8.12: Solutions

1.

1. 244
2. 15
3. 50

3.

N(244,1550)N(244,1550)" role="presentation" style="position: relative;">$N\left(244, \frac{15}{\sqrt{50}}\right)$ $N\left(244, \frac{15}{\sqrt{50}}\right)$

5.

As the sample size increases, there will be less variability in the mean, so the interval size decreases.

7.

X is the time in minutes it takes to complete the U.S. Census short form. X¯X¯" role="presentation" style="position: relative;">$\overline{X}$ $\overline{X}$ is the mean time it took a sample of 200 people to complete the U.S. Census short form.

9.

CI: (7.9441, 8.4559)

This is a normal distribution curve. The peak of the curve coincides with the point 8.2 on the horizontal axis. A central region is shaded between points 7.94 and 8.46.

Figure 8.11

EBM = 0.26

11.

The level of confidence would decrease because decreasing *n* makes the confidence interval wider, so at the same error bound, the confidence level decreases.

13.

1. x¯x¯" role="presentation" style="position: relative;">$\bar{x}$ $\bar{x}$ = 2.2
2. σ = 0.2
3. *n* = 20

15.

X¯X¯" role="presentation" style="position: relative;">$\overline{X}$ $\overline{X}$ is the mean weight of a sample of 20 heads of lettuce.

17.

EBM = 0.07
CI: (2.1264, 2.2736)

This is a normal distribution curve. The peak of the curve coincides with the point 2.2 on the horizontal axis. A central region is shaded between points 2.13 and 2.27.

Figure 8.12

19.

The interval is greater because the level of confidence increased. If the only change made in the analysis is a change in confidence level, then all we are doing is changing how much area is being calculated for the normal distribution. Therefore, a larger confidence level results in larger areas and larger intervals.

21.

The confidence level would increase.

23.

30.4

25.

σ

27.

μ

29.

normal

31.

0.025

33.

(24.52,36.28)

35.

We are 95% confident that the true mean age for Winger Foothill College students is between 24.52 and 36.28.

37.

The error bound for the mean would decrease because as the CL decreases, you need less area under the normal curve (which translates into a smaller interval) to capture the true population mean.

39.

X is the number of hours a patient waits in the emergency room before being called back to be examined. X¯X¯" role="presentation" style="position: relative;">$\overline{X}$ $\overline{X}$ is the mean wait time of 70 patients in the emergency room.

41.

CI: (1.3808, 1.6192)

This is a normal distribution curve. The peak of the curve coincides with the point 1.5 on the horizontal axis. A central region is shaded between points 1.38 and 1.62.

Figure 8.13

EBM = 0.12

43.

1. x¯x¯" role="presentation" style="position: relative;">$\bar{x}$ $\bar{x}$ = 151
2. sxsx" role="presentation" style="position: relative;">s_x s_x = 32

3. n = 108
4. n – 1 = 107

45.

X¯X¯" role="presentation" style="position: relative;">$\overline{X}$ $\overline{X}$ is the mean number of hours spent watching television per month from a sample of 108 Americans.

47.

CI: (142.92, 159.08)

This is a normal distribution curve. The peak of the curve coincides with the point 151 on the horizontal axis. A central region is shaded between points 142.92 and 159.08.

Figure 8.14

EBM = 8.08

49.

1. 3.26
2. 1.02
3. 39

51.

μ

53.

t_{38}

55.

0.025

57.

(2.93, 3.59)

59.

We are 95% confident that the true mean number of colors for national flags is between 2.93 colors and 3.59 colors.

60.

The error bound would become EBM = 0.245. This error bound decreases because as sample sizes increase, variability decreases and we need less interval length to capture the true mean.

63.

The sample size needed would increase. As the confidence level increases, αα" role="presentation" style="position: relative;">$\alpha\alpha$ decreases and z(a2)z(a2)" role="presentation" style="position: relative;">$z_{\left(\frac{a}{2}\right)}$ $z_{\left(\frac{a}{2}\right)}$ increases. To maintain the same error bound, the size of the sample needs to increase.

65.

