5.4: The Exponential Distribution

The exponential probability density function is built upon the general exponential function where the variable is an exponent: $f(x) = a(b)^x$. This equation can be converted to a natural system of logarithms with a base e that has an approximate value of 2.71828 . The exponential probability density function is valuable with a number of practical applications. The exponential distribution is often concerned with the amount of time until some specific event occurs. For example, the amount of time (beginning now) until an earthquake occurs has an exponential distribution. Other examples include the length of time, in minutes, of long-distance business telephone calls and the amount of time, in months, a car battery lasts. It can be shown, too, that the value of the change that you have in your pocket or purse approximately follows an exponential distribution.

Values for an exponential random variable occur in the following way. There are fewer large values and more small values. For example, marketing studies have shown that the amount of money customers spend in one trip to the supermarket follows an exponential distribution. There are more people who spend small amounts of money and fewer people who spend large amounts of money.

Exponential distributions are commonly used in calculations of product reliability, or the length of time a product lasts. A firm will use the exponential distribution analysis to set the number of months they will provide a warranty based upon the probability of a specific time until the failure of the product.

The random variable for the exponential distribution is continuous and often measures a passage of time, although it can be used in other continuous random variable applications. Typical questions may be "What is the probability that some event will occur within the next x hours or days?" or "What is the probability that some event will occur between x_1 hours and x_2 hours?" or "What is the probability that the event will take more than x_1 hours to perform?" In short, the random variable X equals (a) the time between events or (b) the passage of time to complete an action (e.g., wait on a customer). Be sure to differentiate the exponential distribution random variable and that of the Poisson distribution. The Poisson has a discrete random variable that gives us the probability that x number of occurrences will occur in the next time period. The continuous exponential distribution provides us the probability that an X-long time period will occur between now and the next occurrence. We will explore more this link between the exponential distribution and the Poisson distribution.

The exponent probability density function is given by:

$$f(x) = \frac{1}{\mu} e^{\frac{x}{\mu}} \tag{5.4.1}$$

where μ is the historical average waiting time.
The exponential distribution has a mean and standard deviation both equal to μ.
To find a probability with a continuous random variable, the cumulative distribution is integrated between the relevant values of x. The definite integral of a function provides the area under the function between two values of x.

For the exponential distribution, there are three cases that depend upon the question asked. Case I would answer the question "What is the probability that the time to completion is less than X ?" The probability question asked would be written as $P(X \leq x)$.
For example, to calculate the probability that a customer will be served within the next hour given that the historical waiting time was 30 minutes, we integrate the exponential probability distribution function $\frac{1}{\mu} e^{\frac{x}{\mu}}$ from $x = 0$ to $x = 60$.

$$F(x) = P(X \leq x) = \int_0^x \frac{1}{\mu} e^{\frac{x}{\mu}} \, dx = 1 - e^{\frac{x}{\mu}} \tag{5.4.2}$$

Case I graphs the exponential probability density function for this example. The area under the function from zero to 60 minutes is the probability for the question asked. Note that the exponential probability distribution at zero is at the value of $1/\mu$. Solving for the probability density function equation above where $\mu = 30$ is the historical waiting time, and $x = 60$, which is the time we are interested in.

$$P(x \leq 60) = 1 - e^{\frac{x}{\mu}} = 1 - e^{\frac{60}{30}} = 0.865 \tag{5.4.3}$$

With a historical waiting time of 30 minutes, certainly we would expect the high probability of 0.865 of being waited on in an hour's time. (A probability of 0.865 is almost a certainty.)

Case II would answer questions as "What is the probability more than X ?" What is the probability that a person will have to wait MORE THAN x minutes? The relevant probability density function would be

$$P\left(X \geq x_\alpha\right) = 1 - \left(1 - e^{\frac{x}{\mu}}\right) = e^{\frac{x}{\mu}} \tag{5.4.4}$$

and is presented in Figure 5.4.1.

