

Math 121: Support for Introductory Probability and Statistics

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CHAPTER OVERVIEW

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Thumbnail: pixabay.com/photos/diary-journal-pen-notebook-january-614149/

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1.1: Prelude to Managing Your Time and Priorities



Figure 1.1.1: Our devices can be helpful tools for managing time, but they can also lead to distraction.

Student Survey

How do you feel about your time management abilities? Take this quick survey to figure it out, ranking questions on a scale of 1–4, 1 meaning “least like me” and 4 meaning “most like me.” These questions will help you determine how the chapter concepts relate to you right now. As you are introduced to new concepts and practices, it can be informative to reflect on how your understanding changes over time. We’ll revisit these questions at the end of the chapter to see whether your feelings have changed.

1. I regularly procrastinate completing tasks that don't interest me or seem challenging.
2. I use specific time management strategies to complete tasks.
3. I find it difficult to prioritize tasks because I am not sure what is really important.
4. I am pleased with my ability to manage my time.

You can also take the [Chapter survey](#) anonymously online.

STUDENT PROFILE

"Before I started college, I had heard that the amount of work would be overwhelming, and that it would be much harder than high school. That was true, but after being in college for a couple of weeks, I felt that people made it seem scarier than it actually was. I had some homework assignments here, some essays, some hard classes, but it wasn't that bad..until Midterms and Finals came knocking. I had so much to study and so little time. The pressure was unimaginable. And since there was so much material to learn, I kept procrastinating. The nights before the exams were a disaster.

"After the semester, I realized that I needed to do something differently. Instead of crashing before midterms and finals, I would study throughout the semester. I would review notes after class, do a few practice problems in the book even if homework wasn't assigned, and try to ask professors questions during their office hours if I was confused. This continual effort helped me do better on exams because I built up my understanding and was able to get a good night's sleep before the big test. I still studied hard, but the material was in reach and understanding it became a reasonable goal, not an impossibility. I also felt more confident going into the exams, because I knew that I had a deeper knowledge — I could recall things more easily. Most importantly, I now had peace of mind throughout the day and during the tests themselves, since I knew that I was better prepared."

—**Nachum Sash**, Actuarial Science Major, City University of New York

About This Chapter

In this chapter you will learn about two of the most valuable tools used for academic success: prioritizing and time management. By the time you complete this chapter, you should be able to do the following:

- Articulate the ways in which time management differs from high school to college.
- Outline reasons and effects of procrastination, and provide strategies to overcome it.
- Describe ways to evaluate your own time management skills.
- Discuss the importance and the process of prioritization.
- Articulate the importance of goal setting and motivation.
- Detail strategies and specific tactics for managing your time.

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1.2: The Benefits of Time Management

“Poor time management can set into motion a series of events that can seriously jeopardize a student’s success.”

A very unfortunate but all-too-common situation in higher education is the danger students face from poor time management. Many college administrators that work directly with students are aware that a single mishap or a case of poor time management can set into motion a series of events that can seriously jeopardize a student’s success. In some of the more extreme instances, the student may even fail to graduate because of it.

To better understand how one instance of poor time management can trigger a cascading situation with disastrous results, imagine that a student has an assignment due in a business class. She knows that she should be working on it, but she isn’t quite in the mood. Instead she convinces herself that she should think a little more about what she needs to complete the assignment and decides to do so while looking at social media or maybe playing a couple more rounds of a game on her phone. In a little while, she suddenly realizes that she has become distracted and the evening has slipped away. She has little time left to work on her assignment. She stays up later than usual trying to complete the assignment but cannot finish it. Exhausted, she decides that she will work on it in the morning during the hour she had planned to study for her math quiz. She knows there will not be enough time in the morning to do a good job on the assignment, so she decides that she will put together what she has and hope she will at least receive a passing grade.

At this point in our story, an evening of procrastination has not only resulted in a poorly done business class assignment, but now she is going to take a math quiz that she has not studied for. She will take the quiz tired from staying up too late the night before. Her lack of time management has now raised potential issues in two courses. Imagine that each of these issues also causes additional problems, such as earning low scores on *both* the assignment and the quiz. She will now have to work harder in both courses to bring her grades up. Any other problems she has with future assignments in either course could cause a domino effect of circumstances that begins to overwhelm her.

In our imagined situation, you can see how events set into motion by a little procrastination can quickly spiral out of control. You can probably think of similar experiences in your own life, when one small bit of poor time management set off a chain of events that threatened to cause big problems.

The High Cost of Poor Time Management

It’s not just your academic performance that can be affected by cascading events that have a domino effect on your college path. And dropping out of school is not your only danger. There are other consequences that affect the financial cost to you as a student if your lack of time management skills causes you to delay when you finish college.

Based on independent research, a *Washington Post* article details the financial impact delaying graduation by two semesters can have on a student.¹ (Spending a Few Extra Years in College May Cost You More Than You Think, Danielle Douglas-Gabriel, June 21, 2016)

According to the article, there is a significant cost associated with delaying graduation from college by only one year (by dropping and retaking courses, taking less than a full credit load, etc.). Not only will you pay for additional tuition, textbooks, and other fees associated with going to school, but if you are using student loans, you will also accumulate interest on those loans. On average this would come to an extra \$12,557 in actual costs and \$6,040 in interest at a public university, or \$18,992 in tuition and fees and \$7,823 in interest (over 10 years) at a private school. That’s a lot of extra cost to you!

“In the long run, just two extra semesters of college can cost you almost \$150,000.”

While a loss of \$26,815 may seem like a lot of money, it pales in comparison to the other financial areas impacted by a single extra year in school. The *Washington Post* article estimates that one year’s delay of graduation would cost you an additional \$46,355 based on average lost earnings. To make matters worse, like the story of the student that procrastinates finishing her business assignment, there is a spiraling effect that takes place with loss of income when it comes to retirement investments. The figure cited by the *Washington Post* as lost retirement earnings for taking five years instead of four years to graduate is \$82,074. That brings the average total cost for only two extra semesters to over \$150,000. Measured by the financial cost to you, even a slight delay of graduation can have a serious impact.

Table 1.2.1 Credit: Washington Post. Note the numbers in the table above have been averaged between the two scenarios described.

Average Cost of an Additional Year of College	
Tuition, textbooks, and fees	\$15,774
Interest on student loans	\$6,932
Lost wages	\$46,335
Lost retirement earnings	\$82,074
Average total loss:	\$151,115

It is worth noting that any situation that brings about a delay in graduation has the potential to increase the cost of college. This also includes attending school on a part-time basis. While in some instances responsibilities may make it impossible to go to school full-time, from a financial perspective you should do all you can to graduate as soon as you can.

While it may not be possible to prevent life challenges while you are in college, you can do a great deal to prevent the chaos and the chain reaction of unfortunate events that they can cause. This can be accomplished through thoughtful prioritization and time management efforts.

What follows in the rest of this chapter is a close look at the nature of time management and prioritization in ways that can help keep you on track to graduate college on time.

ANALYSIS QUESTION

Can you identify any areas in your life that might be a potential problem if there were a temporary setback (e.g., temporary loss of transportation, temporary loss of housing, an illness that lasted more than a week, etc.)? What could you do for a backup plan if something did happen?

Footnotes

- 1 <https://www.washingtonpost.com/news/...=.f06be365e5d6>

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1.3: Time Management in College

Questions to consider:

- Is time management different in college from what I am used to?
- How different is college schoolwork from high school work?

You may find that time management in college is very different from anything you have experienced previously. For the last 12 years, almost all your school time was managed by educators and your parents. What you did and when you did it was controlled by others. In many cases, even after-school time was set by scheduled activities (such as athletics) and by nightly homework that was due the next day.

In the workplace, the situation is not very different, with activities and time on task being monitored by the company and its management. This is so much a part of the working environment that many companies research how much time each task should take, and they hold employees accountable for the time spent on these job functions. In fact, having these skills will help you stand out on the job and in job interviews.

K-12	College
Many class activities are planned.	Class time is given to receiving information.
Homework is often similar for each student.	You may have freedom in homework choices.
Time is managed by others more often.	Time is managed by the student.

In college, there is a significant difference because a great deal of time management is left up to you. While it is true that there are assignment due dates and organized classroom activities, learning at the college level requires more than just the simple completion of work. It involves decision-making and the ability to evaluate information. This is best accomplished when you are an active partner in your own learning activities.



Figure 1.3.1: Students may set aside specific times and specific places to study.

As an example of how this works, think about a college assignment that involves giving a classroom presentation. To complete the assignment, you are given time to research and reflect on the information found. As a part of the assignment, you must reach your own conclusions and determine which information that you have found is best suited for the presentation. While the date of the actual presentation and how long it will last are usually determined by the instructor, how much time you spend gathering information, the sources you use, and how you use them are left to you.

WHAT STUDENTS SAY

1. How difficult is it for you to keep track of multiple tasks over the course of a term?
 - a. Extremely easy
 - b. Somewhat easy

- c. Somewhat difficult
- d. Extremely difficult

2. Do you use a particular app to help you manage your time?

- a. I use Google calendar
- b. I use the calendar on my phone
- c. I use a paper/notebook planner
- d. I use the calendar on my learning management system
- e. I use another app or system
- f. I don't use any type of planner or app

3. Rank the following in terms of what you would most like to improve regarding your time management skills.

- a. My ability to predict how much time my tasks will take.
- b. My ability to balance various obligations.
- c. My ability to avoid procrastination.
- d. My ability to limit distractions.

You can also take the anonymous [What Students Say](#) surveys to add your voice to this textbook. Your responses will be included in updates.

Students offered their views on these questions, and the results are displayed in the graphs below.

How difficult is it for you to keep track of multiple tasks over the course of a term?

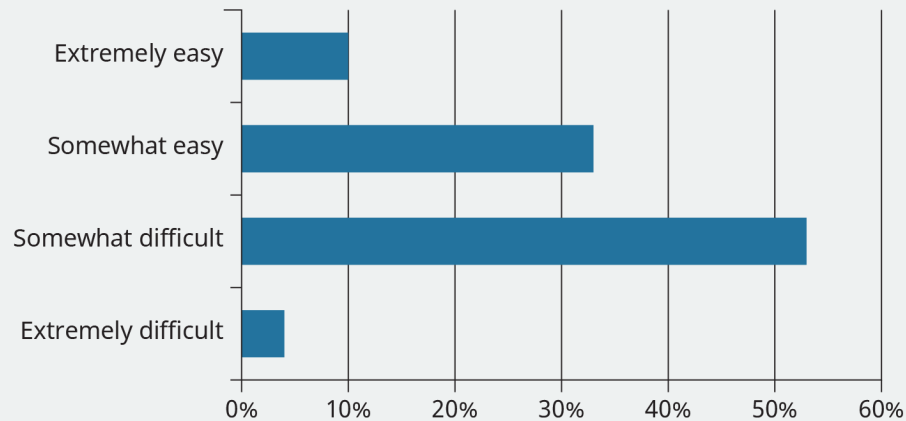


Figure 1.3.2

Do you use a particular app to help you manage your time?

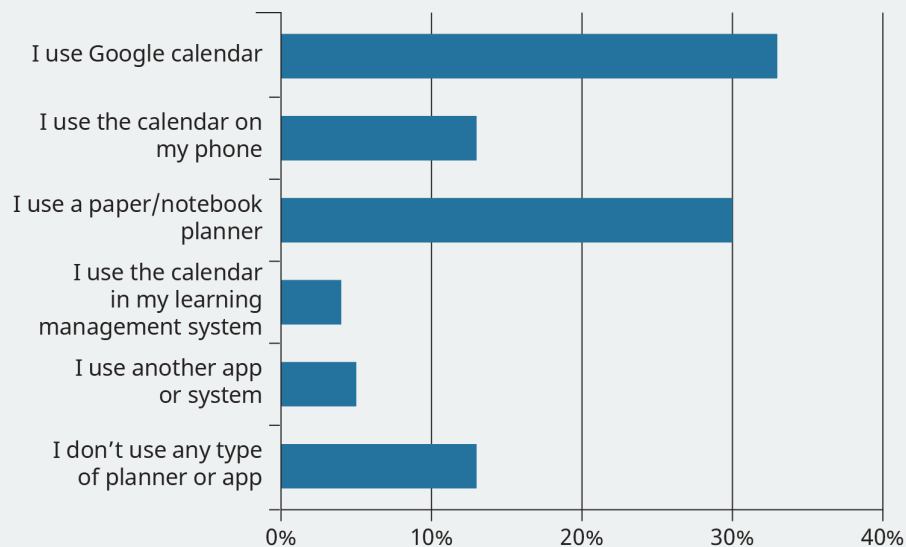


Figure 1.3.3

Rank the following in terms of what you would most like to improve regarding your time management skills.

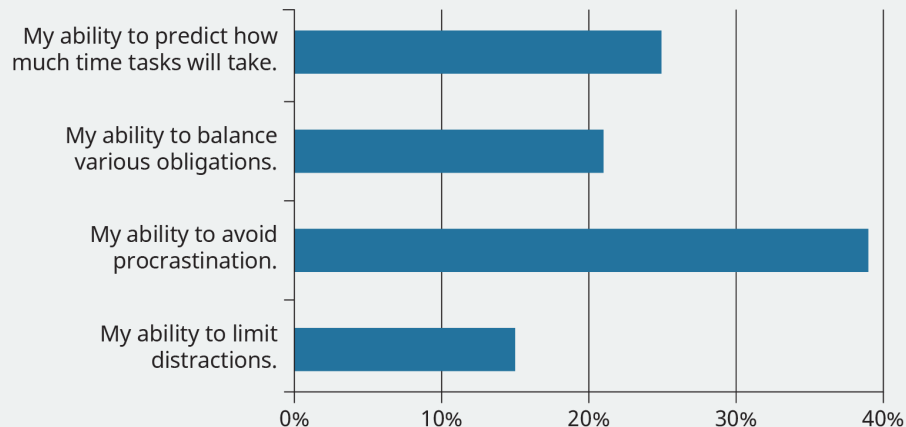


Figure 1.3.4

You Have Lots of Time to Manage

For college-level learning, this approach is important enough that you can expect to spend much more time on learning activities outside the classroom than you will in the classroom. In fact, the estimated time you should spend will be at least two hours of outside learning for every one hour of lecture. Some weeks may be more intense, depending on the time of the semester and the courses you are taking. If those hours are multiplied over several courses in a given session, you can see how there is a significant amount of time to manage. Unfortunately, many students do not always take this into consideration, and they spend far less time than is needed to be successful. The results of poor time management are often a shock to them.

“In college, as an active participant in your own education, what you do and when you do it is largely determined by you.”

The Nature of What You Have to Do Has Changed

Returning to our example of the classroom-presentation assignment, you can see that the types of learning activities in college can be very different from what you have experienced previously. While there may have been similar assignments in high school, such as presentations or written papers, the level of expectation with length and depth is significantly different in college. This point is made very clear when comparing facts about the requirements of high school work to the type of work students produce in college. One very strong statistic that underscores this comes from a study conducted by the Pew Research Center. They found that 82 percent of teens report that their typical high school writing assignments were only a single paragraph to one page in length.² (Writing Technology and Teens, 2004, Pew Research Center) This is in stark contrast to a number of sources that say that writing assignments in lower-level college courses are usually 5–7 pages in length, while writing assignments in upper-level courses increase to 15–20 pages.

It is also interesting to note that the amount of writing done by a college student can differ depending on their program of study. The table below indicates the estimated average amount of writing assigned in several disciplines. To estimate the number of pages of assigned writing, the average number of writing assignments of a given page length was multiplied by an approximate number of pages for the assignment type (see **Estimating Number of Pages Written** for calculation details).

Table 1.3.5 Credit: Updated NSSE (Since 2013)³

Writing Assignments Vary in Length	
Discipline	Number of Pages Assigned in Introductory Course
Arts & Humanities	49
Biological Sciences, Agriculture, & Natural Resources	47
Physical Sciences, Mathematics, & Computer Science	44
Social Sciences	52

Writing Assignments Vary in Length	
Business	48
Communications, Media, & Public Relations	50
Education	46
Engineering	46
Health Professions	43
Social Service Professions	47

High school homework often consists of worksheets or tasks based on reading or classroom activities. In other words, all the students are doing the same tasks, at relatively the same time, with little autonomy over their own education.

Using the earlier example of the presentation assignment, not only will what you do be larger in scale, but the depth of understanding and knowledge you will put into it will be significantly more than you may have encountered in previous assignments. This is because there are greater expectations required of college graduates in the workplace. Nearly any profession that requires a college degree has with it a level of responsibility that demands higher-level thinking and therefore higher learning. An often-cited example of this is the healthcare professional. The learning requirements for that profession are strict because we depend on those graduates for our health and, in some cases, our lives. While not every profession may require the same level of study needed for healthcare, most do require that colleges maintain a certain level of academic rigor to produce graduates who are competent in their fields.

Footnotes

- [2 http://www.pewinternet.org/2008/04/2...ogy-and-teens/](http://www.pewinternet.org/2008/04/2...ogy-and-teens/)
- [3 nsse.indiana.edu/html/sample...of_writing.cfm](http://nsse.indiana.edu/html/sample...of_writing.cfm)

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1.4: Procrastination- The Enemy Within

Questions to consider:

- Why do we procrastinate?
- What are the effects of procrastination?
- How can we avoid procrastination?



Figure 1.4.1: We can think of many creative ways to procrastinate, but the outcome is often detrimental. (Credit: University of the Fraser Valley / Flickr / Attribution 2.0 Generic (CC BY 2.0))

Simply put, procrastination is the act of delaying some task that needs to be completed. It is something we all do to greater and lesser degrees. For most people, a little minor procrastination is not a cause for great concern. But there are situations where procrastination can become a serious problem with a lot of risk. These include: when it becomes a chronic habit, when there are a number of tasks to complete and little time, or when the task being avoided is very important.

Because we all procrastinate from time to time, we usually do not give it much thought, let alone think about its causes or effects. Ironically, many of the psychological reasons for why we avoid a given task also keep us from using critical thinking to understand why procrastination can be extremely detrimental, and in some cases difficult to overcome.

To succeed at time management, you must understand some of the hurdles that may stand in your way. Procrastination is often one of the biggest. What follows is an overview of procrastination with a few suggestions on how to avoid it.

The Reasons Behind Procrastination

There are several reasons we procrastinate, and a few of them may be surprising. On the surface we often tell ourselves it is because the task is something we do not want to do, or we make excuses that there are other things more important to do first. In some cases this may be true, but there can be other contributors to procrastination that have their roots in our physical well-being or our own psychological motivations.

Lack of Energy

Sometimes we just do not feel up to a certain task. It might be due to discomfort, an illness, or just a lack of energy. If this is the case, it is important to identify the cause and remedy the situation. It could be something as simple as a lack of sleep or improper diet. Regardless, if a lack of energy is continually causing you to procrastinate to the point where you are beginning to feel stress over not getting things done, you should definitely assess the situation and address it.

Lack of Focus

Much like having low physical energy, a lack of mental focus can be a cause of procrastination. This can be due to mental fatigue, being disorganized, or allowing yourself to be distracted by other things. Again, like low physical energy, this is something that may have farther-reaching effects in your life that go beyond the act of simply avoiding a task. If it is something that is recurring, you should properly assess the situation.

Fear of Failure

This cause of procrastination is not one that many people are aware of, especially if they are the person avoiding tasks because of it. To put it in simple words, it is a bit of trickery we play on ourselves by avoiding a situation that makes us psychologically uncomfortable. Even though they may not be consciously aware of it, the person facing the task is afraid that they cannot do it or will not be able to do it well. If they fail at the task, it will make them appear incompetent to others or even to themselves. Where the self-trickery comes in is by avoiding the task. In the person's mind, they can rationalize that the reason they failed at the task was because they ran out of time to complete it, not that they were incapable of doing it in the first place.

It is important to note that a fear of failure may not have anything to do with the actual ability of the person suffering from it. They could be quite capable of doing the task and performing well, but it is the fear that holds them back.

ANALYSIS QUESTION

Consider something right now that you may be procrastinating about. Are you able to identify the cause?

The Effects of Procrastination

In addition to the causes of procrastination, you must also consider what effects it can have. Again, many of these effects are obvious and commonly understood, but some may not be so obvious and may cause other issues.

Loss of Time

The loss of time as an effect of procrastination is the easiest to identify since the act of avoiding a task comes down to not using time wisely. Procrastination can be thought of as using the time you have to complete a task in ways that do not accomplish what needs to be done.

Loss of Goals

Another of the more obvious potentially adverse effects of procrastination is the loss of goals. Completing a task leads to achieving a goal. These can be large or small (e.g., from doing well on an assignment to being hired for a good job). Without goals you might do more than delay work on a task—you may not complete it at all. The risk for the loss of goals is something that is very impactful.

Loss of Self-Esteem

Often, when we procrastinate we become frustrated and disappointed in ourselves for not getting important tasks completed. If this continues to happen, we can begin to develop a low opinion of ourselves and our own abilities. We begin to suffer from low self-esteem and might even begin to feel like there is something wrong with us. This can lead to other increasingly negative mental factors such as anger and depression. As you can see, it is important for our own well-being to avoid this kind of procrastination effect.

Stress

Procrastination causes stress and anxiety, which may seem odd since the act of procrastination is often about avoiding a task we think will be stressful in itself! Anyone who has noticed that nagging feeling when they know there is something else they should be doing is familiar with this.

On the other hand, some students see that kind of stress as a boost of mental urgency. They put off a task until they feel that surge of motivation. While this may have worked in the past, they quickly learn that procrastinating when it comes to college work almost always includes an underestimation of the tasks to be completed— sometimes with disastrous results.

Strategies for Psyching Ourselves Out and Managing Procrastination

Now that you understand a few of the major problems procrastination can produce, let's look at methods to manage procrastination and get you on to completing the tasks, no matter how unpleasant you think they might be.

Get Organized

Much of this chapter is dedicated to defining and explaining the nature of time management. The most effective way to combat procrastination is to use time and project management strategies such as schedules, goal setting, and other techniques to get tasks accomplished in a timely manner.

Put Aside Distractions

Several of the methods discussed in this chapter deal specifically with distractions. Distractions are time-killers and are the primary way people procrastinate. It is too easy to just play a video game a little while longer, check out social media, or finish watching a movie when we are avoiding a task. Putting aside distractions is one of the primary functions of setting priorities.

Reward Yourself

Rewarding yourself for the completion of tasks or meeting goals is a good way to avoid procrastination. An example of this would be rewarding yourself with the time to watch a movie you would enjoy *after* you have finished the things you need to do, rather than using the movie to keep yourself from getting things done.

Be Accountable—Tell Someone Else

A strong motivational tool is to hold ourselves accountable by telling someone else we are going to do something and when we are going to do it. This may not seem like it would be very effective, but on a psychological level we feel more compelled to do something if we tell someone else. It may be related to our need for approval from others, or it might just serve to set a level of commitment. Either way, it can help us stay on task and avoid procrastination—especially if we take our accountability to another person seriously enough to warrant contacting that person and apologizing for not doing what we said we were going to do.

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1.5: How to Manage Time

Questions to consider:

- How can I use time-on-task estimates to improve time management?
- What behaviors can help or hinder when it comes to managing time?

In this next section you will learn about managing time and prioritizing tasks. This is not only a valuable skill for pursuing an education, but it can become an ability that follows you through the rest of your life, especially if your career takes you into a leadership role.



Figure 1.5.1: An online calendar is a very useful tool for keeping track of classes, meetings, and other events. Most learning management systems contain these features, or you can use a calendar application.

ANALYSIS QUESTION

Read each statement in the brief self-evaluation tool below, and check the answer that best applies to you. There are no right or wrong answers.

Table 1.5.2

	Always	Usually	Sometimes	Rarely	Never
I like to be given strict deadlines for each task. It helps me stay organized and on track.					
I would rather be 15 minutes early than 1 minute late.					
I like to improvise instead of planning everything out ahead of time.					

	Always	Usually	Sometimes	Rarely	Never
I prefer to be able to manage when and how I do each task.					
I have a difficult time estimating how long a task will take.					
I have more motivation when there is an upcoming deadline. It helps me focus.					
I have difficulty keeping priorities in the most beneficial order.					

This exercise is intended to help you recognize some things about your own time management style. The important part is for you to identify any areas where you might be able to improve and to find solutions for them. This chapter will provide some solutions, but there are many others that can be found by researching time management strategies.

After you have decided your best response to each statement, think about what they may mean in regard to potential strengths and/or challenges for you when it comes to time management in college. If you are a person that likes strict deadlines, what would you do if you took a course that only had one large paper due at the end? Would you set yourself a series of mini deadlines that made you more comfortable and that kept things moving along for you? Or, if you have difficulty prioritizing tasks, would it help you to make a list of the tasks to do and order them, so you know which ones must be finished first?

How to Manage Time

The simplest way to manage your time is to accurately plan for how much time it will take to do each task, and then set aside that amount of time. How you divide the time is up to you. If it is going to take you five hours to study for a final exam, you can plan to spread it over five days, with an hour each night, or you can plan on two hours one night and three hours the next. What you would not want to do is plan on studying only a few hours the night before the exam and find that you fell very short on the time you estimated you would need. If that were to happen, you would have run out of time before finishing, with no way to go back and change your decision. In this kind of situation, you might even be tempted to “pull an all-nighter,” which is a phrase that has been used among college students for decades. In essence it means going without sleep for the entire night and using that time to finish an assignment. While this method of trying to make up for poor planning is common enough to have a name, rarely does it produce the best work.

Table 1.5.3 Sample Time Estimate Table

Activity	Estimated Time	Actual Time
Practice Quiz	5 minutes	15 minutes
Lab Conclusions	20 minutes	35 minutes
Food shopping	45 minutes	30 minutes
Drive to work	20 minutes	20 minutes
Physical Therapy	1 hour	50 minutes

Of all the parts of time management, accurately predicting how long a task will take is usually the most difficult—and the most elusive. Part of the problem comes from the fact that most of us are not very accurate timekeepers, especially when we are busy applying ourselves to a task. The other issue that makes it so difficult to accurately estimate time on task is that our estimations must also account for things like interruptions or unforeseen problems that cause delays.

When it comes to academic activities, many tasks can be dependent upon the completion of other things first, or the time a task takes can vary from one instance to another, both of which add to the complexity and difficulty of estimating how much time and effort are required.

For example, if an instructor assigned three chapters of reading, you would not really have any idea how long each chapter might take to read until you looked at them. The first chapter might be 30 pages long while the second is 45. The third chapter could be only 20 pages but made up mostly of charts and graphs for you to compare. By page count, it might seem that the third chapter would take the least amount of time, but actually studying charts and graphs to gather information can take longer than regular reading.

To make matters even more difficult, when it comes to estimating time on task for something as common as reading, not all reading takes the same amount of time. Fiction, for example, is usually a faster read than a technical manual. But something like the novel *Finnegan's Wake* by James Joyce is considered so difficult that most readers never finish it.

ACTIVITY

To better understand how much time different kinds of material can take to read, try this experiment. You will use two examples of famous texts that are very close to being the same number of words: *The Gettysburg Address* and the opening paragraphs from *A Christmas Carol*. Before you begin, estimate how long it will take you to read each, and predict which you think will take longer. When you do the reading, use a stopwatch function on a device such as a phone or some other timer to see how long it actually takes.

Make certain that you are reading for understanding, not just skimming over words. If you must reread a section to better comprehend what is being said, that is appropriate. The goal here is to compare reading of different texts, not to see how fast you can sight-read the words on a page.

After you have finished *The Gettysburg Address*, read and time *A Christmas Carol* and compare both of your times.

The Gettysburg Address

Abraham Lincoln

Gettysburg, Pennsylvania November 19, 1863

Word count: 278

Four score and seven years ago our fathers brought forth on this continent, a new nation, conceived in Liberty, and dedicated to the proposition that all men are created equal.

Now we are engaged in a great civil war, testing whether that nation, or any nation so conceived and so dedicated, can long endure. We are met on a great battle-field of that war. We have come to dedicate a portion of that field, as a final resting place for those who here gave their lives that that nation might live. It is altogether fitting and proper that we should do this.

But, in a larger sense, we can not dedicate -- we can not consecrate -- we can not hallow -- this ground. The brave men, living and dead, who struggled here, have consecrated it, far above our poor power to add or detract. The world will little note, nor long remember what we say here, but it can never forget what they did here. It is for us the living, rather, to be dedicated here to the unfinished work which they who fought here have thus far so nobly advanced. It is rather for us to be here dedicated to the great task remaining before us -- that from these honored dead we take increased devotion to that cause for which they gave the last full measure of devotion -- that we here highly resolve that these dead shall not have died in vain -- that this nation, under God, shall have a new birth of freedom -- and that government of the people, by the people, for the people, shall not perish from the earth.

A Christmas Carol

Charles Dickens

Chapman & Hall, 1843

Word count: 260

Marley was dead: to begin with. There is no doubt whatever about that. The register of his burial was signed by the clergyman, the clerk, the undertaker, and the chief mourner. Scrooge signed it: and Scrooge's name was good upon 'Change, for anything he chose to put his hand to. Old Marley was as dead as a door-nail.

Mind! I don't mean to say that I know, of my own knowledge, what there is particularly dead about a door-nail. I might have been inclined, myself, to regard a coffin-nail as the deadest piece of ironmongery in the trade. But the wisdom of our ancestors is in the simile; and my unhallowed hands shall not disturb it, or the Country's done for. You will therefore permit me to repeat, emphatically, that Marley was as dead as a door-nail.

Scrooge knew he was dead? Of course he did. How could it be otherwise? Scrooge and he were partners for I don't know how many years. Scrooge was his sole executor, his sole administrator, his sole assign, his sole residuary legatee, his sole friend, and sole mourner. And even Scrooge was not so dreadfully cut up by the sad event, but that he was an excellent man of business on the very day of the funeral, and solemnised it with an undoubted bargain.

The mention of Marley's funeral brings me back to the point I started from. There is no doubt that Marley was dead. This must be distinctly understood, or nothing wonderful can come of the story I am going to relate.

In comparing the two, was one or the other easier to understand or faster to read? Was it the piece you predicted you would read faster?

It is important to note that in this case both readings were only three paragraphs long. While there may have only been half a minute or so between the reading of each, that amount of time would multiply greatly over an entire chapter.

Knowing Yourself

While you can find all sorts of estimates online as to how long a certain task may take, it is important to know these are only averages. People read at different speeds, people write at different speeds, and those numbers even change for each individual depending on the environment.

If you are trying to read in surroundings that have distractions (e.g., conversations, phone calls, etc.), reading 10 pages can take you a lot longer than if you are reading in a quiet area. By the same token, you may be reading in a quiet environment (e.g., in bed after everyone in the house has gone to sleep), but if you are tired, your attention and retention may not be what it would be if you were refreshed.

In essence, the only way you are going to be able to manage your time accurately is to know yourself and to know how long it takes you to do each task. But where to begin?

Below, you will find a table of common college academic activities. This list has been compiled from a large number of different sources, including colleges, publishers, and professional educators, to help students estimate their own time on tasks. The purpose of this table is to both give you a place to begin in your estimates and to illustrate how different factors can impact the actual time spent.

You will notice that beside each task there is a column for the *unit*, followed by the average *time on task*, and a column for notes. The *unit* is whatever is being measured (e.g., pages read, pages written, etc.), and the *time on task* is an average time it takes students to do these tasks. It is important to pay attention to the notes column, because there you will find factors that influence the time on task. These factors can dramatically change the amount of time the activity takes.

Table 1.5.4 Time on task for common college activities.

Time on Task			
Activity	Unit	Time on task	Notes
General academic reading (textbook, professional journals)	1 page	5–7 minutes	Be aware that your personal reading speed may differ and may change over time.
Technical reading (math, charts and data)	1 page	10–15 minutes	Be aware that your personal reading speed may differ and may change over time.

Time on Task			
Simple Quiz or homework question: short answer—oriented toward recall or identification type answers	Per question	1–2 minutes	Complexity of question will greatly influence the time required.
Complex Quiz or homework question: short answer—oriented toward application, evaluation, or synthesis of knowledge	Per question	2–3 minutes	Complexity of question will greatly influence the time required.
Math problem sets, complex	Per question	15 minutes	For example, algebra, complex equations, financial calculations
Writing: short, no research	Per page	60 minutes	Short essays, single-topic writing assignments, summaries, freewriting assignments, journaling—includes drafting, writing, proofing, and finalizing
Writing: research paper	Per page	105 minutes	Includes research time, drafting, editing, proofing, and finalizing (built into per-page calculation)
Study for quiz	Per chapter	60 minutes	45–90 minutes per chapter, depending upon complexity of material
Study for exam	Per exam	90 minutes	1–2 hours, depending upon complexity of material

Again, these are averages, and it does not mean anything if your times are a little slower or a little faster. There is no “right amount of time,” only the time that it takes you to do something so you can accurately plan and manage your time.

There is also another element to look for in the table. These are differentiations in the similar activities that will also affect the time you spend. A good example of this can be found in the first four rows. Each of these activities involves reading, but you can see that depending on the material being read and its complexity, the time spent can vary greatly. Not only do these differences in time account for the different types of materials you might read (as you found in the comparative reading exercise earlier in this chapter), but also they also take into consideration the time needed to think about what you are reading to truly understand and comprehend what it is saying.

GET CONNECTED

Which apps help you best prepare for success when managing your time?

Do you have trouble keeping track of multiple tasks over the course of a term?

[Trello](#) lets you organize all your obligations in helpful boards. You can share them with others (project collaborators), set alerts as reminders, and mark tasks off as you complete them.

Do you use a particular app to help you manage your time?

Sticky note apps are available for PC, Mac, and mobile devices. They let you post quick reminders, reorganize them as needed, and view them separately or as a full to-do list.

What do you wish you could improve about your time management skills?

[Toggl](#) helps you keep track of how and where you are spending your time so you can budget better and make time management changes that free you up for the really important stuff.

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1.6: Prioritization- Self-Management of What You Do and When You Do It

1. Email Professor Raymond
2. Post in poly sci discussion forum
3. Psych project
4. Kyleigh's bday!!!

Stop at financial aid – fseog

Figure 1.6.1: Numbered lists are useful and easy tools to create.

Questions to consider:

- Why is prioritization important?
- What are the steps involved in prioritization?
- How do I deal with situation where others' priorities are not the same as my own?
- What do I do when priorities conflict?
- What are the best ways to make sure I complete tasks?

Prioritization: Self-Management of What You Do and When You Do It

Another key component in time management is that of prioritization. Prioritization can be thought of as ordering tasks and allotting time for them based on their identified needs or value.

This next section provides some insight into not only helping prioritize tasks and actions based on need and value, but also how to better understand the factors that contribute to prioritization.

How to Prioritize

The enemy of good prioritization is panic, or at least making decisions based on strictly emotional reactions. It can be all too easy to immediately respond to a problem as soon as it pops up without thinking of the consequences of your reaction and how it might impact other priorities. It is very natural for us to want to remove a stressful situation as soon as we can. We want the adverse emotions out of the way as quickly as possible. But when it comes to juggling multiple problems or tasks to complete, prioritizing them first may mean the difference between completing everything satisfactorily and completing nothing at all.

Make Certain You Understand the Requirements of Each Task

One of the best ways to make good decisions about the prioritization of tasks is to understand the requirements of each. If you have multiple assignments to complete and you assume one of those assignments will only take an hour, you may decide to put it off until the others are finished. Your assumption could be disastrous if you find, once you begin the assignment, that there are several extra components that you did not account for and the time to complete will be four times as long as you estimated. Or, one of the assignments may be dependent on the results of another—like participating in a study and then writing a report on the results. If you are not aware that one assignment depends upon the completion of the other before you begin, you could inadvertently do the assignments out of order and have to start over. Because of situations like this, it is critically important to understand exactly what needs to be done to complete a task before you determine its priority.

Make Decisions on Importance, Impact on Other Priorities, and Urgency

After you are aware of the requirements for each task, you can then decide your priorities based on the importance of the task and what things need to be finished in which order.

To summarize: *the key components to prioritization are making certain you understand each task and making decisions based on importance, impact, and urgency.*

ACTIVITY

To better see how things may need to be prioritized, some people make a list of the tasks they need to complete and then arrange them in a quadrant map based on importance and urgency. Traditionally this is called the Eisenhower Decision Matrix. Before becoming the 34th president of the United States, Dwight Eisenhower served as the Allied forces supreme commander during World War II and said he used this technique to better prioritize the things he needed to get done.

In this activity you will begin by making a list of things you need or want to do today and then draw your own version of the grid below. Write each item in one of the four squares; choose the square that best describes it based on its urgency and its importance. When you have completed writing each the tasks in its appropriate square, you will see a prioritization order of your tasks. Obviously, those listed in the Important and Urgent square will be the things you need to finish first. After that will come things that are “important but not urgent,” followed by “not important, but urgent,” and finally “not urgent and not important.”

	Urgent	Not Urgent
Important	Urgent and Important <ul style="list-style-type: none"> • Paper due tomorrow • Apply for internship by deadline 	Not Urgent but Important <ul style="list-style-type: none"> • Exam next week • Flu shot
Not Important	Urgent but Not Important <ul style="list-style-type: none"> • Amazon sale • Laundry 	Not Urgent and Not Important <ul style="list-style-type: none"> • Check social • TV show

Figure 1.6.2: The Eisenhower Matrix can help organize priorities and ensure that you focus on the correct tasks.

Who Is Driving Your Tasks?

Another thing to keep in mind when approaching time management is that while you may have greater autonomy in managing your own time, many of your tasks are being driven by a number of different individuals. These individuals are not only unaware of the other things you need to do, but they often have goals that are in conflict with your other tasks. This means that different instructors, your manager at work, or even your friends may be trying to assert their needs into your priorities. An example of this might be a boss that would like for you to work a few hours of overtime, but you were planning on using that time to do research for a paper.

Just like assessing the requirements and needs for each priority, doing the same with how others may be influencing your available time can be an important part of time management. In some cases, keeping others informed about your priorities may help avert possible conflicts (e.g., letting your boss know you will need time on a certain evening to study, letting your friends know you plan to do a journal project on Saturday but can do something on Sunday, etc.).

It will be important to be aware of how others can drive your priorities and for you to listen to your own good judgment. In essence, time management in college is as much about managing all the elements of your life as it is about managing time for class and to complete assignments.

Making the Tough Decision When It Is Needed

Occasionally, regardless of how much you have planned or how well you have managed your time, events arise where it becomes almost impossible to accomplish everything you need to by the time required. While this is very unfortunate, it simply cannot be helped. As the saying goes, “things happen.”

Finding yourself in this kind of situation is when prioritization becomes most important. You may find yourself in the uncomfortable position of only being able to complete one task or another in the time given. When this occurs with college assignments, the dilemma can be extremely stressful, but it is important to not feel overwhelmed by the anxiety of the situation so that you can make a carefully calculated decision based on the value and impact of your choice.

“What do you do when faced with priority conflicts?”