X is the number of “successes” where the woman makes the majority of the purchasing decisions for the household. *P*′ is the percentage of households sampled where the woman makes the majority of the purchasing decisions for the household.

67.

CI: (0.5321, 0.6679)

This is a normal distribution curve. The peak of the curve coincides with the point 0.6 on the horizontal axis. A central region is shaded between points 0.5321 and 0.6679.

Figure 8.15

EBM: 0.0679

69.

X is the number of “successes” where an executive prefers a truck. *P*′ is the percentage of executives sampled who prefer a truck.

71.

CI: (0.19432, 0.33068)

This is a normal distribution curve. The peak of the curve coincides with the point 0.26 on the horizontal axis. A central region is shaded between points 0.1943 and 0.3307.

Figure 8.16

EBM: 0.0707

73.

The sampling error means that the true mean can be 2% above or below the sample mean.

75.

P′ is the proportion of voters sampled who said the economy is the most important issue in the upcoming election.

77.

CI: (0.62735, 0.67265)

EBM: 0.02265

79.

The number of girls, ages 8 to 12, in the 5 P.M. Monday night beginning ice-skating class.

81.

1. x = 64
2. n = 80
3. p' = 0.8

83.

p

85.

P′~N(0.8,(0.8)(0.2)80)P′~N(0.8,(0.8)(0.2)80)" role="presentation" style="position: relative;">$P' \sim N\left(0.8, \sqrt{\frac{(0.8)(0.2)}{80}}\right)$ $P' \sim N\left(0.8, \sqrt{\frac{(0.8)(0.2)}{80}}\right)$. (0.72171, 0.87829).

87.

0.04

89.

(0.72; 0.88)

91.

With 92% confidence, we estimate the proportion of girls, ages 8 to 12, in a beginning ice-skating class at the Ice Chalet to be between 72% and 88%.

93.

The error bound would increase. Assuming all other variables are kept constant, as the confidence level increases, the area under the curve corresponding to the confidence level becomes larger, which creates a wider interval and thus a larger error.

95.

1. 1. 71
   2. 3
   3. 48
2. X is the height of a Swiss male, and is the mean height from a sample of 48 Swiss males.
3. Normal. We know the standard deviation for the population, and the sample size is greater than 30.
4. 1. CI: (70.151, 71.49)
   2. Figure 8.17
   3. *EBM* = 0.849
5. The confidence interval will decrease in size, because the sample size increased. Recall, when all factors remain unchanged, an increase in sample size decreases variability. Thus, we do not need as large an interval to capture the true population mean.

97.

1. 1. x¯x¯" role="presentation" style="position: relative;">$\bar{x}\bar{x}$ = 23.6
   2. σσ" role="presentation" style="position: relative;">$\sigma\sigma$ = 7
   3. n = 100
2. *X* is the time needed to complete an individual tax form. X¯X¯" role="presentation" style="position: relative;">$\overline{X}\,\overline{X}$ is the mean time to complete tax forms from a sample of 100 customers.
3. N(23.6,7100)N(23.6,7100)" role="presentation" style="position: relative;">$N\left(23.6, \frac{7}{\sqrt{100}}\right) N\left(23.6, \frac{7}{\sqrt{100}}\right)$ because we know sigma.
4. 1. (22.228, 24.972)
   2. Figure 8.18
   3. *EBM* = 1.372
5. It will need to change the sample size. The firm needs to determine what the confidence level should be, then apply the error bound formula to determine the necessary sample size.
6. The confidence level would increase as a result of a larger interval. Smaller sample sizes result in more variability. To capture the true population mean, we need to have a larger interval.
7. According to the error bound formula, the firm needs to survey 206 people. Since we increase the confidence level, we need to increase either our error bound or the sample size.

99.

1. 1. 7.9
   2. 2.5
   3. 20
2. X is the number of letters a single camper will send home. X¯X¯" role="presentation" style="position: relative;">$\overline{X}\,\overline{X}$ is the mean number of letters sent home from a sample of 20 campers.
3. *N* 7.9(2.520)7.9(2.520)" role="presentation" style="position: relative;">$7.9\left(\frac{2.5}{\sqrt{20}}\right) 7.9\left(\frac{2.5}{\sqrt{20}}\right)$
4. 1. CI: (6.98, 8.82)

2.