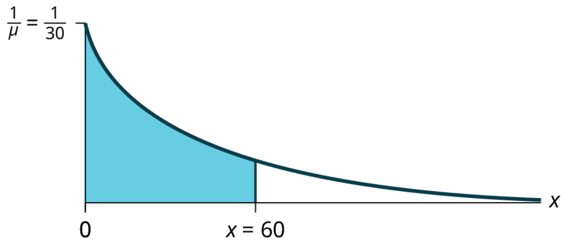

Figure 5.4.1:

Case II

Imagine an electric component is guaranteed to last two years. If it fails within the two-year guarantee, the purchaser will receive their money back in full. The manufacture knows by careful monitoring of their products the historical life of this component is 750 days, or two-and-one-half years. Are they engaged in a risky business policy? Here we can calculate the probability that a component will last longer than 730 days (two years) and thus no payment is required by the firm. The historical mean is 750 days, and the relevant time period is x=730x=730 days. Using the exponential probability density function, it is found that the probability this electric component will last more than two years is only 0.378, as shown on Figure 5.4.2.

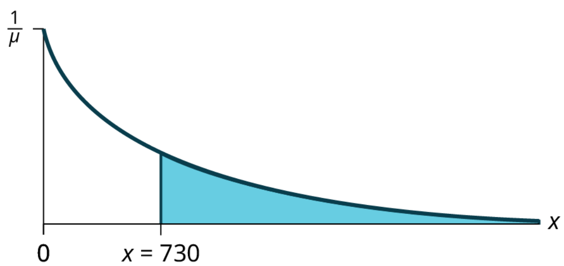

Figure 5.4.2: Copy and Paste Caption here. (Copyright; author via source)

$$P(X \geq x) = e^{\frac{x}{\mu}} = e^{\frac{730}{750}} = 0.378 \tag{5.4.5}$$

Because the working life of each of the electric components is an independent random variable, we can say that approximately 38 percent of these components survive two years and thus 62 percent of purchasers of these components will be asking for a full refund. In short, the low production quality and thus high refund requirements will result in significant financial costs. We could expect that the two-year guarantee will be eliminated soon.

Case III allows for knowing the probability that an occurrence will occur within a window of time. For example, when asking for the arrival time of an airplane, one might be given a range of time such as "plus or minus from the historical expected arrival time measured in hours from the time of take-off." In this case, the mean is the historical expected arrival time given the historical travel time. Assume that the historical travel time between these two points is four hours. Case III requires calculating the probability between two points. Figure 5.4.3: Case III shows the two points that are one hour longer and one hour sooner than the historical arrival time, which is midway between these points.

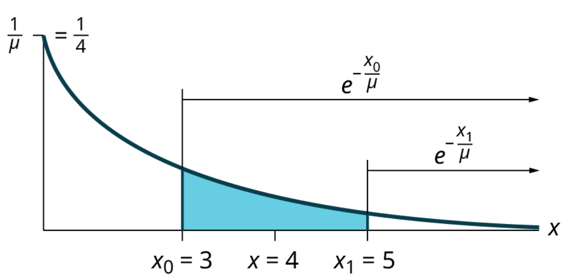

Figure 5.4.3:

If the historical travel time is four hours, then x_o is three hours of travel time and x_1 is five hours of travel time. The mathematical expression is:

$$P\left(x_0 \leq x \leq x_1\right)=P\left(X \geq x_0\right)-P\left(X \geq x_1\right)=e^{\frac{x_0}{\mu}}-e^{\frac{x_1}{\mu}}$$
$$P(3 \leq x \leq 5)=e^{\frac{3}{4}}-e^{\frac{5}{4}}=0.186 \tag{5.4.6}$$

The probability the flight will arrive plus or minus the historical arrival time of four hours from takeoff is 0.186 . Intuitively, this seems a very small probability to hit such a broad window. At the core, the standard deviation of average flight time must be very large.

Exponential Distribution and Decay Factor

The core of the exponential distribution is e, the natural logarithm in an exponential function with the variable $-\frac{x}{\mu}$ in the exponent. The exponent is negative, and thus it describes an exponentially declining function as we have seen in Figure $5.4.1$, Figure $5.4.2$, and Figure $5.4.3$ above.

This gives rise to an alternative formula for the exponential probability distribution:

$$f(x)=me^{-m}$$
$$F(x)=\int_0^x me^{-mt}dt=1-e^{-mx} \tag{5.4.7}$$

Where $m=\frac{1}{\mu}$ is given the name decay factor and measures the speed of decay. The decay factor simply measures how rapidly the probability of an event declines as the random variable X increases.