As an illustration, imagine a situation where you think you can only complete one of two assignments that are both important and urgent, and you must make a choice of which one you will finish and which one you will not. This is when it becomes critical to understand all the factors involved. While it may seem that whichever assignment is worth the most points to your grade is how you make the choice, there are actually a number of other attributes that can influence your decision in order to make the most of a bad situation. For example, one of the assignments may only be worth a minimal number of points toward your total grade, but it may be foundational to the rest of the course. Not finishing it, or finishing it late, may put other future assignments in jeopardy as well. Or the instructor for one of the courses might have a “late assignment” policy that is more forgiving—something that would allow you to turn in the work a little late without too much of a penalty.

If you find yourself in a similar predicament, the first step is to try to find a way to get everything finished, regardless of the challenges. If that simply cannot happen, the next immediate step would be to communicate with your instructors to let them know about the situation. They may be able to help you decide on a course of action, or they may have options you had not thought of. Only then can you make the choices about prioritizing in a tough situation.

The key here is to make certain you are aware of and understand all the ramifications to help make the best decision when the situation dictates you make a hard choice among priorities.

Completing the Tasks

Another important part of time management is to develop approaches that will help you complete tasks in a manner that is efficient and works for you. Most of this comes down to a little planning and being as informed about the specifics of each task as you can be.

Knowing What You Need to Do

As discussed in previous parts of this chapter, many learning activities have multiple components, and sometimes they must occur in a specific order. Additionally, some elements may not only be dependent on the order they are completed, but can also be dependent on how they are completed. To illustrate this we will analyze a task that is usually considered to be a simple one: *attending a class session*. In this analysis we will look at not only what must be accomplished to get the most out of the experience, but also at how each element is dependent upon others and must be done in a specific order. The graphic below shows the interrelationship between the different activities, many of which might not initially seem significant enough to warrant mention, but it becomes obvious that other elements depend upon them when they are listed out this way.

Element or Task Needed for Success	Task it Depends on
Pre-class Prep	
<ul style="list-style-type: none"> • Completing previous homework • Reading appropriate material for lecture • Taking notes on areas that need clarification 	<ul style="list-style-type: none"> • Understanding homework assigned from previous class • Making certain appropriate reading material is identified • Reading appropriate material for lecture
↓	
During Class	
<ul style="list-style-type: none"> • Understanding lecture • Taking notes on lecture • Asking questions for clarification • Taking part in class discussion • Receive assignments for next class 	<ul style="list-style-type: none"> • Reading appropriate material • Understanding lecture • Reading appropriate material, Understanding lecture • Reading appropriate material, Understanding lecture
↓	
Post-Class	
<ul style="list-style-type: none"> • Understanding homework assigned • Making certain appropriate reading material is identified • Ask questions for clarification • Reviewing and rewriting notes 	<ul style="list-style-type: none"> • Receive assignments for next class • Receive assignments for next class • Receive assignments for next class

Figure 1.6.3: Many of your learning activities are dependent on others, and some are the gateways to other steps.

As you can see from the graphic above, even a task as simple as “going to class” can be broken down into a number of different elements that have a good deal of dependency on other tasks. One example of this is preparing for the class lecture by reading materials ahead of time in order to make the lecture and any complex concepts easier to follow. If you did it the other way around, you might miss opportunities to ask questions or receive clarification on the information presented during the lecture.

Understanding what you need to do and when you need to do it can be applied to any task, no matter how simple or how complex. Knowing what you need to do and planning for it can go a long way toward success and preventing unpleasant surprises.

Knowing How You Will Get It Done

After you have a clear understanding of what needs to be done to complete a task (or the component parts of a task), the next step is to create a plan for completing everything.

This may not be as easy or as simple as declaring that you will finish part one, then move on to part two, and so on. Each component may need different resources or skills to complete, and it is in your best interest to identify those ahead of time and include them as part of your plan.

A good analogy for this sort of planning is to think about it in much the same way you would preparing for a lengthy trip. With a long journey you probably would not walk out the front door and then decide how you were going to get where you were going. There are too many other decisions to be made and tasks to be completed around each choice. If you decided you were going by plane, you would need to purchase tickets, and you would have to schedule your trip around flight times. If you decided to go by car, you would need gas money and possibly a map or GPS device. What about clothes? The clothes you will need are dependent on how long will you be gone and what the climate will be like. If it far enough away that you will need to speak another language, you may need to either acquire that skill or at least come with something or someone to help you translate.

What follows is a planning list that can help you think about and prepare for the tasks you are about to begin.

What Resources Will You Need?

The first part of this list may appear to be so obvious that it should go without mention, but it is by far one of the most critical and one of the most overlooked. Have you ever planned a trip but forgotten your most comfortable pair of shoes or neglected to book a hotel room? If a missing resource is important, the entire project can come to a complete halt. Even if the missing resource is a minor component, it may still dramatically alter the end result.

Learning activities are much the same in this way, and it is also important to keep in mind that resources may not be limited to physical objects such as paper or ink. Information can be a critical resource as well. In fact, one of the most often overlooked aspects in planning by new college students is just how much research, reading, and information they will need to complete assignments.



Figure 1.6.4: Allowing time to think is an important part of learning. Credit: Juhan Sonin / Flickr / Attribution 2.0 Generic (CC BY 2.0))

For example, if you had an assignment in which you were supposed to compare and contrast a novel with a film adapted from that novel, it would be important to have access to both the movie and the book as resources. Your plans for completing the work could quickly fall apart if you learned that on the evening you planned to watch the film, it was no longer available.

What Skills Will You Need?

Poor planning or a bad assumption in this area can be disastrous, especially if some part of the task has a steep learning curve. No matter how well you planned the other parts of the project, if there is some skill needed that you do not have and you have no idea how long it will take to learn, it can be a bad situation.

Imagine a scenario where one of your class projects is to create a poster. It is your intent to use some kind of imaging software to produce professional-looking graphics and charts for the poster, but you have never used the software in that way before. It seems easy enough, but once you begin, you find the charts keep printing out in the wrong resolution. You search online for a solution, but the only thing you can find requires you to recreate them all over again in a different setting. Unfortunately, that part of the project will now take twice as long.

It can be extremely difficult to recover from a situation like that, and it could have been prevented by taking the time to learn how to do it correctly before you began or by at least including in your schedule some time to learn and practice.

Set Deadlines

Of course, the best way to approach time management is to set realistic deadlines that take into account which elements are dependent on which others and the order in which they should be completed. Giving yourself two days to write a 20- page work of fiction is not very realistic when even many professional authors average only 6 pages per day. Your intentions may be well founded, but your use of unrealistic deadlines will not be very successful.

Setting appropriate deadlines and sticking to them is very important—so much so that several sections in the rest of this chapter touch on effective deadline practices.

Be Flexible

It is ironic that the item on this list that comes just after a strong encouragement to make deadlines and stick to them is the suggestion to be flexible. The reason that *being flexible* has made this list is because even the best-laid plans and most accurate time management efforts can take an unexpected turn. The idea behind being flexible is to readjust your plans and deadlines when something does happen to throw things off. The worst thing you could do in such a situation is panic or just stop working because the next step in your careful planning has suddenly become a roadblock. The moment when you see that something in your plan may become an issue is when to begin readjusting your plan.

Adjusting a plan along the way is incredibly common. In fact, many professional project managers have learned that it seems something *always* happens or there is always some delay, and they have developed an approach to deal with the inevitable need for some flexibility. In essence, you could say that they are even planning for problems, mistakes, or delays from the very beginning, and they will often add a little extra time for each task to help ensure an issue does not derail the entire project or that the completion of the project does not miss the final due date.

“As you work through tasks, make certain you are always monitoring and adapting to ensure you complete them.”

STUDENT PROFILE

"While in college, I recall an instance where I was awake for two nights in a row trying to cram for upcoming midterms. I quickly learned that trying to navigate through college while working full time posed a significant challenge. Because of inability to manage my responsibilities, my first year of college was quite miserable. I went through a lot of trial and error to find out that *time management* was the key. From my experiences, I have extrapolated three important components to this skill. First, knowing your *values* is imperative. Values will serve as a guide, which will help you to determine which actions bring you closer to your goals and those that don't. Second, know your *constraints*. Constraints (in form of time or other responsibilities) can help you set the parameter within which you can function efficiently. The last component is *action*. This component was the hardest for me to master, but it was the most fruitful. Because knowing values and limitations without engaging in appropriate actions does not serve any meaningful purpose. I strongly believe that learning time management can contribute greatly towards positive university experience."

—Firdavs Khaydarov, Psychology Major, Minnesota State University, Mankato

The Importance of Where You Do Your Work



Figure 1.6.5: Where you do work can be as important as when. (Credit: Mads Bodker / Flickr / Attribution 2.0 Generic (CC BY 2.0))

A large part of ensuring that you can complete tasks on time comes to setting up conditions that will allow you to do the work well. Much of this has to do with the environment where you will do your work. This not only includes physical space such as a work area, but other conditions like being free from distractions and your physical well-being and mental attitude.

The Right Space

Simple things, like where you are set up to do your work, can not only aid in your efficiency but also affect how well you can work or even if you can get the work completed at all. One example of this might be typing on a laptop. While it might seem more comfortable to lie back on a couch and type a long paper, sitting up at a desk or table actually increases your typing speed and reduces the number of mistakes. Even the kind of mouse you use can impact how you work, and using one you are comfortable with can make a big difference.

There are a host of other factors that can come into play as well. Do you have enough space? Is the space cluttered, or do you have the room to keep reference materials and other things you might need within arm's reach? Are there other ways you could work that might be even more efficient? For example, buying an inexpensive second monitor—even secondhand—might be the key to decreasing the amount of time you spend when you can have more than one document displayed at a time.

The key is to find what works for you and to treat your work space as another important resource needed to get the task finished.

Distraction Free

Few things are more frustrating than trying to do work while distractions are going on around you. If other people are continually interrupting you or there are things that keep pulling your attention from the task at hand, everything takes longer and you are more prone to mistakes.⁴

Many people say they work better with distractions—they prefer to leave the television or the radio on—but the truth is that an environment with too many interruptions is rarely helpful when focus is required. Before deciding that the television or talkative roommates do not bother you when you work, take an honest accounting of the work you produce with interruptions compared to work you do without.

If you find that your work is better without distractions, it is a good idea to create an environment that reduces interruptions. This may mean you have to go to a private room, use headphones, or go somewhere like a library to work. Regardless, the importance of a distraction-free environment cannot be emphasized enough.

Working at the Right Time

Most people are subject to their own rhythms, cycles, and preferences throughout their day. Some are alert and energetic in the mornings, while others are considered “night owls” and prefer to work after everyone else has gone to sleep. It can be important to be aware of your own cycles and to use them to your advantage. Rarely does anyone do their best work when they are exhausted, either physically or mentally. Just as it can be difficult to work when you are physically ill, it can also be a hindrance to try to learn or do mental work when you are tired or emotionally upset.

Your working environment definitely includes your own state of mind and physical well-being. Both have a significant influence on your learning and production ability. Because of this, it is not only important to be aware of your own condition and work preferences, but to actually try to create conditions that help you in these areas. One approach is to set aside a specific time to do certain kinds of work. You might find that you concentrate better after you have eaten a meal. If that is the case, make it a habit of doing homework every night after dinner. Or you might enjoy reading more after you are ready for bed, so you do your reading assignments just before you go to sleep at night. Some people find that they are more creative during a certain time of the day or that they are more comfortable writing with subtle lighting. It is worth taking the time to find the conditions that work best for you so that you can take advantage of them.

ANALYSIS QUESTION

Student Survey on Work Environment

Analysis: Take the time to think about where you will do your work and when. What can you do to help ensure your working environment will be helpful rather than harmful? What do you know doesn't work for you? What will you do to prevent those adverse conditions from creeping into your work environment?

Below is a quick survey to help you determine your own preferences in regard to your work space, the time you work, and distractions. Rank each option: 1–4, 1 meaning “least like me” and 4 meaning “most like me.”

- I like my workspace to be organized and clean.
- There are certain places where I am more comfortable when I work.
- I prefer to be alone when I work on certain things.
- I find it difficult to read with other sounds or voices around me.
- There are certain times of the day when I can be more focused.
- My moods or emotions can interfere with my ability to concentrate

Footnotes

- [4 https://en.calameo.com/read/00009178915b8f5b352ba](https://en.calameo.com/read/00009178915b8f5b352ba)

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1.7: Goal Setting and Motivation

Questions to consider:

- How do I set motivational goals?
- What are SMART goals?
- What's the importance of an action plan?
- How do I keep to my plan?

Motivation often means the difference between success and failure. That applies to school, to specific tasks, and to life in general. One of the most effective ways to keep motivated is to set goals.

Goals can be big or small. A goal can range from *I am going to write one extra page tonight*, to *I am going to work to get an A in this course*, all the way to *I am going to graduate in the top of my class so I can start my career with a really good position*. The great thing about goals is that they can include and influence a number of other things that all work toward a much bigger picture. For example, if your goal is to get an A in a certain course, all the reading, studying, and every assignment you do for that course contributes to the larger goal. You have motivation to do each of those things and to do them well.

Setting goals is something that is frequently talked about, but it is often treated as something abstract. Like time management, goal setting is best done with careful thought and planning. This next section will explain how you can apply tested techniques to goal setting and what the benefits of each can be.

Set Goals That Motivate You

The first thing to know about goal setting is that a goal is a specific end result you desire. If the goal is not something you are really interested in, there is little motivational drive to achieve it. Think back to when you were much younger and some well-meaning adult set a goal for you—something that didn't really appeal to you at all. How motivated were you to achieve the goal? More than likely, if you were successful at all in meeting the goal, it was because you were motivated by earning the approval of someone or receiving a possible reward, or you were concerned with avoiding something adverse that might happen if you did not do what you were told. From an honest perspective in that situation, your real goal was based on something else, not the meeting of the goal set for you. To get the most from the goals you set, make sure they are things that you are interested in achieving.

That is not to say you shouldn't set goals that are supported by other motivations (e.g., If I finish studying by Friday, I can go out on Saturday), but the idea is to be intellectually honest with your goals.

Set SMART Goals

Goals should also be SMART. In this case, the word *smart* is not only a clever description of the type of goal, but it is also an acronym that stands for Specific, Measurable, Attainable, Relevant, and Time-bound. The reason these are all desirable traits for your goals is because they not only help you plan how to meet the goal, but they can also contribute to your decision-making processes during the planning stage.

What does it mean to create SMART goals?

- **Specific**—For a goal to be specific, it must be defined enough to actually determine the goal. A goal of *get a good job when I graduate* is too general. It doesn't define what a good job is. In fact, it doesn't even necessarily include a job in your chosen profession. A more specific goal would be something like *be hired as a nurse in a place of employment where it is enjoyable to work and that has room for promotion*.
- **Measurable**—The concept of *measurable* is one that is often overlooked when setting goals. What this means is that the goal should have clearly defined outcomes that are detailed enough to measure and can be used for planning of how you will achieve the goal. For example, setting a goal of *doing well in school* is a bit undefined, but making a goal of *graduating with a GPA above 3.0* is measurable and something you can work with. If your goal is measurable, you can know ahead of time how many points you will have to earn on a specific assignment to stay in that range or how many points you will need to make up in the next assignment if you do not do as well as you planned.
- **Attainable**—*Attainable* or *achievable* goals means they are reasonable and within your ability to accomplish. While a goal of *make an extra one million dollars by the end of the week* is something that would be nice to achieve, the odds that you could make that happen in a single week are not very realistic.

- Relevant—For goal setting, *relevant* means it applies to the situation. In relation to college, a goal of *getting a horse to ride* is not very relevant, but *getting dependable transportation* is something that would contribute to your success in school.
- Time-bound—Time-bound means you set a specific time frame to achieve the goal. *I will get my paper written by Wednesday* is time-bound. You know when you have to meet the goal. *I will get my paper written sometime soon* does not help you plan how and when you will accomplish the goal.

In the following table you can see some examples of goals that do and do not follow the SMART system. As you read each one, think about what elements make them SMART or how you might change those that are not.

Table 1.7.1

Goal	Is it SMART?	
I am going to be rich someday.	No	There is nothing really specific, measurable, or time-bound in this goal.
I will graduate with my degree, on time.	Yes	The statement calls out specific, measurable, and time-bound details. The other attributes of attainable and relevant are implied.
I am going to save enough money to buy a newer car by June.	Yes	All SMART attributes are covered in this goal.
I would like to do well in all my courses next semester.	No	While this is clearly time-bound and meets most of the SMART goal attributes, it is not specific or measurable without defining what “do well” means.
I am going to start being a nicer person.	No	While most of the SMART attributes are implied, there is nothing really measurable in this goal.
I will earn at least a 3.0 GPA in all my courses next semester.	Yes	All of the SMART attributes are present in this goal.
I am going to start being more organized.	No	While most of the SMART attributes are implied, there is nothing really measurable in this goal.

APPLICATION

Try writing two SMART goals—something with a one-week time frame and something that you will accomplish over the next year. Make certain that you include all the appropriate elements—Specific, Measurable, Attainable, Relevant, and Time-bound.

Make an Action Plan

Like anything else, making a step-by-step action plan of how you will attain your goals is the best way to make certain you achieve them. It doesn’t matter if it is a smaller goal with immediate results (e.g., finish all your homework due by Friday) or something bigger that takes several years to accomplish (graduate with my degree in the proper amount of time).

The planning techniques you use for time management and achieving goals can be similar. In fact, accurate goal setting is very much a part of time management if you treat the completion of each task as a goal.

What follows is an example of a simple action plan that lists the steps for writing a short paper. You can use something like this or modify it in a way that would better suit your own preferences.

Table 1.7.2

Action Plan		
Task	Objective	When

Action Plan		
Choose topic.	Select something interesting.	Needs to be done by Monday!
Write outline, look for references.	Create structure of paper and outline each part.	Monday, 6:00 p.m.
Research references to support outline, look for good quotes.	Strengthen paper and resources.	Tuesday, 6:00 p.m.
Write paper introduction and first page draft.	Get main ideas and thesis statement down.	Wednesday, 7:00 p.m.
Write second page and closing draft.	Finish main content and tie it all together.	Thursday, 6:00 p.m.
Rewrite and polish final draft.	Clean up for grammar, writing style, and effective communication.	Friday, 5:00 p.m.

Another useful approach to goal setting is to create SMART goals and then write them down. For most people there is a higher level of commitment when we write something down. If you have your goals written out, you can refer to each component of the SMART acronym and make certain you are on track to achieve it.

Stick with It!

As with anything else, the key to reaching goals is to keep at it, keep yourself motivated, and overcome any obstacles along the way. In the following graphic you will find seven methods that highly successful people use to accomplish this.



Figure 1.7.3: These seven ways to stay motivated are good suggestions from highly successful people. What other strategies would you suggest?

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1.8: Enhanced Strategies for Time and Task Management

Questions to consider:

- What strategy helps me prioritize my top tasks?
- How do I make the best use of my time when prioritizing?
- How do I make sure I tackle unpleasant tasks instead of putting them off?
- What's the best way to plan for long-term tasks?
- How do I find time in a busy schedule?

Over the years, people have developed a number of different strategies to manage time and tasks. Some of the strategies have proven to be effective and helpful, while others have been deemed not as useful.

The good news is that the approaches that do not work very well or do not really help in managing time do not get passed along very often. But others, those which people find of value, do. What follows here are three unique strategies that have become staples of time management. While not everyone will find that all three work for them in every situation, enough people have found them beneficial to pass them along with high recommendations.

Daily Top Three

The idea behind the *daily top three* approach is that you determine which three things are the most important to finish that day, and these become the tasks that you complete. It is a very simple technique that is effective because each day you are finishing tasks and removing them from your list. Even if you took one day off a week and completed no tasks on that particular day, a *daily top three* strategy would have you finishing 18 tasks in the course of a single week. That is a good amount of things crossed off your list.

analysis question

Analysis: Think about what would be your top three tasks for today? What would you have on the list tomorrow?

Pomodoro Technique



Figure 1.8.1: The Pomodoro Technique is named after a type of kitchen timer, but you can use any clock or countdown timer. (Marco Verch /Flickr / Attribution 2.0 Generic (CC BY 2.0))

The Pomodoro Technique was developed by Francesco Cirillo. The basic concept is to use a timer to set work intervals that are followed by a short break. The intervals are usually about 25 minutes long and are called *pomodoros*, which comes from the Italian word for tomato because Cirillo used a tomato-shaped kitchen timer to keep track of the intervals.

In the original technique there are six steps:

1. Decide on the task to be done.
2. Set the timer to the desired interval.
3. Work on the task.
4. When the timer goes off, put a check mark on a piece of paper.
5. If you have fewer than four check marks, take a short break (3–5 minutes), then go to Step 1 or 2 (whichever is appropriate).
6. After four pomodoros, take a longer break (15–30 minutes), reset your check mark count to zero, and then go to Step 1 or 2.

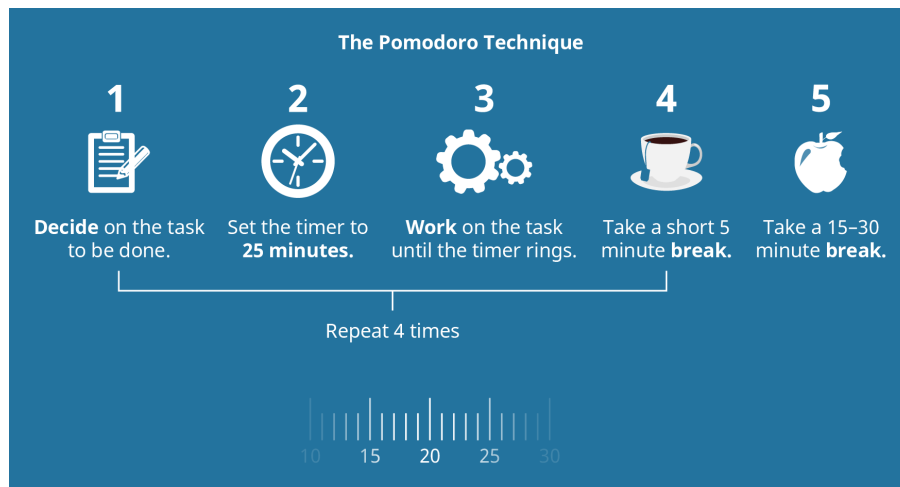


Figure 1.8.2: The Pomodoro Technique contains five defined steps.

There are several reasons this technique is deemed effective for many people. One is the benefit that is derived from quick cycles of work and short breaks. This helps reduce mental fatigue and the lack of productivity caused by it. Another is that it tends to encourage practitioners to break tasks down to things that can be completed in about 25 minutes, which is something that is usually manageable from the perspective of time available. It is much easier to squeeze in three 25-minute sessions of work time during the day than it is to set aside a 75- minute block of time.

Eat the Frog

Of our three quick strategies, *eat the frog* probably has the strangest name and may not sound the most inviting. The name comes from a famous quote, attributed to Mark Twain: “Eat a live frog first thing in the morning and nothing worse will happen to you the rest of the day.” *Eat the Frog* is also the title of a best-selling book by Brian Tracy that deals with time management and avoiding procrastination.

How this applies to time and task management is based on the concept that if a person takes care of the biggest or most unpleasant task first, everything else will be easier after that.

Although stated in a humorous way, there is a good deal of truth in this. First, we greatly underestimate how much worry can impact our performance. If you are continually distracted by anxiety over a task you are dreading, it can affect the task you are working on at the time. Second, not only will you have a sense of accomplishment and relief when the task you are concerned with is finished and out of the way, but other tasks will seem lighter and not as difficult.

Application: Try Three Time Management Strategies

Over the next two weeks, try each of these three methods to see which ones might work for you. Is there one you favor over the others? Might each of these three approaches serve you better in different situations or with different tasks? Do you have a creative alternative or possibly a way to use some combination of these techniques?

In addition to these three strategies, you could also develop whole new approaches from suggestions found earlier in this chapter. For example, you could apply some of the strategies for avoiding procrastination or for setting appropriate priorities and see how they work in combination with these techniques or on their own.

The key is to find which system works best for you.

Breaking Down the Steps and Spreading Them over Shorter Work Periods

Above, you read about several different tried-and-tested strategies for effective time management—approaches that have become staples in the professional world. In this section you will read about two more creative techniques that combine elements from these other methods to handle tasks when time is scarce and long periods of time are a luxury you just do not have.

The concept behind this strategy is to break tasks into smaller, more manageable units that do not require as much time to complete. As an illustration of how this might work, imagine that you are assigned a two-page paper that is to include references. You estimate that to complete the paper—start to finish—would take you between four and a half and five hours. You look at your

calendar over the next week and see that there simply are no open five-hour blocks (unless you decided to only get three hours of sleep one night). Rightly so, you decide that going without sleep is not a good option. While looking at your calendar, you do see that you can squeeze in an hour or so every night. Instead of trying to write the entire paper in one sitting, you break it up into much smaller components as shown in the table below:

Table 1.8.3: Breaking Down Projects into Manageable-Sized Tasks

Day/Time	Task	Time
Monday, 6:00 p.m.	Write outline; look for references.	60 minutes
Tuesday, 6:00 p.m.	Research references to support outline; look for good quotes.	60 minutes
Wednesday, 7:00 p.m.	Write paper introduction and first page draft.	60 minutes
Thursday, 6:00 p.m.	Write second page and closing draft.	60 minutes
Friday, 5:00 p.m.	Rewrite and polish final draft.	60 minutes
Saturday, 10:00 a.m.	<i>Only if needed—finish or polish final draft.</i>	60 minutes?

While this is a simple example, you can see how it would redistribute tasks to fit your available time in a way that would make completing the paper possible. In fact, if your time constraints were even more rigid, it would be possible to break these divided tasks down even further. You could use a variation of the Pomodoro Technique and write for three 20-minute segments each day at different times. The key is to look for ways to break down the entire task into smaller steps and spread them out to fit your schedule.

Table 1.8.4

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
8:00–10:00		Work		Work			
10:00–12:00	Algebra	Work	Algebra	Work	Algebra	10 a.m.–11 a.m. <i>Only if needed</i>	Work
12:00–2:00	Lunch/study	1 p.m. English Comp	Lunch/study	1 p.m. English Comp	Lunch/study	Family picnic	Work
2:00–4:00	History	English Comp	History	English Comp	History	Family picnic	
4:00–6:00	Study for Algebra quiz.	Grocery	Study for History exam.	Study for History exam.	5 p.m.–6 p.m. Rewrite and polish final draft.	Family picnic	Laundry
6:00–7:00	Write outline; look for references.	Research references to support outline; look for good quotes.	Research presentation project.	Write second page and closing draft	Create presentation.	Meet with Darcy.	Prepare school stuff for next week.
7:00–8:00	Free time	Free time	Write paper introduction and first page draft.	Research presentation project.	Create presentation.		Free time

STUDENT PROFILE

"Time management is probably one of the hardest things I had to pick up when I got to college. For starters, I didn't have anyone to come wake me up if I forgot to set an alarm or to tell me to get out of bed so that I wouldn't be late. I had to start placing my phone far away from my bed; so that way, I would have to get out of bed in order to turn the alarm off. Accomplishing work on time can also be difficult. It's tough to find the fine balance between when you have to stay in and work on assignments and when is acceptable to go out and do leisure activities.

"I learned the 8-8-8 rule. Every day you spend eight hours working on school work or going to class, eight hours of free time to do what you want, and then eight hours to sleep at night so that you will get enough rest. Sleep is crucial for time management. I learned very quickly that you cannot focus or be productive if you are struggling to keep your head from falling over because you are so tired. Basically, I've learned that if you want to be successful in college, then you have to be on top of your game when it comes to time. It's something thing you cannot make up once it's gone."

—Preston Allen, University of Central Arkansas

Analyzing Your Schedule and Creating Time to Work

Of all the strategies covered in this chapter, this one may require the most discipline, but it can also be the most beneficial in time management. The fact is most of us waste time throughout the day. Some of it is due to a lack of awareness, but it can also be caused by the constraints of our current schedules. An example of this is when we have 15 to 20 minutes before we must leave to go somewhere. We don't do anything with that time because we are focused on leaving or where we are going, and we might not be organized enough to accomplish something in that short of a time period. In fact, a good deal of our 24-hour days are spent a few minutes at a time waiting for the next thing scheduled to occur. These small units of time add up to a fair amount each day.

The intent of this strategy is to recapture those lost moments and use them to your advantage. This may take careful observation and consideration on your part, but the results of using this as a method of time management are more than worth it.

The first step is to look for those periods of time that are wasted or that can be repurposed. In order to identify them, you will need to pay attention to what you do throughout the day and how much time you spend doing it. The example of waiting for the next thing in your schedule has already been given, but there are many others. How much time do you spend in activities after you have really finished doing them but are still lingering because you have not begun to do something else (e.g., letting the next episode play while binge-watching, reading social media posts or waiting for someone to reply, surfing the Internet, etc.)? You might be surprised to learn how much time you use up each day by just adding a few unproductive minutes here and there.

If you set a limit on how much time you spend on each activity, you might find that you can recapture time to do other things. An example of this would be limiting yourself to reading news for 30 minutes. Instead of reading the main things that interest you and then spending an additional amount of time just looking at things that you are only casually interested in because that is what you are doing at the moment, you could stop after a certain allotted period and use the extra time you have gained on something else.

After you identify periods of lost time, the next step will be to envision how you might restructure your activities to bring those extra minutes together into useful blocks of time. Using the following scenario as an illustration, we will see how this could be accomplished.



Figure 1.8.5: Sarah has to balance a lot of obligations.

On Tuesday nights, Sarah has a routine: After work, she does her shopping for the week (2 hours driving and shopping) and then prepares and eats dinner (1 hour). After dinner, she spends time on homework (1 hour) and catching up with friends, reading the news, and other Internet activities (1 hour), and then she watches television or reads before going to bed (1 hour). While it may seem that there is very little room for improvement in her schedule without cutting out something she enjoys, limiting the amount of time she spends on each activity and rethinking how she goes about each task can make a significant difference.

In this story, Sarah's Tuesday-night routine includes coming home from work, taking stock of which items in her home she might need to purchase, and then driving to the store. While at the store, she spends time picking out and selecting groceries as she plans for meals she will eat during the rest of the week. Then, after making her purchases, she drives home. Instead, if she took the time to make a list and plan for what she needed at the store before she arrived, she would not spend as much time looking for inspiration in each aisle. Also, if she had a prepared list, not only could she quickly pick up each item, but she could stop at the store on the way home from work, thus cutting out the extra travel time. If purchasing what she needed took 30 minutes less because she was more organized and she cut out an additional 20 minutes of travel time by saving the extra trip to the store from her house, she could recapture a significant amount of her Tuesday evening. If she then limited the time she spent catching up with friends and such to 30 minutes or maybe did some of that while she prepared dinner, she would find that she had added almost an extra hour and a half to the time available to her on that evening, without cutting out anything she needed to do or enjoys. If she decided to spend her time on study or homework, this would more than double the time she previously had available in her schedule for homework.

Analysis Question: Reflection

Analysis: Identify areas in the way you spend your day where you may be able to recapture and repurpose time. Are there things you can move around to gain more time? Are there ways you can combine tasks or reduce travel time?

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1.9: Summary

This chapter began by pointing out the dangers of poor time management, both in cost and even the potential risk to graduation. After presenting why time management is important, sections of the text covered how time management for college can be different from what students may have experienced before. Following this, the chapter contained several sections on how to effectively manage time (including predicting time on task), how to use technology to your advantage, and how to prioritize tasks. Other topics included goal setting and motivation, some specific strategies for time and task management, and avoiding procrastination.

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1.10: Career Connection

Rick says: I've wanted to work in radio since I was in high school and had great opportunities in college to learn at the campus station. I interned for a semester at a local Top 40 station and, after graduation, was offered a position as the producer of the station's morning show.

The only problem: I had to be at the radio station by 4:45 a.m. I couldn't do it. I tried everything—alarms on my phone, clock radio alarms, friends calling me. This is not a job you can be late for—dead air is a radio DJ's greatest nightmare. But no matter what I tried, I could not wake up on time. The third time I arrived late, the radio station let me go.

Reflection question: How might you have handled the situation differently? How might this aspiring radio DJ have managed his time differently to ensure he was not late for work?

For discussion: Is the Internet responsible for most of our wasted work time? Read through this article. What do you think?

<https://openstax.org/l/whowastestime>

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1.11: Rethinking

Revisit the questions you answered at the beginning of the chapter, and consider one option you learned in this chapter that might change your answer to one of them.

1. I regularly procrastinate completing tasks that don't interest me or seem challenging.
 2. I use specific time management strategies to complete tasks.
 3. I find it difficult to prioritize tasks because I am not sure what is really important.
 4. I am pleased with my ability to manage my time.
-

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1.12: Where do you go from here?

Refining your time management skills based on an honest assessment is something that should never stop. The benefits of good time management skills are something that will apply to the rest of your life. What would you like to learn more about? Choose a topic from the list below, and create an annotated bibliography that would direct further research.

- Psychological reasons for procrastinating
 - Technology and social media as distractions
 - Additional time management strategies
 - Time management strategies that successful people use
-

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CHAPTER OVERVIEW

2: Reading and Notetaking

- [2.1: Prelude to Reading and Notetaking](#)
- [2.2: The Nature and Types of Reading](#)
- [2.3: Effective Reading Strategies](#)
- [2.4: Taking Notes](#)
- [2.5: Summary](#)
- [2.6: Career Connection](#)
- [2.7: Rethinking](#)
- [2.8: Where do you go from here?](#)

Thumbnail: pixabay.com/photos/startup-meeting-brainstorming-594090/

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2.1: Prelude to Reading and Notetaking



Figure 2.1.1: Each of us reads and records information in our own way.

Student Survey

These questions will help you determine how the chapter concepts relate to you right now. As we are introduced to new concepts and practices, it can be informative to reflect on how your understanding changes over time. We'll revisit these questions at the end of the chapter to see whether your feelings have changed.

On a scale of 1 (I need significant improvement) to 4 (I'm doing great), reflect on how you're doing right now on these statements:

1. I am reading on a college level.
2. I take good notes that help me study for exams.
3. I understand how to manage all the reading I need to do for college.
4. I recognize the need for different notetaking strategies for different college subjects.

You can also take the [Chapter survey](#) anonymously online.

STUDENT PROFILE

"Before I came to college, I always loathed reading from the textbook, taking notes during class, and even listening to lectures. I've since learned that in most cases I should do what my teacher suggests. I have a course that requires me to read two textbook chapters each week. Taking notes on the chapters is optional, making it easy to brush off these assignments. But there are reasons that professors tell students to read and do other classwork. They believe it is valuable information for a student to learn. Note taking in class may become tedious and, in some cases, feel redundant. But you can't recall a whole class from memory. There is not much time to learn the contents of a class in one semester, and it can feel overwhelming. It's important to take notes because writing them helps you remember."

—**Christopher Naldini**, Westchester Community College

About this Chapter

In this chapter we will explore two skills you probably think you already understand—reading and notetaking. But the goal is to make sure you've honed these skills well enough to lead you to success in college. By the time you finish this chapter, you should be able to do the following:

- Discuss the way reading differs in college and how to successfully adapt to that change.
- Demonstrate the usefulness of strong notetaking for college students.

Reading and consuming information are increasingly important today because of the amount of information we encounter. Not only do we need to read critically and carefully, but we also need to read with an eye to distinguishing fact from opinion and identifying

solid sources. Reading helps us make sense of the world—from simple reminders to pick up milk to complex treatises on global concerns, we read to comprehend, and in so doing, our brains expand. An interesting study from Emory University in Atlanta, Georgia, used MRI scans to track the brain conductivity while participants were reading. The researchers assert that a biological change to your brain actually happens when you read, and it lingers. If you want to read the study, published in the journal *Brain Connectivity*, you can find it online at <https://openstax.org/1/brainconnectivity>.

In academic settings, as we deliberately work to become stronger readers and better notetakers, we are both helping our current situation and enhancing our abilities to be successful in the future. Seems like a win-win. Take advantage of all the study aids you have at hand, including human, electronic, and physical resources, to increase your performance in these crucial skill sets.

Why? You need to read. It improves your thinking, your vocabulary, and your ability to make connections between disparate parts, which are all parts of critical thinking. Educational researchers Anne Cunningham and Keith Stanovich discovered after extensive study with college students that “reading volume [how much you read] made a significant contribution to multiple measures of vocabulary, general knowledge, spelling, and verbal fluency.”

Research continues to assess and support the fact that one of the most significant learning skills necessary for success in any field is reading. You may have performed this skill for decades already, but learning to do it more effectively and practicing the skill consistently is critical to how well you do in all subjects. If reading *isn't your thing*, strive to make that your challenge. Your academic journey, your personal well-being, and your professional endeavors will all benefit from your reading. Put forth the effort and make it your thing. The long-term benefits will far outweigh the sacrifices you make now.

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2.2: The Nature and Types of Reading

Questions to consider

- What are the pros and cons of online reading?
- How can distinguishing between reading types help you academically and personally?
- How can you best prepare to read for college?

Research supports the idea that reading is good for you. Students who read at or above reading level throughout elementary and secondary school have a higher chance of starting—and more importantly, finishing—college. Educational researchers convincingly claim that reading improves everything from grades to vocabulary (Cunningham 2).

If you don't particularly enjoy reading, don't despair. We read for a variety of reasons, and you may just have to step back and take a bigger picture of your reading habits to understand why you avoid engaging in this important skill. The myriad distractions we now face as well as the intense information overload we can suffer on a daily basis in all aspects of our lives can combine to make it difficult to slow down to read, an activity that demands at least a modicum of attention in a way that most television and music do not. You may need to adjust your schedule for more reading time, especially in college, because every class you take will expect you to read more pages than you probably have in the past.

Types of Reading

We may read small items purely for immediate information, such as notes, e-mails, or directions to an unfamiliar location. You can find all sorts of information online about how to fix a faucet or tie a secure knot. You won't have to spend too much time reading these sorts of texts because you have a specific goal in mind for them, and once you have accomplished that goal, you do not need to prolong the reading experience. These encounters with texts may not be memorable or stunning, but they don't need to be. When we consider why we read longer pieces—outside of reading for pleasure—we can usually categorize the reasons into about two categories: 1) reading to introduce ourselves to new content, and 2) reading to more fully comprehend familiar content.



Figure 2.2.1: A bookstore or library can be a great place to explore. Aside from books and resources you need, you may find something that interests you or helps with your course work.

Reading to Introduce New Content

Glenn felt uncomfortable talking with his new roommates because he realized very quickly that he didn't know anything about their major—architecture. Of course he knew that it had something to do with buildings and construction sites, but the field was so different from his discipline of biology that he decided he needed to find out more so he could at least engage in friendly conversation with his roommates. Since he would likely not go into their field, he didn't need to go into full research mode. When we read to introduce new content, we can start off small and increase to better and more sophisticated sources. Much of our further study and reading depends on the sources we originally read, our purpose for finding out about this new topic, and our interest level.