Figure 8.19

3. *EBM*: 0.92

5. The error bound and confidence interval will decrease.

101.

1. x¯x¯" role="presentation" style="position: relative;">$\bar{x}\bar{x}$ = \$568,873
2. *CL* = 0.95 α = 1 – 0.95 = 0.05 zα2zα2" role="presentation" style="position: relative;">$z_{\frac{\alpha}{2}}\, z_{\frac{\alpha}{2}}$ = 1.96

   EBM = z0.025σnz0.025σn" role="presentation" style="position: relative;">$z_{0.025}\frac{\sigma}{\sqrt{n}}\, z_{0.025}\frac{\sigma}{\sqrt{n}}$ = 1.96 9092004090920040" role="presentation" style="position: relative;">$\frac{909200}{\sqrt{40}}\ \frac{909200}{\sqrt{40}}$ = \$281,764
3. x¯x¯" role="presentation" style="position: relative;">$\bar{x}\bar{x}$ − *EBM* = 568,873 − 281,764 = 287,109

   x¯x¯" role="presentation" style="position: relative;">$\bar{x}\bar{x}$ + *EBM* = 568,873 + 281,764 = 850,637

   Alternate solution:

   ### Using the TI-83, 83+, 84, 84+ Calculator

   1. Press `STAT` and arrow over to `TESTS` .
   2. Arrow down to `7:ZInterval` .
   3. Press `ENTER` .
   4. Arrow to Stats and press `ENTER` .
   5. Arrow down and enter the following values:
      - σ : 909,200
      - x¯x¯" role="presentation">$\bar{x}\bar{x}$: 568,873
      - *n*: 40
      - *CL*: 0.95
   6. Arrow down to Calculate and press `ENTER` .
   7. The confidence interval is (\$287,114, \$850,632).
   8. Notice the small difference between the two solutions–these differences are simply due to rounding error in the hand calculations.
4. We estimate with 95% confidence that the mean amount of contributions received from all individuals by House candidates is between \$287,109 and \$850,637.

103.

Use the formula for *EBM*, solved for *n*:

n= z2σ2EBM2n= z2σ2EBM2" role="presentation" style="position: relative;">$n = \ \frac{z^2\sigma^2}{EBM^2}\, n = \frac{z^2\sigma^2}{EBM^2}$

From the statement of the problem, you know that σ = 2.5, and you need *EBM* = 1.

$z = z_{0.035}$ = 1.812

(This is the value of *z* for which the area under the density curve to the ***right*** of *z* is 0.035.)

n= z2σ2EBM2=1.81222.5212 ≈ 20.52n= z2σ2EBM2=1.81222.5212 ≈ 20.52" role="presentation" style="position: relative;">$n = \ \frac{z^2\sigma^2}{EBM^2} = n = \frac{z^2\sigma^2}{EBM^2} = \frac{1.812^2 2.5^2}{1^2} \approx 20.52$

You need to measure at least 21 male students to achieve your goal.

105.

1. 1. 8629
   2. 6944
   3. 35
   4. 34
2. t34t34" role="presentation" style="position: relative;">$t_{34}\, t_{34}$
3. 1. CI: (6244, 11,014)

2.

Figure 8.20

3. EB = 2385

4. It will become smaller

107.

1. 1. x¯x¯" role="presentation" style="position: relative;">$\bar{x}\bar{x}$ = 2.51
   2. sxsx" role="presentation" style="position: relative;">$s_x s_x$ = 0.318
   3. n = 9
   4. n - 1 = 8
2. the effective length of time for a tranquilizer
3. the mean effective length of time of tranquilizers from a sample of nine patients
4. We need to use a Student's-t distribution, because we do not know the population standard deviation.
5. 1. CI: (2.27, 2.76)
   2. Answers may vary.
   3. *EBM*: 0.25
6. If we were to sample many groups of nine patients, 95% of the samples would contain the true population mean length of time.