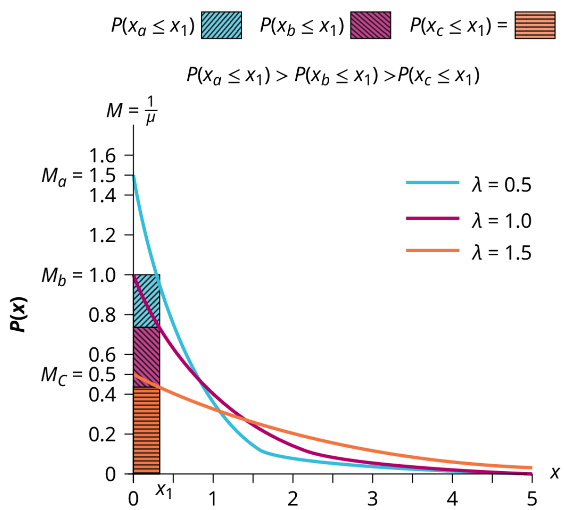

Figure $5.4.4$: Copy and Paste Caption here. (Copyright; author via source)

Figure $5.4.4$ shows the exponential distribution for three different decay factors. As before, the intercept on the vertical axis, the probability density, is the decay factor. A decay factor of 1.5 , as in the first exponential function, is three time the speed of the third exponential distribution with a decay factor of 0.5 . Looking at the probability for each of the three functions at the same value of x_1, it is clear that the first function has a significantly greater probability of occurrence at x_1 than the other two (i.e., $P\left(X_\alpha \leq x_1\right)>P\left(X_b \leq x_1\right)>P\left(X_c \leq x_1\right)$ comparing the relevant probabilities for each of the functions $\mathrm{a, b, c}$ in order). Finally, the exponential distribution shows with drama the true meaning of "exponential." In each unit, a change in x in an exponential distribution has a significant change in probability. For example, if the simple exponential distribution function $Y=2^x$ changes from $x=4$ to $x=5$, the y-value increases from 16 to 32 , but a one-unit change from $x=10$ to $x=11$ causes the y-value to increase from 1,024 to 2,048 . Likewise, in an exponential function with a negative exponent such as the exponential probability distribution, the changes are startling for each unit change at higher levels of X. At $X=-4$ to $x=-5$, the y-value changes from 0.063 to 0.031 , while a one-unit change from $X=-10$ to $X=-11$ results in a change in the y-value from 0.00097 to 0.00048 .

The conclusion is that probabilities in an exponential probability distribution at large x-values are scarce almost to the point of nonexistence.

? Exercise $5.4.1$

Let $X=$ amount of time (in minutes) a postal clerk spends with a customer. The time is known from historical data to have an average amount of time equal to four minutes.

It is given that $\mu=4$ minutes, that is, the average time the clerk spends with a customer is 4 minutes. Remember that we are still doing probability and thus we have to be told the population parameters such as the mean. To do any calculations, we need to know the mean of the distribution: the historical time to provide a service, for example. Knowing the historical mean allows the calculation of the decay parameter, m.

$$m = \frac{1}{\mu}. \text{ Therefore, } m = \frac{1}{4} = 0.25 \tag{5.4.8}$$

When the notation used the decay parameter, m, the probability density function is presented as $f(x) = me^{-mx}$, which is simply the original formula with m substituted for $\frac{1}{\mu}$, or $f(x) = \frac{1}{\mu} e^{-\frac{1}{\mu}x}$.

To calculate probabilities for an exponential probability density function, we need to use the cumulative density function. As shown below, the curve for the cumulative density function is $f(x) = 0.25e^{-0.25x}$ where x is at least zero and $m = 0.25$.

For example, $f(5) = 0.25e^{(-0.25)(5)} = 0.072$. In other words, the function has a value of .072 when $x = 5$.
The graph is as follows:

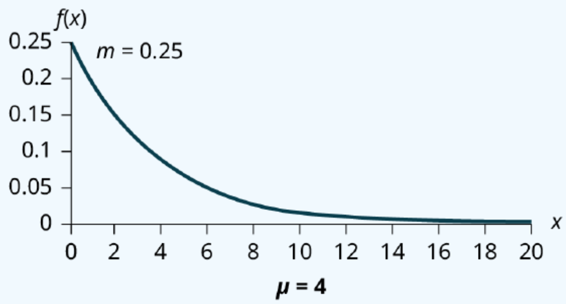

Figure 5.4.5:

Notice the graph is a declining curve. When $x = 0$, $f(x) = 0.25e^{(-0.25)(0)} = (0.25)(1) = 0.25 = m$. The maximum value on the y-axis is always m, one divided by the mean.