Chances are, you have done this sort of exploratory reading before. You may read reviews of a new restaurant or look at what people say about a movie you aren't sure you want to spend the money to see at the theater. This reading helps you decide. In academic settings, much of what you read in your courses may be relatively new content to you. You may have heard the word *volcano* and have a general notion of what it means, but until you study geology and other sciences in depth, you may not have a full understanding of the environmental origins, ecological impacts, and societal and historic responses to volcanoes. These perspectives will come from reading and digesting various material. When you are working with new content, you may need to schedule more time for reading and comprehending the information because you may need to look up unfamiliar terminology and you may have to stop more frequently to make sure you are truly grasping what the material means. When you have few ways to connect new material to your own prior knowledge, you have to work more diligently to comprehend it.

Application

Try an experiment with a group of classmates. Without looking on the Internet, try to brainstorm a list of 10 topics about which all of you may be interested but for which you know very little or nothing at all. Try to make the topics somewhat obscure rather than ordinary—for example, the possibility of the non-planet Pluto being reclassified again as opposed to something like why we need to drink water.

After you have this random list, think of ways you could find information to read about these weird topics. Our short answer is always: Google. But think of other ways as well. How else could you read about these topics if you don't know anything about them? You may well be in a similar circumstance in some of your college classes, so you should listen carefully to your classmates on this one. Think beyond pat answers such as "I'd go to the library," and press for what that researcher would do once at the library. What types of articles or books would you try to find? One reason that you should not always ignore the idea of doing research at the physical library is because once you are there and looking for information, you have a vast number of other sources readily available to you in a highly organized location. You also can tap into the human resources represented by the research librarians who likely can redirect you if you cannot find appropriate sources.

Reading to Comprehend Familiar Content

Reading about unfamiliar content is one thing, but what if you do know something about a topic already? Do you really still need to keep reading about it? Probably. For example, what if during the brainstorming activity in the previous section, you secretly felt rather smug because you know about the demotion of the one-time planet Pluto and that there is currently quite the scientific debate going on about that whole de-planet-ation thing. Of course, you didn't say anything during the study session, mostly to spare your classmates any embarrassment, but you are pretty familiar with Pluto-gate. So now what? Can you learn anything new?

Again—probably. When did Pluto's qualifications to be considered a planet come into question? What are the qualifications for being considered a planet? Why? Who even gets to decide these things? Why was it called *Pluto* in the first place? On Amazon alone, you can find hundreds of books about the once-planet Pluto (not to be confused with the Disney dog also named Pluto). A Google search brings up over 34 million options for your reading pleasure. You'll have plenty to read, even if you do know something or quite a bit about a topic, but you'll approach reading about a familiar topic and an unfamiliar one differently.

With familiar content, you can do some initial skimming to determine what you already know in the book or article, and mark what may be new information or a different perspective. You may not have to give your full attention to the information you know, but you will spend more time on the new viewpoints so you can determine how this new data meshes with what you already know. Is this writer claiming a radical new definition for the topic or an entirely opposite way to consider the subject matter, connecting it to other topics or disciplines in ways you have never considered?

When college students encounter material in a discipline-specific context and have some familiarity with the topic, they sometimes can allow themselves to become a bit overconfident about their knowledge level. Just because a student may have read an article or two or may have seen a TV documentary on a subject such as the criminal mind, that does not make them an expert. What makes an expert is a person who thoroughly studies a subject, usually for years, and understands all the possible perspectives of a subject as well as the potential for misunderstanding due to personal biases and the availability of false information about the topic.

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2.3: Effective Reading Strategies

Questions to consider

- What methods can you incorporate into your routine to allow adequate time for reading?
- What are the benefits and approaches to active reading?
- Do your courses or major have specific reading requirements?

Allowing Adequate Time for Reading

You should determine the reading requirements and expectations for every class very early in the semester. You also need to understand why you are reading the particular text you are assigned. Do you need to read closely for minute details that determine cause and effect? Or is your instructor asking you to skim several sources so you become more familiar with the topic? Knowing this reasoning will help you decide your timing, what notes to take, and how best to undertake the reading assignment.



Figure 2.3.1: If you plan to make time for reading while you commute, remember that unexpected events like delays and cancellations could impact your concentration.

Depending on the makeup of your schedule, you may end up reading both primary sources—such as legal documents, historic letters, or diaries—as well as textbooks, articles, and secondary sources, such as summaries or argumentative essays that use primary sources to stake a claim. You may also need to read current journalistic texts to stay current in local or global affairs. A realistic approach to scheduling your time to allow you to read and review all the reading you have for the semester will help you accomplish what can sometimes seem like an overwhelming task.

When you allow adequate time in your hectic schedule for reading, you are investing in your own success. Reading isn't a magic pill, but it may seem like it when you consider all the benefits people reap from this ordinary practice. Famous successful people throughout history have been voracious readers. In fact, former U.S. president Harry Truman once said, "Not all readers are leaders, but all leaders are readers." Writer of the U.S. Declaration of Independence, inventor, and also former U.S. president Thomas Jefferson claimed "I cannot live without books" at a time when keeping and reading books was an expensive pastime. Knowing what it meant to be kept from the joys of reading, 19th-century abolitionist Frederick Douglass said, "Once you learn to read, you will be forever free." And finally, George R. R. Martin, the prolific author of the wildly successful *Game of Thrones* empire, declared, "A reader lives a thousand lives before he dies . . . The man who never reads lives only one."

You can make time for reading in a number of ways that include determining your usual reading pace and speed, scheduling active reading sessions, and practicing recursive reading strategies.

Determining Reading Speed and Pacing

To determine your reading speed, select a section of text—passages in a textbook or pages in a novel. Time yourself reading that material for exactly 5 minutes, and note how much reading you accomplished in those 5 minutes. Multiply the amount of reading you accomplished in 5 minutes by 12 to determine your average reading pace (5 times 12 equals the 60 minutes of an hour). Of

course, your reading pace will be different and take longer if you are taking notes while you read, but this calculation of reading pace gives you a good way to estimate your reading speed that you can adapt to other forms of reading.

Example Reading Times			
Reader	Pages Read in 5 Minutes	Pages per Hour	Approximate Hours to Read 500 Pages
Marta	4	48	10 hours, 30 minutes
Jordi	3	36	13 hours
Estevan	5	60	8 hours, 20 minutes

So, for instance, if Marta was able to read 4 pages of a dense novel for her English class in 5 minutes, she should be able to read about 48 pages in one hour. Knowing this, Marta can accurately determine how much time she needs to devote to finishing the novel within a set amount of time, instead of just guessing. If the novel Marta is reading is 497 pages, then Marta would take the total page count (497) and divide that by her hourly reading rate (48 pages/hour) to determine that she needs about 10 to 11 hours overall. To finish the novel spread out over two weeks, Marta needs to read a little under an hour a day to accomplish this goal.

Calculating your reading rate in this manner does not take into account days where you're too distracted and you have to reread passages or days when you just aren't in the mood to read. And your reading rate will likely vary depending on how dense the content you're reading is (e.g., a complex textbook vs. a comic book). Your pace may slow down somewhat if you are not very interested in what the text is about. What this method *will* help you do is be realistic about your reading time as opposed to waging a guess based on nothing and then becoming worried when you have far more reading to finish than the time available.

Chapter 3, "Time Management and Prioritization," offers more detail on how best to determine your speed from one type of reading to the next so you are better able to schedule your reading.

Scheduling Set Times for Active Reading

Active reading takes longer than reading through passages without stopping. You may not need to read your latest sci-fi series actively while you're lounging on the beach, but many other reading situations demand more attention from you. Active reading is particularly important for college courses. You are a scholar actively engaging with the text by posing questions, seeking answers, and clarifying any confusing elements. Plan to spend at least twice as long to read actively than to read passages without taking notes or otherwise marking select elements of the text.

To determine the time you need for active reading, use the same calculations you use to determine your traditional reading speed and double it. Remember that you need to determine your reading pace for all the classes you have in a particular semester and multiply your speed by the number of classes you have that require different types of reading.

Example Active Reading Times				
Reader	Pages Read in 5 Minutes	Pages per Hour	Approximate Hours to Read 500 Pages	Approximate Hours to Actively Read 500 Pages
Marta	4	48	10 hours, 30 minutes	21 hours
Jordi	3	36	13 hours	26 hours
Estevan	5	60	8 hours, 20 minutes	16 hours, 40 minutes

Practicing Recursive Reading Strategies

One fact about reading for college courses that may become frustrating is that, in a way, it never ends. For all the reading you do, you end up doing even more rereading. It may be the same content, but you may be reading the passage more than once to detect the emphasis the writer places on one aspect of the topic or how frequently the writer dismisses a significant counterargument. This rereading is called recursive reading.

For most of what you read at the college level, you are trying to make sense of the text for a specific purpose—not just because the topic interests or entertains you. You need your full attention to decipher everything that's going on in complex reading material—

and you even need to be considering what the writer of the piece may *not* be including and why. This is why reading for comprehension is recursive.

Specifically, this boils down to seeing reading not as a formula but as a process that is far more circular than linear. You may read a selection from beginning to end, which is an excellent starting point, but for comprehension, you'll need to go back and reread passages to determine meaning and make connections between the reading and the bigger learning environment that led you to the selection—that may be a single course or a program in your college, or it may be the larger discipline, such as all biologists or the community of scholars studying beach erosion.

People often say writing is rewriting. For college courses, reading is rereading.

Strong readers engage in numerous steps, sometimes combining more than one step simultaneously, but knowing the steps nonetheless. They include, not always in this order:

- bringing any prior knowledge about the topic to the reading session,
- asking yourself pertinent questions, both orally and in writing, about the content you are reading,
- inferring and/or implying information from what you read,
- learning unfamiliar discipline-specific terms,
- evaluating what you are reading, and eventually,
- applying what you're reading to other learning and life situations you encounter.

Let's break these steps into manageable chunks, because you are actually doing quite a lot when you read.

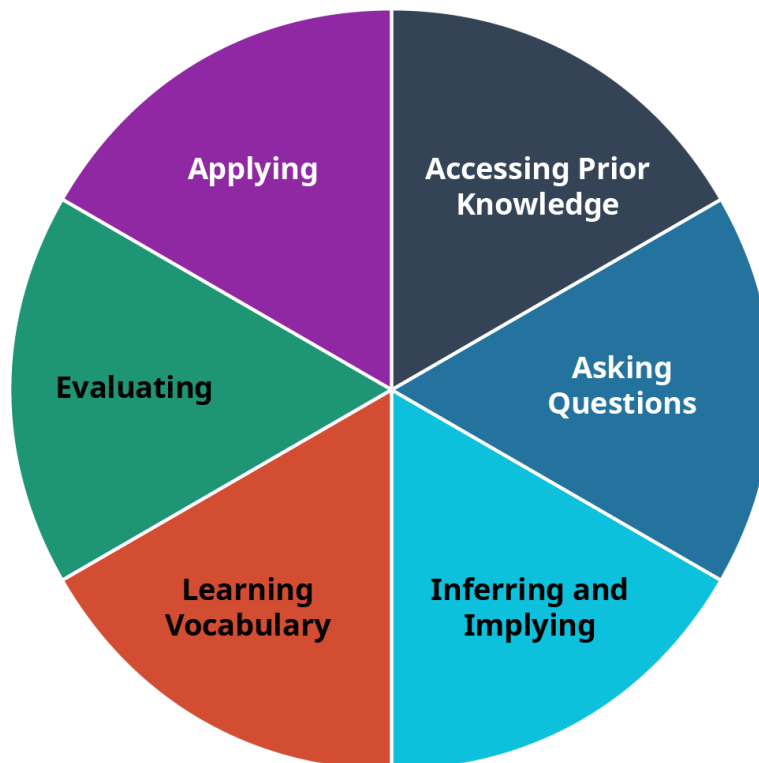


Figure 2.3.2: The six elements of recursive reading should be considered as a circular, not linear, process.

Asking Questions

Humans are naturally curious beings. As you read actively, you should be asking questions about the topic you are reading. Don't just say the questions in your mind; write them down. You may ask: Why is this topic important? What is the relevance of this topic currently? Was this topic important a long time ago but irrelevant now? Why did my professor assign this reading?

You need a place where you can actually write down these questions; a separate page in your notes is a good place to begin. If you are taking notes on your computer, start a new document and write down the questions. Leave some room to answer the questions when you begin and again after you read.

Inferring and Implying

When you read, you can take the information on the page and *infer*, or conclude responses to related challenges from evidence or from your own reasoning. A student will likely be able to infer what material the professor will include on an exam by taking good notes throughout the classes leading up to the test.

Writers may *imply* information without directly stating a fact for a variety of reasons. Sometimes a writer may not want to come out explicitly and state a bias, but may imply or hint at his or her preference for one political party or another. You have to read carefully to find implications because they are indirect, but watching for them will help you comprehend the whole meaning of a passage.

Learning Vocabulary

Vocabulary specific to certain disciplines helps practitioners in that field engage and communicate with each other. Few people beyond undertakers and archeologists likely use the term *sarcophagus* in everyday communications, but for those disciplines, it is a meaningful distinction. Looking at the example, you can use context clues to figure out the meaning of the term *sarcophagus* because it is something undertakers and/or archeologists would recognize. At the very least, you can guess that it has something to do with death. As a potential professional in the field you're studying, you need to know the lingo. You may already have a system in place to learn discipline-specific vocabulary, so use what you know works for you. Two strong strategies are to look up words in a dictionary (online or hard copy) to ensure you have the exact meaning for your discipline and to keep a dedicated list of words you see often in your reading. You can list the words with a short definition so you have a quick reference guide to help you learn the vocabulary.

Evaluating

Intelligent people always question and evaluate. This doesn't mean they don't trust others; they just need verification of facts to understand a topic well. It doesn't make sense to learn incomplete or incorrect information about a subject just because you didn't take the time to evaluate all the sources at your disposal. When early explorers were afraid to sail the world for fear of falling off the edge, they weren't stupid; they just didn't have all the necessary data to evaluate the situation.

When you evaluate a text, you are seeking to understand the presented topic. Depending on how long the text is, you will perform a number of steps and repeat many of these steps to evaluate all the elements the author presents. When you evaluate a text, you need to do the following:

- Scan the title and all headings.
- Read through the entire passage fully.
- Question what main point the author is making.
- Decide who the audience is.
- Identify what evidence/support the author uses.
- Consider if the author presents a balanced perspective on the main point.
- Recognize if the author introduced any biases in the text.

When you go through a text looking for each of these elements, you need to go beyond just answering the surface question; for instance, the audience may be a specific field of scientists, but could anyone else understand the text with some explanation? Why would that be important?

Analysis Question

Think of an article you need to read for a class. Take the steps above on how to evaluate a text, and apply the steps to the article. When you accomplish the task in each step, ask yourself and take notes to answer the question: Why is this important? For example, when you read the title, does that give you any additional information that will help you comprehend the text? If the text were written for a different audience, what might the author need to change to accommodate that group? How does an author's bias distort an argument? This deep evaluation allows you to fully understand the main ideas and place the text in context with other material on the same subject, with current events, and within the discipline.

Applying

When you learn something new, it always connects to other knowledge you already have. One challenge we have is applying new information. It may be interesting to know the distance to the moon, but how do we apply it to something we need to do? If your

biology instructor asked you to list several challenges of colonizing Mars and you do not know much about that planet's exploration, you may be able to use your knowledge of how far Earth is from the moon to apply it to the new task. You may have to read several other texts in addition to reading graphs and charts to find this information.

That was the challenge the early space explorers faced along with myriad unknowns before space travel was a more regular occurrence. They had to take what they already knew and could study and read about and apply it to an unknown situation. These explorers wrote down their challenges, failures, and successes, and now scientists read those texts as a part of the ever-growing body of text about space travel. Application is a sophisticated level of thinking that helps turn theory into practice and challenges into successes.

Preparing to Read for Specific Disciplines in College

Different disciplines in college may have specific expectations, but you can depend on all subjects asking you to read to some degree. In this college reading requirement, you can succeed by learning to read actively, researching the topic and author, and recognizing how your own preconceived notions affect your reading. Reading for college isn't the same as reading for pleasure or even just reading to learn something on your own because you are casually interested.

In college courses, your instructor may ask you to read articles, chapters, books, or *primary sources* (those original documents about which we write and study, such as letters between historic figures or the Declaration of Independence). Your instructor may want you to have a general background on a topic before you dive into that subject in class, so that you know the history of a topic, can start thinking about it, and can engage in a class discussion with more than a passing knowledge of the issue.

If you are about to participate in an in-depth six-week consideration of the U.S. Constitution but have never read it or anything written about it, you will have a hard time looking at anything in detail or understanding how and why it is significant. As you can imagine, a great deal has been written about the Constitution by scholars and citizens since the late 1700s when it was first put to paper (that's how they did it then). While the actual document isn't that long (about 12–15 pages depending on how it is presented), learning the details on how it came about, who was involved, and why it was and still is a significant document would take a considerable amount of time to read and digest. So, how do you do it all? Especially when you may have an instructor who drops hints that you may also *love* to read a historic novel covering the same time period . . . in your *spare time*, not required, of course! It can be daunting, especially if you are taking more than one course that has time-consuming reading lists. With a few strategic techniques, you can manage it all, but know that you must have a plan and schedule your required reading so you *are* also able to pick up that recommended historic novel—it may give you an entirely new perspective on the issue.

Strategies for Reading in College Disciplines

No universal law exists for how much reading instructors and institutions expect college students to undertake for various disciplines. Suffice it to say, it's a LOT.

For most students, it is the volume of reading that catches them most off guard when they begin their college careers. A full course load might require 10–15 hours of reading per week, some of that covering content that will be more difficult than the reading for other courses.

You cannot possibly read word-for-word every single document you need to read for all your classes. That doesn't mean you give up or decide to only read for your favorite classes or concoct a scheme to read 17 percent for each class and see how that works for you. You need to learn to skim, annotate, and take notes. All of these techniques will help you comprehend more of what you read, which is why we read in the first place. We'll talk more later about annotating and notetaking, but for now consider what you know about skimming as opposed to active reading.

Skimming

Skimming is not just glancing over the words on a page (or screen) to see if any of it sticks. Effective skimming allows you to take in the major points of a passage without the need for a time-consuming reading session that involves your active use of notations and annotations. Often you will need to engage in that painstaking level of active reading, but skimming is the first step—not an alternative to deep reading. The fact remains that neither do you need to read everything nor could you possibly accomplish that given your limited time. So learn this valuable skill of skimming as an accompaniment to your overall study tool kit, and with practice and experience, you will fully understand how valuable it is.

When you skim, look for guides to your understanding: headings, definitions, pull quotes, tables, and context clues. Textbooks are often helpful for skimming—they may already have made some of these skimming guides in bold or a different color, and chapters

often follow a predictable outline. Some even provide an overview and summary for sections or chapters. Use whatever you can get, but don't stop there. In textbooks that have some reading guides, or especially in text that does not, look for introductory words such as *First* or *The purpose of this article . . .* or summary words such as *In conclusion . . .* or *Finally*. These guides will help you read only those sentences or paragraphs that will give you the overall meaning or gist of a passage or book.

Now move to the meat of the passage. You want to take in the reading as a whole. For a book, look at the titles of each chapter if available. Read each chapter's introductory paragraph and determine why the writer chose this particular order. Depending on what you're reading, the chapters may be only informational, but often you're looking for a specific argument. What position is the writer claiming? What support, counterarguments, and conclusions is the writer presenting?

Don't think of skimming as a way to buzz through a boring reading assignment. It is a skill you should master so you can engage, at various levels, with all the reading you need to accomplish in college. End your skimming session with a few notes—terms to look up, questions you still have, and an overall summary. And recognize that you likely will return to that book or article for a more thorough reading if the material is useful.

Active Reading Strategies

Active reading differs significantly from skimming or reading for pleasure. You can think of active reading as a sort of conversation between you and the text (maybe between you and the author, but you don't want to get the author's personality too involved in this metaphor because that may skew your engagement with the text).

When you sit down to determine what your different classes expect you to read and you create a reading schedule to ensure you complete all the reading, think about when you should read the material strategically, not just how to *get it all done*. You should read textbook chapters and other reading assignments *before* you go into a lecture about that information. Don't wait to see how the lecture goes before you read the material, or you may not understand the information in the lecture. Reading before class helps you put ideas together between your reading and the information you hear and discuss in class.

Different disciplines naturally have different types of texts, and you need to take this into account when you schedule your time for reading class material. For example, you may look at a poem for your world literature class and assume that it will not take you long to read because it is relatively short compared to the dense textbook you have for your economics class. But reading and understanding a poem can take a considerable amount of time when you realize you may need to stop numerous times to review the separate word meanings and how the words form images and connections throughout the poem.

The SQ3R Reading Strategy

You may have heard of the **SQ3R** method for active reading in your early education. This valuable technique is perfect for college reading. The title stands for **S**urvey, **Q**uestion, **R**ead, **R**ecite, **R**evue, and you can use the steps on virtually any assigned passage. Designed by Francis Pleasant Robinson in his 1961 book *Effective Study*, the active reading strategy gives readers a systematic way to work through any reading material.

Survey is similar to skimming. You look for clues to meaning by reading the titles, headings, introductions, summary, captions for graphics, and keywords. You can survey almost anything connected to the reading selection, including the copyright information, the date of the journal article, or the names and qualifications of the author(s). In this step, you decide what the general meaning is for the reading selection.

Question is your creation of questions to seek the main ideas, support, examples, and conclusions of the reading selection. Ask yourself these questions separately. Try to create valid questions about what you are about to read that have come into your mind as you engaged in the Survey step. Try turning the headings of the sections in the chapter into questions. Next, how does what you're reading relate to you, your school, your community, and the world?

Read is when you actually read the passage. Try to find the answers to questions you developed in the previous step. Decide how much you are reading in chunks, either by paragraph for more complex readings or by section or even by an entire chapter. When you finish reading the selection, stop to make notes. Answer the questions by writing a note in the margin or other white space of the text.

You may also carefully underline or highlight text in addition to your notes. Use caution here that you don't try to rush this step by haphazardly circling terms or the other extreme of underlining huge chunks of text. Don't over-mark. You aren't likely to remember what these cryptic marks mean later when you come back to use this active reading session to study. The text is the source of information—your marks and notes are just a way to organize and make sense of that information.

Recite means to speak out loud. By reciting, you are engaging other senses to remember the material—you read it (visual) and you said it (auditory). Stop reading momentarily in the step to answer your questions or clarify confusing sentences or paragraphs. You can recite a summary of what the text means to you. If you are not in a place where you can verbalize, such as a library or classroom, you can accomplish this step adequately by *saying* it in your head; however, to get the biggest bang for your buck, try to find a place where you can speak aloud. You may even want to try explaining the content to a friend.

Review is a recap. Go back over what you read and add more notes, ensuring you have captured the main points of the passage, identified the supporting evidence and examples, and understood the overall meaning. You may need to repeat some or all of the SQR3 steps during your review depending on the length and complexity of the material. Before you end your active reading session, write a short (no more than one page is optimal) summary of the text you read.

Reading Primary and Secondary Sources

Primary sources are original documents we study and from which we glean information; primary sources include letters, first editions of books, legal documents, and a variety of other texts. When scholars look at these documents to understand a period in history or a scientific challenge and then write about their findings, the scholar's article is considered a secondary source. Readers have to keep several factors in mind when reading both primary and secondary sources.

Primary sources may contain dated material we now know is inaccurate. It may contain personal beliefs and biases the original writer didn't intend to be openly published, and it may even present fanciful or creative ideas that do not support current knowledge. Readers can still gain great insight from primary sources, but readers need to understand the context from which the writer of the primary source wrote the text.

Likewise, secondary sources are inevitably another person's perspective on the primary source, so a reader of secondary sources must also be aware of potential biases or preferences the secondary source writer inserts in the writing that may persuade an incautious reader to interpret the primary source in a particular manner.

For example, if you were to read a secondary source that is examining the U.S. Declaration of Independence (the primary source), you would have a much clearer idea of how the secondary source scholar presented the information from the primary source if you also read the Declaration for yourself instead of trusting the other writer's interpretation. Most scholars are honest in writing secondary sources, but you as a reader of the source are trusting the writer to present a balanced perspective of the primary source. When possible, you should attempt to read a primary source in conjunction with the secondary source. The Internet helps immensely with this practice.

WHAT STUDENTS SAY

1. What is the most influential factor in how thoroughly you read the material for a given course?
 - a. How engaging the material is or how much I enjoy reading it.
 - b. Whether or not the course is part of my major.
 - c. Whether or not the instructor assesses knowledge from the reading (through quizzes, for example), or requires assignments based on the reading.
 - d. Whether or not knowledge or information from the reading is required to participate in lecture.
2. What best describes your reading approach for required texts/materials for your classes?
 - a. I read all of the assigned material.
 - b. I read most of the assigned material.
 - c. I skim the text and read the captions, examples, or summaries.
3. What best describes your notetaking style?
 - a. I use a systematic method such as the Cornell method or something similar.
 - b. I highlight or underline all the important information.
 - c. I create outlines and/or note-cards.
 - d. I use an app or program.
 - e. I write notes in my text (print or digital).
 - f. I don't have a style. I just write down what seems important.
 - g. I don't take many notes.

You can also take the anonymous [What Students Say](#) surveys to add your voice to this textbook. Your responses will be included in updates.

Students offered their views on these questions, and the results are displayed in the graphs below.

What is the most influential factor in how thoroughly you read the material for a given course?

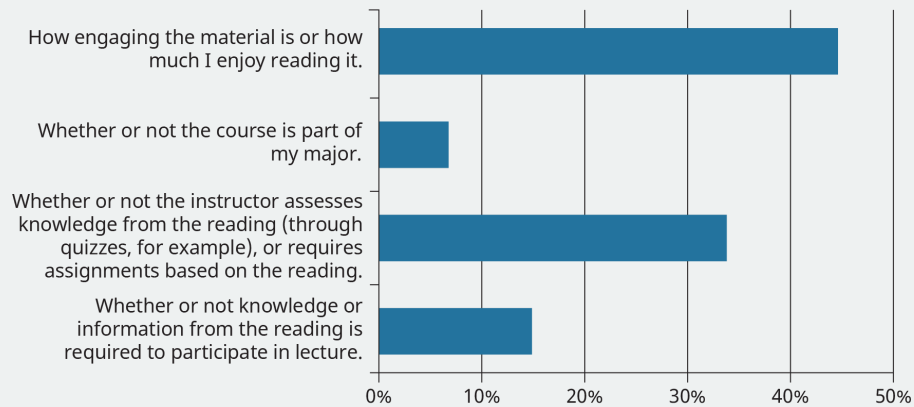


Figure 2.3.5

What best describes your reading approach for required texts/materials for your classes?

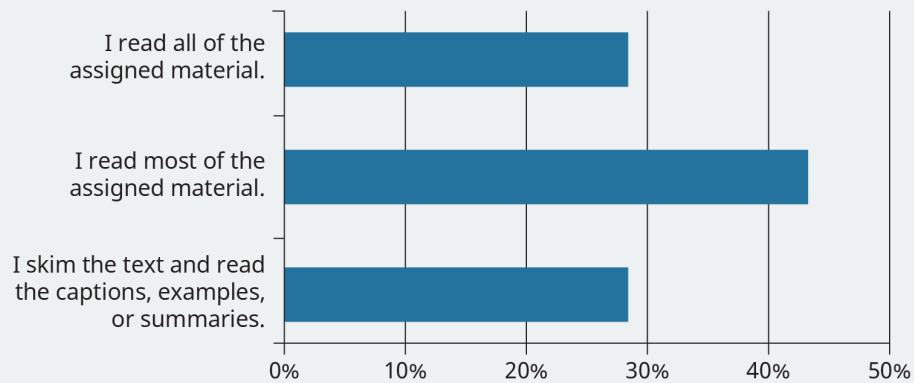


Figure 2.3.6

What best describes your notetaking style?

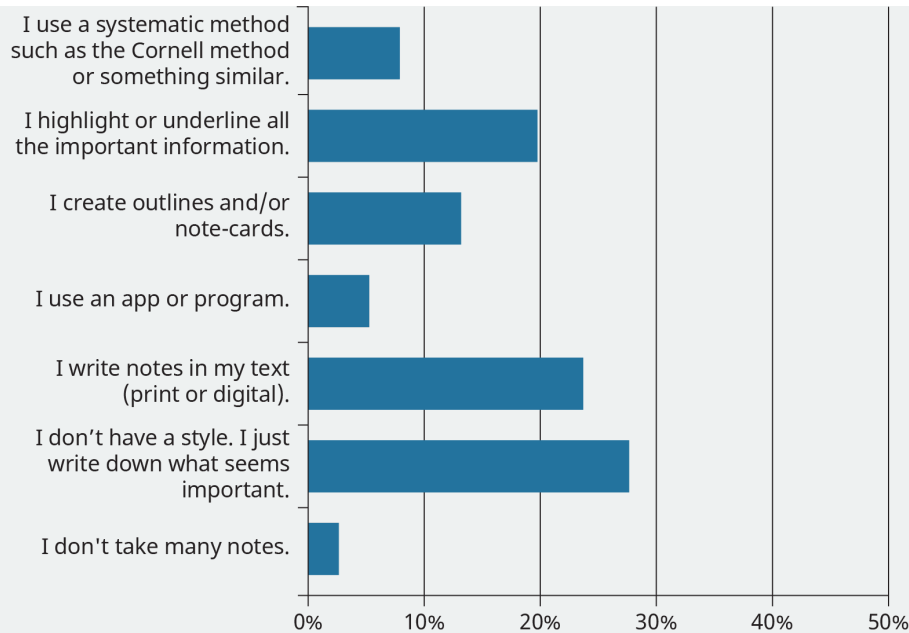


Figure 2.3.3

Researching Topic and Author

During your preview stage, sometimes called pre-reading, you can easily pick up on information from various sources that may help you understand the material you're reading more fully or place it in context with other important works in the discipline. If your selection is a book, flip it over or turn to the back pages and look for an author's biography or note from the author. See if the book itself contains any other information about the author or the subject matter.

The main things you need to recall from your reading in college are the topics covered and how the information fits into the discipline. You can find these parts throughout the textbook chapter in the form of headings in larger and bold font, summary lists, and important quotations pulled out of the narrative. Use these features as you read to help you determine what the most important ideas are.

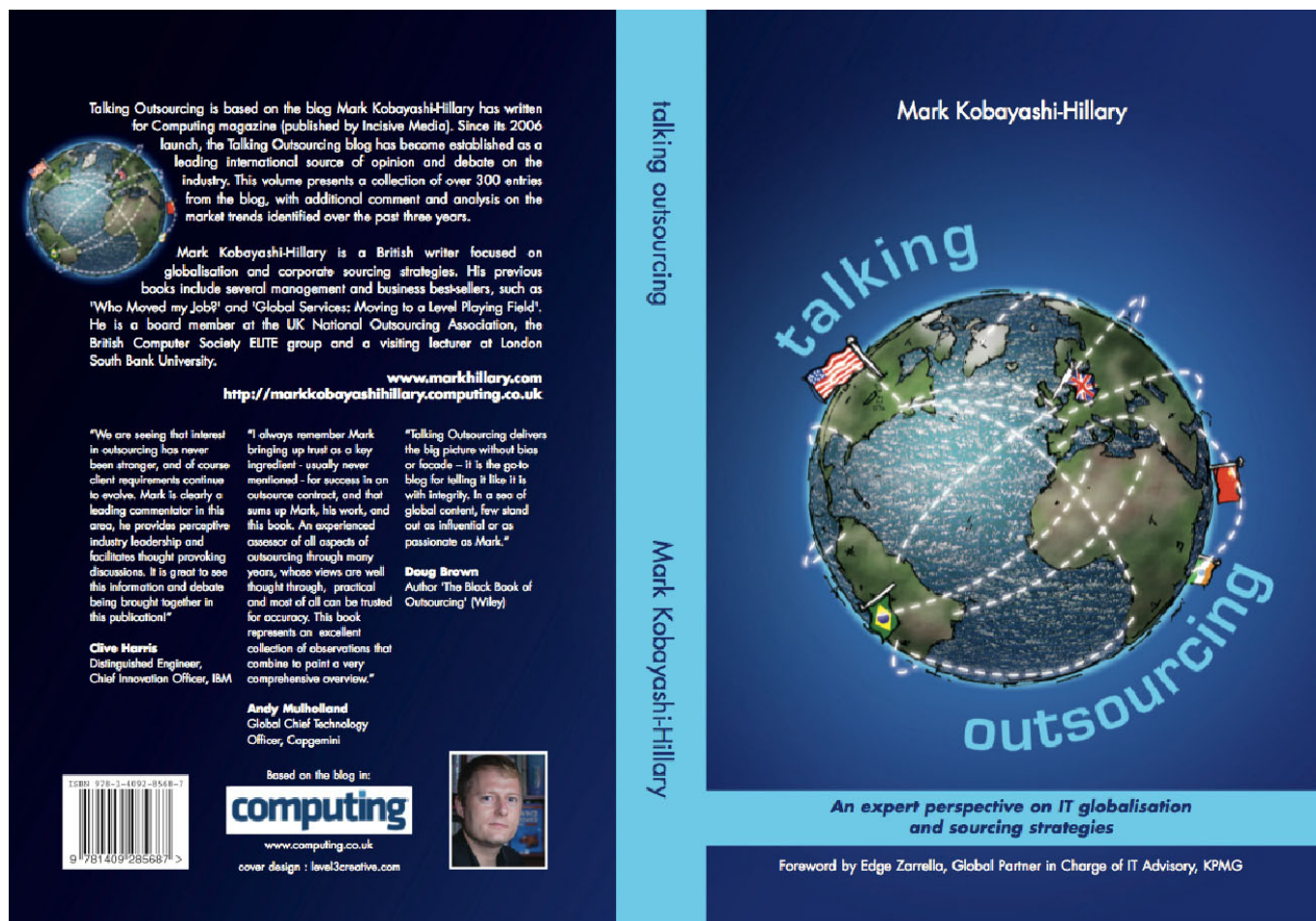


Figure 2.3.4: Learning about the book you're reading can provide good context and information. Look for an author's biography and forward on the back cover or in the first few pages. (Credit: Mark Hillary / Flickr / Attribution 2.0 Generic (CC-BY 2.0))

Remember, many books use quotations about the book or author as testimonials in a marketing approach to sell more books, so these may not be the most reliable sources of unbiased opinions, but it's a start. Sometimes you can find a list of other books the author has written near the front of a book. Do you recognize any of the other titles? Can you do an Internet search for the name of the book or author? Go beyond the search results that want you to buy the book and see if you can glean any other relevant information about the author or the reading selection. Beyond a standard Internet search, try the library article database. These are more relevant to academic disciplines and contain resources you typically will not find in a standard search engine. If you are unfamiliar with how to use the library database, ask a reference librarian on campus. They are often underused resources that can point you in the right direction.

Understanding Your Own Preset Ideas on a Topic

Laura really enjoys learning about environmental issues. She has read many books and watched numerous televised documentaries on this topic and actively seeks out additional information on the environment. While Laura's interest can help her understand a new reading encounter about the environment, Laura also has to be aware that with this interest, she also brings forward her preset ideas and biases about the topic. Sometimes these prejudices against other ideas relate to religion or nationality or even just tradition. Without evidence, thinking the way we always have is not a good enough reason; evidence can change, and at the very least it needs honest review and assessment to determine its validity. Ironically, we may not want to learn new ideas because that may mean we would have to give up old ideas we have already mastered, which can be a daunting prospect.

With every reading situation about the environment, Laura needs to remain open-minded about what she is about to read and pay careful attention if she begins to ignore certain parts of the text because of her preconceived notions. Learning new information can be very difficult if you balk at ideas that are different from what you've always thought. You may have to force yourself to listen to a different viewpoint multiple times to make sure you are not closing your mind to a viable solution your mindset does not currently allow.

Analysis Question

Can you think of times you have struggled reading college content for a course? Which of these strategies might have helped you understand the content? Why do you think those strategies would work?

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2.4: Taking Notes

Questions to consider

- How can you prepare to take notes to maximize the effectiveness of the experience?
- What are some specific strategies you can employ for better notetaking?
- Why is annotating your notes after the notetaking session a critical step to follow?

Beyond providing a record of the information you are reading or hearing, notes help you organize the ideas and help you make meaning out of something about which you may not be familiar, so notetaking and reading are two compatible skill sets. Taking notes also helps you stay focused on the question at hand. Nanami often takes notes during presentations or class lectures so she can follow the speaker's main points and condense the material into a more readily usable format. Strong notes build on your prior knowledge of a subject, help you discuss trends or patterns present in the information, and direct you toward areas needing further research or reading.



Figure 2.4.1: Strong notes build on your prior knowledge of a subject, help you discuss trends or patterns present in the information, and direct you toward areas needing further research or reading.

It is not a good habit to transcribe every single word a speaker utters—even if you have an amazing ability to do that. Most of us don't have that court-reporter-esque skill level anyway, and if we try, we would end up missing valuable information. Learn to listen for main ideas and distinguish between these main ideas and details that typically support the ideas. Include examples that explain the main ideas, but do so using understandable abbreviations.

Think of all notes as potential study guides. In fact, if you only take notes without actively working on them after the initial notetaking session, the likelihood of the notes helping you is slim. Research on this topic concludes that without active engagement after taking notes, most students forget 60–75 percent of material over which they took the notes—within two days! That sort of defeats the purpose, don't you think? This information about memory loss was first brought to light by 19th-century German psychologist Hermann Ebbinghaus. Fortunately, you do have the power to thwart what is sometimes called the Ebbinghaus Forgetting Curve by reinforcing what you learned through review at intervals shortly after you take in the material and frequently thereafter.

If you are a musician, you'll understand this phenomenon well. When you first attempt a difficult piece of music, you may not remember the chords and notes well at all, but after frequent practice and review, you generate a certain muscle memory and cognitive recall that allows you to play the music more easily.

Notetaking may not be the most glamorous aspect of your higher-education journey, but it is a study practice you will carry throughout college and into your professional life. Setting yourself up for successful notetaking is almost as important as the actual taking of notes, and what you do after your notetaking session is equally significant. Well-written notes help you organize your thoughts, enhance your memory, and participate in class discussion, and they prepare you to respond successfully on exams. With all that riding on your notes, it would behoove you to learn how to take notes properly and continue to improve your notetaking skills.

Analysis Question

Do you currently have a preferred way to take notes? When did you start using it? Has it been effective? What other strategy might work for you?

Preparing to Take Notes

Preparing to take notes means more than just getting out your laptop or making sure you bring pen and paper to class. You'll do a much better job with your notes if you understand why we take notes, have a strong grasp on your preferred notetaking system, determine your specific priorities depending on your situation, and engage in some version of efficient shorthand.

Like handwriting and fingerprints, we all have unique and fiercely independent notetaking habits. These understandably and reasonably vary from one situation to the next, but you can only improve your skills by learning more about ways to take effective notes and trying different methods to find a good fit.

