6.2: The Standard Normal Distribution

The **standard normal distribution** is a normal distribution of **standardized values called z-scores. A z-score is measured in units of the standard deviation.**

The mean for the standard normal distribution is zero, and the standard deviation is one. What this does is dramatically simplify the mathematical calculation of probabilities. Take a moment and substitute zero and one in the appropriate places in the above formula and you can see that the equation collapses into one that can be much more easily solved using integral calculus. The transformation $z = \frac{x-\mu}{\sigma}$ produces the distribution $Z \sim N(0,1)$. The value x in the given equation comes from a known normal distribution with known mean μ and known standard deviation σ. The z-score tells how many standard deviations a particular x is away from the mean.

Z-Scores

If X is a normally distributed random variable and $X \sim N(\mu, \sigma)$, then the z-score for a particular x is:

$$z = \frac{x-\mu}{\sigma} \tag{6.2.1}$$

The z-score tells you how many standard deviations the value x is above (to the right of) or below (to the left of) the mean, μ. Values of x that are larger than the mean have positive z-scores, and values of x that are smaller than the mean have negative z-scores. If x equals the mean, then x has a z-score of zero.

Exercise 6.2.1

Suppose $X \sim N(5,6)$. This says that X is a normally distributed random variable with mean $\mu = 5$ and standard deviation $\sigma = 6$. Suppose $x = 17$. Then:

$$z = \frac{x-\mu}{\sigma} = \frac{17-5}{6} = 2 \tag{6.2.2}$$

This means that $x = 17$ is **two standard deviations** (2σ) above or to the right of the mean $\mu = 5$.

Now suppose $x = 1$. Then: $z = \frac{x-\mu}{\sigma} = \frac{1-5}{6} = -0.67$ (rounded to two decimal places)

This means that $x = 1$ is 0.67 standard deviations (-0.67σ) below or to the left of the mean $\mu = 5$.

Try It 6.2.1

Fill in the blanks.

Jerome averages 16 points a game with a standard deviation of four points. $X \sim N(16,4)$. Suppose Jerome scores ten points in a game. The z–score when $x = 10$ is –1.5. This score tells you that $x = 10$ is _____ standard deviations to the ______(right or left) of the mean______(What is the mean?).

The Empirical Rule

If X is a random variable and has a normal distribution with mean μ and standard deviation σ, then the Empirical Rule states the following:

- About 68% of the x values lie between -1σ and $+1\sigma$ of the mean μ (within one standard deviation of the mean).
- About 95% of the x values lie between -2σ and $+2\sigma$ of the mean μ (within two standard deviations of the mean).
- About 99.7% of the x values lie between -3σ and $+3\sigma$ of the mean μ (within three standard deviations of the mean). Notice that almost all the x values lie within three standard deviations of the mean.
- The z-scores for $+1\sigma$ and -1σ are +1 and -1 , respectively.
- The z-scores for $+2\sigma$ and -2σ are +2 and -2 , respectively.
- The z-scores for $+3\sigma$ and -3σ are +3 and -3 respectively.

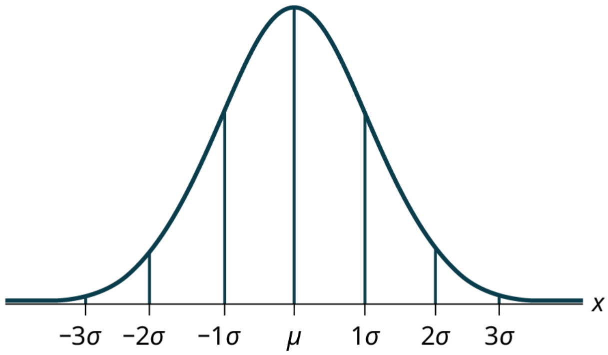

Figure 6.2.1

Exercise 6.2.2

Suppose x has a normal distribution with mean 50 and standard deviation 6 .

- About 68% of the x values lie within one standard deviation of the mean. Therefore, about 68% of the x values lie between $-1\sigma = (-1)(6) = -6$ and $1\sigma = (1)(6) = 6$ of the mean 50 . The values $50 - 6 = 44$ and $50 + 6 = 56$ are within one standard deviation from the mean 50 . The z-scores are -1 and +1 for 44 and 56, respectively.
- About 95% of the x values lie within two standard deviations of the mean. Therefore, about 95% of the x values lie between $-2\sigma = (-2)(6) = -12$ and $2\sigma = (2)(6) = 12$. The values $50 - 12 = 38$ and $50 + 12 = 62$ are within two standard deviations from the mean 50. The z-scores are -2 and +2 for 38 and 62, respectively.
- About 99.7% of the x values lie within three standard deviations of the mean. Therefore, about 99.7% of the x values lie between $-3\sigma = (-3)(6) = -18$ and $3\sigma = (3)(6) = 18$ of the mean 50 . The values $50 - 18 = 32$ and $50 + 18 = 68$ are within three standard deviations from the mean 50 . The z-scores are -3 and +3 for 32 and 68 , respectively.

Try It 6.2.2

Suppose *X* has a normal distribution with mean 25 and standard deviation five. Between what values of *x* do 68% of the values lie?

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