Try It 5.4.1

The amount of time spouses shop for anniversary cards can be modeled by an exponential distribution with the average amount of time equal to eight minutes. Write the distribution, state the probability density function, and graph the distribution.

? Exercise 5.4.2

Using the information in Exercise ? 5.4.1, find the probability that a clerk spends four to five minutes with a randomly selected customer.

Answer

Find $P(4 < x < 5)$.

The cumulative distribution function (CDF) gives the area to the left.

$$\begin{gathered} P(x < x) = 1 - e^{-mx} \\ P(x < 5) = 1 - e^{(-0.25)(5)} = 0.7135 \text{ and } P(x < 4) = 1 - e^{(-0.25)(4)} = 0.6321 \\ P(4 < x < 5) = 0.7135 - 0.6321 = 0.0814 \end{gathered} \tag{5.4.9}$$

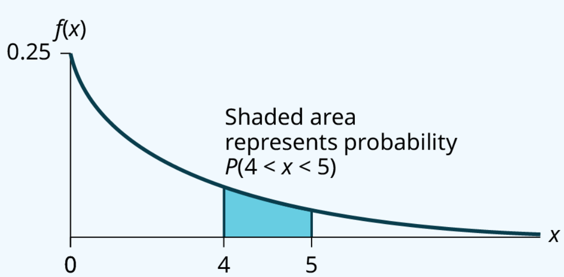

Figure 5.4.6:

Try It 5.4.2

The number of days ahead travelers purchase their airline tickets can be modeled by an exponential distribution with the average amount of time equal to 15 days. Find the probability that a traveler will purchase a ticket fewer than ten days in advance. How many days do half of all travelers wait?

Exercise 5.4.3a

On the average, a certain computer part lasts ten years. The length of time the computer part lasts is exponentially distributed.

What is the probability that a computer part lasts more than 7 years?

Answer

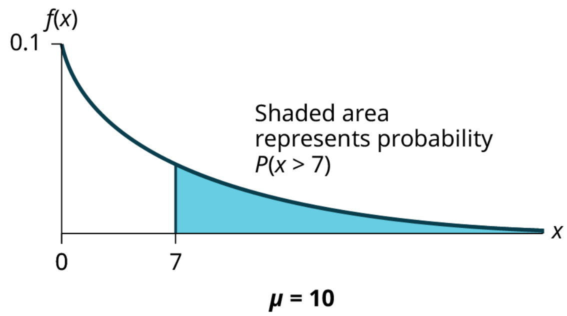

Figure 5.4.7:

Exercise 5.4.3b

On the average, how long would five computer parts last if they are used one after another?

Answer

b. On the average, one computer part lasts ten years. Therefore, five computer parts, if they are used one right after the other would last, on the average, (5)(10) = 50 years.

Exercise 5.4.3c

What is the probability that a computer part lasts between nine and 11 years?

Answer

d. Find $P(9 < x < 11)$. Draw the graph.

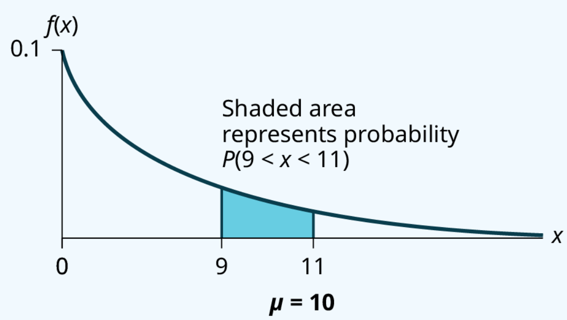

Figure 5.4.8:

$$P(9 < x < 11) = P(x < 11) - P(x < 9) = \left(1 - e^{(-0.1)(11)}\right) - \left(1 - e^{(-0.1)(9)}\right) = 0.6671 - 0.5934 = 0.0737. \quad (5.4.10)$$

The probability that a computer part lasts between nine and 11 years is 0.0737 .