The very best notes are the ones you take in an organized manner that encourages frequent review and use as you progress through a topic or course of study. For this reason, you need to develop a way to organize all your notes for each class so they remain together and organized. As old-fashioned as it sounds, a clunky three-ring binder is an excellent organizational container for class notes. You can easily add to previous notes, insert handouts you may receive in class, and maintain a running collection of materials for each separate course. If the idea of carrying around a heavy binder has you rolling your eyes, then transfer that same structure into your computer files. If you don't organize your many documents into some semblance of order on your computer, you will waste significant time searching for improperly named or saved files.

You may be interested in relatively new research on what is the more effective notetaking strategy: handwriting versus typing directly into a computer. While individuals have strong personal opinions on this subject, most researchers agree that the format of student notes is less important than what students do with the notes they take afterwards. Both handwriting notes and using a computer for notetaking have pros and cons.

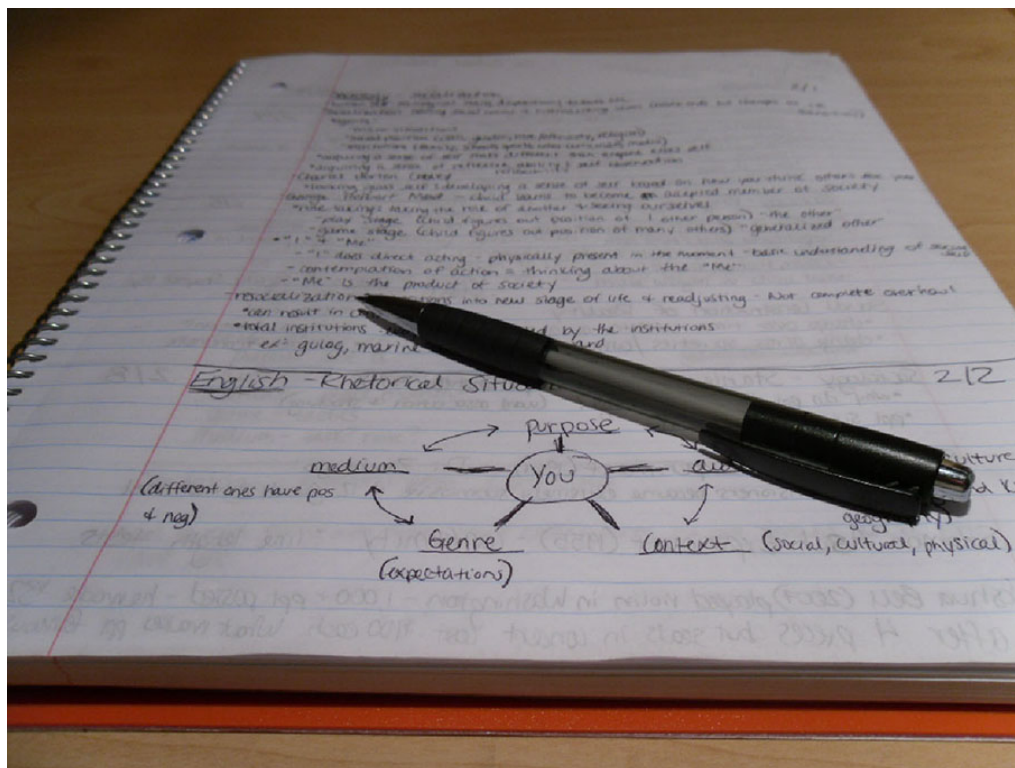


Figure 2.4.2: The best notes are the ones you take in an organized manner. Frequent review and further annotation are important to build a deep and useful understanding of the material. (Credit: English106 / Flickr / Attribution 2.0 Generic (CC-BY 2.0))

Managing Notetaking Systems (Computer, Paper/Pen, Note Cards, Textbook)

Whichever of the many notetaking systems you choose (and new ones seem to come out almost daily), the very best one is the one that you will use consistently. The skill and art of notetaking is not automatic for anyone; it takes a great deal of practice, patience, and continuous attention to detail. Add to that the fact that you may need to master multiple notetaking techniques for different classes, and you have some work to do. Unless you are specifically directed by your instructor, you are free to combine the best parts of different systems if you are most comfortable with that hybrid system.

Just to keep yourself organized, all your notes should start off with an identifier, including at the very least the date, the course name, the topic of the lecture/presentation, and any other information you think will help you when you return to use the notes for further study, test preparation, or assignment completion. Additional, optional information may be the number of notetaking sessions about this topic or reminders to cross-reference class handouts, textbook pages, or other course materials. It's also always a good idea to leave some blank space in your notes so you can insert additions and questions you may have as you review the material later.

Notetaking Strategies

You may have a standard way you take all your notes for all your classes. When you were in high school, this one-size-fits-all approach may have worked. Now that you're in college, reading and studying more advanced topics, your general method may still work some of the time, but you should have some different strategies in place if you find that your method isn't working as well with college content. You probably will need to adopt different notetaking strategies for different subjects. The strategies in this section represent various ways to take notes in such a way that you are able to study after the initial notetaking session.

Cornell Method

One of the most recognizable notetaking systems is called the *Cornell Method*, a relatively simple way to take effective notes devised by Cornell University education professor Dr. Walter Pauk in the 1940s. In this system, you take a standard piece of note paper and divide it into three sections by drawing a horizontal line across your paper about one to two inches from the bottom of the page (the summary area) and then drawing a vertical line to separate the rest of the page above this bottom area, making the left side about two inches (the recall column) and leaving the biggest area to the right of your vertical line (the notes column). You may want to make one page and then copy as many pages as you think you'll need for any particular class, but one advantage of this

Once you are satisfied with your notes and recall columns, summarize this page of notes in two or three sentences using the summary area at the bottom of the sheet. This is an excellent time to get with another classmate or a group of students who all heard the same lecture to make sure you all understood the key points. Now, before you move onto something else, cover the large notes column, and quiz yourself over the key ideas you recorded in the recall column. Repeat this step often as you go along, not just immediately before an exam, and you will help your memory make the connections between your notes, your textbook reading, your in-class work, and assignments that you need to succeed on any quizzes and exams.

Academic Essay Elements	
Topic	Topic <ul style="list-style-type: none"> – Establishes context – Limits scope of essay – Introduces Issue/Problem
Thesis	Thesis <ul style="list-style-type: none"> – Central argument or point of paper – Arrives early in paper—usually toward end of first paragraph (maybe a bit later in longer papers) – Focused, clear, and specific – Reflects writer's position on the topic/issue
Supporting Details	Supporting Detail Paragraphs <ul style="list-style-type: none"> – Each paragraph has a specific topic – Clarify, explain, illustrate, expand on topic – Provide EVIDENCE—quotes, data, references <p><i>Cite everything properly!</i></p>
Conclusion	Conclusion <ul style="list-style-type: none"> – Tie back to intro/thesis – Show how details supported the argument – Why is it important? – Point to implications/outcomes, but don't introduce entirely new ideas
<p>Use the structure, but don't follow it too rigidly. The most important pieces are a strong thesis and good evidence to back it up. The conclusion should not just summarize—take it a little further.</p>	

Figure 2.4.4: This sample set of notes in the Cornell Method is designed to make sense of a large amount of information. The process of organizing the notes can help you retain the information more effectively than less consistent methods.

The main advantage of the Cornell Method is that you are setting yourself up to have organized, workable notes. The neat format helps you move into study-mode without needing to re-copy less organized notes or making sense of a large mass of information you aren't sure how to process because you can't remember key ideas or what you meant. If you write notes in your classes without any sort of system and later come across something like "Napoleon—short" in the middle of a glob of notes, what can you do at this point? Is that important? Did it connect with something relevant from the lecture? How would you possibly know? You are your best advocate for setting yourself up for success in college.

Outlining

Other note organizing systems may help you in different disciplines. You can take notes in a formal outline if you prefer, using Roman numerals for each new topic, moving down a line to capital letters indented a few spaces to the right for concepts related to the previous topic, then adding details to support the concepts indented a few more spaces over and denoted by an Arabic numeral. You can continue to add to a formal outline by following these rules.

You don't absolutely have to use the formal numerals and letter, but you have to then be careful to indent so you can tell when you move from a higher level topic to the related concepts and then to the supporting information. The main benefit of an outline is how

organized it is. You have to be on your toes when you are taking notes in class to ensure you keep up the organizational format of the outline, which can be tricky if the lecture or presentation is moving quickly or covering many diverse topics.

The following formal outline example shows the basic pattern:

- I. Dogs (*main topic—usually general*)
 - A. German Shepherd (*concept related to main topic*)
 - 1. Protection (*supporting info about the concept*)
 - 2. Assertive
 - 3. Loyal
 - B. Weimaraner (*concept related to main topic*)
 - 1. Family-friendly (*supporting info about the concept*)
 - 2. Active
 - 3. Healthy
- II. Siamese

You would just continue on with this sort of numbering and indenting format to show the connections between main ideas, concepts, and supporting details. Whatever details you do not capture in your notetaking session, you can add after the lecture as you review your outline.

Chart or table

Similar to creating an outline, you can develop a chart to compare and contrast main ideas in a notetaking session. Divide your paper into four or five columns with headings that include either the main topics covered in the lecture or categories such as How?, What?, When used?, Advantages/Pros, Disadvantages/Cons, or other divisions of the information. You write your notes into the appropriate columns as that information comes to light in the presentation.

Example of a Chart to Organize Ideas and Categories				
	Structure	Types	Functions in Body	Additional Notes
Carbohydrates				
Lipids				
Proteins				
Nucleic Acid				

This format helps you pull out the salient ideas and establishes an organized set of notes to study later. (If you haven't noticed that this *reviewing later* idea is a constant across all notetaking systems, you should...take note of that.) Notes by themselves that you never reference again are little more than scribbles. That would be a bit like compiling an extensive grocery list so you stay on budget when you shop, work all week on it, and then just throw it away before you get to the store. You may be able to recall a few items, but likely won't be as efficient as you could be if you had the notes to reference. Just as you cannot read all the many books, articles, and documents you need to peruse for your college classes, you cannot remember the most important ideas of all the notes you will take as part of your courses, so you must review.

Concept Mapping and Visual Notetaking

One final notetaking method that appeals to learners who prefer a visual representation of notes is called *mapping* or sometimes *mind mapping* or *concept mapping*, although each of these names can have slightly different uses. Variations of this method abound, so you may want to look for more versions online, but the basic principles are that you are making connections between main ideas through a graphic depiction; some can get rather elaborate with colors and shapes, but a simple version may be more useful at least to begin. Main ideas can be circled or placed in a box with supporting concepts radiating off these ideas shown with a connecting line and possibly details of the support further radiating off the concepts. You can present your main ideas vertically or horizontally, but turning your paper long-ways, or in landscape mode, may prove helpful as you add more main ideas.

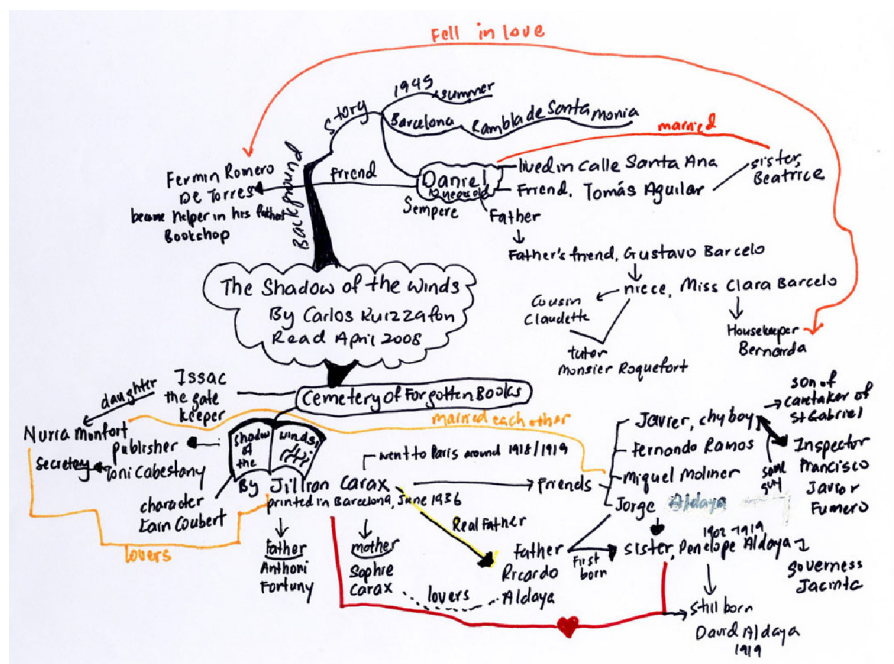


Figure 2.4.5: Concept mapping, sometimes referred to as mind mapping, can be an effective and very personalized approach to capturing information. (Credit: ArtistIvanChew / Flickr / Attribution 2.0 Generic (CC-BY 2.0))

You may be interested in trying visual notetaking or adding pictures to your notes for clarity. Sometimes when you can't come up with the exact wording to explain something or you're trying to add information for complex ideas in your notes, sketching a rough image of the idea can help you remember. According to educator Sherrill Knezel in an article entitled "The Power of Visual Notetaking," this strategy is effective because "When students use images and text in notetaking, it gives them two different ways to pull up the information, doubling their chances of recall." Don't shy away from this creative approach to notetaking just because you believe you aren't an artist; the images don't need to be perfect. You may want to watch [Rachel Smith's TEDx Talk called "Drawing in Class"](#) to learn more about visual notetaking.

You can play with different types of notetaking suggestions and find the method(s) you like best, but once you find what works for you, stick with it. You will become more efficient with the method the more you use it, and your notetaking, review, and test prep will become, if not easier, certainly more organized, which can decrease your anxiety.

Practicing Decipherable Shorthand

Most college students don't take a class in shorthand, once the domain of secretaries and executive assistants, but maybe they should. That almost-lost art in the age of computers could come in very handy during intense notetaking sessions. Elaborate shorthand systems do exist, but you would be better served in your college notetaking adventures to hone a more familiar, personalized form of shorthand to help you write more in a shorter amount of time. Seemingly insignificant shortcuts can add up to ease the stress notetaking can induce—especially if you ever encounter an "I'm not going to repeat this" kind of presenter! Become familiar with these useful abbreviations:

Shortcut symbol	Meaning
w/, w/o, w/in	with, without, within
&	and
#	number
b/c	because
X, √	incorrect, correct
Diff	different, difference
etc.	and so on

ASAP	as soon as possible
US, UK	United States, United Kingdom
info	information
Measurements: ft, in, k, m	foot, inch, thousand, million
¶	paragraph or new paragraph
Math symbols: =, +, >, <, ÷	equal, plus, greater, less, divided by
WWI, WWII	World Wars I and II
impt	important
?, !, **	denote something is very significant; don't over use

Do you have any other shortcuts or symbols that you use in your notes? Ask your parents if they remember any that you may be able to learn.

Annotating Notes After Initial Notetaking Session

Annotating notes after the initial notetaking session may be one of the most valuable study skills you can master. Whether you are highlighting, underlining, or adding additional notes, you are reinforcing the material in your mind and memory.

Admit it—who can resist highlighting markers? Gone are the days when yellow was the star of the show, and you had to be very careful not to press too firmly for fear of obliterating the words you were attempting to emphasize. Students now have a veritable rainbow of highlighting options and can color-code notes and text passages to their hearts' content. Technological advances may be important, but highlighter color choice is monumental! Maybe.

The only reason to highlight anything is to draw attention to it, so you can easily pick out that ever-so-important information later for further study or reflection. One problem many students have is not knowing when to stop. If what you need to recall from the passage is a particularly apt and succinct definition of the term important to your discipline, highlighting the entire paragraph is less effective than highlighting just the actual term. And if you don't rein in this tendency to color long passages (possibly in multiple colors) you can end up with a whole page of highlighted text. Ironically, that is no different from a page that is not highlighted at all, so you have wasted your time. Your mantra for highlighting text should be *less is more*. Always read your text selection first before you start highlighting anything. You need to know what the overall message is before you start placing emphasis in the text with highlighting.

Another way to annotate notes after initial notetaking is underlying significant words or passages. Albeit not quite as much fun as its colorful cousin highlighting, underlining provides precision to your emphasis.

Some people think of annotations as only using a colored highlighter to mark certain words or phrases for emphasis. Actually, annotations can refer to anything you do with a text to enhance it for your particular use (either a printed text, handwritten notes, or other sort of document you are using to learn concepts). The annotations may include highlighting passages or vocabulary, defining those unfamiliar terms once you look them up, writing questions in the margin of a book, underlining or circling key terms, or otherwise marking a text for future reference. You can also annotate some electronic texts.

Realistically, you may end up doing all of these types of annotations at different times. We know that repetition in studying and reviewing is critical to learning, so you may come back to the same passage and annotate it separately. These various markings can be invaluable to you as a study guide and as a way to see the evolution of your learning about a topic. If you regularly begin a reading session writing down any questions you may have about the topic of that chapter or section and also write out answers to those questions at the end of the reading selection, you will have a good start to what that chapter covered when you eventually need to study for an exam. At that point, you likely will not have time to reread the entire selection especially if it is a long reading selection, but with strong annotations in conjunction with your class notes, you won't need to do that. With experience in reading discipline-specific texts and writing essays or taking exams in that field, you will know better what sort of questions to ask in your annotations

When did Lincoln die? April 15, 1865

The Gettysburg Address

Where is Gettysburg? Pennsylvania
What happened there? Civil War battle of Gettysburg, July 1-3, 1863 - Union victory, but largest # of dead in entire war

President Abraham Lincoln

November 19, 1863

$80(4 \times 20) + 7 = 87$ 1776

"Fourscore and seven years ago our fathers brought forth on this continent a new nation, conceived in liberty and dedicated to the proposition that all men are created equal. ^{from US Constitution?} ^{No - Dec of Independence}

^{proposal}

"Now we are engaged in a great civil war, testing whether that nation, or any nation so ^{formed} conceived and so ^{last} dedicated, can long endure. We are met on a great battlefield of that war. We have come to ^{cemetery} dedicate a portion of that field as a final resting-place for those who here gave their lives that that nation might live. It is altogether fitting and proper that we should do this. But, in a larger sense, we ^{make holy} cannot dedicate — we ^{make holy} cannot consecrate — we ^{death} cannot hallow — this ground. The brave men, living and dead, who struggled here have consecrated it, far above our poor power to add or detract. The world will little note, nor long remember what we say here, but it can never forget what they did here. It is for us the living, rather, to be ^{like royalty} dedicated here to the unfinished work which they who fought here have thus far so nobly advanced. It is rather for us to be here ^{for no reason} dedicated to the great task remaining before us — that from these honored dead we take increased devotion to that cause for which they gave the last full measure of devotion — that we here highly resolve that these dead shall not have died in vain — that ^{phrases from Constitution?} this nation, under God, shall have a new birth of freedom and that ^{repetition} government of the people, by the people, for the people, shall not ^{die} perish from the earth."

Figure 2.4.6: Annotations may include highlighting important topics, defining unfamiliar terms, writing questions in, underlining or circling key terms, or otherwise marking a text for future reference. Whichever approach you choose, try not to overdo it; neat, organized, and efficient notes are more effective than crowded or overdone notes.

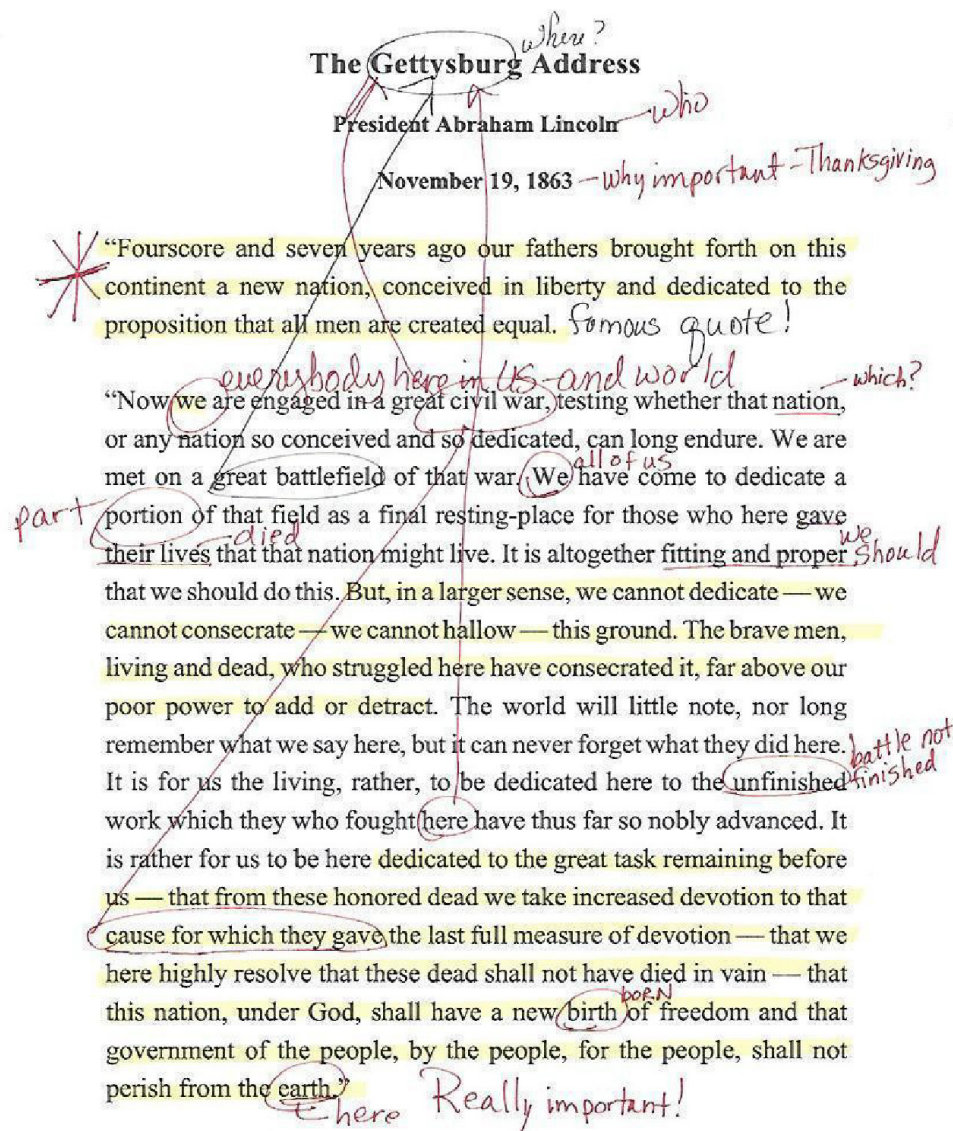


Figure 2.4.7: While these notes may be meaningful to the person who took them, they are neither organized nor consistent. For example, note that some of the more commonly used terms, like “we” and “unfinished,” are defined, but less common ones -- “consecrate” and “hallow” -- are not.

What you have to keep in the front of your mind while you are annotating, especially if you are going to conduct multiple annotation sessions, is to not overdo whatever method you use. Be judicious about what you annotate and how you do it on the page, which means you must be neat about it. Otherwise, you end up with a mess of either color or symbols combined with some cryptic notes that probably took you quite a long time to create, but won’t be worth as much to you as a study aid as they could be. This is simply a waste of time and effort.

You cannot eat up every smidgen of white space on the page writing out questions or summaries and still have a way to read the original text. If you are lucky enough to have a blank page next to the beginning of the chapter or section you are annotating, use this, but keep in mind that when you start writing notes, you aren’t exactly sure how much space you’ll need. Use a decipherable shorthand and write only what you need to convey the meaning in very small print. If you are annotating your own notes, you can make a habit of using only one side of the paper in class, so that if you need to add more notes later, you could use the other side. You can also add a blank page to your notes before beginning the next class date in your notebook so you’ll end up with extra paper for annotations when you study.

Professional resources may come with annotations that can be helpful to you as you work through the various documentation requirements you’ll encounter in college as well. Purdue University’s Online Writing Lab ([OWL](https://www.purdue.edu/owl/)) provides an annotated sample for

how to format a college paper according to guidelines in the Modern Language Association (MLA) manual that you can see, along with other annotations.

Adding Needed Additional Explanations to Notes

Marlon was totally organized and ready to take notes in a designated course notebook at the beginning of every philosophy class session. He always dated his page and indicated what the topic of discussion was. He had various colored highlighters ready to denote the different note purposes he had defined: vocabulary in pink, confusing concepts in green, and note sections that would need additional explanations later in yellow. He also used his own shorthand and an impressive array of symbols to indicate questions (red question mark), highly probable test material (he used a tiny bomb exploding here), additional reading suggestions, and specific topics he would ask his instructor before the next class. Doing everything so precisely, Marlon's methods seemed like a perfect example of how to take notes for success. Inevitably though, by the end of the hour-and-a-half class session, Marlon was frantically switching between writing tools, near to tears, and scouring his notes as waves of yellow teased him with uncertainty. What went wrong?

As with many of us who try diligently to do everything we know how to do for success or what we think we know because we read books and articles on success in between our course work, Marlon is suffering from trying to do too much simultaneously. It's an honest mistake we can make when we are trying to save a little time or think we can multitask and kill two birds with one stone.

Unfortunately, this particular error in judgement can add to your stress level exponentially if you don't step back and see it for what it is. Marlon attempted to take notes in class as well as annotate his notes to get them ready for his test preparation. It was too much to do at one time, but even if he could have done all those things during class, he's missing one critical point about notetaking.

As much as we may want to hurry and get it over with, notetaking in class is just the beginning. Your instructor likely gave you a pre-class assignment to read or complete before coming to that session. The intention of that preparatory lesson is for you to come in with some level of familiarity for the topic under consideration and questions of your own. Once you're in class, you may also need to participate in a group discussion, work with your classmates, or perform some other sort of lesson-directed activity that would necessarily take you away from taking notes. Does that mean you should ignore taking notes for that day? Most likely not. You may just need to indicate in your notes that you worked on a project or whatever other in-class event you experienced that date.

Very rarely in a college classroom will you engage in an activity that is not directly related to what you are studying in that course. Even if you enjoyed every minute of the class session and it was an unusual format for that course, you still need to take some notes. Maybe your first note could be to ask yourself why you think the instructor used that unique teaching strategy for the class that day. Was it effective? Was it worth using the whole class time? How will that experience enhance what you are learning in that course?

If you use an ereader or ebooks to read texts for class or read articles from the Internet on your laptop or tablet, you can still take effective notes. Depending on the features of your device, you have many choices. Almost all electronic reading platforms allow readers to highlight and underline text. Some devices allow you to add a written text in addition to marking a word or passage that you can collect at the end of your notetaking session. Look into the specific tools for your device and learn how to use the features that allow you to take notes electronically. You can also find apps on devices to help with taking notes, some of which you may automatically have installed when you buy the product. Microsoft's OneNote, Google Keep, and the Notes feature on phones are relatively easy to use, and you may already have free access to those.

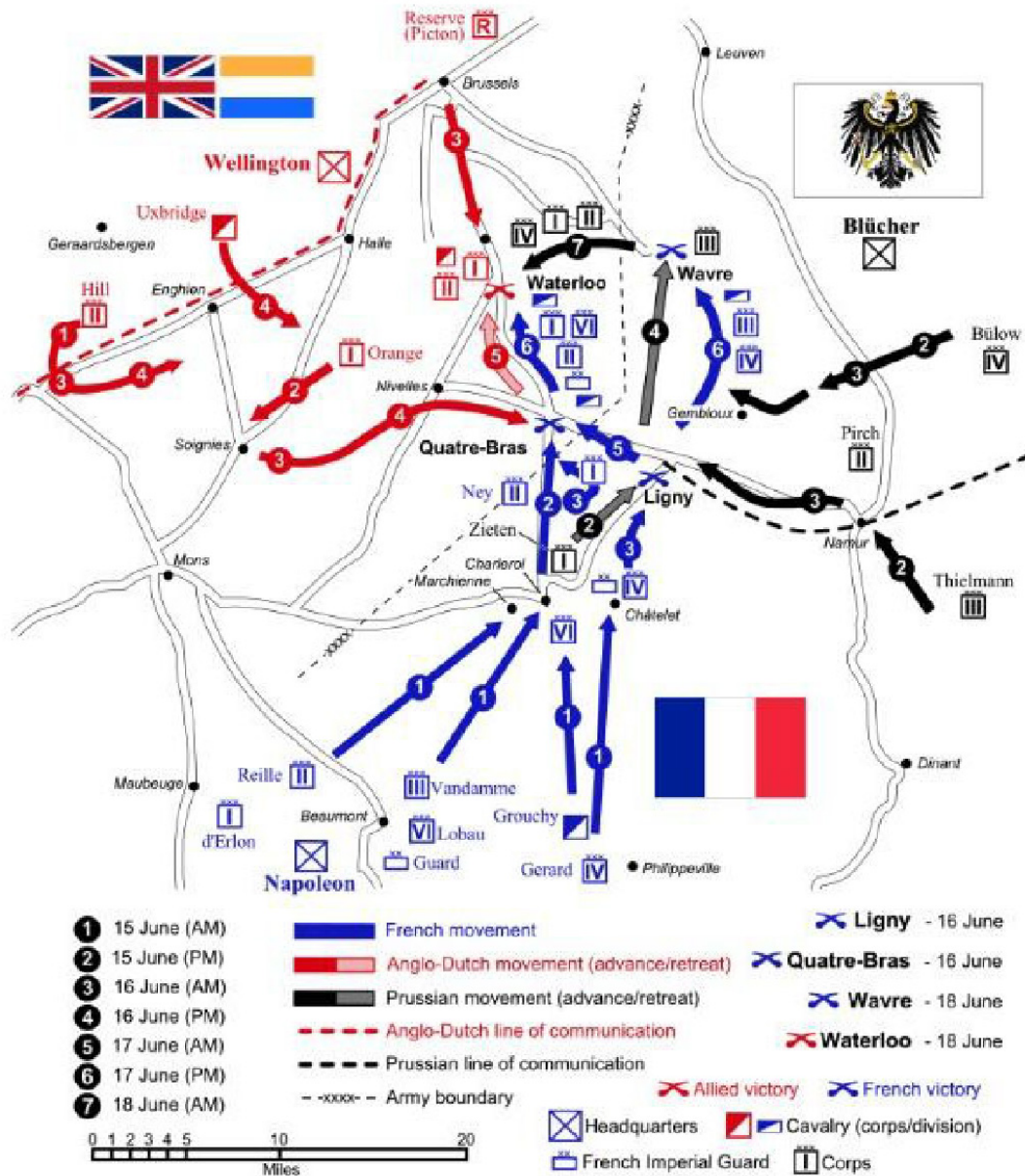
Taking Notes on Non-Text Items (i.e., Tables, Maps, Figures, etc.)

You may also encounter situations as you study and read textbooks, primary sources, and other resources for your classes that are not actually texts. You can still take notes on maps, charts, graphs, images, and tables, and your approach to these non-text features is similar to when you prepare to take notes over a passage of text. For example, if you are looking at the following map, you may immediately come up with several questions. Or it may initially appear overwhelming. Start by asking yourself these questions:

What is the main point of this map?

- Who is the intended audience?
- Where is it?
- What time period does it depict?
- What does the map's legend (the explanation of symbols) include?
- What other information do I need to make sense of this map?

Order of Napoleon's Battle of Waterloo



Source: Wikipedia Creative Commons, https://en.wikipedia.org/wiki/Order_of_battle_of_the_Waterloo_campaign

Figure 2.4.8: Graphics, charts, graphs, and other visual items are also important to annotate. Not only do they often convey important information, but they may appear on exams or in other situations where you'll need to use or demonstrate knowledge. Credit: "Lpankonin" / Wikipedia Commons / Attribution 3.0 Generic (CC BY 3.0)

You may want to make an extra copy of a graphic or table before you add annotations if you are dealing with a lot of information. Making sense of all the elements will take time, and you don't want to add to the confusion.

Returning to Your Notes

Later, as soon as possible after the class, you can go back to your notes and add in missing parts. Just as you may generate questions as you're reading new material, you may leave a class session or lecture or activities with many questions. Write those

down in a place where they won't get lost in all your other notes.

The exact timing of when you get back to the notes you take in class or while you are reading an assignment will vary depending on how many other classes you have or what other obligations you have in your daily schedule. A good starting place that is also easy to remember is to make every effort to review your notes within 24 hours of first taking them. Longer than that and you are likely to have forgotten some key features you need to include; much less time than that, and you may not think you need to review the information you so recently wrote down, and you may postpone the task too long.

Use your phone or computer to set reminders for all your note review sessions so that it becomes a habit and you keep on top of the schedule.

Your personal notes play a significant role in your test preparation. They should enhance how you understand the lessons, textbooks, lab sessions, and assignments. All the time and effort you put into first taking the notes and then annotating and organizing the notes will be for naught if you do not formulate an effective and efficient way to use them before sectional exams or comprehensive tests.

The whole cycle of reading, notetaking in class, reviewing and enhancing your notes, and preparing for exams is part of a continuum you ideally will carry into your professional life. Don't try to take short cuts; recognize each step in the cycle as a building block. Learning doesn't end, which shouldn't fill you with dread; it should help you recognize that all this work you're doing in the classroom and during your own study and review sessions is ongoing and cumulative. Practicing effective strategies now will help you be a stronger professional.

Activity

What resources can you find about reading and notetaking that will actually help you with these crucial skills? How do you go about deciding what resources are valuable for improving your reading and notetaking skills?

The selection and relative value of study guides and books about notetaking vary dramatically. Ask your instructors for recommendations and see what the library has available on this topic. The following list is not comprehensive, but will give you a starting point for books and articles on notetaking in college.

- *College Rules!: How to Study, Survive, and Succeed in College*, by Sherri Nist-Olejnuk and Jodi Patrick Holschuh. More than just notetaking, this book covers many aspects of transitioning into the rigors of college life and studying.
- *Effective Notetaking*, by Fiona McPherson. This small volume has suggestions for using your limited time wisely before, during, and after notetaking sessions.
- *How to Study in College*, by Walter Pauk. This is the book that introduced Pauk's notetaking suggestions we now call the Cornell Method. It is a bit dated (from the 1940s), but still contains some valuable information.
- *Learn to Listen, Listen to Learn 2: Academic Listening and Note-taking*, by Roni S. Lebauer. The main point of this book is to help students get the most from college lectures by watching for clues to lecture organization and adapting this information into strong notes.
- *Study Skills: Do I Really Need this Stuff?*, by Steve Piscitelli. Written in a consistently down-to-earth manner, this book will help you with the foundations of strong study skills, including time management, effective notetaking, and seeing the big picture.
- "What Reading Does for the Mind," by Anne Cunningham and Keith Stanovich, 1998, <https://www.aft.org/sites/default/files/periodicals/cunningham.pdf>
- Adler, Mortimer J. and Charles Van Doren. *How to Read a Book: The Classic Guide to Intelligent Reading*. NY: Simon & Schuster, 1940.
- Berns, Gregory S., Kristina Blaine, Michael J. Prietula, and Brandon E. Pye. *Brain Connectivity*. Dec 2013. ahead of print <http://doi.org/10.1089/brain.2013.0166>

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2.5: Summary

Reading and notetaking are major elements of college studying and learning. The expectations in college is that you read considerable amounts of text for each subject. You may encounter reading situations, such as professional journal articles and long textbook chapters, that are more difficult to understand than texts you have read previously. As you progress through your college courses, you can employ reading strategies to help you complete your college reading assignments. Likewise, you will take notes in college that need to be complete so you can study and recall the information you learn in lectures and lab sessions. With so much significant information that you need to collect, study, and recall for your college courses, you need to be deliberate in your reading and notetaking.

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2.6: Career Connection

Sanvi is a pre-nursing student who is having trouble between all the reading she is expected to complete, her general dislike of reading, and her need to comprehend both her reading assignments and her own notes to be successful in nursing school. She has spoken with several of her instructors and a tutor at the Student Success Center on campus, and their advice centers around Sanvi's reluctance to read in general. She is working on how to manage her time so she has more dedicated time to read her assignments in between her classes and her work schedule.

That is helping some, but Sanvi is still worried because she knows one problem is that she doesn't exactly know what types of reading or notetaking she would need to know how to do as a professional nurse. This confusion makes her doubt that the extra reading she is doing now is really beneficial. After some reflection on what was holding her back, Sanvi mentioned this aspect of her studying to one of her instructors who had been a hospital RN for years before coming to the college to teach. She recalled that the first time she read a patient chart in the hospital, she had to think quickly about how to get all the meaning out of the chart in the same way she would have read a complex textbook chapter.

Sanvi's nursing instructor reminded her that all professions need their personnel to read. They may not all need to read books or articles, but all jobs involve reading to some extent. For example, consider this list of disciplines and the typical types of reading they do. You may be surprised that not all reading is in text form.

Nurses/doctors	Patient charts, prescription side effects, medical articles
Teachers	Student work, lesson plans, educational best practices
Architects	Blueprints, construction contracts, permit manuals
Accountants	Financial spreadsheets, tax guidelines, invoices, trend diagrams
Beauticians	Client hair and facial features, best practices articles, product information
Civil engineers	Work site maps, government regulations, financial spreadsheets
Auto mechanics	Car engines, auto manuals, government regulations

As this incomplete list shows, not every job you pursue will require you to read text-based documents, but all jobs require some reading.

- How could Sanvi and her instructor use this list to make more sense of how college reading will prepare Sanvi to be a stronger nurse?
- How would understanding the types of professional reading help you complete your reading assignments?
- If your chosen field of study is not listed above, can you think of what sort of reading those professionals would need to do?

Think about the questions that opened this chapter and what you have read. How do you feel about your reading and notetaking skills now that you have some more strategies?

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2.7: Rethinking

Revisit the questions you answered at the beginning of the chapter, and consider one option you learned in this chapter that might change your answer to one of them.

1. I am reading on a college level
 2. I take good notes that help me study for exams
 3. I understand how to manage all the reading I need to do for college
 4. I recognize the need for different notetaking strategies for different college subjects
-

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2.8: Where do you go from here?

Reading is such a part of our everyday lives that we sometimes take it for granted. And even we don't formally write down our thoughts, we take notes in our heads far more often than we use our notetaking skills to make sense of a textbook passage or a graphic. Honing these fundamental skills can only help you succeed in college and beyond. What else about reading and note taking would you like to learn more about? Choose topics from the list below to research more.

- How to maximize e-readers to comprehend texts.
 - How professional use reading and note taking in their careers.
 - Is speed-reading a myth or a viable strategy?
 - Compare reading and notetaking strategies from different countries to those you use
-

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CHAPTER OVERVIEW

3: Studying, Memory, and Test Taking

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Thumbnail: pixabay.com/photos/laptop-woman-education-study-young-3087585/

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3.1: Prelude to Studying, Memory, and Test Taking



Figure 3.1.1: How we study is as important as what we study. The environment is a critical element of success.

