer treatment for individuals over age 65.
   1. Fill in the row for all other races.
b. Find the probability that a randomly selected individual was a White male.
c. Find the probability that a randomly selected individual was a Black female.
d. Find the probability that a randomly selected individual was Black
e. Find the probability that a randomly selected individual was male.
f. Out of the individuals over age 65, find the probability that a randomly selected individual was a Black or White male.

*Use the following information to answer the next two exercises.* The table of data obtained from *www.baseball-almanac.com* shows hit information for four well known baseball players. Suppose that one hit from the table is randomly selected.

| NAME | Single | Double | Triple | Home Run | TOTAL HITS |
|---|---|---|---|---|---|
| Babe Ruth | 1,517 | 506 | 136 | 714 | 2,873 |
| Jackie Robinson | 1,054 | 273 | 54 | 137 | 1,518 |
| Ty Cobb | 3,035 | 724 | 295 | 117 | 4,189 |
| Hank Aaron | 2,294 | 624 | 98 | 755 | 3,771 |
| TOTAL | 7,918 | 2,127 | 583 | 1,723 | 12,351 |

116. Find $P$ (hit was made by Babe Ruth).

a. $\frac{1518}{2873}$
b. $\frac{2873}{12351}$
c. $\frac{583}{12351}$
d. $\frac{4189}{12351}$

117. Find $P$ (hit was made by Ty Cobb | The hit was a Home Run).

a. $\frac{4189}{12351}$
b. $\frac{117}{1723}$
c. $\frac{1723}{4189}$
d. $\frac{117}{12351}$

118. The table below identifies a group of children by one of four hair colors, and by type of hair.

| Hair Type | Brown | Blond | Black | Red | Totals |
|---|---|---|---|---|---|
| Wavy | 20 | | 15 | 3 | 43 |
| Straight | 80 | 15 | | 12 | |
| Totals | | 20 | | | 215 |

a. Complete the table.
b. What is the probability that a randomly selected child will have wavy hair?
c. What is the probability that a randomly selected child will have either brown or blond hair?
d. What is the probability that a randomly selected child will have wavy brown hair?
e. What is the probability that a randomly selected child will have red hair, given that they have straight hair?
f. If *B* is the event of a child having brown hair, find the probability of the complement of *B*.
g. In words, what does the complement of *B* represent?

119.

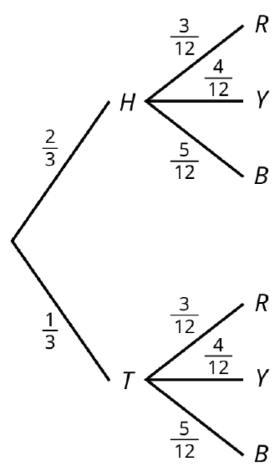
In a previous year, the weights of the members of the **San Francisco 49ers** and the **Dallas Cowboys** were published in the *San Jose Mercury News*. The factual data were compiled into the following table.

| Shirt# | ≤ 210 | 211–250 | 251–290 | > 290 |
|---|---|---|---|---|
| 1–33 | 21 | 5 | 0 | 0 |
| 34–66 | 6 | 18 | 7 | 4 |
| 66–99 | 6 | 12 | 22 | 5 |

For the following, suppose that you randomly select one player from the 49ers or Cowboys.

a. Find the probability that his shirt number is from 1 to 33.
b. Find the probability that he weighs at most 210 pounds.
c. Find the probability that his shirt number is from 1 to 33 AND he weighs at most 210 pounds.
d. Find the probability that his shirt number is from 1 to 33 OR he weighs at most 210 pounds.
e. Find the probability that his shirt number is from 1 to 33 GIVEN that he weighs at most 210 pounds.

Use the following information to answer the next two exercises. This tree diagram shows the tossing of an unfair coin followed by drawing one bead from a cup containing three red $(R)$, four yellow $(Y)$ and five blue $(B)$ beads. F the coin, $P(H)=\frac{2}{3}$ and $P(T)=\frac{1}{3}$ where $H$ is heads and $T$ is tails.




120. Find $P$ (tossing a Head on the coin AND a Red bead)

a. $\frac{2}{3}$
b. $\frac{5}{15}$

c. $\frac{6}{36}$
d. $\frac{5}{36}$

121. Find $P$ (Blue bead).

a. $\frac{15}{36}$
b. $\frac{10}{36}$
c. $\frac{10}{12}$
d. $\frac{6}{36}$

$\frac{6}{36}$

122

A box of cookies contains three chocolate and seven butter cookies. Miguel randomly selects a cookie and eats it. Then he randomly selects another cookie and eats it. (How many cookies did he take?)

a. Draw the tree that represents the possibilities for the cookie selections. Write the probabilities along each branch of the tree.
b. Are the probabilities for the flavor of the SECOND cookie that Miguel selects independent of his first selection? Explain.
c. For each complete path through the tree, write the event it represents and find the probabilities.
d. Let S be the event that both cookies selected were the same flavor. Find $P(S)$.
e. Let $T$ be the event that the cookies selected were different flavors. Find $P(T)$ by two different methods: by using the complement rule and by using the branches of the tree. Your answers should be the same with both methods.
f. Let $U$ be the event that the second cookie selected is a butter cookie. Find $P(U)$.