Try It 5.4.3

On average, a pair of running shoes can last 18 months if used every day. The length of time running shoes last is exponentially distributed. What is the probability that a pair of running shoes last more than 15 months? On average, how long would six pairs of running shoes last if they are used one after the other? Eighty percent of running shoes last at most how long if used every day?

Exercise 5.4.4

Suppose that the length of a phone call, in minutes, is an exponential random variable with decay parameter $\frac{1}{12}$. The decay p[parameter is another way to view $1/\lambda$. If another person arrives at a public telephone just before you, find the probability that you will have to wait more than five minutes. Let $X =$ the length of a phone call, in minutes.

What is m, μ, and σ ? The probability that you must wait more than five minutes is _______ .

Answer

- $m = \frac{1}{12}$
- $\mu = 12$
- $\sigma = 12$

$$P(x > 5) = 0.6592 \tag{5.4.11}$$

Try It 5.4.4

Suppose that the distance, in miles, that people are willing to commute to work is an exponential random variable with a decay parameter 120120. Let *X* = the distance people are willing to commute in miles. What is *m*, *μ*, and *σ*? What is the probability that a person is willing to commute more than 25 miles?

Exercise 5.4.5

The time spent waiting between events is often modeled using the exponential distribution. For example, suppose that an average of 30 customers per hour arrive at a store and the time between arrivals is exponentially distributed.

Problem

a. On average, how many minutes elapse between two successive arrivals?
b. When the store first opens, how long on average does it take for three customers to arrive?
c. After a customer arrives, find the probability that it takes less than one minute for the next customer to arrive.
d. After a customer arrives, find the probability that it takes more than five minutes for the next customer to arrive.
e. Is an exponential distribution reasonable for this situation?

Answer

a. Since we expect 30 customers to arrive per hour (60 minutes), we expect on average one customer to arrive every two minutes on average.

b. Since one customer arrives every two minutes on average, it will take six minutes on average for three customers to arrive.

c. Let $X =$ the time between arrivals, in minutes. By part a, $\mu = 2$, so $m = \frac{1}{2} = 0.5$.

The cumulative distribution function is $P(X < x) = 1 - e^{(-0.5)(x)}$
Therefore $P(X < 1) = 1 - \mathrm{e}^{(-0.5)(1)} = 0.3935$.

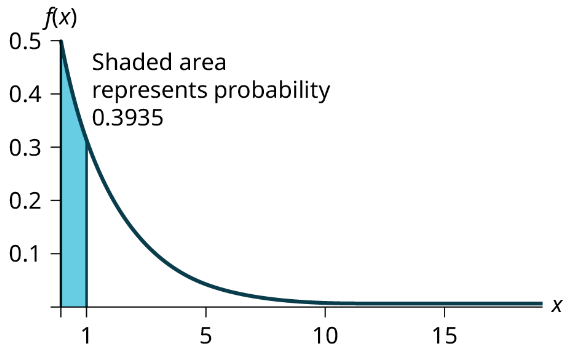

Figure 5.4.9:

d. $P(X>5)=1-P(X<5)=1-\left(1-e^{(-0.5)(5)}\right)=e^{-2.5}\approx 0.0821$.

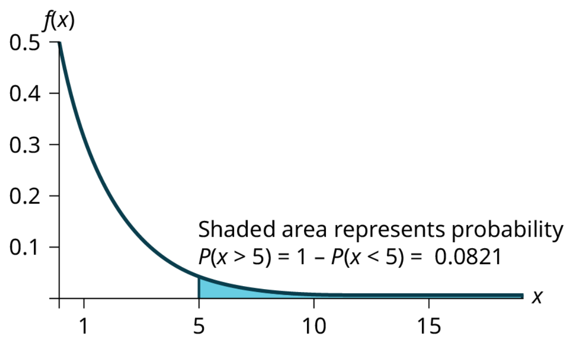

Figure 5.4.10:

e. This model assumes that a single customer arrives at a time, which may not be reasonable since people might shop in groups, leading to several customers arriving at the same time. It also assumes that the flow of customers does not change throughout the day, which is not valid if some times of the day are busier than others.