Student Survey

How confident are you in preparing for and taking tests? Take this quick survey to figure it out, ranking questions on a scale of 1–4, 1 meaning “least like me” and 4 meaning “most like me.” These questions will help you determine how the chapter concepts relate to you right now. As you are introduced to new concepts and practices, it can be informative to reflect on how your understanding changes over time. We’ll revisit these questions at the end of the chapter to see whether your feelings have changed.

1. I set aside enough time to prepare for tests.
2. If I don’t set aside enough time, or if life gets in the way, I can usually cram and get positive results.
3. I prefer to pull all-nighters. The adrenaline and urgency help me remember what I need come test time.
4. I study my notes, highlight book passages, and use flash cards, but I still don’t feel like I’m as successful as I should be on tests.

You can also take the [Chapter Survey](#) anonymously online.

STUDENT PROFILE

“I didn’t have to study much for tests in high school, but I learned really quick that you have to for college. One of the best strategies is to test yourself over the material. This will help you improve your retrieval strength and help you remember more when it comes to the test. I also learned about reviewing your graded tests. This will help you see where you went wrong and why. Being able to see your mistakes and correct them helps the storage and retrieval strength as well as building those dendrites. Getting a question wrong will only improve those things helping you remember the next time it comes up.”

—**Lilli Branstetter**, University of Central Arkansas

About this Chapter

By the time you finish this chapter, you should be able to do the following:

- Outline the importance of memory when studying, and note some opportunities to strengthen memory.
- Discuss specific ways to increase the effectiveness of studying.
- Articulate test-taking strategies that minimize anxiety and maximize results.

Kerri didn’t need to study in high school. She made good grades, and her friends considered her lucky because she never seemed to sweat exams or cram. In reality, Kerri did her studying during school hours, took excellent notes in class, asked great questions, and read the material before class meetings—all of these are excellent strategies. Kerri just seemed to do them without much fuss.

Then when she got to college, those same skills weren’t always working as well. Sound familiar? She discovered that, for many classes, she needed to read paragraphs and textbook passages more than once for comprehension. Her notes from class sessions

were longer and more involved—the subject material was more complicated and the problems more complex than she had ever encountered. College isn't high school, as most students realize shortly after enrolling in a higher ed program. Some old study habits and test-taking strategies may serve as a good foundation, but others may need major modification.

It makes sense that, the better you are at studying and test taking, the better results you'll see in the form of high grades and long-term learning and knowledge acquisition. And the more experience you have using your study and memorization skills and employing success strategies during exams, the better you'll get at it. But you have to keep it up—maintaining these skills and learning better strategies as the content you study becomes increasingly complex is crucial to your success. Once you transition into a work environment, you will be able to use these same skills that helped you be successful in college as you face the problem-solving demands and expectations of your job. Earning high grades is one goal, and certainly a good one when you're in college, but true learning means committing content to long-term memory.

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3.2: Memory

Questions to consider

- How does working memory work, exactly?
- What's the difference between working and short-term memory?
- How does long-term memory function?
- What obstacles exist to remembering?
- When and how should you memorize things?

In what situations is it best to memorize, and what do you memorize?

What can you do consistently to improve both your short- and long-term memory?

Memory is one of those cherished but mysterious elements in life. Everyone has memories, and some people are very good at rapid recall, which is an enviable skill for test takers. We know that we seem to lose the capacity to remember things as we age, and scientists continue to study how we remember some things but not others and what memory means, but we don't know that much about memory, really.

Nelson Cowan is one researcher who is working to explain what we do know about memory. His article “What Are the Differences between Long-Term, Short-Term, and Working Memory?” breaks down the different types of memory and what happens when we recall thoughts and ideas. When we remember something, we actually do quite a lot of thinking.¹

We go through three basic steps when we remember ideas or images: we encode, store, and retrieve that information. Encoding is how we first perceive information through our senses, such as when we smell a lovely flower or a putrid trash bin. Both make an impression on our minds through our sense of smell and probably our vision. Our brains encode, or label, this content in short-term memory in case we want to think about it again.

If the information is important and we have frequent exposure to it, the brain will store it for us in case we need to use it in the future in our aptly named long-term memory. Later, the brain will allow us to recall or retrieve that image, feeling, or information so we can do something with it. This is what we call remembering.



Figure 3.2.1

Analysis Question

Take a few minutes to list ways you create memories on a daily basis. Do you think about how you make memories? Do you do anything that helps you keep track of your memories?

Foundations of Memory

William Sumrall et al. in the *International Journal of Humanities and Social Science* explain the foundation of memory by noting: “Memory is a term applied to numerous biological devices by which living organisms acquire, retain, and make use of skills and knowledge. It is present in all forms of higher order animals. The most evolutionary forms of memory have taken place in human beings. Despite much research and exploration, a complete understanding of human memory does not exist.”²

Working Memory

Working memory is a type of short-term memory, but we use it when we are actively performing a task. For example, nursing student Marilyn needs to use her knowledge of chemical reactions to suggest appropriate prescriptions in various medical case studies. She does not have to recall every single fact she learned in years of chemistry classes, but she does need to have a working memory of certain chemicals and how they work with others. To ensure she can make these connections, Marilyn will have to review and study the relevant chemical details for the types of drug interactions she will recommend in the case studies.

In working memory, you have access to whatever information you have stored in your memory that helps you complete the task you are performing. For instance, when you begin to study an assignment, you certainly need to read the directions, but you must also remember that in class your professor reduced the number of problem sets the written instructions indicated you needed to finish. This was an oral addition to the written assignment. The change to the instructions is what you bring up in working memory when you complete the assignment.

Short-Term Memory

Short-term memory is a very handy thing. It helps us remember where we set our keys or where we left off on a project the day before. Think about all the aids we employ to help us with short-term memory: you may hang your keys in a particular place each evening so you know exactly where they are supposed to be. When you go grocery shopping, do you ever choose a product because you recall an advertising jingle? You see the box of cereal and you remember the song on the TV commercial. If that memory causes you to buy that product, the advertising worked. We help our memory along all the time, which is perfectly fine. In fact, we can modify these everyday examples of memory assistance for purposes of studying and test taking. The key is deliberate use of strategies that are not so elaborate that they are too difficult to remember in our short-term memory.

Activity

Consider this list of items. Look at the list for no more than 30 seconds. Then, cover up the list and use the spaces below to complete an activity.

Baseball	Picture frame	Tissue	Paper clip
Bread	Pair of dice	Fingernail polish	Spoon
Marble	Leaf	Doll	Scissors
Cup	Jar of sand	Deck of cards	Ring
Blanket	Ice	Marker	String

Without looking at the list, write down as many items as you can remember.

Now, look back at your list and make sure that you give yourself credit for any that you got right. Any items that you misremembered, meaning they were not in the original list, you won't count in your total. TOTAL ITEMS REMEMBERED

_____.

There were 20 total items. Did you remember between 5 and 9 items? If you did, then you have a typical short-term memory and you just participated in an experiment, of sorts, to prove it.

Harvard psychology professor George A. Miller in 1956 claimed humans can recall about five to nine bits of information in our short-term memory at any given time. Other research has come after this claim, but this concept is a popular one. Miller's article is entitled "The Magical Number Seven, Plus or Minus Two" and is easily accessible online if you're interested in learning more about this seminar report.³

Considering the vast amount of knowledge available to us, five to nine bits isn't very much to work with. To combat this limitation, we clump information together, making connections to help us stretch our capacity to remember. Many factors play into how much we can remember and how we do it, including the subject matter, how familiar we are with the ideas, and how interested we are in the topic, but we certainly cannot remember absolutely everything, for a test or any other task we face. As such, we have to use effective strategies, like those we cover later in this chapter, to get the most out of our memories.

Activity

Now, let's revisit the items above. Go back to them and see if you can organize them in a way that you would have about five groups of items. See below for an example of how to group them.

Row 1: Items found in a kitchen

Row 2: Items that a child would play with

Row 3: Items of nature

Row 4: Items in a desk drawer/school supplies

Row 5: Items found in a bedroom

Cup	Spoon	Ice	Bread	
Baseball	Marble	Pair of dice	Doll	Deck of cards
Jar of sand	Leaf			
Marker	String	Scissors	Paper clip	
Ring	Picture frame	Fingernail polish	Tissue	Blanket

Now that you have grouped items into categories, also known as chunking, you can work on remembering the categories and the items that fit into those categories, which will result in remembering more items. Check it out below by covering up the list of items again and writing down what you can remember.

Now, look back at your list and make sure that you give yourself credit for any that you got right. Any items that you misremembered, meaning they were not in the original list, you won't count in your total. TOTAL ITEMS REMEMBERED _____. Did you increase how many items you could remember?

Long-Term Memory

Long-term memory is exactly what it sounds like. These are things you recall from the past, such as the smell of your elementary school cafeteria or how to pop a wheelie on a bicycle. Our brain keeps a vast array of information, images, and sensory experiences in long-term memory. Whatever it is we are trying to keep in our memories, whether a beautiful song or a list of chemistry vocabulary terms, must first come into our brains in short-term memory. If we want these fleeting ideas to transfer into long-term memory, we have to do some work, such as causing frequent exposure to the information over time (such as studying the terms every day for a period of time or the repetition you performed to memorize multiplication tables or spelling rules) and some relevant manipulation for the information.

According to Alison Preston of the University of Texas at Austin's Center for Learning and Memory, "A short-term memory's conversion to a long-term memory requires changes within the brain . . . and result[s] in changes to neurons (nerve cells) or sets of neurons. . . . For example, new synapses—the connections between neurons through which they exchange information—can form to allow for communication between new networks of neurons. Alternatively, existing synapses can be strengthened to allow for increased sensitivity in the communication between two neurons."⁴

When you work to convert your thoughts into memories, you are literally *changing your mind*. Much of this brain work begins in the part of the brain called the *hippocampus*. Preston continues, "Initially, the hippocampus works in concert with sensory-processing regions distributed in the neocortex (the outermost layer of the brain) to form the new memories. Within the neocortex, representations of the elements that constitute an event in our life are distributed across multiple brain regions according to their content. . . . When a memory is first formed, the hippocampus rapidly combines this distributed information into a single memory, thus acting as an index of representations in the sensory-processing regions. As time passes, cellular and molecular changes allow for the strengthening of direct connections among the neocortical regions, enabling access to the memory independent of the hippocampus."

We learn the lyrics of a favorite song by singing and/or playing the song over and over. That alone may not be enough to get that song into the coveted long-term memory area of our brain, but if we have an emotional connection to the song, such as a painful breakup or a life-changing proposal that occurred while we were listening to the song, this may help. Think of ways to make your study session memorable and create connections with the information you need to study. That way, you have a better chance of keeping your study material in your memory so you can access it whenever you need it.

Analysis Question

What are some ways you convert short-term memories into long-term memories?

Do your memorization strategies differ for specific courses (e.g., how you remember for math or history)?

Obstacles to Remembering

If remembering things we need to know for exams or for learning new disciplines were easy, no one would have problems with it, but students face several significant obstacles to remembering, including a persistent lack of sleep and an unrealistic reliance on cramming. Life is busy and stressful for all students, so you have to keep practicing strategies to help you study and remember successfully, but you also must be mindful of obstacles to remembering.

Lack of Sleep

Let's face it, sleep and college don't always go well together. You have so much to do! All that reading, all those papers, all those extra hours in the science lab or tutoring center or library! And then we have the social and emotional aspects of going to school, which may not be the most critical aspect of your life as you pursue more education but are a significant part of who you are. When you consider everything you need to attend to in college, you probably won't be surprised that sleep is often the first thing we give up as we search for more time to accomplish everything we're trying to do. That seems reasonable—just wake up an hour earlier or stay up a little later. But you may want to reconsider picking away at your precious sleep time.

Sleep benefits all of your bodily functions, and your brain needs sleep time to dream and rest through the night. You probably can recall times when you had to do something without adequate sleep. We say things like “I just can't wake up” and “I'm walking around half asleep.”

In fact, you may actually be doing just that. Lack of sleep impairs judgment, focus, and our overall mood. Do you know anyone who is always grumpy in the morning? A fascinating medical study from the University of California Los Angeles (UCLA) claims that sleep deprivation is as dangerous as being drunk, both in what it does to our bodies and in the harm we may cause to ourselves and others in driving and performing various daily tasks.⁵⁶

If you can't focus well because you didn't get enough sleep, then you likely won't be able to remember whatever it is you need to recall for any sort of studying or test-taking situation. Most exams in a college setting go beyond simple memorization, but you still have a lot to remember for exams. For example, when Saanvi sits down to take an exam on introductory biology, she needs to recall all the subject-specific vocabulary she read in the textbook's opening chapters, the general connections she made between biological studies and other scientific fields, and any biology details introduced in the unit for which she is taking the exam.

Trying to make these mental connections on too little sleep will take a large mental toll because Saanvi has to concentrate even harder than she would with adequate sleep. She isn't merely tired; her brain is not refreshed and primed to conduct difficult tasks. Although not an exact comparison, think about when you overtax a computer by opening too many programs simultaneously. Sometimes the programs are sluggish or slow to respond, making it difficult to work efficiently; sometimes the computer shuts down completely and you have to reboot the entire system. Your body is a bit like that on too little sleep.

On the flip side, though, your brain on adequate sleep is amazing, and sleep can actually assist you in making connections, remembering difficult concepts, and studying for exams. The exact reasons for this is still a serious research project for scientists, but the results all point to a solid connection between sleep and cognitive performance.

If you're interested in learning more about this research, the American Academy of Sleep Medicine (AASM) is a good place to start. One article is entitled “College Students: Getting Enough Sleep Is Vital to Academic Success.”

Analysis Question

How long do you sleep every night on average? Do you see a change in your ability to function when you haven't had enough sleep? What could you do to limit the number of nights with too little sleep?

Downside of Cramming

At least once in their college careers, most students will experience the well-known pastime called *cramming*. See if any of this is familiar: Shelley has lots of classes, works part-time at a popular restaurant, and is just amazingly busy, so she puts off serious study sessions day after day. She isn't worried because she has set aside time she would have spent sleeping to cram just before the exam. That's the idea anyway. Originally, she planned to stay up a little late and study for four hours from 10 p.m. to 2 a.m. and still get several hours of refreshing sleep. But it's Dolphin Week or Beat State Day or whatever else comes up, and her study session doesn't start until midnight—she'll pull an *all-nighter* (to be more precise, this is actually an *all-really-early-morning-er*, but it doesn't quite have the same ring to it). So, two hours after her original start time, she tries to *cram* all the lessons, problems,

and information from the last two weeks of lessons into this one session. Shelley falls asleep around 3 a.m. with her notes and books still on her bed. After her late night, she doesn't sleep well and goes into the morning exam tired.

Shelley does OK but not great on the exam, and she is not pleased with her results. More and more research is showing that the stress Shelley has put on her body doing this, combined with the way our brains work, makes cramming a seriously poor choice for learning.

One sleep researcher, Dr. Susan Redline from Boston, says, "Sleep deficiency can affect mood and the ability to make memories and learn, but it also affects metabolism, appetite, blood pressure, levels of inflammation in the body and perhaps even the immune response."⁷

Your brain simply refuses to cooperate with cramming—it sounds like a good idea, but it doesn't work. Cramming causes stress, which can lead to paralyzing test anxiety; it erroneously supposes you can remember and understand something fully after only minimal exposure; and it overloads your brain, which, however amazing it is, can only focus on one concept at a time and a limited number of concepts all together for learning and retention.

Leading neuroscientist John Medina claims that the brain begins to wander at about 10 minutes, at which point you need a new stimulus to spark interest.⁸ That doesn't mean you can't focus for longer than 10 minutes; you just have to switch gears a lot to keep your brain engaged. Have you ever heard a speaker drone on about one concept for, say, 30 minutes without somehow changing pace to engage the listeners? It doesn't take much to re-engage—pausing to ask the listeners questions or moving to a different location in the room will do it—but without these subtle attention markers, listeners start thinking of something else. The same thing happens to you if you try to cram all reading, problem-solving, and note reviewing into one long session; your brain will wander.

WHAT STUDENTS SAY

1. Which of the following is your most common method of studying?
 - a. Reading or rereading the text or my class notes.
 - b. Watching videos of my instructor's lecture or other people discussing the topics.
 - c. Taking practice quizzes/tests.
 - d. Creating/using study tools (flashcards, mnemonic devices, etc.)
 - e. Working with a study group, tutor, or academic support.
2. Which of the following do you have the most difficulty remembering?
 - a. Vocabulary and facts (such as Biology vocab, Historical facts.)
 - b. Problem-solving methods (such as in Math)
 - c. Details from text and literature
 - d. Skills and processes (such as a lab technique or a building process)
 - e. Computer functions/locations/processes
 - f. Which formulas, processes, or categories to apply in situations (such as in Physics or Accounting)
3. How much anxiety do you feel when an exam or other major course evaluation is approaching?
 - a. A great deal
 - b. A lot
 - c. A moderate amount
 - d. A little
 - e. None at all

You can also take the anonymous What [Students Say](#) surveys to add your voice to this textbook. Your responses will be included in updates.

Students offered their views on these questions, and the results are displayed in the graphs below.

Which of the following is your most common method of studying?

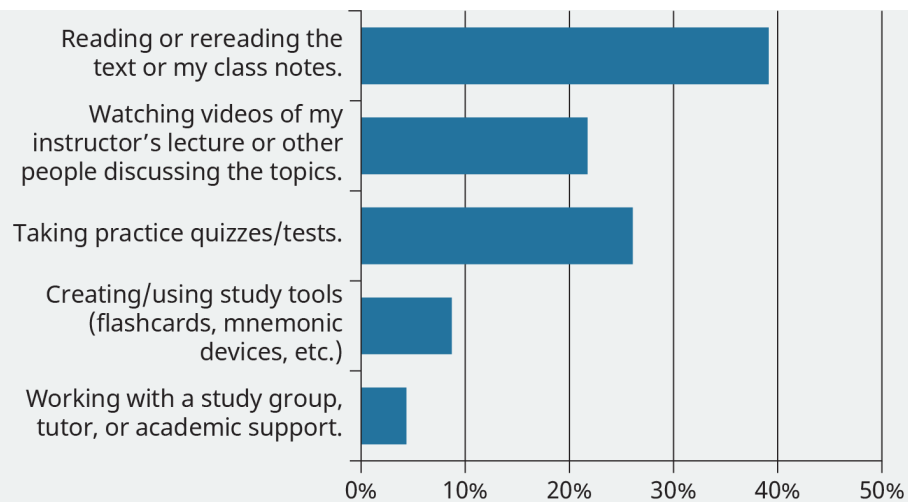


Figure 3.2.2

Which of the following do you have the most difficulty remembering?

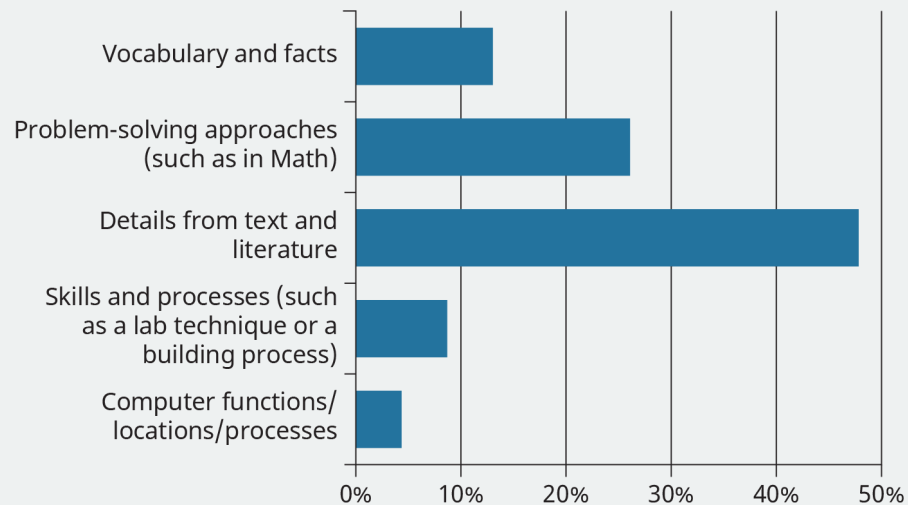


Figure 3.2.3

How much anxiety do you feel when an exam or other major course evaluation is approaching?

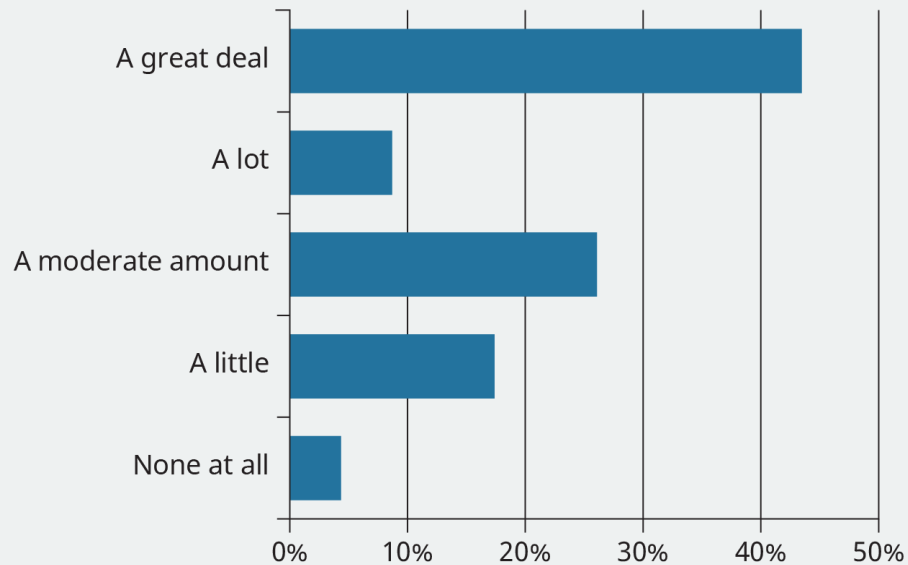


Figure 3.2.4

Determining When/What to Memorize

In the realm of learning and studying, some conditions warrant memorization as the most effective way to work with information. For instance, if you are expected to have a working knowledge of conversational French or Spanish, you will have to memorize some words. Simply knowing a long list of terms isn't going to help you order food in a café or ask for directions in a foreign country because you also need to understand the other language's grammar and have some sort of context for what needs to be said from your vocabulary list. But you cannot say the words in a different language if you cannot remember your vocabulary. From this scenario, you can assume that memorization is a good fit for some parts of language acquisition.

A worthwhile book on memory, thinking, and learning is a short study called *Make It Stick: The Science of Successful Learning* by Peter Brown, Henry Roediger, and Mark McDaniel. The authors conclude, after extensive research, that our attempts to speed up learning and make studying easier are not good ideas. Studying is hard work, and it should be. For learning to *stick*, we need to work hard to pull the information out of our memory and use it by continually pushing ourselves to accomplish increasingly difficult tasks.⁹

Footnotes

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3.3: Studying

Questions to consider

- How do you prepare yourself and your environment for successful studying?
- What study strategies will be most beneficial to you?
- What are learning preferences and strategies, and how can you leverage those to your advantage?

Preparing to Study

Studying is hard work, but you can still learn some techniques to help you be a more effective learner. Two major and interrelated techniques involve avoiding distractions to the best of your ability and creating a study environment that works to help you concentrate.

Avoiding Distractions

We have always had distractions—video games, television shows, movies, music, friends—even housecleaning can distract us from doing something else we need to do, like study for an exam. That may seem extreme, but sometimes vacuuming is the preferred activity to buckling down and working through calculus problems! Cell phones, tablets, and portable computers that literally bring a world of possibilities to us anywhere have brought *distraction* to an entirely new level. When was the last time you were with a large group of people when you didn't see at least a few people on devices?



Figure 3.3.1: Video games are a common distraction, but we need to be aware that even tedious activities like cleaning can be a distraction from studying.

When you study, your biggest challenge may be to block out all the competing noise. And letting go of that connection to our friends and the larger world, even for a short amount of time, can be difficult. Perhaps the least stressful way to allow yourself a distraction-free environment is to make the study session a definite amount of time: long enough to get a significant amount of studying accomplished but short enough to hold your attention.

You can increase that attention time with practice and focus. Pretend it is a professional appointment or meeting during which you cannot check e-mail or texts or otherwise engage with your portable devices. We have all become very attached to the ability to check in—anonously on social media or with family and friends via text, chat, and calls. If you set a specific amount of time to study without interruptions, you can convince your wandering mind that you will soon be able to return to your link to the outside world. Start small and set an alarm—a 30-minute period to review notes, then a brief break, then another 45-minute study session to quiz yourself on the material, and so on.

When you prepare for your optimal study session, remember to do these things:

- Put your phone out of sight—in another room or at least some place where you will not see or hear it vibrate or ring. Just flipping it over is not enough.
- Turn off the television or music (more on that in the next section).

- Unless you are deliberately working with a study group, study somewhere alone if possible or at least away from others enough to not hear them talking.

If you live with lots of other people or don't have access to much privacy, see if you can negotiate some space alone to study. Ask others to leave one part of the house or an area in one room as a quiet zone during certain hours. Ask politely for a specific block of time; most people will respect your educational goals and be willing to accommodate you. If you're trying to work out quiet zones with small children in the house, the bathtub with a pillow can make a fine study oasis.

Study Environment

You may not always be in the mood or inspired to study. And if you have a long deadline, maybe you can blow off a study session on occasion, but you shouldn't get into the habit of ignoring a strong study routine. Jane Austen once wrote in a letter, "I am not at all in a humor for writing; I must write on till I am." Sometimes just starting is the hard part; go ahead and begin. Don't wait around for your study muse; start working, and she'll show up.

Sometimes you just need to plop down and study whenever and wherever you can manage—in the car waiting for someone, on the bus, at the Little League field as you cheer on your shortstop. And that's OK if this is the exception. For long-term success in studying, though, you need a better study setting that will help you get the most out of your limited study time. Whatever your space limitations, carve out a place that you can dedicate to reading, writing, note taking, and reviewing. This doesn't need to be elaborate and expensive—all you truly need is a flat surface large enough to hold either your computer or writing paper, book or notes, pens/pencils/markers, and subject-specific materials you may need (e.g., stand-alone calculators, drawing tools, and notepads). Your space should be cool or warm enough for you to be comfortable as you study. What do you have now that you consider your study space? Is it set up for your optimal success?



Figure 3.3.2: Which is before, and which is after? (Credit: Ali West / Flickr / Attribution 2.0 Generic (CC-BY 2.0))

If it is at all possible, try to make this area exclusive to your study sessions and something you can leave set up all the time and a place out of the way of family or roommate traffic. For example, Martina thought setting up her study station on the dining room table was a good idea at first. The view was calming, and the table was big enough to spread out and could even hold all her materials to study architectural drawings, her favorite subject. But then she needed the table for a small family dinner party, so she had to find a cubbyhole to hide away her supplies with some needing to go into a closet in the next room. Now she was spread out over multiple study spaces. And the family TV was in an adjacent room, not visible from the table but certainly an auditory distraction. Martina ultimately decided to forgo her view and create a smaller station in an unused bedroom so she could leave her supplies out and have a quieter area. You may have to try out numerous places to determine what works best for you.

Wherever you study, try to make it a welcoming place you want to be in—not an uncomfortable environment that makes you want to just do the minimum you must complete and leave. You should include the basics: a good chair, a work surface, and whatever materials, books, notes, and other supplies you need for the subject you are studying. If you want to make it even more of a productive place, you can look in magazines for ideas or search the web to see how others have set up simple areas or more elaborate arrangements. Don't let decorating your workspace be an excuse to get out of studying!

You don't need an elaborate setting, but you may want to consider including a few effective additions if you have the space:

- small bulletin board for often-used formulas
- encouraging quotes or pictures of your goal
- whiteboard for brainstorming
- sticky notes for reminders in texts and notes
- file holder for most-used documents

- bookshelf for reference books

Activity

Describe every element in your ideal study environment and explain why it's there as well as how it will make more efficient use of your time, limit distractions, or in some other way strengthen your ability to study.

After you have described your ideal study environment, think about how you can adapt that environment if you cannot be in your favorite place to study. How do you *make your own space* in the library, a student lounge, or a dedicated space on campus for student studying?

Debunking Study Myths

MYTH #1: You can multitask while studying.

How many times do you eat in the car? Watch TV while you write out a grocery list? Listen to music while you cook dinner? What about type an e-mail while you're on the phone with someone else and jot down notes about the call? The common term for this attempt to do more than one thing at a time is multitasking, and almost everyone does it at some point. On some days, you simply cannot accomplish all that you want to get done, so you double up. The problem is, multitasking doesn't really work. Of course, it exists, and we do it. For instance, we walk and chew gum or drive and talk, but we are not really thinking about two or more distinct things or doing multiple processes simultaneously.

MYTH #2: Highlighting main points of a text is useful.

Another myth of studying that seems to have a firm hold is that the idea of highlighting text—in and of itself—is the best way to review study material. It is one way, and you can get some benefit from it, but don't trick yourself into spending too much time on this surface activity and consider your study session complete. Annotating texts or notes is a first-step type of study practice. If you allow it to take up all your time, you may want to think you are fully prepared for an exam because you put in the time. Actually, you need much more time reviewing and retrieving your lessons and ideas from the text or class lecture as well as quizzing yourself to accomplish your goal of learning so you can perform well on the exam. Highlighting is a task you can do rather easily, and it makes you feel good because you are actively engaging with your text, but true learning needs more steps.

MYTH #3: Studying effectively is effortless.

There is nothing effortless, or even pleasant at times, about studying. This is why so many students don't put in the time necessary to learn complex material: it takes time, effort, and, in some cases, a little drudgery. This is not to say that the outcome, learning—and maybe making an A—is not pleasant and rewarding. It is just that when done right, learning takes focus, deliberate strategies, and time. Think about a superstar athlete who puts in countless hours of drills and conditioning so that she makes her work on the field look easy. If you can also *enjoy* the studying, the skill development, and the knowledge building, then you will most likely be more motivated to do the work.

Analysis Question

When are you most liable to multitask? How could you be more aware of this practice and try to eliminate it, especially when it comes to studying? How can you make your initial text highlighting more time efficient so you can include other study practices?

Study Strategies

Everyone wishes they had a better memory or a stronger way to use memorization. You can make the most of the memory you have by making some conscious decisions about how you study and prepare for exams. Incorporate these ideas into your study sessions:

Practicing effective memorization is when you use a trick, technique, or strategy to recall something—for another class, an exam, or even to bring up an acquaintance's name in a social situation. Really whatever works for you to recall information is a good tool to have. You can create your own quizzes and tests to go over material from class. You can use mnemonics to jog your memory. You can work in groups to develop unique ways to remember complex information. Whatever methods you choose to enhance your memory, keep in mind that repetition is one of the most effective tools in any memory strategy. Do whatever you do over and over for the best results.

Using Mnemonics

Mnemonics (pronounced new-monics) are a way to remember things using reminders. Did you learn the points of the compass by remembering NEWS (north, east, west, and south)? Or the notes on the music staff as FACE or EGBDF (every good boy does fine)? These are mnemonics. When you're first learning something and you aren't familiar with the foundational concepts, these help you bring up the information quickly, especially for multistep processes or lists. After you've worked in that discipline for a while, you likely don't need the mnemonics, but you probably won't forget them either.

Here are some familiar mnemonics you may find useful:



Figure 3.3.3

You can certainly make up your own mnemonics, but be careful that your reminder isn't so complex and convoluted that it is more difficult to remember than the information you were relating it to!

Analysis Question

Do you have other mnemonics that help you remember difficult material? What are they? How have they helped you with remembering important things?

Practicing Concept Association

When you study, you're going to make connections to other things—that's a good thing. It shows a highly intelligent ability to make sense of the world when you can associate like and even somewhat unlike components. If, for instance, you were reading Martin Luther King Jr.'s "Letter from a Birmingham Jail," and you read the line that he had been in Birmingham, you may remember a trip you took with your family last summer through Alabama and that you passed by and visited the civil rights museum in Birmingham. This may remind you of the different displays you saw and the discussions you had with your family about what had happened concerning civil rights in the 1950s, '60s, and '70s in the United States.

This is a good connection to make, but if your assignment is to critique the literary aspects of King's long epistle, you need to be able to come back to the actual words of the letter and see what trends you can see in his writing and why he may have used his actual words to convey the powerful message. The connection is fine, but you can't get lost in going down rabbit holes that may or may not be what you're supposed to be doing at the time. Make a folder for this assignment where you can put things such as a short summary of your trip to Alabama. You may eventually include notes from this summary in your analysis. You may include something from a website that shows you information about that time period. Additionally, you could include items about Martin Luther King Jr.'s life and death and his work for civil rights. All of these elements may help you understand the significance of this one letter, but you need to be cognizant of what you're doing at the time and remember it is not usually a good idea to just try to

keep it all in your head. The best idea is to have a way to access this information easily, either electronically or in hard copy, so that if you are able to use it, you can find it easily and quickly.

Generating Idea Clusters

Like mnemonics, idea clusters are nothing more than ways to help your brain come up with ways to recall specific information by connecting it to other knowledge you already have. For example, Andrea is an avid knitter and remembers how to create complicated stitches by associating them with nursery rhymes she read as a child. A delicate stitch that requires concentration because it makes the yarn look like part of it is hiding brings to mind Red Riding Hood, and connecting it to that character helps Andrea recall the exact order of steps necessary to execute the design. You can do the same thing with song lyrics, lines from movies, or favorite stories where you draw a connection to the well-known phrase or song and the task you need to complete.

Application

Choose one of the following, and create an idea cluster to group and organize:

- Example A: aviation jobs in North America
- Example B: the use of analytics in sports to determine team rosters
- Example C: how social media affects political campaigns

Start the idea cluster with the topic circled in the middle of the page. For Example A, you might make one cluster off the main circle for specific positions; you could add another cluster for salary ranges and another for geographic regions.

Three Effective Study Strategies

There are more than three study strategies, but focusing on the most effective strategies will make an enormous difference in how well you will be able to demonstrate learning (also known as “acing your tests”). Here is a brief overview of each of the three strategies:

- Spacing—This has to do with *when* you study. Hint: Don’t cram; study over a period of days, preferably with “breaks” in between.
- Interleaving—This has to do with *what* you study. Hint: Don’t study just one type of content, topic, chapter, or unit at a time; instead, mix up the content when you study.
- Practice testing—This has to do with *how* you study. Hint: Don’t just reread content. You must quiz or test your ability to retrieve the information from your brain.

Spacing

We all know that cramming is not an effective study strategy, but do we know why? Research on memory suggests that giving yourself time in between study sessions actually helps you forget the information. And forgetting, which sounds like it would be something you *don’t* want to do, is actually good for your ability to remember information long-term. That’s because every time you forget something, you need to relearn it, leading to gains in your overall understanding and “storage” of the material. The table below demonstrates how spacing works. Assume you are going to spend about four hours studying for a Sociology exam. Cramming would have you spending most of those four hours the night before the exam. With spacing, on the other hand, you would study a little bit each day.

Table 3.3.4

Spacing							
	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Cramming				Study for 1 hour	Study for 3		
Spacing	Study for 1 hour		Study for 30 minutes	Study for 1 hour	Study for 90 minutes		

Interleaving

One particular studying technique is called interleaving, which calls for students to mix up the content that is being studied. This means not just spending the entire study session on one sort of problem and then moving on to a different sort of problem at a later

time.

If you take the schedule we used for the spacing example above, we can add the interleaving concepts to it. Notice that interleaving includes revisiting material from a previous chapter or unit or revisiting different types of problems or question sets. The benefit is that your brain is “mixing up” the information, which can sometimes lead to short-term forgetting but can lead to long-term memory and learning.

Table 3.3.5

Interleaving					
Sunday	Monday	Tuesday	Wednesday	Thursday	Friday
Reread Sociology, Chapter 1. Reorganize notes		Reread Sociology, Chapter 1 and 2 Take Ch 1 online quiz. Create Chapter 2 concept map	Reread Sociology, Chapters 1-3. Take online quizzes for chapters 2 and 3. Reorganize notes. Create practice test	Reread notes. Review items missed on online quizzes. Take practice test and review challenge areas.	TEST in sociology, Chapters 1-3

Practice Testing

You can do a practice “test” in two ways. One is to test yourself as you are reading or taking in information. This is a great way to add a little variety to your studying. You can ask yourself what a paragraph or text section means as you read. To do this, read a passage in a text, cover up the material, and ask yourself, “What was the main idea of this section?” Recite aloud or write down your answer, and then check it against the original information.

Another, more involved, way to practice test is to create flashcards or an actual test by writing a test. This takes more time, but there are online programs such as Quizlet that make it a little easier. Practice testing is an effective study strategy because it helps you practice *retrieving* information, which is what you want to be able to do when you are taking the real test.

One of the best ways to learn something is to teach it to someone else, so ask a friend or family member if you can explain something to them, and *teach* them the lesson. You may find you know more about the subject than you thought . . . or you may realize quickly that you need to do more studying. Why does teaching someone else rank as one of the most effective ways to learn something? It is a form of practice testing that requires you to demonstrate you know something in front of someone else! No one wants to look like they don’t know what they are talking about, even if it your audience is another classmate.

Recognizing Strengths/Weaknesses of Preferred Study Approaches

Most children don’t learn to ride a bicycle by reading a manual; they learn by watching other kids, listening to instructions, and getting up on the seat and learning to balance—sometimes with training wheels or a proud parent holding on, but ultimately without any other support. They may fall over and feel insecure, but usually, they learn to make the machine go. Most of us employ multiple methods of study all the time. You usually only run into trouble if you stubbornly rely on just one way to learn or study and the material you’re studying or the task you want to accomplish doesn’t lend itself to that preference. You can practice specific strategies to help you learn in your preferred learning approach. Can you think of a time when the way you usually study a situation didn’t work?

When deciding on a study approach, consider what you know about the material and the type of knowledge it involves. Is it a group of concepts related to problem-solving methods, such as those you’d find in a physics class? Or is it a literary analysis of a novel? Consider as many elements as possible about the material -- and the way the material will be assessed -- to help choose a study approach.

You should also consider your instructor’s preferred method of teaching and learning. Watching the way they teach lessons or convey necessary course information to the class. Do they almost always augment lessons with video clips to provide examples or create a memorable narrative? Do they like to show you how something works by demonstrating and working with their hands—for instance, assembling a piece of equipment by taking it apart and putting it back together again? Echoing their teaching approach may help your study. That doesn’t mean you have to change your entire learning approach to match your instructors’ methods. Many instructors understand that their students will have different ways of learning and try to present information in multiple ways.