109.

x¯=$251,854.23x¯=$251,854.23" role="presentation" style="position: relative;">$\bar{x} = \$251,854.23\bar{x} = \$251,854.23$

s= $521,130.41s= $521,130.41" role="presentation" style="position: relative;">$s = \ \$521,130.41 s = \$521,130.41$

Note that we are not given the population standard deviation, only the standard deviation of the sample.

There are 30 measures in the sample, so n = 30, and df = 30 - 1 = 29

CL = 0.96, so α = 1 - CL = 1 - 0.96 = 0.04

α2=0.02tα2=t0.02α2=0.02tα2=t0.02" role="presentation" style="position: relative;">$\frac{\alpha}{2} = 0.02 t_{\frac{\alpha}{2}} = t_{0.02} \frac{\alpha}{2} = 0.02 t_{\frac{\alpha}{2}} = t_{0.02}$ = 2.150

EBM=tα2(sn)=2.150(521,130.4130) ~ $204,561.66EBM=tα2(sn)=2.150(521,130.4130) ~ $204,561.66" role="presentation" style="position: relative;">$EBM = t$

$EBM = t_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right) = 2.150\left(\frac{521,130.41}{\sqrt{30}}\right)$ ~$\$204,561.66$

x¯x¯" role="presentation" style="position: relative;">$\bar{x}\bar{x}$ - *EBM* = $251,854.23 - $204,561.66 = $47,292.57

x¯x¯" role="presentation" style="position: relative;">$\bar{x}\bar{x}$ + *EBM* = $251,854.23+ $204,561.66 = $456,415.89

We estimate with 96% confidence that the mean amount of money raised by all Leadership PACs during the specific election cycle lies between $47,292.57 and $456,415.89.

Alternate Solution

Using the TI-83, 83+, 84, 84+ Calculator

Enter the data as a list.

Press `STAT` and arrow over to `TESTS` .

Arrow down to `8:TInterval` .

Press `ENTER` .

Arrow to Data and press `ENTER` .

Arrow down and enter the name of the list where the data is stored.

Enter `Freq` : 1

Enter `C-Level` : 0.96

Arrow down to `Calculate` and press `Enter` .

The 96% confidence interval is ($47,262, $456,447).

The difference between solutions arises from rounding differences.

111.

1. 1. x¯x¯" role="presentation" style="position: relative;">$\bar{x}\bar{x}$ = 11.6
   2. sxsx" role="presentation" style="position: relative;">$s_x s_x$ = 4.1
   3. *n* = 225
   4. *n* - 1 = 224
2. *X* is the number of unoccupied seats on a single flight. X¯X¯" role="presentation" style="position: relative;">$\overline{X}\,\overline{X}$ is the mean number of unoccupied seats from a sample of 225 flights.
3. We will use a Student's-t distribution, because we do not know the population standard deviation.
4. 1. CI: (11.12 , 12.08)
   2. Answers may vary.
   3. *EBM*: 0.48

113.

1. 1. CI: (7.64 , 9.36)
   2. Figure 8.21
   3. *EBM*: 0.86
2. The sample should have been increased.
3. Answers will vary.
4. Answers will vary.
5. Answers will vary.

115.

b

117.

1. 1,068
2. The sample size would need to be increased since the critical value increases as the confidence level increases.

119.

1. *X* = the number of people who feel that the president is doing an acceptable job;

   P′ = the proportion of people in a sample who feel that the president is doing an acceptable job.
2. N(0.61,(0.61)(0.39)1200)N(0.61,(0.61)(0.39)1200)" role="presentation" style="position: relative;">$N\left(0.61, \sqrt{\frac{(0.61)(0.39)}{1200}}\right) N\left(0.61, \sqrt{\frac{(0.61)(0.39)}{1200}}\right)$
3. 1. CI: (0.59, 0.63)
   2. Answers may vary.
   3. *EBM*: 0.02

121.