Try It 5.4.5

Suppose that on a certain stretch of highway, cars pass at an average rate of five cars per minute. Assume that the duration of time between successive cars follows the exponential distribution.

a. On average, how many seconds elapse between two successive cars?
b. After a car passes by, how long on average will it take for another seven cars to pass by?
c. Find the probability that after a car passes by, the next car will pass within the next 20 seconds.
d. Find the probability that after a car passes by, the next car will not pass for at least another 15 seconds. Try

Memorylessness of the Exponential Distribution

Recall that the amount of time between customers for the postal clerk discussed earlier is exponentially distributed with a mean of two minutes. Suppose that five minutes have elapsed since the last customer arrived. Since an unusually long amount of time has now elapsed, it would seem to be more likely for a customer to arrive within the next minute. With the exponential distribution, this is not the case-the additional time spent waiting for the next customer does not depend on how much time has already elapsed since the last customer. This is referred to as the memoryless property. The exponential and geometric probability density functions are the only probability functions that have the memoryless property. Specifically, the memoryless property says that
$P(X>r+t\mid X>r)=P(X>t)\quad$ for all $r\geq 0$ and $t\geq 0$

For example, if five minutes have elapsed since the last customer arrived, then the probability that more than one minute will elapse before the next customer arrives is computed by using $r=5$ and $t=1$ in the foregoing equation.

$$P(X>5+1\mid X>5)=P(X>1)=e^{(-0.5)(1)}=0.6065. \tag{5.4.12}$$

This is the same probability as that of waiting more than one minute for a customer to arrive after the previous arrival.

The exponential distribution is often used to model the longevity of an electrical or mechanical device. In Example 5.4.3, the lifetime of a certain computer part has the exponential distribution with a mean of ten years. The memoryless property says that knowledge of what has occurred in the past has no effect on future probabilities. In this case it means that an old part is not any more likely to break down at any particular time than a brand new part. In other words, the part stays as good as new until it suddenly breaks. For example, if the part has already lasted ten years, then the probability that it lasts another seven years is $P(X>17 \mid X>10)=P(X>7)=0.4966$, where the vertical line is read as "given".

? Exercise 5.4.6

Refer back to the postal clerk again where the time a postal clerk spends with a customer has an exponential distribution with a mean of four minutes.

Suppose a customer has spent four minutes with a postal clerk. What is the probability that the customer will spend at least an additional three minutes with the postal clerk?

The decay parameter of X is $m=\frac{1}{4}=0.25$, so $X \sim \operatorname{Exp}(0.25)$.

The cumulative distribution function is $P(X<x)=1-e^{-0.25 x}$.

We want to find $P(X>7 \mid X>4)$. The memoryless property says that $P(X>7 \mid X>4)=P(X>3)$, so we just need to find the probability that a customer spends more than three minutes with a postal clerk.

This is $P(X>3)=1-P(X<3)=1-\left(1-e^{-0.25 \cdot 3}\right)=e^{-0.75} \approx 0.4724$.

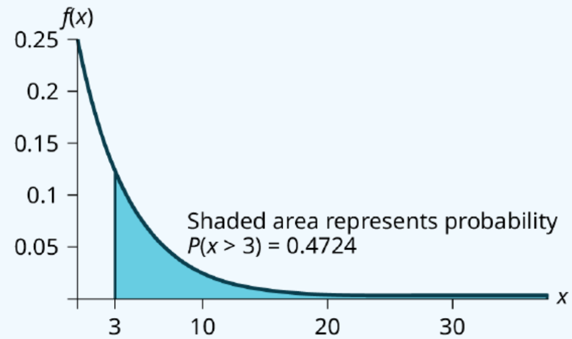

Figure 5.4.11:

Try It 5.4.6

Suppose that the longevity of a light bulb is exponential with a mean lifetime of eight years. If a bulb has already lasted 12 years, find the probability that it will last a total of over 19 years.

Relationship between the Poisson and the Exponential Distribution

There is an interesting relationship between the exponential distribution and the Poisson distribution. Suppose that the time that elapses between two successive events follows the exponential distribution with a mean of μ units of time. Also assume that these times are independent, meaning that the time between events is not affected by the times between previous events. If these assumptions hold, then the number of events per unit time follows a Poisson distribution with mean μ. Recall that if X has the Poisson distribution with mean μ, then $P(X=x)=\frac{\mu^x e^{-\mu}}{x!}$.