Practicing Active Continuous Improvement for All Preferences

You can certainly learn through specific approaches or according to specific preferences, but you will also need to adapt to different situations, skills, and subject areas. Don't limit yourself to thinking you can *only* learn one way or another. That mindset induces anxiety when you encounter a learning situation that doesn't match your preference. What if your instructor *only* uses a spoken lecture to teach concepts in your chemistry class, and you consider yourself a visual learner? Or what if the only method presented to you for learning mathematical computations is to see videos of others working problems, and you're more hands-on? You may have to concentrate in a different way or devise other strategies to learn, but you can do it. In fact, you should sometimes work on the styles/preferences that you feel are your least favorite; it will actually strengthen your overall ability to learn and retain information.¹⁰

Dr. Stephen Covey, famous leadership coach and businessman, called this attention to knowing and honing all your skill sets, not just your favorites, *sharpening the saw*. He advised that people should be aware of their strengths but should always hone their weaknesses by saying, "We must never become too busy sawing to take time to sharpen the saw."¹¹ For instance, in the chemistry lecture example, you may need to take good notes from the spoken lecture and then review those notes as you sketch out any complex ideas or formulas. If the math videos are not enough for you to grasp difficult problems, you may ask for or find your own problems for additional practice covering that particular mathematical concept to solve on your own.

Footnotes

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3.4: Test Taking

Questions to consider

- What are the differences between test prep and taking the actual test?
- How can you take a *whole person* approach to test taking?
- What can you do on test day to increase your confidence and success?
- What should you know about test anxiety?

Once you are practicing good study habits, you'll be better prepared for actual test taking. Since studying and test taking are both part of learning, honing your skills in one will help you in the other.

Probably the most obvious differences between your preparation for an exam and the actual test itself is your level of urgency and the time constraints. A slight elevation in your stress level can actually be OK for testing—it keeps you focused and *on your game* when you need to bring up all the information, thinking, and studying to show what you've learned. Properly executed, test preparation mixed in with a bit of stress can significantly improve your actual test-taking experience.

Preparation vs. Actual Test

You can replicate the effective sense of urgency an actual test produces by including timed writing into your study sessions. You don't need all of your study time to exactly replicate the test, but you would be well served to find out the format of the exam in advance and practice the skills you'll need to use for the various test components. On one early exam in history, Stuart learned the prof was going to include several short-answer essay questions—one for each year of the time period covered. Stuart set up practice times to write for about 15 to 20 minutes on significant events from his notes because he estimated that would be about how much time he could devote out of the hour-long testing session to write one or two required short-answer questions. He would write a prompt from his notes, set a timer, and start writing. If you're ready and you have practiced and know the material, 20 minutes is adequate to prepare, draft, and revise a short response, but you don't have a lot of extra time.

Likewise, in a math exam, you will need to know what kinds of problems you will have to solve and to what extent you'll need to show your computational work on the exam. If you are able to incorporate this sort of timed problem-solving into your study time, you'll be more prepared and confident when you actually come to the exam. Making yourself adhere to a timed session during your study can only help. It puts a sense of urgency on you, and it will help you to find out what types of problems you need to practice more than ones that perhaps you're more comfortable solving.

Leveraging Study Habits for Test Prep

In your mind, you probably know what you need to do to be prepared for tests. Occasionally, something may surprise you—emphasis on a concept you considered unimportant or a different presentation of a familiar problem. But those should be exceptions. You can take all your well-honed study habits to get ready for exams. Here's a checklist for study and test success for your consideration:



Figure 3.4.1:

Read this list with each separate class in mind, and check off the items you already do. Give yourself one point for every item you checked. If you always take the success steps—congratulations! They are not a guarantee, but doing the steps mindfully will give you a nice head start. If you do fewer than five of the steps—you have some work to do. But recognition is a good place to start, and you can incorporate these steps starting now.

As strange as this may sound, you can find some interesting research articles online about using the taste or smell of peppermint to increase memory, recall, and focus. Read more at: <http://naturalsociety.com/mint-scent...gnition-memory>. While sucking on a peppermint disk won't replace studying, why not experiment with this relatively easy idea that seems to be gaining some scientific traction?

Whole Person Approach to Testing

Just because you are facing a major exam in your engineering class (or math or science or English class) doesn't mean everything else in your life comes to a stop. Perhaps that's somewhat annoying, but that's reality. Allergies still flare up, children still need to eat, and you still need to sleep. You must see your academic life as one segment of who you are—it's an important segment, but just one aspect of who you are as a whole person. Neela tries to turn off everything else when she has exams coming up in her nursing program, which is pretty often. She ignores her health, puts off her family, tries to reschedule competing work tasks, and focuses all her energy on the pending exam. On the surface, that sounds like a reasonable approach, but if she becomes really sick by ignoring a minor head cold, or if she misses an important school deadline for one of her children, Neela risks making matters worse by attempting to compartmentalize so strictly. Taking care of her own health by eating and sleeping properly; asking for help

in other aspects of her busy life, such as attending to the needs of her children; and seeing the big picture of how it all fits together would be a better approach. Pretending otherwise may work sporadically, but it is not sustainable for the long run.

A whole person approach to testing takes a lot of organization, scheduling, and attention to detail, but the life-long benefits make the effort worthwhile.

Establishing Realistic Expectations for Test Situations

Would you expect to make a perfect pastry if you've never learned how to bake? Or paint a masterpiece if you've never tried to work with paints and brushes? Probably not. But often we expect ourselves to perform at much higher levels of achievement than that for which we've actually prepared. If you become very upset and stressed if you make any score lower than the highest, you probably need to reevaluate your own expectations for test situations. Striving to always do your best is an admirable goal. Realistically knowing that your current *best* may not achieve the highest academic ratings can help you plot your progress.

Realistic continuous improvement is a better plan, because people who repeatedly attempt challenges for which they have not adequately prepared and understandably fail (or at least do not achieve the desired highest ranking) often start moving toward the goal in frustration. They simply quit. This doesn't mean you settle for mediocre grades or refrain from your challenges. It means you become increasingly aware of yourself and your current state and potential future. Know yourself, know your strengths and weaknesses, and be honest with yourself about your expectations.

Understanding Accommodations and Responsibilities

As with so many parts of life, some people take exams in stride and do just fine. Others may need more time or change of location or format to succeed in test-taking situations. With adequate notice, most faculty will provide students with reasonable accommodations to assist students in succeeding in test situations. If you feel that you would benefit from receiving these sorts of accommodations, first speak with your instructor. You may also need to talk to a student services advisor for specific requirements for accommodations at your institution.

If you need accommodations, you are responsible for understanding what your specific needs are and communicating your needs with your instructors. Before exams in class, you may be allowed to have someone else take notes for you, receive your books in audio form, engage an interpreter, or have adaptive devices in the classroom to help you participate. Testing accommodations may allow for additional time on the test, the use of a scribe to record exam answers, the use of a computer instead of handwriting answers, as well as other means to make the test situation successful. Talk to your instructors if you have questions about testing accommodations.

Prioritizing Time Surrounding Test Situations

Keep in mind that you don't have any more or less time than anyone else, so you can't *make time* for an activity. You can only use the time everyone gets wisely and realistically. Exams in college classes are important, but they are not the only significant events you have in your classes. In fact, everything leading up to the exam, the exam itself, and the post-exam activities are all one large continuum. Think of the exam as an event with multiple phases, more like a long-distance run instead of a 50-yard dash. Step back and look at the big picture of this timeline. Draw it out on paper. What needs to happen between now and the exam so you feel comfortable, confident, and ready?

If your instructor conducts some sort of pre-exam summary or prep session, make sure to attend. These can be invaluable. If this instructor does not provide that sort of formal exam prep, create your own with a group of classmates or on your own. Consider everything you know about the exam, from written instructions to notes you took in class, including any experiential notes you may have from previous exams, such as the possibility of bonus points for answering an extra question that requires some time management on your part. You can read more about time management in Chapter 3.

Get Connected

Which apps can help you study for a test, increase your memory, and even help you overcome test anxiety?

Look for a free online gaming app to reduce stress and anxiety. The games retrain your brain to think more positively, reducing stress to help you focus on the experiences around you.

Games like solitaire, hangman, and Simon Says all build on your memory, keeping it sharp and active. There are loads of fun, free online memory games you can use to make time wasting a little less wasteful. For more than 250 options, visit the [Memory Improvement Tips](#) website.

YouTube might be able to help you dig into a research topic or find additional content to help you if you're struggling with a course. Their library of free lectures and content comes from some of the most highly respected universities around the world.

Chegg Prep is a flashcard-based self-quizzing resource. It provides millions of pre-made flashcards and decks organized by course and topic, which you can search, sort, bookmark, and use in a variety of ways. The service is free and does not require a login unless you'd like to save or create your own cards.

Test Day

Once you get to the exam session, try your best to focus on nothing but the exam. This can be very difficult with all the distractions in our lives. But if you have done all the groundwork to attend the classes, completed the assignments, and scheduled your exam prep time, you are ready to focus intently for the comparatively short time most exams last.

Arriving to class:

Don't let yourself be sidetracked right at the end. Beyond the preparation we've discussed, give yourself some more advantages on the actual test day:

- Get to the testing location a few minutes early so you can settle into your place and take a few relaxing breaths.
- Don't let other classmates interrupt your calmness at this point.
- Just get to your designated place, take out whatever supplies and materials you are allowed to have, and calm your mind.

Taking the test:

Once the instructor begins the test:

- Listen carefully for any last-minute oral directions that may have changed some detail on the exam, such as the timing or the content of the questions.
- As soon as you receive the exam sheet or packet, make a quick scan over the entire test.
- Don't spend a lot of time on this initial glance, but make sure you are familiar with the layout and what you need to do.
- Using this first review, decide how you will allocate your available time for each section.
- You can even jot down how many minutes you can allow for the different sections or questions.

Then for each section, if the exam is divided this way, be sure you **read the section directions** very carefully so you don't miss an important detail. For example, instructors often offer options—so you may have four short-answer questions from which to choose, but you only need to answer two of them. If you had not read the directions for that section, you may have thought you needed to provide answers to all four prompts. Working on extra questions for which you likely will receive no credit would be a waste of your limited exam time. The extra time you spend at the beginning is like an investment in your overall results.

Answer every required question on the exam. Even if you don't complete each one, you may receive some credit for partial answers. Whether or not you can receive partial credit would be an excellent question to ask before the exam during the preparation time. If you are taking an exam that contains multiple-choice questions, go through and answer the questions about which you are the most confident first.

Read the entire question carefully even if you think you know what the stem (the introduction of the choices) says, and read all the choices. Skip really difficult questions or ones where your brain goes blank. Then you can go back and concentrate on those skipped ones later after you have answered the majority of the questions confidently. Sometimes a later question will trigger an idea in your mind that will help you answer the skipped questions.

And, in a similar fashion to spending a few minutes right at the beginning of the test time to read the directions carefully and identify the test elements, **allow yourself a few minutes at the end of the exam session to review your answers.** Depending on what sort of exam it is, you can use this time to check your math computations, review an essay for grammatical and content errors, or answer the difficult multiple-choice questions you skipped earlier. Finally, **make sure you have completed the entire test:** check the backs of pages, and verify that you have a corresponding answer section for every question section on the exam. It can be easy to skip a section with the idea you will come back to it but then forget to return there, which can have a significant impact on your test results.

After the Test

As you leave the exam room, the last thing you may want to think about is that particular test. You probably have numerous other assignments, projects, and life obligations to attend to, especially if you pushed some of those off to study for this completed exam.

Give yourself some space from this exam, but only for the duration of the time when your instructor is grading your exam. Once you have your results, study them—whether you did really well (Go, you!) or not as well as you had hoped (Keep your spirits up!). Both scenarios hold valuable information if you will use it.

Thandie had a habit of going all-out for exams before she took them, and she did pretty well usually, but once the instructor passed back the graded tests, she would look at the letter grade, glance half-heartedly at the instructor's comments, and toss the exam away, ready to move on to the next chapter, section, or concept. A better plan would be to learn from her exam results and analyze both what she did well and where she struggled. After a particularly unimpressive exam outing in her statistics class, Thandie took her crumpled-up exam to the campus tutoring center, where the tutor reviewed the test with her section by section. Together they discovered that Thandie did particularly well on the computational sections, which she admitted were her favorites, and not well at all on the short-answer essay questions that she did not expect to find in a stats class, which in her experience had been more geared toward the mathematical side of solving statistical problems.

Going forward in this class, Thandie should practice writing out her explanations of how to compute the problems and talk to her instructor about ways to hone this skill. This tutoring session also proved to Thandie the benefit of holding on to important class papers—either electronically or in hard copy, depending on the class setup—for future reference. For some classes, you probably don't need to keep every scrap of paper (or file) associated with your notes, exams, assignments, and projects, but for others, especially for those in your major, those early class materials may come in very handy in your more difficult later undergraduate courses or even in grad school when you need a quick refresher on the basic concepts.

Test Anxiety



Figure 3.4.2: Text anxiety can be a common occurrence, but you can use strategies to manage it.

Test anxiety is very real. You may know this firsthand. Almost everyone gets a little nervous before a major exam, in the same way most people get slightly anxious meeting a new potential date or undertaking an unfamiliar activity. We second-guess whether we're ready for this leap, if we prepared adequately, or if we should postpone this potentially awkward situation. And in most situations, testing included, that reasonable level of nervous anticipation can be a good thing—enhancing your focus and providing you with a bit of bravado to get you through a difficult time.

Test anxiety, however, can cause us to doubt ourselves so severely that we underperform or overcompensate to the point that we do not do well on the exam. Don't despair; you can still succeed if you suffer from test anxiety. The first step is to understand what it is and what it is not, and then to practice some simple strategies to cope with your anxious feelings relative to test taking. Whatever you do, don't use the label *test anxiety* to keep you from your dreams of completing your education and pursuing whatever career you have your eyes on. You are bigger than any anxiety.

Understanding Test Anxiety

If someone tries to tell you that test anxiety is *all in your head*, they're sort of right. Our thinking is a key element of anxiety of any sort. On the other hand, test anxiety can manifest itself in other parts of our bodies as well. You may feel queasy or light-headed if you are experiencing test anxiety. Your palms may sweat, or you may become suddenly very hot or very cold for no apparent reason. At its worst, test anxiety can cause its sufferers to experience several unpleasant conditions including nausea, diarrhea, and shortness of breath. Some people may feel as though they may throw up, faint, or have a heart attack, none of which would make going into a testing situation a pleasant idea. You can learn more about symptoms of test anxiety from the Anxiety and Depression Association of America that conducts research on this topic.¹²

Back to our minds for a minute. We think constantly, and if we have important events coming up, such as exams, but other significant events as well, we tend to think about them seemingly all the time. Almost as if we have a movie reel looping in our heads, we can anticipate everything that may happen during these events—both sensational results and catastrophic endings. What if you oversleep on the test day? What if you're hit by a bus on the way to campus? What if you get stung by a mysterious insect and have to save the world on the very day of your exam?

How about the other way? You win the lottery! Your screenplay is accepted by a major publisher! You get a multimillion-dollar record deal! It could happen. Typically, though, life falls somewhere in between those two extremes, unless you live in an action movie. Our minds, however, (perhaps influenced by some of those action movies or spy novels we've seen and read) often gravitate to those black-and-white, all-or-nothing results. Hence, we can become very nervous when we think about taking an exam because if we do really poorly, we think, we may have to face consequences as dire as dropping out of school or never graduating. Usually, this isn't going to happen, but we can literally make ourselves sick with anxiety if we dwell on those slight possibilities. You actually may encounter a few tests in your academic careers that are so important that you have to alter your other life plans temporarily, but truly, this is the exception, not the rule. Don't let the most extreme and severe result take over your thoughts. Prepare well and do your best, see where you land, and then go from there.

Using Strategies to Manage Test Anxiety

You have to work hard to control test anxiety so it does not take an unhealthy hold on you every time you face a test situation, which for many of you will last well into your careers. One of the best ways to control test anxiety is to be prepared for the exam. You can control that part. You can also learn effective relaxation techniques including controlled breathing, visualization, and meditation. Some of these practices work well even in the moment: at your test site, take a deep breath, close your eyes, and smile—just bringing positive thoughts into your mind can help you meet the challenges of taking an exam without anxiety taking over.

The tests in the corporate world or in other career fields may not look exactly like the ones you encounter in college, but professionals of all sorts take tests routinely. Again, being prepared helps reduce or eliminate this anxiety in all these situations. Think of a presentation or an explanation you have provided well numerous times—you likely are not going to feel anxious about this same presentation if asked to provide it again. That's because you are prepared and know what to expect. Try to replicate this feeling of preparation and confidence in your test-taking situations.

Many professions require participants to take frequent licensing exams to prove they are staying current in their rapidly changing work environments, including nursing, engineering, education, and architecture, as well as many other occupations. You have tools to take control of your thinking about tests. Better to face it head-on and let test anxiety know who's in charge!

Footnotes

- Retegui, Jo-Ann. "Relationship between anxiety and standardized patient test performance in the medicine clerkship." *Journal of general internal medicine* vol. 21,5 (2006): 415-8. doi:10.1111/j.1525-1497.2006.00419.

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3.5: Summary

Studying and taking tests will always be a large part of college, so learning now to do these well can only help you be more successful. Experts provide us with many tools, techniques, and ideas to use when we determine how best to study, use our memories effectively, and prepare to take exams. You can help yourself by taking these guidelines seriously and tracking your progress. If one strategy works better for you in some classes and another is more suited to a different course, keep that in mind when you begin to study. Use all the resources available to you, and you'll be well on your way to success in college.

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3.6: Career Connection

Studies have shown that parents contribute to test anxiety in children by drawing students' attention to the test day and increasing pressure to perform well. Do you think that worrying about an upcoming test is as harmful as anxiety while taking the test? What do you think can be done to minimize worry?

This [article](#) discusses how to help with test anxiety.

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3.7: Rethinking

Revisit the questions you answered at the beginning of the chapter, and consider one option you learned in this chapter that might change your answer to one of them.

How confident are you in your skills at preparing for and taking tests? Take this quick survey to figure it out, ranking questions on a scale of 1–4, 1 meaning “least like me” and 4 meaning “most like me.”

1. I believe I set aside enough time to prepare for tests.
 2. If I don’t set aside enough time, or if life gets in the way, I can usually cram and get similar results.
 3. I prefer to pull all-nighters. The adrenaline and urgency help me remember what I need come test time.
 4. I study my notes, highlight book passages, and use flash cards, but I still don’t feel like I’m as successful as I should be on tests.
-

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3.8: Where do you go from here?

Studying and test taking skills often need to evolve to meet the needs of college responsibilities. What would you like to learn more about? Choose a topic from the list below and create an annotated bibliography that would direct further research.

- the importance of memory in learning new material
 - strategies to increase memory
 - strategies to increase the effectiveness of studying
 - test anxiety
-

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CHAPTER OVERVIEW

4: Evaluate and Move Ahead

[4.1: Evaluate Your Learning](#)

[4.2: Develop a Growth Mindset](#)

[4.3: Put the Growth Mindset into Practice](#)

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4.1: Evaluate Your Learning

“We all need people who will give us feedback. That’s how we improve.” – **Bill Gates**

During the learning process, we have many opportunities to receive feedback about the quality of our learning and work. In the university environment, this often comes in the form of grades and instructor comments on assignments and exams. By using this feedback to evaluate your learning strategies in light of your goals, you will be able to make adjustments to move you towards your goals in current and future courses.

Reflecting mid-semester

An excellent time for self-evaluation is after you have received feedback on your first midterm exam or major assignment. Consider the following reflection questions at this stage in your course:

- What grade do I hope to achieve in this course? _____
- To what extent am I meeting my goal for the course at this point?
- What about my exam/assignment preparation worked well?
- What about my exam/assignment preparation did not work well? What do I want to change?
- How will what I have learned help me in the second half of the course?



Use Evaluation to Support Planning

Consider your use of learning resources. These include instructor office hours, online resources that supplement your textbook, peer tutors, and Learning Strategist consultations. Use the Stop-Start-Continue method to make your plan. If any of your current strategies are ineffective, you may wish to stop them and replace them with other study methods. Continue strategies that are currently effective, and start new strategies that you feel will support your success.

	Learning Strategies	Learning Resources
Stop		
Start		
Continue		



Image Credit: Graeme Robinson-Clogg

Reflecting at the End of a Course

The completion of a course is also an excellent time for reflection and evaluation. In addition to the questions in the midterm evaluation, consider the following:

1. How will what I have learned help me in my next courses?
2. How will I use what I have learned in my future career and other aspects of my life?

By reflecting on feedback and evaluating your learning regularly, you will avoid getting stuck in unproductive patterns. You will contribute to your own ongoing personal growth and development, supporting your success in future courses and other life endeavours.

[\[1\]](#) [\[2\]](#)

Try it!

[Download the evaluation template](#) to support you in the process of reflecting and moving ahead.

-
1. Chen, P., Chavez, O., Ong, D. C., & Gunderson, B. (2017). Strategic resource use for learning: A self-administered intervention that guides self-reflection on effective resource use enhances academic performance. *Psychological Science*, 28(6), 774–785. <https://doi.org/10.1177/0956797617696456>; ↵
 2. Tanner, K. D. (2012). Promoting student metacognition. *Cell Biology Education*, 11(2), 113–120. <https://doi.org/10.1187/cbe.12-03-0033>↵
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4.2: Develop a Growth Mindset

In any academic endeavor, you will encounter times when you are faced with obstacles or difficulties. Perhaps you are taking a course that you are finding particularly difficult. Perhaps you received some difficult feedback in a grade that was lower than you expected. How can you move ahead in a way that prepares you for success?

Everyone encounters setbacks at times. When this happens, you have a choice of possible responses. Some people respond to setbacks by concluding that they may lack the ability to complete the course successfully. Others respond by concluding that the course or instructor is unfair, and blame their setback on an external force beyond their control. These responses are associated with what is called a *fixed mindset*.

Others respond to setbacks and negative feedback by asking what they can learn from the experience. Their focus is less on achieving a specific grade or result, and more on learning as much as possible from their experiences in university. Individuals with this mindset, which is called a *growth mindset* are more able to recover from setbacks and to go on to achieve greater success.

How do these two mindsets compare?

Growth mindset	Fixed mindset
<ul style="list-style-type: none">• Intelligence is not fixed, but it can be developed over time• Difficult tasks are worth pursuing• Feedback, even if it offers correction, is beneficial to support future growth	<ul style="list-style-type: none">• Intelligence is fixed, and cannot be changed• If a task is difficult, it should be discontinued• Negative feedback should be avoided or minimized

A growth mindset is associated with successful learning. Why? The growth mindset principles are supported by what we know about the brain and learning. Adult brains continue to develop over time by through learning. Working to master complex material results in the development of additional neural connections. In other words, by learning difficult material, you can actually become smarter. If you believe that you are able to succeed by working hard, you are more able to persevere through the difficult moments in learning, and continue to make progress towards your learning goals. ^[2] ^[3]

[Try it!](#)

[Download the Growth Mindset Evaluation worksheet.](#)



Growth Mindset Evaluation

What mindset do you have? Indicate whether you agree or disagree with the following statements:

	Agree	Disagree
You cannot change the amount of intelligence you have.		
Even though you can learn new things, this doesn't change how intelligent you are.		
You are able to change the amount of intelligence you have throughout your life.		
Even though you are a certain kind of person now, you can still change the important parts of who you are.		

The first and second statements in this chart are characteristic ways of thinking of you have a fixed mindset. The third and fourth reflect a growth mindset.

1. Which mindset do you have now?

2. How can you continue to move towards a growth mindset?

(Adapted from: Dweck, C. S. (2008). *Mindset: the new psychology of success*. New York: Ballantine Books.)



Image Credit: Graeme Robinson-Clogg

1. Adapted from: Dweck, C. S. (2008). *Mindset: the new psychology of success*. New York: Ballantine Books. ↩
2. Paunesku, D., Walton, G. M., Romero, C., Smith, E. N., Yeager, D. S., & Dweck, C. S. (2015). Mind-set interventions are a scalable treatment for academic underachievement. *Psychological Science*, 26(6), 784–793. <https://doi.org/10.1177/0956797615571017> ↩
3. Yeager, D. S., & Dweck, C. S. (2012). Mindsets that promote resilience: When students believe that personal characteristics can be developed. *Educational Psychologist*, 47(4), 302–314. <https://doi.org/10.1080/00461520.2012.722805> ↩

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4.3: Put the Growth Mindset into Practice

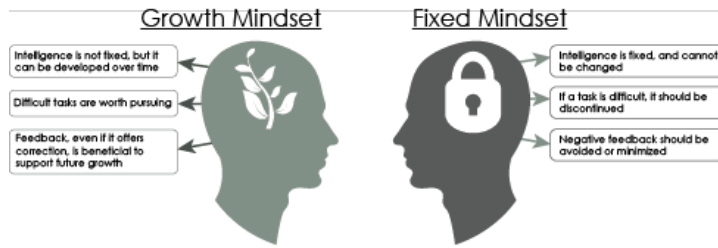


Image Credit: Graeme Robinson-Clogg

If you have discovered that you have a fixed mindset, consider how changing your thinking towards a growth mindset can influence your opportunities for successful learning and growth.

- (1) Adjust your self-talk. A key sentence to remember is “*I can’t do it — yet*”. Consider feedback as information to help you as you continue to grow towards a goal, not as a final evaluation of your ability to learn and achieve.
- (2) Respond to feedback differently. Rather than viewing feedback as criticism of who you are, consider feedback as an opportunity to grow.
- (3) Rather than quitting when you face setbacks, use them as an opportunity to adjust your approach. You may be learning that your current approach to learning is not leading to the success you desire. Seek out support from others, and try new ways of learning. Setbacks are an opportunity to learn about yourself and to discover what ways of working will be most effective for you.
- (4) Embrace challenging opportunities. Though it may at first seem easier to avoid situations that might be difficult or perhaps risk failure, embracing challenges leads to success in the long term. Consider how accepting challenges will help you become the person you want to be in the future. ^[1]

Try it!

Apply the thinking strategies above to a situation you are currently facing.

1. What was a challenging situation I faced this semester?
2. How can I think about it differently using a growth mindset?
3. What are the benefits to me of adopting this new way of thinking?

Extend Your Learning

Developing a growth mindset can make a powerful difference in your lifelong learning. Explore the following resources to deepen your understanding of this concept.

1. Are you interested in understanding more about your mindset? [Try this online assessment to identify whether you currently have a growth mindset.](#)
2. Explore [The Mindset Continuum infographic](#). As its author, James Anderson, emphasizes, fixed and growth mindsets are the end points on a spectrum of perspectives. Use this infographic to explore where you currently are on the mindset continuum, and identify areas for future growth.

1. Adapted from: UNSW Sydney. (n.d.). *Growth mindset*. Retrieved from <https://student.unsw.edu.au/growth-mindset>↗

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CHAPTER OVERVIEW

5: Decimals, Fractions and Percents

This chapter is from the [Support Course for Elementary Statistics](#) by Larry Green from Lake Tahoe Community College and includes sections from other open-source textbooks.

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5.1: Comparing Fractions, Decimals, and Percents

Learning Outcomes

1. Compare two fractions
2. Compare two numbers given in different forms

In this section, we will go over techniques to compare two numbers. These numbers could be presented as fractions, decimals or percents and may not be in the same form. For example, when we look at a histogram, we can compute the fraction of the group that occurs the most frequently. We might be interested in whether that fraction is greater than 25% of the population. By the end of this section we will know how to make this comparison.

Comparing Two Fractions

Whether you like fractions or not, they come up frequently in statistics. For example, a probability is defined as the number of ways a sought after event can occur over the total number of possible outcomes. It is commonly asked to compare two such probabilities to see if they are equal, and if not, which is larger. There are two main approaches to comparing fractions.

Approach 1: Change the fractions to equivalent fractions with a common denominator and then compare the numerators

The procedure of approach 1 is to first find the common denominator and then multiply the numerator and the denominator by the same whole number to make the denominators common.

Example 5.1.1

Compare: $\frac{2}{3}$ and $\frac{5}{7}$

Solution

A common denominator is the product of the two: $3 \times 7 = 21$. We convert:

$$\frac{2}{3} \frac{7}{7} = \frac{14}{21}$$

and

$$\frac{5}{7} \frac{3}{3} = \frac{15}{21}$$

Next we compare the numerators and see that $14 < 15$, hence

$$\frac{2}{3} < \frac{5}{7}$$

Example 5.1.2

In statistics, we say that two events are independent if the probability of the second occurring is equal to the probability of the second occurring given that the first occurs. The probability of rolling two dice and having the sum equal to 7 is $\frac{6}{36}$. If you know that the first die lands on a 4, then the probability that the sum of the two dice is a 7 is $\frac{1}{6}$. Are these events independent?

Solution

We need to compare $\frac{6}{36}$ and $\frac{1}{6}$. The common denominator is 36. We convert the second fraction to

$$\frac{1}{6} \frac{6}{6} = \frac{6}{36}$$

Now we can see that the two fractions are equal, so the events are independent.

Approach 2: Use a calculator or computer to convert the fractions to decimals and then compare the decimals

If it is easy to build up the fractions so that we have a common denominator, then Approach 1 works well, but often the fractions are not simple, so it is easier to make use of the calculator or computer.

Example 5.1.3

In computing probabilities for a uniform distribution, fractions come up. Given that the number of ounces in a medium sized drink is uniformly distributed between 15 and 26 ounces, the probability that a randomly selected medium sized drink is less than 22 ounces is $\frac{7}{11}$. Given that the weight of in a medium sized American is uniformly distributed between 155 and 212 pounds, the probability that a randomly selected medium sized American is less than 195 pounds is $\frac{40}{57}$. Is it more likely to select a medium sized drink that is less than 22 ounces or to select a medium sized American who is less than 195 pounds?

Solution

We could get a common denominator and build the fractions, but it is much easier to just turn both fractions into decimal numbers and then compare. We have:

$$\frac{7}{11} \approx 0.6364$$

and

$$\frac{40}{57} \approx 0.7018$$

Notice that

$$0.6364 < 0.7018$$

Hence, we can conclude that it is less likely to pick the medium sized 22 ounce or less drink than to pick the 195 pound or lighter medium sized person.

Exercise

If you guess on 10 true or false questions, the probability of getting at least 9 correct is $\frac{11}{1024}$. If you guess on six multiple choice questions with three choices each, then the probability of getting at least five of the six correct is $\frac{7}{729}$. Which of these is more likely?

Comparing Fractions, Decimals and Percents

When you want to compare a fraction to a decimal or a percent, it is usually easiest to convert to a decimal number first, and then compare the decimal numbers.

Example 5.1.4

Compare 0.52 and $\frac{7}{13}$.

Solution

We first convert $\frac{7}{13}$ to a decimal by dividing to get 0.5385. Now notice that

$$0.52 < 0.5385$$

Thus

$$0.52 < \frac{7}{13}$$

Example 5.1.5

When we perform a hypothesis test in statistics, we have to compare a number called the p-value to another number called the level of significance. Suppose that the p-value is calculated as 0.0641 and the level of significance is 5%. Compare these two numbers.

Solution

We first convert the level of significance, 5%, to a decimal number. Recall that to convert a percent to a decimal, we move the decimal over two places to the right. This gives us 0.05. Now we can compare the two decimals:

$$0.0641 > 0.05$$

Therefore, the p-value is greater than the level of significance.



This is an application of comparing fractions to probability.

- [Example: Comparing Fractions with Different Denominators using Inequality Symbols](#)
- [Ex: Compare Fractions and Decimals using Inequality Symbols](#)
- <https://youtu.be/ISzNkQjcfEU>

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5.2: Converting Between Fractions, Decimals and Percents

Learning Outcomes

1. Given a decimal, convert it to a percent
2. Given a percent, convert it to a decimal
3. Convert a fraction to a decimal and percent

In this section, we will convert from decimals to percents and back. We will also start with a fraction and convert it to a decimal and a percent. In statistics we are often given a number as a percent and have to do calculations on it. To do so, we must first convert it to a percent. Also, the computer or calculator shows numbers as decimals, but for presentations, percents are friendlier. It is also much easier to compare decimals than fractions, thus converting to a decimal is helpful.

For example, we often want to see if a probability is greater than 5%. A computer will display the probability as a decimal such as 0.04836. To make the comparison we will first change it to a percent and then compare it to 5%.

Transforming a Decimal to a Percent

We have all heard of percents before. "You only have a 20% chance of winning the game", "Just 38% of all Americans approve of Congress", and "I am 95% confident that my answer is correct" are just a few of the countless examples of percents as they come up in statistics.

Definition: Percent

Percent means parts per hundred.

Thus if we are given a decimal and want to convert it to a percent, we multiply the decimal by 100. In practice, this means we move the decimal point two places to the right.

Example 5.2.1

Convert the number 0.1738 to a percent.

Solution

We move the decimal over two to the right as shown below.

0.1738



We get: 17.38% for the answer.

Example 5.2.2

Convert 0.7 to a percent.

Solution

We want to move the decimal two places to the right, but there is only one digit to the right of the decimal place. The good news is that we can always add a 0 to the right of the last digit. We write:

$$0.7 = 0.70$$

Now move the decimal place two digits to the right to get 70%.

Example 5.2.3

In regression analysis, an important number that is calculated is called R-Squared. It helps us determine how helpful one variable is in predicting another variable. The computer and calculator always display it as a decimal, but it is more meaningful

as a percent. Suppose that the R-Squared value that relates the amount of studying students do to prepare for a final exam and the score on the exam is: $r^2 = 0.8971$. Convert this to a percent rounded to the nearest whole number percent.

Solution

We move the decimal 0.8971 two places to the right to get 89.71%

Now round to the nearest whole number percent. Note that the digit to the left of the whole number is $7 \geq 5$. Thus we add 1 to the whole number, 89. This gives us 90%.

Try It

A standard goal in statistics is to come up with a range of values that a population proportion is likely to lie. This range is called a confidence interval. Suppose that we want to interpret a confidence interval for the percent of patients who experience side effects from an experimental cancer treatment. The computer calculates it as the decimal range: [0.023, 0.029]. What is the likely range for the percent of patients who experience side effects from the experimental cancer treatment?

Answer

The decimal range is from 0.023 to 0.029. Moving the decimal 2 places to the right for both values, we get 2.3% to 2.9%.

The likely range for the percent of patients who experience side effects from the experimental cancer treatment is 2.3% to 2.9%.

Transforming a Percent to a Decimal

To convert a decimal to a percent, we multiply the decimal by 100 which is equivalent to moving the decimal two places to the right. Not surprisingly, to convert a percent to a decimal, we do exactly the opposite. We divide the number by 100 which is equivalent to moving the decimal two places to the left.

Example 5.2.4

Convert the percent 89.4% to a decimal.

Solution

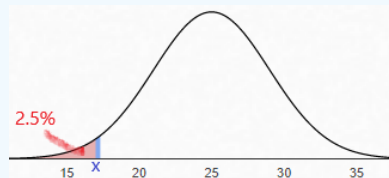
We move the decimal over two to the left as shown below.

89.4

We get: 0.894 for the answer.

Example 5.2.5

Suppose that you want to find the value of x such that 2.5% of the entire area under the Normal curve lies to the left of x . The first step will be to convert the 2.5% to a decimal. What decimal is equivalent to 2.5%?



Solution

We want to move the decimal 2.5 two places to the left, but since there is only one digit to the left of the decimal, we add a zero first: 02.5. Now move the decimal two places to the left to get 0.025.

Converting a Fraction to a Decimal and a Percent

Often in probability it is natural to represent probabilities as fractions, but it is easier to make comparisons as decimals. Thus, we need to be able to convert fractions to decimals. To do so we just divide.

Example 5.2.6

Convert the fraction $\frac{4}{7}$ to a decimal, rounding to the nearest hundredth.

Solution

We use long division:

$$\begin{array}{r} .571 \\ 7 \overline{)4.000} \\ \underline{35} \\ 50 \\ \underline{49} \\ 10 \end{array} \quad (5.2.1)$$

Next round to the nearest hundredth to get 0.57.

Although everyone's favorite thing to do is to perform long division by hand, in most statistics classes you will have a calculator or computer to use. Thus you just have to remember to perform the division with the calculator or computer and then round.

Example 5.2.7

In statistics we need to find basic probabilities and create a table for them. Suppose that you roll two six-sided dice, what percent of the time will the sum equal to a 4? Round to the nearest whole number percent.

Solution

First, notice that there are 36 total possibilities for rolling the dice, since there are 6 faces on the first die and for each value of the first die roll, there are 6 possibilities for the second die roll. Multiplying: $6 \times 6 = 36$. This will be the denominator. To find the numerator, we list all the possible outcome where the sum is 4:

(1,3), (2,2), and (3,1)

There are three possible outcomes with the sum equaling a 4. Thus:

$$P(\text{sum} = 4) = 3/36$$

Now we divide:

$$\frac{3}{36} = 0.08333...$$

Next to convert this decimal to a percent, we move the decimal two places to the right to get: 8.333...%

We are asked to round to the nearest whole number percent. The digit to the right of the whole number (8) is a 3. Since $3 < 5$, we can just erase everything to the left of the 8 and leave the 8 unchanged to get 8%. Thus there is an 8% chance of getting a sum of 4 if you roll two sixsided dice.

Additional video resources:

- [Convert Percentages to Decimals](#)
- [Relating Fractions, Decimals, and Percents](#)
- [Statistics Application of Converting Decimals to Percents](#)

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5.3: Decimals- Rounding

Learning Outcomes

1. Understand what it means to have a number rounded to a certain number of decimal places.
2. Round a number to a fixed number of digits.

In this section, we will go over how to round decimals to the nearest whole number, nearest tenth, nearest hundredth, etc. In most statistics applications that you will encounter, the numbers will not come out evenly, and you will need to round the decimal.

For example, suppose that you used a calculator to find the probability that a randomly selected day in July will have a high temperature of over 90°. Your calculator gives the answer: 0.4987230156. This is far too many digits for practical use, so it makes sense to round to just a few digits. By the end of this section you will be able to perform the rounding that is necessary to make unmanageable numbers manageable.

Brief Review of Decimal Language

Consider the decimal number: 62.5739. There is a defined way to refer to each of the digits.

- The digit 6 is in the "Tens Place"
- The digit 2 is in the "Ones Place"
- The digit 5 is in the "Tenths Place"
- The digit 7 is in the "Hundredths Place"
- The digit 3 is in the "Thousandths Place"
- The digit 9 is in the "Ten-thousandths Place"
- We also say that 62 is the "Whole Number" part.

62.5739

Tens
Ones
Tenths
Hundredths
Thousandths
Ten thousandths

Keeping this example in mind will help you when you are asked to round to a specific place value.

Example 5.3.1

It is reported that the mean number of classes that college students take each semester is 3.2541. Then the digit in the *hundredths place* is 5.

Rules of Rounding

Now that we have reviewed place values of numbers, we are ready to go over the process of rounding to a specified place value. When asked to round to a specified place value, the answer will erase all the digits after the specified digit. To decide whether to keep the specified digit the same or round it up, you need to look at the *test digit*, or the digit to the right of the specified digit. The process to deal with the other digits is best shown by examples.

Example 5.3.2: Case 1 - The Test Digit is Less Than 5

Round 3.741 to the nearest tenth.

Solution

3.741
Tenths Test Digit

Since the test digit (4) is less than 5, we just erase everything to the right of the tenths digit, 7. The answer is: 3.7.

Example 5.3.3: Case 2 - The Test Digit is 5 or Greater

Round 8.53692 to the nearest hundredth.

