7.5: Finite Population Correction Factor

We saw that the sample size has an important effect on the variance and thus the standard deviation of the sampling distribution. Also of interest is the proportion of the total population that has been sampled. We have assumed that the population is extremely large and that we have sampled a small part of the population. As the population becomes smaller and we sample a larger number of observations the sample observations are not independent of each other. To correct for the impact of this, the Finite Correction Factor can be used to adjust the variance of the sampling distribution. It is appropriate when more than 5% of the population is being sampled and the population has a known population size. There are cases when the population is known, and therefore the correction factor must be applied. The issue arises for both the sampling distribution of the means and the sampling distribution of proportions. The **Finite Population Correction Factor** for the variance of the means shown in the standardizing formula is:

$$Z=\frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}} \cdot \sqrt{\frac{N-n}{N-1}}} \tag{7.5.1}$$

and for the variance of proportions is:

$$\sigma_{\mathrm{p}}, =\sqrt{\frac{p(1-p)}{n}} \times \sqrt{\frac{N-n}{N-1}} \tag{7.5.2}$$

The following examples show how to apply the factor. Sampling variances get adjusted using the above formula.

Exercise 7.5.1

It is learned that the population of White German Shepherds in the USA is 4,000 dogs, and the mean weight for German Shepherds is 75.45 pounds. It is also learned that the population standard deviation is 10.37 pounds.

Problem

If the sample size is 225 dogs, then find the probability that a sample will have a mean that differs from the true probability mean by less than 2 pounds.

Answer

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\begin{array}{l}
  \begin{array}{c}
  N=4000, \quad n=225, \quad \sigma=10.37, \quad \mu=75.45, \quad(\bar{x}-\mu)= \pm 2 \\
  Z=\dfrac{\bar{x}-\mu}{\dfrac{\sigma}{\sqrt{n}} \cdot \sqrt{\dfrac{N-\mathrm{n}}{N-1}}}=\dfrac{ \pm 2}{\dfrac{10.37}{\sqrt{225}} \cdot \sqrt{\dfrac{4000-225}{4000-1}}} \approx 2.978 \\
  f(Z) \approx 2.978 \cdot 2=0.997
  \end{array}
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\end{array}
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Note

Note that "differs by less" references the area on both sides of the mean within 2 pounds right or left.

Try It 7.5.1

For the 5,000 students in University A, the average height of the students is 170 cm. The population standard deviation is 5 cm.

If the sample size is 100 students, find the probability that the sample mean is within 1 cm of the true mean.

Exercise 7.5.2

When a customer places an order with Rudy's On-Line Office Supplies, a computerized accounting information system (AIS) automatically checks to see if the customer has exceeded their credit limit. Past records indicate that the probability of customers exceeding their credit limit is .06.

Problem

Suppose that on a given day, 3,000 orders are placed in total. If we randomly select 360 orders, what is the probability that between 10 and 20 customers will exceed their credit limit?

Answer

$$\begin{gathered}
N=3000, \quad n=360, \quad p=0.06 \\
\sigma_{\mathrm{p}'}=\sqrt{\frac{p(1-p)}{n}} \times \sqrt{\frac{N-n}{N-1}}=\sqrt{\frac{0.06(1-0.06)}{360}} \times \sqrt{\frac{3000-360}{3000-1}}=0.0117 \\
p_1=\frac{10}{360}=0.0278, \quad p_2=\frac{20}{360}=0.0556 \\
Z=\frac{p, -p}{\sqrt{\frac{p(1-p)}{n}} \cdot \sqrt{\frac{N-n}{N-1}}}=\frac{0.0278-0.06}{0.011744}=-2.74 \\
Z=\frac{p, -p}{\sqrt{\frac{p(1-p)}{n}} \cdot \sqrt{\frac{N-n}{N-1}}}=\frac{0.0556-0.06}{0.011744}=-0.38 \\
p\left(\frac{0.0278-0.06}{0.011744}<z<\frac{0.0556-0.06}{0.011744}\right)=p(-2.74<z<-0.38)=0.4969-0.1480=0.3489
\end{gathered} \tag{7.5.3}$$

Try It 7.5.2

There is a 0.01 probability that a student gets food poisoning eating food in a cafeteria. Suppose there are 2,000 orders in a day in the cafeteria. If we select 200 orders randomly, what is the probability that between 5 and 10 students will get food poisoning?

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