The formula for the exponential distribution: $P(X=x)=m e^{-m x}=\frac{1}{\mu} e^{-\frac{1}{\mu} x}$ Where $m=$ the rate parameter, or μ = average time between occurrences.

We see that the exponential is the cousin of the Poisson distribution and they are linked through this formula. There are important differences that make each distribution relevant for different types of probability problems.

First, the Poisson has a discrete random variable, x, where time; a continuous variable is artificially broken into discrete pieces. We saw that the number of occurrences of an event in a given time interval, x, follows the Poisson distribution.

For example, the number of times the telephone rings per hour. By contrast, the time between occurrences follows the exponential distribution. For example. The telephone just rang, how long will it be until it rings again? We are measuring length of time of the interval, a continuous random variable, exponential, not events during an interval, Poisson.

The Exponential Distribution v. the Poisson Distribution

A visual way to show both the similarities and differences between these two distributions is with a time line.

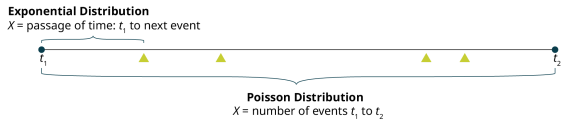

Figure 5.4.12:

The random variable for the Poisson distribution is discrete and thus counts events during a given time period, t_1 to t_2 on Figure 5.24, and calculates the probability of that number occurring. The number of events, four in the graph, is measured in counting numbers; therefore, the random variable of the Poisson is a discrete random variable.

The exponential probability distribution calculates probabilities of the passage of time, a continuous random variable. In Figure 5.24 this is shown as the bracket from t_1 to the next occurrence of the event marked with a triangle.

Classic Poisson distribution questions are "how many people will arrive at my checkout window in the next hour?".

Classic exponential distribution questions are "how long it will be until the next person arrives," or a variant, "how long will the person remain here once they have arrived?".

Again, the formula for the exponential distribution is:

$$f(x)=me^{-mx} \text{ or } f(x)=\frac{1}{\mu}e^{-\frac{1}{\mu}x} \tag{5.4.13}$$

We see immediately the similarity between the exponential formula and the Poisson formula.

$$P(x)=\frac{\mu^x e^{-\mu}}{x!} \tag{5.4.14}$$

Both probability density functions are based upon the relationship between time and exponential growth or decay. The " e " in the formula is a constant with the approximate value of 2.71828 and is the base of the natural logarithmic exponential growth formula. When people say that something has grown exponentially this is what they are talking about.

An example of the exponential and the Poisson will make clear the differences been the two. It will also show the interesting applications they have.

Poisson Distribution

Suppose that historically 10 customers arrive at the checkout lines each hour. Remember that this is still probability so we have to be told these historical values. We see this is a Poisson probability problem.

We can put this information into the Poisson probability density function and get a general formula that will calculate the probability of any specific number of customers arriving in the next hour.

The formula is for any value of the random variable we chose, and so the x is put into the formula. In this example, $\mu=10$ because we expect 10 customers in line each hour. This is the formula:

$$f(x)=\frac{10^x e^{-10}}{x!} \tag{5.4.15}$$

As an example, the probability of 15 people arriving at the checkout counter in the next hour would be

$$P(x=15)=\frac{10^{15}e^{-10}}{15!}=0.0347 \tag{5.4.16}$$

Here we have inserted $x=15$ and calculated the probability that in the next hour 15 people will arrive is 0.035 .

Exponential Distribution

If we keep the same historical facts that 10 customers arrive each hour, but we now are interested in the service time a person spends at the counter, then we would use the exponential distribution. The exponential probability function for any value of x, the random variable, for this particular checkout counter historical data is:

$$f(x)=\frac{1}{.1}e^{-x/1}=10e^{-10x} \tag{5.4.17}$$

To calculate μ, the historical average service time, we simply divide the number of people that arrive per hour, 10 , into the time period, one hour, and have $\mu=0.1$. Historically, people spend 0.1 of an hour at the checkout counter, or 6 minutes. This explains the .1 in the formula.