Solution

8.53692
Hundredths Test Digit

Since the test digit (6) is 5 or greater, we add one to the hundredths digit and erase everything to the right of the hundredths digit, 3. Thus the 3 becomes a 4. The answer is: 8.54.

Example 5.3.4: Case 3 - The Test Digit is 5 or Greater and the rounding position digit is a 9

Round 0.014952 to four decimal places.

Solution

0.014952
Rounding Position Test Digit

The test digit is 5, so we must round up. The rounding position is a 9 and adding 1 gives 10, which is not a single digit number. Instead look at the two digits to the left of the test digit: 49. If we add 1 to 49, we get 50. Thus the answer is 0.0150.

Applications

Rounding is used in most areas of statistics, since the calculator or computer will produce numerical answers with far more digits than are useful. If you are not told how many decimal places to round to, then you often want to think about the smallest number of decimals to keep so that no important information is lost. For example suppose you conducted a sample to find the proportion of college students who receive financial aid and the calculator presented 0.568429314. You could turn this into a percent at 56.8429314%. There are no applications where keeping this many decimal places is useful. If, for example, you wanted to present this finding to the student government, you might want to round to the nearest whole number. In this case the ones digit is 6 and the test digit is 8. Since $8 \geq 5$, you add 1 to the ones digit. You can tell the student government that 57% of all college students receive financial aid.

Example 5.3.5

Suppose that you found out that the probability that a randomly selected person who has misused prescription opioids will transition to heroin is 0.04998713. Round this number to four decimal places.

Solution

The first four decimal places are 0.0499 and the test digit is 8. Since $8 \geq 5$, we would like to add 1 to the fourth digit. Since this is a 9, we go to the next digit to the left. This is also a 9, so we go to the next one which is a 4. We can think of adding 0499 +

1 = 0500. Thus the answer is 0.0500. Note that we keep the last two 0's after the 5 to emphasize that this is accurate to the fourth decimal place.

Rounding and Arithmetic

Many times, we have to do arithmetic on numbers with several decimal places and want the answer rounded to a smaller number of decimal places. One question you might ask is should you round before you perform the arithmetic or after. For the most accurate result, you should always round **after** you perform the arithmetic if possible.

When asked to do arithmetic and present your answer rounded to a fixed number of decimal places, only round after performing the arithmetic.

Example 5.3.6

Suppose you pick three cards from a 52-card deck with replacement and want to find the probability of event A, that none of the three cards will be a 2 through 7 of hearts. This probability is:

$$P(A) = (0.8846)^3$$

Round the answer to 2 decimal places.

Solution

Note that we have to first perform the arithmetic. With a computer or calculator we get:

$$0.8846^3 = 0.69221467973$$

Now we round to two decimal places. Notice that the hundredths digit is a 9 and the test digit is a 2. Thus the 9 remains unchanged and everything to the right of the 9 goes away. the result is

$$P(A) \approx 0.69$$

If we mistakenly rounded 0.8846 to two decimal places (0.88) and then cubed the answer we would have gotten 0.68 which is not the correct answer.

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5.4: Decimals- Scientific Notation

Learning Outcomes

1. Convert from scientific notation to decimal notation and back.

In this section, we will look at how to read scientific notation. A very common error that statistics students make is not noticing that the calculator is giving an answer in scientific notation.

Scientific Notation

When a calculator presents a number in scientific notation, we must pay attention to what this represents. The standard way of writing a number in scientific notation is writing the number as a product of a number greater than or equal 1 but less than 10 followed by a power of 10. For example:

$$602,000,000,000,000,000,000,000 = 6.02 \times 10^{23}$$

The main purpose of scientific notation is to allow us to write very large numbers or very small numbers close to 0 without having to use so many digits. Most calculators and computers use a different notation for scientific notation, most likely because the superscript is difficult to render on a screen. For example, with a calculator:

$$0.00000032 = 3.2E - 7$$

Notice that to arrive at 3.2, the decimal needed to be moved 7 places to the right. The "E" represents "times 10 raised to the power of."

Example 5.4.1

A calculator displays:

$$2.0541E6$$

Write this number in decimal form.

Solution

Notice that the number following E is 6. This means move the decimal over 6 places to the right. The first 4 moves is natural, but for the last 2 moves, there are no numbers to move the decimal place past. We can always add extra zeros after the last number to the right of the decimal place:

$$2.0541E6 = 2.054100E6$$

Now we can move the decimal place to the right 6 places to get

$$2.0541E6 = 2.054100E6 = 2,054,100$$

Example 5.4.2

If you use a calculator or computer to find the probability of flipping a coin 27 times and getting all heads, then it will display:

$$7.45E - 9$$

Write this number in decimal form.

Solution

Many students will forget to look for the "E" and just write that the probability is 7.45, but probabilities can never be bigger than 1. You can not have a 745% chance of it occurring. Notice that the number following E is -9. Since the power is negative, this means move the decimal to the left, and in particular 9 places to the left. There is only one digit to the left of the decimal place, so we need to insert 8 zeros:

$$7.45E - 9 = 000000007.45E - 9$$

Now we can move the decimal place to the right 9 places to the left to get

$$7.45E-9 = 000000007.45E-9 = 0.00000000745$$

This is a very small probability and essentially rounds to 0.

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5.5: Using Fractions, Decimals and Percents to Describe Charts

Learning Outcomes

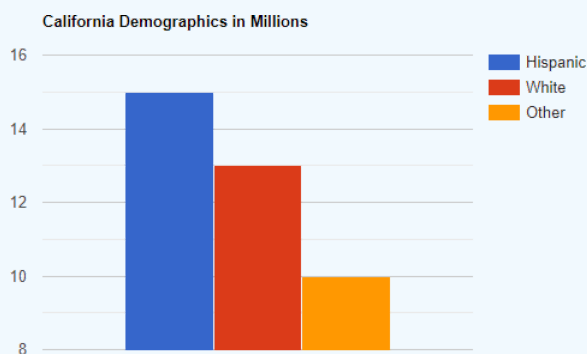
1. Interpret bar charts using fractions, decimals and percents
2. Interpret pie charts using fractions, decimals and percents

Charts, such as bar charts and pie charts are visual ways of presenting data. You can think of each slice of the pie or each bar as a part of the whole. The numerical versions of this are a list of fractions, decimals and percents. By the end of this section we will be able to look at one of these charts and produce the corresponding fractions, decimals, and percents.

Reading a Bar Chart

Bar charts occur frequently and it is definitely required to understand how to read them and interpret them in statistics. Often we want to convert the information of a bar chart to information shown numerically. We need fractions and/or percents to do this.

Example 5.5.1



The above bar chart shows the demographics of California in 2019 where the numbers represent millions of people. Here are some questions that might come up in a statistics class.

- A. What fraction of Californians was Hispanic in 2019?
- B. What proportion of all Californians was White in 2019? Write your answer as a decimal number rounded to four decimal places.
- C. What percent of Californians who were neither Hispanic nor White in 2019? Round your answer to the nearest percent.

Solution

- A. To find the fraction of California that was Hispanic in 2019, the numerator will be the total number of Hispanics and the denominator will be the total number of people in California in 2019. The height of the bar that represents Hispanics is 15. Therefore the numerator is 15. To find the total number of people in California, we add up the heights of the three bars:

$$15 + 13 + 10 = 38$$

Now we can just write down the fraction:

$$\frac{15}{38}$$

To find the proportion of Californians who were White in 2019, we start in the same way. The numerator will be the number of Whites: 13. The denominator will be the total number of Californians which we already computed as 38. Therefore the fraction of Californians who were White is:

$$\frac{13}{38}$$

To convert this to a decimal, we use a calculator to get:

$$\frac{13}{38} \approx 0.342105$$

Next round to four decimal places. Since the digit to the right of the fourth decimal place is $0 < 5$, we round down to:

$$0.3421$$

B. To find the percent of Californians who were neither Hispanic nor White in 2019, we first find the fraction who were neither. The numerator will be the number of "Other" which is: 10. The denominator will be the total which is 38. Thus the fraction is:

$$\frac{10}{38}$$

Next, use a calculator to divide these numbers to get:

$$\frac{10}{38} \approx 0.263158$$

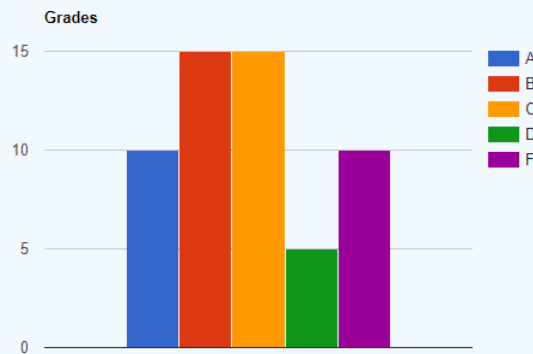
To convert this to a percent we multiply by 100% by moving the decimal two places to the right:

$$0.263158 \times 100\% = 26.3158\%$$

Finally we round to the nearest whole number. Noting that $3 < 5$, we round down to get: 26%

Exercise

The bar chart below shows the grade distribution for a math class.



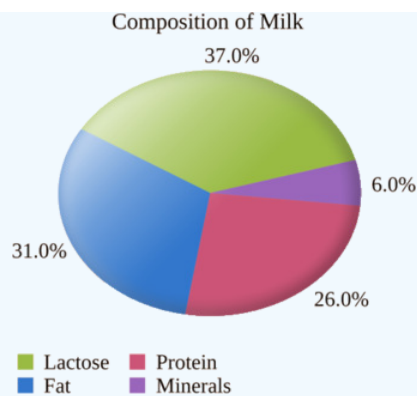
- Find the fraction of students who received a "C" grade.
- Find the proportion of grades below a "C". Write your answer as a decimal number rounded to the nearest hundredth.
- What percent of the students received an "A" grade? Round your answer to the nearest whole number percent.

Reading a Pie Chart

Another important chart that is used to display the components of a whole is a pie chart. With a pie chart, it is very easy to determine the percent of each item.

Example 5.5.2

The pie chart below shows the makeup of milk. Write the proportion of fat contained in milk as a decimal.

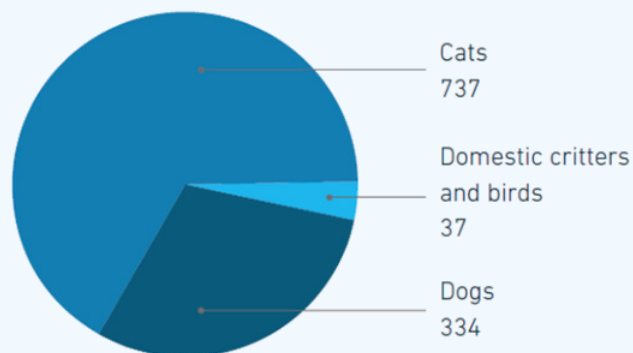


Solution

We see that 31% of milk is fat. To convert a percent to a decimal, we just move the decimal over two places to the left. Thus, 31% becomes 0.31.

Example 5.5.3

Euthanasia by species



The pie chart above shows the number of pets of each type that had to be euthanized by the humane society due to incurable illnesses.

- What fraction of the euthanized pets were dogs?
- What percent of the euthanized pets were cats? Round to the nearest whole number percent.

Solution

- We take the number of dogs over the total. There were 334 euthanized dogs. To find the total we add:

$$737 + 37 + 334 = 1108$$

Therefore, the fraction of euthanized dogs is

$$\frac{334}{1108}$$

- To find the percent of euthanized cats, we first find the fraction. There were 737 cats over a total of 1108 pets. The fraction is

$$\frac{737}{1108}$$

Next use a calculator to get the decimal number: 0.66516. Now multiply by 100% by moving the decimal place two digits to the right to get: 66.516%. Finally, we need to round to the nearest whole number percent. Since $5 \geq 5$, we round up.

Thus the percent of euthanized cats is 67%.

- [Finding Fractions, Decimals and Percents from a Bar Chart](#)
- [Ex: Find the a Percent of a Total Using an Amount in Pie Chart](#)

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5.6: Equivalent Fractions

In this section we deal with fractions, numbers or expressions of the form $\frac{a}{b}$.

Definition: Fraction

A number of the form

$$\frac{a}{b}$$

where a and b are numbers is called a **fraction**. The number a is called the **numerator** of the fraction, while the number b is called the **denominator** of the fraction.

We start our study of fractions with the definition of *equivalent fractions*.

Equivalent Fractions

Two fractions are **equivalent** if they represent the same numerical value.

But how can we tell if two fractions represent the same number? Well, one technique involves some simple visualizations. Consider the image shown in Figure 5.6.1, where the shaded region represents $\frac{1}{3}$ of the total area of the figure (one of three equal regions is shaded).



Figure 5.6.1: The shaded region is $\frac{1}{3}$ of the whole region.

In Figure 5.6.2, we've shaded $\frac{2}{6}$ of the entire region (two of six equal regions are shaded).



Figure 5.6.2: The shaded region is $\frac{2}{6}$ of the whole region.

In Figure 5.6.3, we've shaded $\frac{4}{12}$ of the entire region (four of twelve equal regions are shaded).



Figure 5.6.3: The shaded region is $\frac{4}{12}$ of the whole region.

Let's take the diagrams from Figure 5.6.1, Figure 5.6.2, and Figure 5.6.3 and stack them one atop the other, as shown in Figure 5.6.4.

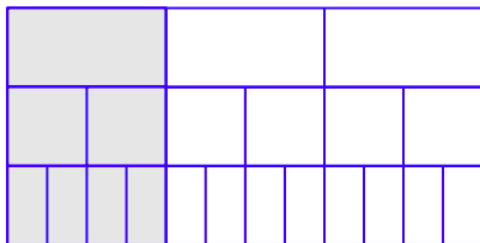


Figure 5.6.4: One of three equals two of six equals four of twelve.

Figure 5.6.4 provides solid visual evidence that the following fractions are equivalent.

$$\frac{1}{3} = \frac{2}{6} = \frac{4}{12}$$

Key Observations

1. If we start with the fraction $\frac{1}{3}$, then multiply both numerator and denominator by 2, we get the following result.

$$\begin{aligned}\frac{1}{3} &= \frac{1 \cdot 2}{3 \cdot 2} && \text{Multiply the numerator and denominator by 2.} \\ &= \frac{2}{6} && \text{Simplify numerator and denominator.}\end{aligned}$$

This is precisely the same thing that happens going from Figure 5.6.1 to 5.6.2, where we double the number of available boxes (going from 3 available to 6 available) and double the number of shaded boxes (going from 1 shaded to 2 shaded).

2. If we start with the fraction $\frac{1}{3}$, then multiply both numerator and denominator by 4, we get the following result.

$$\begin{aligned}\frac{1}{3} &= \frac{1 \cdot 4}{3 \cdot 4} && \text{Multiply numerator and denominator by 4.} \\ &= \frac{4}{12} && \text{Simplify numerator and denominator.}\end{aligned}$$

This is precisely the same thing that happens going from Figure 5.6.1 to 5.6.3, where we multiply the number of available boxes by 4 (going from 3 available to 12 available) and multiply the number of shaded boxes by 4 (going from 1 shaded to 4 shaded).

The above discussion motivates the following fundamental result.

Creating Equivalent Fractions

If you start with a fraction, then multiply both its numerator and denominator by the same number, the resulting fraction is equivalent (has the same numerical value) to the original fraction. In symbols,

$$\frac{a}{b} = \frac{a \cdot x}{b \cdot x}$$

Arguing in Reverse

Reversing the above argument also holds true.

1. If we start with the fraction $\frac{2}{6}$, then divide both numerator and denominator by 2, we get the following result.

$$\begin{aligned}\frac{2}{6} &= \frac{2 \div 2}{6 \div 2} && \text{Divide numerator and denominator by 2.} \\ &= \frac{1}{3} && \text{Simplify numerator and denominator.}\end{aligned}$$

This is precisely the same thing that happens going backwards from Figure 5.6.2 to 5.6.1, where we divide the number of available boxes by 2 (going from 6 available to 3 available) and dividing the number of shaded boxes by 2 (going from 2 shaded to 1 shaded).

2. If we start with the fraction $\frac{4}{12}$, then divide both numerator and denominator by 4, we get the following result.

$$\begin{aligned}\frac{4}{12} &= \frac{4 \div 4}{12 \div 4} && \text{Divide numerator and denominator by 4.} \\ &= \frac{1}{3} && \text{Simplify numerator and denominator.}\end{aligned}$$

This is precisely the same thing that happens going backwards from Figure 5.6.3 to 5.6.1, where we divide the number of available boxes by 4 (going from 12 available to 3 available) and divide the number of shaded boxes by 4 (going from 4 shaded to 1 shaded).

The above discussion motivates the following fundamental result.

Creating Equivalent Fractions

If you start with a fraction, then divide both its numerator and denominator by the same number, the resulting fraction is equivalent (has the same numerical value) to the original fraction. In symbols,

$$\frac{a}{b} = \frac{a \div x}{b \div x}.$$

The Greatest Common Divisor

We need a little more terminology.

Divisor

If d and a are natural numbers, we say that “ d divides a ” if and only if when a is divided by d , the remainder is zero. In this case, we say that “ d is a **divisor** of a .”

For example, when 36 is divided by 4, the remainder is zero. In this case, we say that “4 is a divisor of 36.” On the other hand, when 25 is divided by 4, the remainder is not zero. In this case, we say that “4 is not a divisor of 25.”

Greatest Common Divisor

Let a and b be natural numbers. The common divisors of a and b are those natural numbers that divide both a and b . The **greatest common divisor** is the largest of these common divisors.

Example 5.6.1

Find the greatest common divisor of 18 and 24.

Solution

First list the divisors of each number, the numbers that divide each number with zero remainder.

Divisors of 18 : 1, 2, 3, 6, 9, and 18

Divisors of 24 : 1, 2, 3, 4, 6, 8, 12, and 24

The common divisors are:

Common Divisors : 1, 2, 3, and 6

The greatest common divisor is the largest of the common divisors. That is,

Greatest Common Divisor = 6.

That is, the largest number that divides both 18 and 24 is the number 6.

Try It 5.6.1

Find the greatest common divisor of 12 and 18.

Answer

6

Reducing a Fraction to Lowest Terms

First, a definition.

Lowest Terms

A fraction is said to be **reduced to lowest terms** if the greatest common divisor of both numerator and denominator is 1.

Thus, for example, $\frac{2}{3}$ is reduced to lowest terms because the greatest common divisor of 2 and 3 is 1. On the other hand, $\frac{4}{6}$ is **not** reduced to lowest terms because the greatest common divisor of 4 and 6 is 2.

Example 5.6.2

Reduce the fraction $\frac{18}{24}$ to lowest terms.

Solution

One technique that works well is dividing both numerator and denominator by the greatest common divisor of the numerator and denominator. In Example 5.6.1, we saw that the greatest common divisor of 18 and 24 is 6. We divide both numerator and denominator by 6 to get

$$\begin{aligned}\frac{18}{24} &= \frac{18 \div 6}{24 \div 6} && \text{Divide numerator and denominator by 6.} \\ &= \frac{3}{4} && \text{Simplify numerator and denominator.}\end{aligned}$$

Note that the greatest common divisor of 3 and 4 is now 1. Thus, $\frac{3}{4}$ is reduced to lowest terms.

There is a second way we can show division of numerator and denominator by 6. First, factor both numerator and denominator as follows:

$$\frac{18}{24} = \frac{3 \cdot 6}{4 \cdot 6} \quad \text{Factor out a 6.}$$

You can then show “division” of both numerator and denominator by 6 by “crossing out” or “canceling” a 6 in the numerator for a 6 in the denominator, like this:

$$\begin{aligned}&= \frac{3 \cdot \cancel{6}}{4 \cdot \cancel{6}} && \text{Cancel common factor.} \\ &= \frac{3}{4}\end{aligned}$$

Note that we get the same equivalent fraction, reduced to lowest terms, namely $\frac{3}{4}$.

Try It 5.6.2

Reduce the fraction $\frac{12}{18}$ to lowest terms.

Answer

$$\frac{2}{3}$$

Important Point

In Example 5.6.2 we saw that 6 was both a *divisor* and a *factor* of 18. The words *divisor* and *factor* are equivalent.

We used the following technique in our second solution in Example 5.6.2.

Cancellation Rule

If you express numerator and denominator as a product, then you may cancel common factors from the numerator and denominator. The result will be an equivalent fraction.

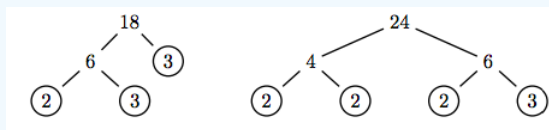
Because of the “Cancellation Rule,” one of the most effective ways to reduce a fraction to lowest terms is to first find prime factorizations for both numerator and denominator, then cancel all common factors.

Example 5.6.3

Reduce the fraction $\frac{18}{24}$ to lowest terms.

Solution

Use factor trees to prime factor numerator and denominator.



Once we’ve factored the numerator and denominator, we cancel common factors.

$$\begin{aligned}
 \frac{18}{24} &= \frac{2 \cdot 3 \cdot 3}{2 \cdot 2 \cdot 2 \cdot 3} && \text{Prime factor numerator and denominator.} \\
 &= \frac{\cancel{2} \cdot \cancel{3} \cdot 3}{\cancel{2} \cdot 2 \cdot 2 \cdot \cancel{3}} && \text{Cancel common factors.} \\
 &= \frac{3}{2 \cdot 2} && \text{Remaining factors.} \\
 &= \frac{3}{4} && \text{Simplify denominator.}
 \end{aligned}$$

Thus, $\frac{18}{24} = \frac{3}{4}$.

Try It 5.6.3

Reduce the fraction $\frac{28}{35}$ to lowest terms.

Answer

$$\frac{4}{5}$$

Example 5.6.4

Reduce the fraction $\frac{28}{42}$ to lowest terms.

Solution

Use factor trees to prime factor numerator and denominator.



Now we can cancel common factors.

$$\begin{aligned}\frac{28}{42} &= \frac{2 \cdot 2 \cdot 7}{2 \cdot 3 \cdot 7} && \text{Prime factor numerator and denominator.} \\ &= \frac{\cancel{2} \cdot 2 \cdot \cancel{7}}{\cancel{2} \cdot 3 \cdot \cancel{7}} && \text{Cancel common factors.} \\ &= \frac{2}{3}\end{aligned}$$

Thus, $\frac{28}{42} = \frac{2}{3}$.

Try It 5.6.4

Reduce the fraction $\frac{36}{60}$ to lowest terms.

Answer

$$\frac{3}{5}$$

Equivalent Fractions in Higher Terms

Sometimes the need arises to find an equivalent fraction with a different, larger denominator.

Example 5.6.5

Express $\frac{3}{5}$ as an equivalent fraction having denominator 20.

Solution

The key here is to remember that multiplying numerator and denominator by the same number produces an equivalent fraction.

To get an equivalent fraction with a denominator of 20, we'll have to multiply numerator and denominator of $\frac{3}{5}$ by 4.

$$\begin{aligned}&= \frac{3 \cdot 4}{5 \cdot 4} && \text{Multiply numerator and denominator by 4.} \\ &= \frac{12}{20} && \text{Simplify numerator and denominator.}\end{aligned}$$

Therefore, $\frac{3}{5} = \frac{12}{20}$.

Try It 5.6.5

Express $\frac{2}{3}$ as an equivalent fraction having denominator 21.

Answer

$$\frac{14}{21}$$

Example 5.6.6

Express 8 as an equivalent fraction having denominator 5.

Solution

The key here is to note that

$$8 = \frac{8}{1} \text{ Understood denominator is 1.}$$

To get an equivalent fraction with a denominator of 5, we'll have to multiply numerator and denominator of $\frac{8}{1}$ by 5.

$$\begin{aligned} &= \frac{8 \cdot 5}{1 \cdot 5} \text{ Multiply numerator and denominator by 5.} \\ &= \frac{40}{5} \text{ Simplify numerator and denominator.} \end{aligned}$$

Therefore, 8 equals $\frac{40}{5}$.

Try It 5.6.6

Express 5 as an equivalent fraction having denominator 7.

Answer

$$\frac{35}{7}$$

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5.7: Multiplying Fractions

Consider the image in Figure 5.7.1, where the vertical lines divide the rectangular region into three equal pieces. If we shade one of the three equal pieces, the shaded area represents $\frac{1}{3}$ of the whole rectangular region.



Figure 5.7.1: The shaded region is $\frac{1}{3}$ of the whole region.

We'd like to visualize taking $\frac{1}{2}$ of $\frac{1}{3}$. To do that, we draw an additional horizontal line which divides the shaded region in half horizontally. This is shown in Figure 5.7.2. The shaded region that represented $\frac{1}{3}$ is now divided into two smaller rectangular regions, one of which is shaded with a different color. This region represents $\frac{1}{2}$ of $\frac{1}{3}$.

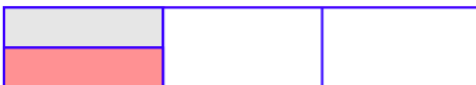


Figure 5.7.2: Shading $\frac{1}{2}$ of $\frac{1}{3}$.

Next, extend the horizontal line the full width of the rectangular region, as shown in Figure 5.7.3.

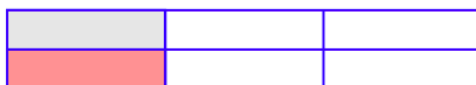


Figure 5.7.3: Shading $\frac{1}{2}$ of $\frac{1}{3}$.

Note that drawing the horizontal line, coupled with the three original vertical lines, has succeeded in dividing the full rectangular region into six smaller but equal pieces, only one of which (the one representing $\frac{1}{2}$ of $\frac{1}{3}$) is shaded in a new color. Hence, this newly shaded piece represents $\frac{1}{6}$ of the whole region. The conclusion of our visual argument is the fact that $\frac{1}{2}$ of $\frac{1}{3}$ equals $\frac{1}{6}$. In symbols,

$$\frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}.$$

Multiplication Rule

In Figure 5.7.3, we saw that $\frac{1}{2}$ of $\frac{1}{3}$ equals $\frac{1}{6}$. Note what happens when we multiply the numerators and multiply the denominators of the fractions $\frac{1}{2}$ and $\frac{1}{3}$.

$$\begin{aligned} \frac{1}{2} \cdot \frac{1}{3} &= \frac{1 \cdot 1}{2 \cdot 3} && \text{Multiply numerators; multiply denominators.} \\ &= \frac{1}{6} && \text{Simplify numerators and denominators.} \end{aligned}$$

We get $\frac{1}{6}$!

This example motivates the following definition.

Definition: Multiplication Rule

To find the product of the fractions $\frac{a}{b}$ and $\frac{c}{d}$, multiply their numerators and denominators. In symbols,

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$$

Example 5.7.1

Multiply: $\frac{1}{5} \cdot \frac{7}{9}$

Solution

Multiply numerators and multiply denominators.

$$\begin{aligned}\frac{1}{5} \cdot \frac{7}{9} &= \frac{1 \cdot 7}{5 \cdot 9} && \text{Multiply numerators; multiply denominators.} \\ &= \frac{7}{45} && \text{Simplify numerators and denominators.}\end{aligned}$$

Try It 5.7.1

Multiply: $\frac{1}{3} \cdot \frac{2}{5}$

Answer

$$\frac{2}{15}$$

Multiply and Reduce

After multiplying two fractions, make sure your answer is reduced to lowest terms (see previous section).

Example 5.7.2

Multiply $\frac{3}{4} \cdot \frac{8}{9}$

Solution

After multiplying, divide numerator and denominator by the greatest common divisor of the numerator and denominator.

$$\begin{aligned}\frac{3}{4} \cdot \frac{8}{9} &= \frac{3 \cdot 8}{4 \cdot 9} && \text{Multiply numerators and denominators.} \\ &= \frac{24}{36} && \text{Simplify numerator and denominator.} \\ &= \frac{24 \div 12}{36 \div 12} && \text{Divide numerator and denominator by GCD.} \\ &= \frac{2}{3} && \text{Simplify numerator and denominator.}\end{aligned}$$

Alternatively, after multiplying, you can prime factor both numerator and denominator, then cancel common factors.

$$\begin{aligned}\frac{3}{4} \cdot \frac{8}{9} &= \frac{24}{36} && \text{Multiply numerators and denominators.} \\ &= \frac{2 \cdot 2 \cdot 2 \cdot 3}{2 \cdot 2 \cdot 3 \cdot 3} && \text{Prime factor numerator and denominator.} \\ &= \frac{\cancel{2} \cdot \cancel{2} \cdot 2 \cdot \cancel{3}}{\cancel{2} \cdot \cancel{2} \cdot 3 \cdot \cancel{3}} && \text{Cancel common factors.} \\ &= \frac{2}{3}\end{aligned}$$

Try It 5.7.2

Multiply: $\frac{3}{7} \cdot \frac{14}{9}$

Answer

$$\frac{2}{3}$$

Multiply and Cancel or Cancel and Multiply

When you are working with larger numbers, it becomes a bit harder to multiply, factor, and cancel. Consider the following argument.

$$\begin{aligned}
 \frac{18}{30} \cdot \frac{35}{6} &= \frac{630}{180} && \text{Multiply numerators; multiply denominators.} \\
 &= \frac{2 \cdot 3 \cdot 3 \cdot 5 \cdot 7}{2 \cdot 2 \cdot 3 \cdot 3 \cdot 5} && \text{Prime factor numerators and denominators.} \\
 &= \frac{\cancel{2} \cdot \cancel{3} \cdot \cancel{3} \cdot \cancel{5} \cdot 7}{2 \cdot \cancel{2} \cdot \cancel{3} \cdot \cancel{3} \cdot \cancel{5}} && \text{Cancel common factors.} \\
 &= \frac{7}{2} && \text{Remaining factors.}
 \end{aligned}$$

There are a number of difficulties with this approach. First, you have to multiply large numbers, and secondly, you have to prime factor the even larger results.

One possible workaround is to not bother multiplying numerators and denominators, leaving them in factored form.

$$\frac{18}{30} \cdot \frac{35}{6} = \frac{18 \cdot 35}{30 \cdot 6} \quad \text{Multiply numerators; multiply denominators.}$$

Finding the prime factorization of these smaller factors is easier.

$$= \frac{(2 \cdot 3 \cdot 3) \cdot (5 \cdot 7)}{(2 \cdot 3 \cdot 5) \cdot (2 \cdot 3)} \quad \text{Prime factor.}$$

Now we can cancel common factors. Parentheses are no longer needed in the numerator and denominator because both contain a product of prime factors, so order and grouping do not matter.

$$\begin{aligned}
 &= \frac{\cancel{2} \cdot \cancel{3} \cdot \cancel{3} \cdot \cancel{5} \cdot 7}{\cancel{2} \cdot \cancel{3} \cdot \cancel{5} \cdot 2 \cdot \cancel{3}} && \text{Cancel common factors.} \\
 &= \frac{7}{2} && \text{Remaining factors.}
 \end{aligned}$$

Another approach is to factor numerators and denominators in place, cancel common factors, then multiply.

$$\begin{aligned}
 \frac{18}{30} \cdot \frac{35}{6} &= \frac{2 \cdot 3 \cdot 3}{2 \cdot 3 \cdot 5} \cdot \frac{5 \cdot 7}{2 \cdot 3} && \text{Factor numerators and denominators.} \\
 &= \frac{\cancel{2} \cdot \cancel{3} \cdot \cancel{3}}{\cancel{2} \cdot \cancel{3} \cdot \cancel{5}} \cdot \frac{\cancel{5} \cdot 7}{2 \cdot \cancel{3}} && \text{Cancel common factors.} \\
 &= \frac{7}{2} && \text{Remaining factors.}
 \end{aligned}$$

Note that this yields exactly the same result, $\frac{7}{2}$.

Cancellation Rule

When multiplying fractions, cancel common factors according to the following rule: “Cancel a factor in a numerator for an identical factor in a denominator.”

Example 5.7.3

Multiply: $\frac{14}{15} \cdot \frac{30}{140}$

Solution

Multiply numerators and multiply denominators. Prime factor, cancel common factors, then multiply.

$$\begin{aligned}
 \frac{14}{15} \cdot \frac{30}{140} &= \frac{14 \cdot 30}{15 \cdot 140} && \text{Multiply numerators; multiply denominators.} \\
 &= \frac{(2 \cdot 7) \cdot (2 \cdot 3 \cdot 5)}{(3 \cdot 5) \cdot (2 \cdot 2 \cdot 5 \cdot 7)} && \text{Prime factor numerators and denominators.} \\
 &= \frac{\cancel{2} \cdot \cancel{7} \cdot \cancel{2} \cdot \cancel{3} \cdot \cancel{5}}{\cancel{3} \cdot 5 \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{5} \cdot \cancel{7}} && \text{Cancel common factors.} \\
 &= \frac{1}{5} && \text{Multiply.}
 \end{aligned}$$

Note: Everything in the numerator cancels because you've divided the numerator by itself. Hence, the answer has a 1 in its numerator.

Try It 5.7.3

Multiply: $\frac{6}{35} \cdot \frac{70}{36}$

Answer

$$\frac{1}{3}$$

When Everything Cancels

When all the factors in the numerator cancel, this means that you are dividing the numerator by itself. Hence, you are left with a 1 in the numerator. The same rule applies to the denominator. If everything in the denominator cancels, you're left with a 1 in the denominator.

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5.8: Dividing Fractions

Suppose that you have four pizzas and each of the pizzas has been sliced into eight equal slices. Therefore, each slice of pizza represents $\frac{1}{8}$ of a whole pizza.

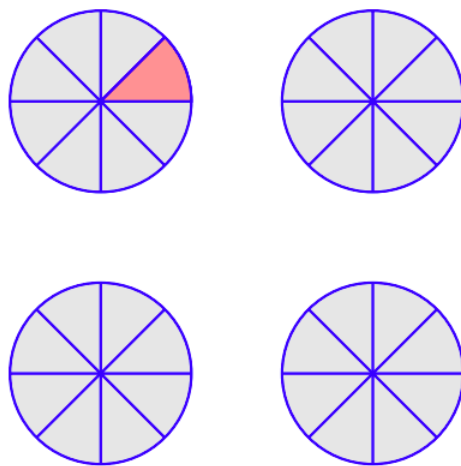


Figure 5.8.1: One slice of pizza is $\frac{1}{8}$ of one whole pizza.

Now for the question: How many one-eighths are there in four? This is a division statement. To find how many one-eighths there are in 4, divide 4 by $\frac{1}{8}$. That is,

$$\text{Number of one-eighths in four} = 4 \div \frac{1}{8}.$$

On the other hand, to find the number of one-eighths in four, Figure 5.8.1 clearly demonstrates that this is equivalent to asking how many slices of pizza are there in four pizzas. Since there are 8 slices per pizza and four pizzas,

$$\text{Number of pizza slices} = 4 \cdot 8.$$

The conclusion is the fact that $4 \div \frac{1}{8}$ is equivalent to $4 \cdot 8$. That is,

$$\begin{aligned} 4 \div \frac{1}{8} &= 4 \cdot 8 \\ &= 32. \end{aligned}$$

Therefore, we conclude that there are 32 one-eighths in 4.

Reciprocals

Multiplicative Inverse Property

Let $\frac{a}{b}$ be any fraction. The number $\frac{b}{a}$ is called the **multiplicative inverse** or **reciprocal** of $\frac{a}{b}$. The product of reciprocals is 1.

$$\frac{a}{b} \cdot \frac{b}{a} = 1$$

Note: To find the multiplicative inverse (reciprocal) of a number, simply invert the number (turn it upside down).

For example, the number $\frac{1}{8}$ is the multiplicative inverse (reciprocal) of 8 because

$$8 \cdot \frac{1}{8} = 1.$$

Note that 8 can be thought of as $\frac{8}{1}$. Invert this number (turn it upside down) to find its multiplicative inverse (reciprocal) $\frac{1}{8}$.

Example 5.8.1

Find the reciprocals of: (a) $\frac{2}{3}$ and (b) 12.

Solution

a. Because $\frac{2}{3} \cdot \frac{3}{2} = 1$, the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.

b. Because $12 \cdot (\frac{1}{12}) = 1$, the reciprocal of 12 is $\frac{1}{12}$. Again, note that we simply inverted the number 12 (understood to equal $\frac{12}{1}$) to get its reciprocal $\frac{1}{12}$.

Try It 5.8.1

Find the reciprocals of: (a) $\frac{3}{7}$ and (b) 15

Answer

(a) $\frac{7}{3}$, (b) $\frac{1}{15}$

Division

Recall that we computed the number of one-eighths in four by doing this calculation:

$$\begin{aligned} 4 \div \frac{1}{8} &= 4 \cdot 8 \\ &= 32. \end{aligned}$$

Note how we inverted the divisor (second number), then changed the division to multiplication. This motivates the following definition of division.

Division Definition

If $\frac{a}{b}$ and $\frac{c}{d}$ are any fractions, then

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}.$$

That is, we invert the divisor (second number) and change the division to multiplication. *Note: We like to use the phrase “invert and multiply” or “multiply by the reciprocal” as a memory aid for this definition.*

Example 5.8.2

Divide $\frac{1}{2}$ by $\frac{3}{5}$.

Solution

To divide $\frac{1}{2}$ by $\frac{3}{5}$, invert the divisor (second number), then multiply.

$$\begin{aligned} \frac{1}{2} \div \frac{3}{5} &= \frac{1}{2} \cdot \frac{5}{3} && \text{Invert the divisor (second number).} \\ &= \frac{5}{6} && \text{Multiply.} \end{aligned}$$

Try It 5.8.2

Divide: $\frac{2}{3} \div \frac{10}{3}$

Answer

$$\frac{1}{5}$$

Example 5.8.3

Simplify the following expressions: (a) $3 \div \frac{2}{3}$ and (b) $\frac{4}{5} \div 5$.

Solution

In each case, invert the divisor (second number), then multiply.

a. Note that 3 is understood to be $\frac{3}{1}$.

$$\begin{aligned} 3 \div \frac{2}{3} &= \frac{3}{1} \cdot \frac{3}{2} \\ &= \frac{9}{2} \end{aligned}$$

Invert the divisor (second number).

Multiply numerators; multiply denominators.

b. Note that 5 is understood to be $\frac{5}{1}$.

$$\begin{aligned} \frac{4}{5} \div 5 &= \frac{4}{5} \cdot \frac{1}{5} \\ &= \frac{4}{25} \end{aligned}$$

Invert the divisor (second number).

Multiply numerators; multiply denominators.

Try It 5.8.3

Divide: $\frac{15}{7} \div 5$

Answer

$$\frac{3}{7}$$

After inverting, you may need to factor and cancel.

Example 5.8.4

Divide $\frac{6}{35}$ by $\frac{33}{55}$.