1. 1. (0.72, 0.82)
   2. (0.65, 0.76)
   3. (0.60, 0.72)
2. Yes, the intervals (0.72, 0.82) and (0.65, 0.76) overlap, and the intervals (0.65, 0.76) and (0.60, 0.72) overlap.
3. We can say that there does not appear to be a significant difference between the proportion of Asian adults who say that their families would welcome a White person into their families and the proportion of Asian adults who say that their families would welcome a Hispanic/Latino person into their families.
4. We can say that there is a significant difference between the proportion of Asian adults who say that their families would welcome a White person into their families and the proportion of Asian adults who say that their families would welcome a Black person into their families.

123.

1. *X* = the number of adult Americans who feel that crime is the main problem; *P′* = the proportion of adult Americans who feel that crime is the main problem
2. Since we are estimating a proportion, given *P′* = 0.2 and *n* = 1000, the distribution we should use is

   N(0.2,(0.2)(0.8)1000)N(0.2,(0.2)(0.8)1000)" role="presentation" style="position: relative;">$N\left(0.2, \sqrt{\frac{(0.2)(0.8)}{1000}}\right) N\left(0.2, \sqrt{\frac{(0.2)(0.8)}{1000}}\right)$.
3. 1. CI: (0.18, 0.22)
   2. Answers may vary.
   3. *EBM*: 0.02
4. One way to lower the sampling error is to increase the sample size.
5. The stated "± 3%" represents the maximum error bound. This means that those doing the study are reporting a maximum error of 3%. Thus, they estimate the percentage of adult Americans who feel that crime is the main problem to be between 18% and 22%.

125.

c

127.

d

129.

a

131.

1. p' = (0.55 + 0.49)2(0.55 + 0.49)2" role="presentation" style="position: relative;">$\frac{(0.55 + 0.49)}{2}$ $\frac{(0.55 + 0.49)}{2}$ = 0.52; *EBP* = 0.55 - 0.52 = 0.03
2. No, the confidence interval includes values less than or equal to 0.50. It is possible that less than half of the population believe this.
3. *CL* = 0.75, so α = 1 – 0.75 = 0.25 and α2=0.125 zα2=1.150α2=0.125 zα2=1.150" role="presentation" style="position: relative;">$\frac{\alpha}{2} = 0.125$ $z_{\frac{\alpha}{2}} = 1.150$ $\frac{\alpha}{2} = 0.125 z_{\frac{\alpha}{2}} = 1.150$. (The area to the right of this z is 0.125, so the area to the left is 1 – 0.125 = 0.875.)

   EBP=(1.150)0.52(0.48)1,000≈0.018EBP=(1.150)0.52(0.48)1,000≈0.018" role="presentation" style="position: relative;">$EBP = (1.150)\sqrt{\frac{0.52(0.48)}{1,000}} \approx 0.018$

   $EBP = (1.150)\sqrt{\frac{0.52(0.48)}{1,000}} \approx 0.018$

   (p' - *EBP*, p' + *EBP*) = (0.52 – 0.018, 0.52 + 0.018) = (0.502, 0.538)

   Alternate Solution

   ### Using the TI-83, 83+, 84, 84+ Calculator

   STAT TESTS `A: 1-PropZinterval` with x = (0.52)(1,000), n = 1,000, CL = 0.75.

   Answer is (0.502, 0.538)
4. Yes – this interval does not fall less than 0.50 so we can conclude that at least half of all American adults believe that major sports programs corrupt education – but we do so with only 75% confidence.

133.

CL = 0.95 α = 1 – 0.95 = 0.05 α2α2" role="presentation" style="position: relative;">$\frac{\alpha}{2}$ $\frac{\alpha}{2}$ = 0.025 zα2zα2" role="presentation" style="position: relative;">$z_{\frac{\alpha}{2}}$ $z_{\frac{\alpha}{2}}$ = 1.96. Use p' = q' = 0.5.

n= zα22p′q′EBP2= 1.962(0.5)(0.5)0.052=384.16n= zα22p′q′EBP2= 1.962(0.5)(0.5)0.052=384.16" role="presentation" style="position: relative;">
$n = \frac{{z_{\frac{\alpha}{2}}}^2 p' q'}{EBP^2} = \frac{1.96^2(0.5)(0.5)}{0.05^2} = 384.16$

You need to interview at least 385 students to estimate the proportion to within 5% at 95% confidence.