There is a natural confusion with μ in both the Poisson and exponential formulas. They have different meanings, although they have the same symbol. The mean of the exponential is one divided by the mean of the Poisson. If you are given the historical number of arrivals you have the mean of the Poisson. If you are given an historical length of time between events you have the mean of an exponential.

Continuing with our example at the checkout clerk; if we wanted to know the probability that a person would spend 9 minutes or less checking out, then we use this formula.

First, we convert to the same time units which are parts of one hour. Nine minutes is 0.15 of one hour. Next we note that we are asking for a range of values. This is always the case for a continuous random variable. We write the probability question as:

$$p(x \leq 9)=1-10 e^{-10 x} \tag{5.4.18}$$

We can now put the numbers into the formula and we have our result.

$$p(x=.15)=1-10 e^{-10(.15)}=0.7769 \tag{5.4.19}$$

The probability that a customer will spend 9 minutes or less checking out is 0.7769 .

We see that we have a high probability of getting out in less than nine minutes and a tiny probability of having 15 customers arriving in the next hour.

Exercise 5.4.7

At a police station in a large city, calls come in at an average rate of four calls per minute. Take note that we are concerned only with the rate at which calls come in, and we are ignoring the time spent on the phone. We must also assume that the times spent between calls are independent. This means that a particularly long delay between two calls does not mean that there will be a shorter waiting period for the next call. We may then deduce that the total number of calls received during a time period has a Poisson distribution.

Problem

a. Find the average time between two successive calls.
b. Find the probability that after a call is received, the next call occurs in less than ten seconds.
c. Find the probability that exactly five calls occur within a minute.
d. Find the probability that fewer than five calls occur within a minute.
e. Find the probability that more than 40 calls occur in an eight-minute period.

Answer

a. On average, four calls occur per minute, so one call occurs every 15 seconds, or $=0.25$ minutes occur between successive calls on average.

b. Let $T=$ time elapsed between calls. From part a, $\mu=0.25$, so $m=4$. Thus, $T \sim \operatorname{Exp}(4)$. The cumulative distribution function is $P(T<t)=1-e-4 t)$.

The probability that the next call occurs in less than ten seconds (ten seconds $=\frac{1}{6}$ minute) is $P\left(T<\frac{1}{6}\right)=1-e^{-4\left(\frac{1}{6}\right)}=0.4866$.

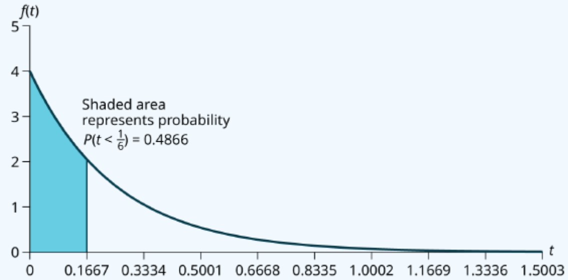

Figure 5.4.13:

c. Let $X=$ the number of calls per minute. As previously stated, the number of calls per minute has a Poisson distribution, with a mean of four calls per minute.

Therefore, $X \sim$ Poisson(4) and so

$$P(X=5)=\frac{4^5 e^{-4}}{5!} \approx 0.1563. \quad 5!=(5)(4)(3)(2)(1) \tag{5.4.20}$$

d. Keep in mind that X must be a whole number, so $P(X<5)=P(X\leq 4)$.

To compute this, we could take

$$P(X=0)+P(X=1)+P(X=2)+P(X=3)+P(X=4) \tag{5.4.21}$$

Using technology, we see that $P(X\leq 4)=0.6288$.

e. Let $\mathrm{Y}=$ the number of calls that occur during an eight-minute period.

Since there is an average of four calls per minute, there is an average of (8)(4) = 32 calls during each eight-minute period.

Hence, $Y\sim$ Poisson (32). Therefore,

$$P(Y>40)=1-P(Y\leq 40)=1-0.9294=0.0707 \tag{5.4.22}$$

Try It 5.4.7

In a small city, the number of automobile accidents occur with a Poisson distribution at an average of three per week.

a. Calculate the probability that there are at most 2 accidents occur in any given week.
b. What is the probability that there is at least two weeks between any 2 accidents?