Solution

Invert, multiply, factor, and cancel common factors.

$$\begin{aligned}
 \frac{6}{35} \div \frac{33}{55} &= \frac{6}{35} \cdot \frac{55}{33} && \text{Invert the divisor (second number).} \\
 &= \frac{6 \cdot 55}{35 \cdot 33} && \text{Multiply numerators; multiply denominators.} \\
 &= \frac{(2 \cdot 3) \cdot (5 \cdot 11)}{(5 \cdot 7) \cdot (3 \cdot 11)} && \text{Factor numerators and denominators.} \\
 &= \frac{2 \cdot \cancel{3} \cdot \cancel{5} \cdot \cancel{11}}{\cancel{5} \cdot 7 \cdot \cancel{3} \cdot \cancel{11}} && \text{Cancel common factors.} \\
 &= \frac{2}{7} && \text{Remaining factors.}
 \end{aligned}$$

Try It 5.8.4

Divide: $\frac{6}{15} \div \left(\frac{42}{35}\right)$

Answer

$$\frac{1}{3}$$

Of course, you can also choose to factor numerators and denominators in place, then cancel common factors.

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5.9: Adding and Subtracting Fractions

Paul and Tony order a pizza which has been cut into eight equal slices. Thus, each slice is $\frac{1}{8}$ of the whole pizza. Paul eats two slices (shaded in light gray in Figure 5.9.1), or $\frac{2}{8}$ of the whole pizza. Tony eats three slices (shaded in light red) in Figure 5.9.1), or $\frac{3}{8}$ of the whole pizza.

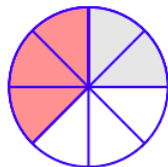


Figure 5.9.1: Paul eats two slices ($\frac{2}{8}$) and Tony eats three slices ($\frac{3}{8}$).

It should be clear that together Paul and Tony eat five slices, or $\frac{5}{8}$ of the whole pizza. This reflects the fact that

$$\frac{2}{8} + \frac{3}{8} = \frac{5}{8}.$$

This demonstrates how to add two fractions with a common (same) denominator. Keep the common denominator and add the numerators. That is,

$$\begin{aligned} \frac{2}{8} + \frac{3}{8} &= \frac{2+3}{8} && \text{Keep denominator; add numerators.} \\ &= \frac{5}{8} && \text{Simplify numerator.} \end{aligned}$$

Adding Fractions with Common Denominators

Let $\frac{a}{c}$ and $\frac{b}{c}$ be two fractions with a common (same) denominator. Their sum is defined as

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

That is, to add two fractions having common denominators, keep the common denominator and add their numerators.

A similar rule holds for subtraction.

Subtracting Fractions with Common Denominators

Let $\frac{a}{c}$ and $\frac{b}{c}$ be two fractions with a common (same) denominator. Their difference is defined as

$$\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}.$$

That is, to subtract two fractions having common denominators, keep the common denominator and subtract their numerators.

Example 5.9.1

Find the sum of $\frac{4}{9}$ and $\frac{3}{9}$.

Solution

Keep the common denominator and add the numerators.

$$\begin{aligned} \frac{4}{9} + \frac{3}{9} &= \frac{4+3}{9} && \text{Keep denominator; add numerators.} \\ &= \frac{7}{9} && \text{Simplify numerator.} \end{aligned}$$

Try It 5.9.1

Add: $\frac{1}{8} + \frac{2}{8}$

Answer

$$\frac{3}{8}$$

Example 5.9.2

Subtract $\frac{5}{16}$ from $\frac{13}{16}$.

Solution

Keep the common denominator and subtract the numerators.

$$\begin{aligned}\frac{13}{16} - \frac{5}{16} &= \frac{13-5}{16} && \text{Keep denominator; subtract numerators.} \\ &= \frac{8}{16} && \text{Simplify numerator.}\end{aligned}$$

Of course, we should always reduce our final answer to lowest terms. One way to accomplish that in this case is to divide numerator and denominator by 8, the greatest common divisor of 8 and 16.

$$\begin{aligned}&= \frac{8 \div 8}{16 \div 8} && \text{Divide numerator and denominator by 8.} \\ &= \frac{1}{2} && \text{Simplify numerator and denominator.}\end{aligned}$$

Try It 5.9.2

Subtract: $\frac{11}{12} - \frac{7}{12}$

Answer

$$\frac{1}{3}$$

Adding Fractions with Different Denominators

Consider the sum

$$\frac{4}{9} + \frac{1}{6}.$$

We cannot add these fractions because they do not have a common denominator. So, what to do?

Goals

In order to add two fractions with different denominators, we need to:

1. Find a common denominator for the given fractions.
2. Make fractions with the common denominator that are equivalent to the original fractions.

If we accomplish the two items in the “Goal,” we will be able to find the sum of the given fractions.

So, how to start? We need to find a common denominator, but not just any common denominator. Let’s agree that we want to keep the numbers as small as possible and find a *least common denominator*.

Definition: Least Common Denominator

The **least common denominator** (LCD) for a set of fractions is the smallest number divisible by each of the denominators of the given fractions.

Consider again the sum we wish to find:

$$\frac{4}{9} + \frac{1}{6}.$$

The denominators are 9 and 6. We wish to find a least common denominator, the smallest number that is divisible by both 9 and 6. A number of candidates come to mind: 36, 54, and 72 are all divisible by 9 and 6, to name a few. But the smallest number that is divisible by both 9 and 6 is 18. This is the least common denominator for 9 and 6.

We now proceed to the second item in “Goal.” We need to make fractions having 18 as a denominator that are equivalent to $\frac{4}{9}$ and $\frac{1}{6}$. In the case of $\frac{4}{9}$, if we multiply both numerator and denominator by 2, we get

$$\begin{aligned}\frac{4}{9} &= \frac{4 \cdot 2}{9 \cdot 2} && \text{Multiply numerator and denominator by 2.} \\ &= \frac{8}{18}. && \text{Simplify numerator and denominator.}\end{aligned}$$

In the case of $\frac{1}{6}$, if we multiply both numerator and denominator by 3, we get

$$\begin{aligned}\frac{1}{6} &= \frac{1 \cdot 3}{6 \cdot 3} && \text{Multiply numerator and denominator by 3.} \\ &= \frac{3}{18}. && \text{Simplify numerator and denominator.}\end{aligned}$$

Typically, we’ll arrange our work as follows.

$$\begin{aligned}\frac{4}{9} + \frac{1}{6} &= \frac{4 \cdot 2}{9 \cdot 2} + \frac{1 \cdot 3}{6 \cdot 3} && \text{Equivalent fractions with LCD = 18.} \\ &= \frac{8}{18} + \frac{3}{18} && \text{Simplify numerators and denominators.} \\ &= \frac{8+3}{18} && \text{Keep common denominator; add numerators.} \\ &= \frac{11}{18} && \text{Simplify numerator.}\end{aligned}$$

Let’s summarize the procedure.

Adding or Subtracting Fractions with Different Denominators

1. Find the LCD, the smallest number divisible by all the denominators of the given fractions.
2. Create fractions using the LCD as the denominator that are equivalent to the original fractions.
3. Add or subtract the resulting equivalent fractions. Simplify, including reducing the final answer to lowest terms.

Example 5.9.3

Simplify: $\frac{3}{5} - \frac{2}{3}$.

Solution

The smallest number divisible by both 5 and 3 is 15.

$$\begin{aligned}\frac{3}{5} - \frac{2}{3} &= \frac{3 \cdot \cancel{3}}{5 \cdot \cancel{3}} - \frac{2 \cdot \cancel{5}}{\cancel{3} \cdot 5} \\ &= \frac{9}{15} - \frac{10}{15} \\ &= \frac{9 - 10}{15} \\ &= \frac{-1}{15}\end{aligned}$$

Equivalent fractions with LCD = 15.

Simplify numerators and denominators.

Keep LCD; subtract numerators.

Simplify numerator.

Although this answer is perfectly acceptable, negative divided by positive gives us a negative answer, so we could also write

$$= -\frac{1}{15}.$$

Try It 5.9.3

Subtract: $\frac{3}{4} - \frac{7}{5}$

Answer

$$-\frac{13}{20}$$

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CHAPTER OVERVIEW

6: The Number Line

This chapter is from the [Support Course for Elementary Statistics](#) by Larry Green from Lake Tahoe Community College.

[6.1: Distance between Two Points on a Number Line](#)

[6.2: Plotting Points and Intervals on the Number Line](#)

[6.3: Represent an Inequality as an Interval on a Number Line](#)

[6.4: The Midpoint](#)

Thumbnail: Demonstration the addition on the line number. (CC BY 3.0 unported; [Stephan Kulla](#)).

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6.1: Distance between Two Points on a Number Line

Learning Outcomes

1. Calculate the distance between two points on a number line when both are non-negative.
2. Calculate the distance between two points on a number line when at least one is negative.

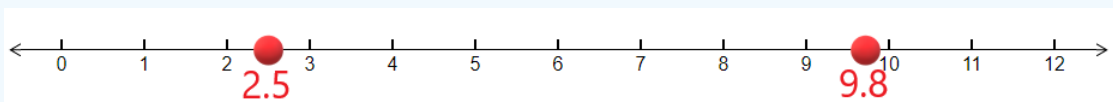
The number line is the main visual base in statistics and we often want to look at two points on the number line and determine the distance between them. This is used to find the base of a rectangle or another figure that lies above the number line. By the end of this section, you will be able to determine the distance between any two points on a number line that comes from a statistics application.

Finding the Distance Between Two Points with Positive Coordinates on a Number Line

The key to finding the distance between two points is to remember that the geometric definition of subtraction is the distance between the two numbers as long as we subtract the smaller number from the larger.

Example 6.1.1

Find the distance between the points 2.5 and 9.8 as shown below on the number line.



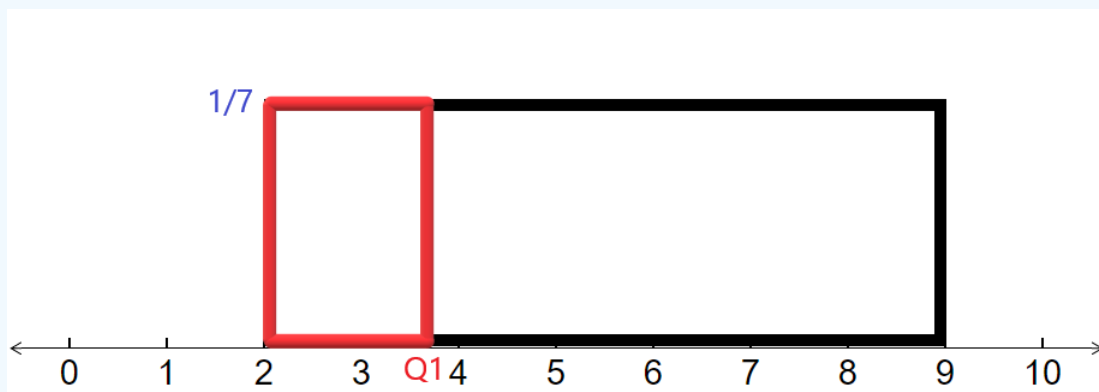
Solution

To find the distance, we just subtract:

$$9.8 - 2.5 = 7.3$$

Example 6.1.2

When finding probabilities involving a uniform distribution, we have to find the base of a rectangle that lies on a number line. Find the base of the rectangle shown below that represents a uniform distribution from 2 to 9.



Solution

We just subtract:

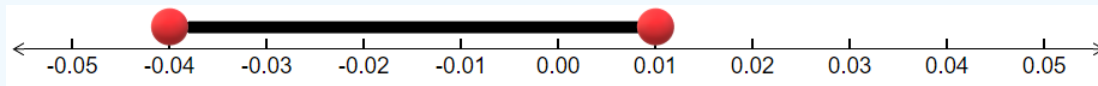
$$9 - 2 = 7$$

Finding the Distance Between Two Points on a Number Line When the Coordinates Are Not Both Positive

In statistics, it is common to have points on a number line where the points are not both positive and we need to find the distance between them.

Example 6.1.3

The diagram below shows the confidence interval for the difference between the proportion of men who are planning on going into the health care profession and the proportion of women. What is the width of the confidence interval?



Solution

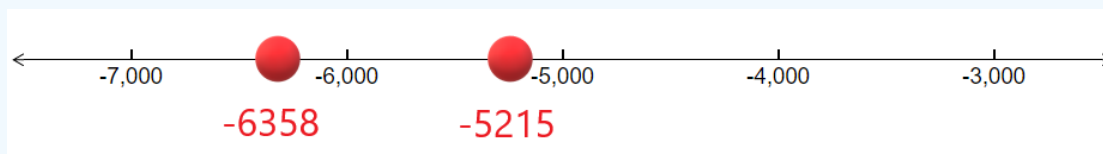
Whenever we want to find the distance between two numbers, we always subtract. Recall that subtracting a negative number is adding.

$$0.01 - (-0.04) = 0.01 + 0.04 = 0.05$$

Therefore the width of the confidence interval is 0.05.

Example 6.1.4

The mean value of credit card accounts is -6358 dollars. A study was done of recent college graduates and found their mean value for their credit card accounts was -5215 dollars. The number line below shows this situation. How far apart are these values?



Solution

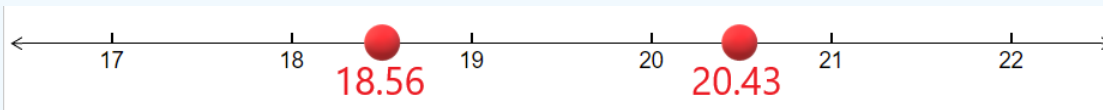
We subtract the two numbers and recall that when we subtract two negative numbers when we are looking at the right minus the left, we make them positive and subtract the positive numbers.

$$-5215 - (-6358) = 6358 - 5215 = 1143$$

Thus the mean credit card balances are \$1143 apart.

Exercise

In statistics, we are asked to find a z-score, which tells us how unusual an event is. The first step in finding a z-score is to calculate the distance a value is from the mean. The number line below depicts the mean of 18.56 and the value of 20.43. Find the distance between these two points.



- [Finding the Distance Between Points on a Number Line](#)
- [Integer Subtraction Using the Number Line](#)

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6.2: Plotting Points and Intervals on the Number Line

Learning Outcomes

1. Plot a point on the number line
2. Plot an interval on the number line

The number line is of fundamental importance and is used repeatedly in statistics. It is a tool to visualize all of the possible outcomes of a study and to organize the results of the study. Often a diagram is placed above the number line to provide us with a picture of the results. By the end of this section, you will be able to plot points and intervals on a number line and use these plots to understand the possible outcomes and actual outcomes of studies.

Drawing Points on a Number Line

A number line is just a horizontal line that is used to display all the possible outcomes. It is similar to a ruler in that it helps us describe and compare numbers. Similar to a ruler that can be marked with many different scales such as inches or centimeters, we get to choose the scale of the number line and where the center is.

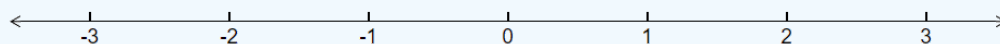
Example 6.2.1

The standard normal distribution is plotted above a number line. The most important values are the integers between -3 and 3. The number 0 is both the mean (average) and median (center).

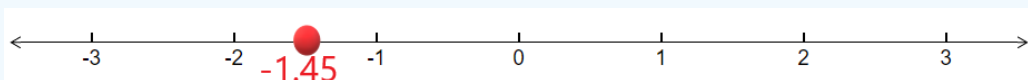
1. Plot the number line that best displays this information.
2. Plot the value -1.45 on this number line.

Solution

1. We sketch a line, mark 0 as the center, and label the numbers -3, -2, -1, 0, 1, 2, 3 from left to right.



2. To plot the point -1.45, we first have to understand that this number is between -1 and -2. It is close to half way between -1 and -2. We put a circle on the number line that is close to halfway between these values as shown below.

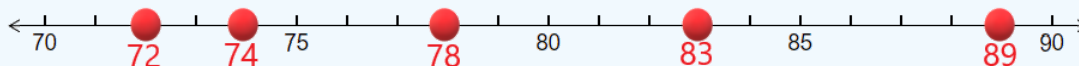


Example 6.2.2

When working with box plots, we need to first set up a number line that labels what is called the five point summary: Minimum, First Quartile, Median, Third Quartile, and Maximum. Suppose the five point summary for height in inches for a basketball team is: 72, 74, 78, 83, 89. Plot these points on a number line

Solution

When plotting points on a number line, we first have to decide what range of the line we want to show in order to best display the points that appear. Technically all numbers are on every number line, but that does not mean we show all numbers. In this example, the numbers are all between 70 and 90, so we certainly don't need to display the number 0. A good idea is to let 70 be on the far left and 90 be on the far right and then plot the points between them. We also have to decide on the spacing of the tick marks. Since the range from 70 to 90 is 20, this may be too many numbers to display. Instead we might want to count by 5's. Below is the number line that shows the numbers 70 to 90 and counts by 5's. The five point summary is plotted on this line.



Exercise

A histogram will be drawn to display the annual income that experienced registered nurses make. The boundaries of the bars of the histogram are: \$81,000, \$108,000, \$135,000, \$162,000, and \$189,000. Plot these points on a number line.

Plotting an Interval on a Number Line

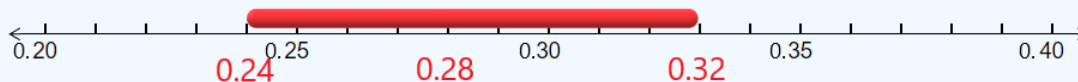
Often in statistics, instead of just having to plot a few points on a number line, we need to instead plot a whole interval on the number line. This is especially useful when we want to exhibit a range of values between two numbers, to the left of a number or to the right of a number.

Example 6.2.3

A 95% confidence interval for the proportion of Americans who work on weekends is found to be 0.24 to 0.32, with the center at 0.28. Use a number line to display this information.

Solution

We just draw a number line, include the three key numbers: 0.24, 0.32, and 0.28 and highlight the part of the interval between 0.24 and 0.32.

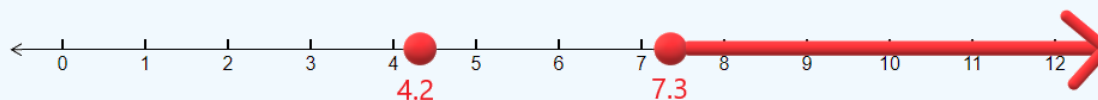


Example 6.2.4: rejection region

In Hypothesis testing, we sketch something called the rejection region which is an interval that goes off to infinity or to negative infinity. Suppose that the mean number of hours to work on the week's homework is 4.2. The rejection region for the hypothesis test is all numbers larger than 7.3 hours. Plot the mean and sketch the rejection region on a number line.

Solution

We plot the point 4.2 on the number line and shade everything to the right of 7.3 on the number line.



- [Plot Integers on the Number Line](#)
- [Intervals: Given an Inequality, Graph the Interval and State Using Interval Notation](#)
- [Plotting Points on a Number Line Application](#)

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6.3: Represent an Inequality as an Interval on a Number Line

Learning Outcomes

1. Graph and inequality on a number line.
2. Graph the complement on a number line for both continuous and discrete variables.

Inequalities come up frequently in statistics and it is often helpful to plot the inequality on the number line in order to visualize the inequality. This helps both for inequalities that involve real numbers and for inequalities that refer to just integer values. As an extension of this idea, we often want to look at the complement of an inequality, that is all numbers that make the inequality false. In this section we will look at examples that accomplish this task.

Sketching an Inequality on a number line where the possible values are real numbers.

There are four different inequalities: $<$, \leq , $>$, \geq . What makes this the most challenging is when they are expressed in words. Here are some of the words that are used for each:

- $<$: "Less Than", "Smaller", "Lower", "Younger"
- \leq : "Less Than or Equal to", "At Most", "No More Than", "Not to Exceed"
- $>$: "Greater Than", "Larger", "Higher", "Bigger", "Older", "More Than"
- \geq : "Greater Than or Equal to", "At Least", "No Less than"

These are the most common words that correspond to the inequalities, but there are others that come up less frequently.

Example 6.3.1

Graph the inequality: $3 < x \leq 5$ on a number line

Solution

First notice that the interval does not include the number 3, but does include the number 5. We can represent not including a number with an open circle and including a number with a closed circle. The number line representation of the inequality is shown below.

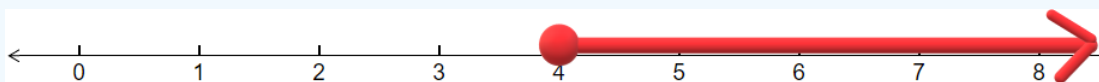


Example 6.3.2

In statistics, we often want to find probabilities of an event being at least as large or no more than a given value. It helps to first plot the interval on a number line. Suppose you want to find the probability that you will have to wait in line for at least 4 minutes. Sketch this inequality on a number line.

Solution

First, notice that "At Least" has the symbol \geq . Thus, we have a closed circle on the number 4. There is no upper bound, so we draw a long arrow from 4 to the right of 4. The solution is shown below

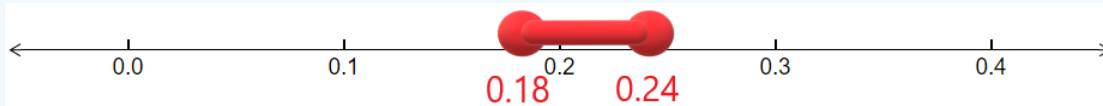


Example 6.3.3.1

Another main topic that comes up in statistics is confidence intervals. For example in recent poll to see the percent of Americans who think that Congress is doing a good job found that a 95% confidence interval had lower bound of 0.18 and an upper bound of 0.24. This can be written as $[0.18, 0.24]$. Sketch this interval on the number line.

Solution

The first thing we need to do is decide on the tick marks to put on the number line. If we counted by 1's, then the interval of interest would be too small to stand out. Instead we will count by 0.1's. The number line is shown below.

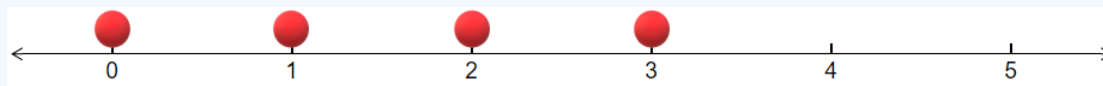


Example 6.3.4

Often in statistics, we deal with discrete variables. Most of the time this will mean that only whole number values can occur. For example, you want to find out the probability that a college student is taking at most three classes. Graph this on a number line.

Solution

First note that the outcomes can only be whole numbers. Second, note that "at most" means \leq . Thus the possible outcomes are: 0, 1, 2, and 3. The number line below displays these outcomes.



Graphing the Complement

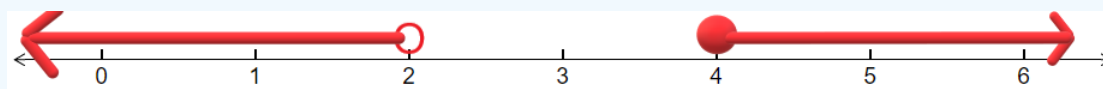
In statistics, we often want to graph the complement of an interval. The complement means everything that is not in the interval.

Example 6.3.5

Graph the complement of the interval $[2, 4)$.

Solution

Notice that the complement of numbers inside the interval between 2 and 4 is the numbers outside that interval. This will consist of the numbers to the left of 2 and to the right of 4. Since the number 2 is included in the original interval, it will not be included in the complement. Since the number 4 is not included in the original interval, it will be included in the complement. The complement is shown on the number line below.

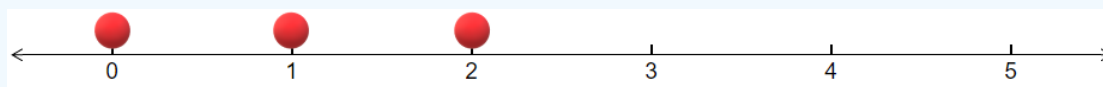


Example 6.3.6

Some calculators can only find probabilities for values less than a certain number. If we want the probability of an interval greater than a number, we need to use the complement. Suppose that you want to find the probability that a person will have traveled to more than two foreign countries in the last twelve months. Find the complement of this and graph it on a number line.

Solution

First notice that only whole numbers are possible since it does not make sense to go to a fractional number of countries. Second note that the lowest number that is more than 2 is 3. If 3 is included in the original list, then 3 will not be included in the complement. Thus, the highest number that is in the complement of "more than 2" is 2. The number line below shows the complement of more than 2.



Exercise

Suppose you want to find the probability that at least 4 people in your class have a last name that contains the letter "W". To make this calculation you will need to first find the complement of "at least 4". Sketch this complement on the number line.

- [Intervals: Given an Inequality, Graph the Interval and State Using Interval Notation](#)
- [Express Inequalities as a Graph and Interval Notation](#)
- [Sketching the Complement of an Interval on a Number Line](#)

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6.4: The Midpoint

Learning Outcomes

1. Find the midpoint between two numbers.
2. Sketch the midpoint of two numbers on a number line.

As the word sounds, "midpoint" means "the point in the middle". Finding a midpoint is not too difficult and has applications in many areas of statistics, from confidence intervals to sketching distributions, to means.

Finding the Midpoint Between Two Numbers

If we are given two numbers, then the midpoint is just the average of the two numbers. To calculate the midpoint, we add them up and then divide the result by 2. The formula is as follows:

Definition: the Midpoint

Let a and b be two numbers. Then the midpoint, M of these two numbers is

$$M = \frac{a+b}{2} \quad (6.4.1)$$

Example 6.4.1

Find the midpoint of the numbers 3.5 and 7.2.

Solution

The most important thing about finding the midpoint is that the addition of the two numbers must occur before the division by 2. We can either do this one step at a time in our calculator or we can enclose the sum in parentheses. In this example we will perform the addition first:

$$3.5 + 7.2 = 10.7$$

Now we are ready to divide by 2:

$$\frac{10.7}{2} = 5.35$$

Thus the midpoint of 3.5 and 7.2 is 5.35.

Example 6.4.2

A major topic in statistics is the confidence interval which tells us the most likely interval that the mean or the proportion will lie in. Often the lower and upper bound of the confidence interval are given, but the midpoint of these two numbers is the best guess for what we are looking for. Suppose a 95% confidence interval for the difference between two means is -1.34 and 2.79. Find the midpoint of these numbers, which is the best guess for the difference between the two means.

Solution

We use the formula for the midpoint (Equation 6.4.1):

$$M = \frac{a+b}{2} = \frac{-1.34 + 2.79}{2}$$

Now let's use a calculator. We will need parentheses around the numerator:

$$(-1.34 + 2.79) \div 2 = 0.725$$

Thus, the midpoint of the numbers -1.34 and 2.79 is 0.725.

Sketching the Midpoint on a Number Line

Visualizing the midpoint can often reveal it much better than just writing down its value. The diagrams are of fundamental importance in statistics.

Example 6.4.3

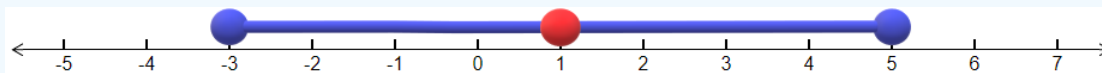
Sketch the points -3, 5 and the midpoint of these two numbers on a number line.

Solution

We start by finding the midpoint using the midpoint formula (Equation 6.4.1):

$$M = \frac{-3 + 5}{2} = (-3 + 5) \div 2 = 1$$

Now we sketch these three points on the number line:

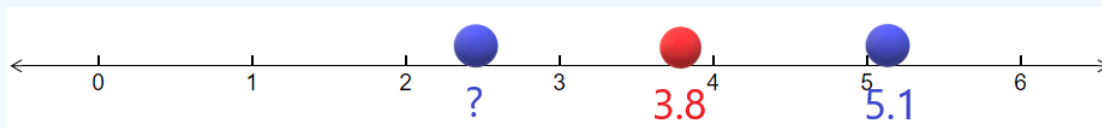


Example 6.4.4: hypothesis testing

Another application of the midpoint involves hypothesis testing. Sometimes we are given the hypothesized mean, which is the midpoint. We are also given the sample mean, which is either the left or right endpoint. The goal is to find the other endpoint. Suppose that the midpoint (hypothesized mean) is at 3.8 and the right endpoint (sample mean) is at 5.1. Find the value of the left endpoint.

Solution

It helps to sketch the diagram on the number line as shown below.



Now since 3.8 is the midpoint, the distance from the left endpoint to the midpoint is equal to the distance from 3.8 to 5.1. The distance from 3.8 to 5.1 is:

$$5.1 - 3.8 = 1.3$$

Therefore the left endpoint is 1.3 to the left of 3.8. This can be found by subtracting the two numbers:

$$3.8 - 1.3 = 2.5$$

Therefore the left endpoint is at 2.5.

Exercise

Suppose that the midpoint (hypothesized proportion) is at 0.31 and the left endpoint (sample proportion) is at 0.28. Find the value of the right endpoint.

- [Midpoint on the Number line](#)
- [Finding the Right Endpoint Given the Left Endpoint and Midpoint](#)

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CHAPTER OVERVIEW

7: Operations on Numbers

This chapter is from the [Support Course for Elementary Statistics](#) by Larry Green from Lake Tahoe Community College.

- [7.1: Area of a Rectangle](#)
- [7.2: Factorials and Combination Notation](#)
- [7.3: Order of Operations](#)
- [7.4: Order of Operations in Expressions and Formulas](#)
- [7.5: Perform Signed Number Arithmetic](#)
- [7.6: Powers and Roots](#)
- [7.7: Using Summation Notation](#)
- [7.8: Solving Equations Using the Subtraction and Addition Properties of Equality](#)
- [7.9: Introduction to Inequalities and Interval Notation](#)
- [7.10: Translating English to Math](#)

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7.1: Area of a Rectangle

Learning Outcomes

- Find the area of a rectangle.
- Find the height of a rectangle given that the area is equal to 1.

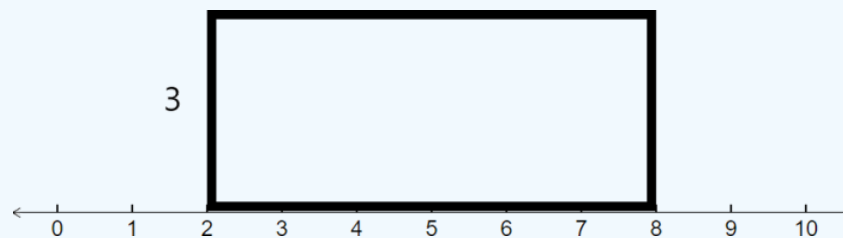
Rectangles are of fundamental importance in the portion of statistics that involves the uniform distribution. Every rectangle has a base and a height and an area. The formula for the area of a rectangle is:

$$\text{Area} = \text{Base} \times \text{Height} \quad (7.1.1)$$

When working with the uniform distribution, the area represents the probability of an event being within the bounds of the base.

Example 7.1.1

Consider the rectangle shown below.



Find the area of this rectangle.

Solution

We use the Area formula (Equation 7.1.1). To find the base, we notice that it runs from 2 to 8, so we subtract these numbers to get the base:

$$\text{Base} = 8 - 2 = 6$$

Next multiply by the height, 3, to get

$$\text{Area} = \text{Base} \times \text{Height} = 6 \times 3 = 18$$

Example 7.1.2

It turns out that the area of the rectangles that equal to 1 will occur the most often for a uniform distribution. Suppose that we know that the area of a rectangle that depicts a uniform distribution is equal to 1 and that the base of the rectangle goes from 4 to 7. Find the height of the rectangle.

Solution

First sketch the rectangle below, labeling the height as h .



Next, find the base of the rectangle that goes from 4 to 7 by subtracting:

$$\text{Base} = 7 - 4 = 3$$

Next, plug in what we know into the area equation:

$$1 = \text{Area} = \text{Base} \times \text{Height} = 3 \times h$$

This tells us that 3 times a number is equal to 1. To find out what the number is, we just divide both sides by 3 to get:

$$h = \frac{1}{3}$$

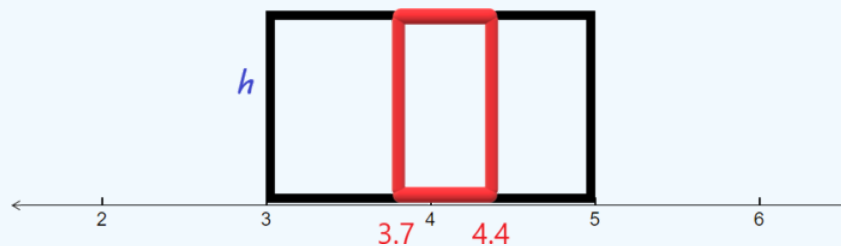
Therefore the height of an area 1 rectangle with base from 4 to 7 is $\frac{1}{3}$.

Example 7.1.3

Suppose that we know that the area of a rectangle that depicts a uniform distribution is equal to 1 and that the base of the rectangle goes from 3 to 5. There is a smaller rectangle within the larger one with the same height, but whose base goes from 3.7 to 4.4. Find the area of the smaller rectangle.

Solution

First, sketch the larger rectangle with the smaller rectangle shaded in.



Next, we find the height of the rectangle. We know that the area of the larger rectangle is 1. The base goes from 3 to 5, so the base is $5 - 3 = 2$. Hence:

$$1 = \text{Area} = \text{Base} \times \text{Height} = 2h$$

Dividing by 2, gives us that the height is $\frac{1}{2}$ or 0.5. Now we are ready to find the area of the smaller rectangle. We first find the base by subtracting:

$$\text{Base} = 4.4 - 3.7 = 0.7$$

Next, use the area formula:

$$\text{Area} = \text{Base} \times \text{Height} = 0.7 \times 0.5 = 0.35$$

Exercise 7.1.1

Suppose that elementary students' ages are uniformly distributed from 5 to 11 years old. The rectangle that depicts this has base from 5 to 11 and area 1. The rectangle that depicts the probability that a randomly selected child will be between 6.5 and 8.6 years old has base from 6.5 to 8.6 and the same height as the larger rectangle. Find the area of the smaller rectangle

- [Ex: Determine the Area of a Rectangle Involving Whole Numbers](#)
- [Area of a Rectangle and the Uniform Distribution](#)

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7.2: Factorials and Combination Notation

Learning Outcomes

1. Evaluate a factorial.
2. Use combination notation for statistics applications.

When we need to compute probabilities, we often need to multiply descending numbers. For example, if there is a deck of 52 cards and we want to pick five of them without replacement, then there are 52 choices for the first pick, 51 choices for the second pick since one card has already been picked, 50 choices for the third, 49 choices for the fourth, and 48 for the fifth. If we want to find out how many different outcomes there are, we can use what we call the multiplication principle and multiply them: $52 \times 51 \times 50 \times 49 \times 48$. If we wanted to pick all 52 of the cards one at a time, then this list would be excessively long. Instead there is a notation that describes multiplying all the way down to 1, called the factorial. It must be exciting, since we use the symbol "!" for the factorial.

Example 7.2.1

Calculate $4!$

Solution

We use the definition which says start at 4 and multiply until we get to 1:

$$4! = 4 \times 3 \times 2 \times 1 = 24$$

Example 7.2.2

If we pick 5 cards from a 52 card deck without replacement and the same two sets of 5 cards, but in different orders, are considered different, how many sets of 5 cards are there?

Solution

From the introduction, the number of sets is just:

$$52 \times 51 \times 50 \times 49 \times 48$$

This is not quite a factorial since it stops at 48; however, we can think of this as $52!$ with $47!$ removed from it. In other words we need to find

$$\frac{52!}{47!}$$

We could just multiply the numbers from the original list, but it is a good idea to practice with your calculator or computer to find this using the ! symbol. When you do use technology, you should get:

$$\frac{52!}{47!} = 311,875,200$$

Combinations

One of the most important applications of factorials is combinations which count the number of ways of selecting a smaller collection from a larger collection when order is not important. For example if there are 12 people in a room and you want to select a team of 4 of them, then the number of possibilities uses combinations. Here is the definition:

Definition: Combinations

The number of ways of selecting k items without replacement from a collection of n items when order does not matter is:

$$\binom{n}{r} = {}_nC_r = \frac{n!}{r!(n-r)!} \quad (7.2.1)$$

Notice that there are a few notations. The first is more of a mathematical notation while the second is the notation that a calculator uses. For example, in the TI 84+ calculator, the notation for the number of combinations when selecting 4 from a collection of 12 is:

$${}_{12}C_4$$

There are many internet sites that will perform combinations. For example the [math is fun](#) site asks you to put in n and r and also state whether order is important and repetition is allowed. If you click to make both "no" then you will get the combinations.

Example 7.2.3

Calculate

$$\binom{15}{11} = {}_{15}C_{11}$$

Solution

Whether you use a hand calculator or a computer you should get the number: 1365

Example 7.2.4

The probability of winning the Powerball lottery if you buy one ticket is:

$$P(\text{win}) = \frac{1}{{}_{69}C_5 \times 26}$$

Calculate this probability.

Solution

First, let's calculate ${}_{69}C_5$. Using a calculator or computer, you should get 11,238,513. Next, multiply by 26 to get

$$11,238,513 \times 26 = 292,201,338$$

Thus, there is a one in 292,201,338 chance of winning the Powerball lottery if you buy a ticket. We can also write this as a decimal by dividing:

$$P(\text{win}) = \frac{1}{292,201,338} = 0.000000003422$$

As you can see, your chances of winning the Powerball are very small.

Exercise

A classroom is full of 28 students and there will be one president of the class and a "Congress" of 4 others selected. The number of different leadership group possibilities is:

$$28 \times {}_{27}C_4$$

Calculate this number to find out how many different leadership group possibilities there are.

Ex 1: Simplify Expressions with Factorials

Combinations

Combinations

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7.3: Order of Operations

Learning Outcomes

1. Use the order of operations to correctly perform multi-step arithmetic.
2. Apply the order of operations to statistics related complex questions.

When we are given multiple arithmetic operations within a calculation, there is an established order that we must do them in based on how the expression is written. Understanding these rules is especially important when using a calculator, since calculators are programmed to strictly follow the order of operations. This comes up in every topic in statistics, so knowing the order of operations is an essential skill for all successful statistics students to have.

PEMDAS

The order of operations are as follows:

1. **P**arentheses
2. **E**xponents
3. **M**ultiplication and **D**ivision, from left to right
4. **A**ddition and **S**ubtraction, from left to right

Notice that Multiplication and division are listed together as item 3. If you see multiplication and division in the same expression the rule is to go in order from left to right. You do not calculate all of the multiplication operations then go back and do the division. Similarly, if you see addition and subtraction in the same expression the rule is to go in order from left to right. The same goes for two of the same arithmetic operators.

Example 7.3.1

Evaluate: $20 - 6 \div 3 + (2 \times 3^2)$

Solution

We start with what is inside the parentheses: 2×3^2 . Since exponents comes before addition, we find $3^2 = 9$ first. We now have

$$20 - 6 \div 3 + (2 \times 9)$$

We continue inside the parentheses and perform the multiplication: $2 \times 9 = 18$.

This gives

$$20 - 6 \div 3 + 18$$

Since division comes before addition and subtraction, we next calculate $6 \div 3 = 2$ to get

$$20 - 2 + 18$$

Since subtraction and addition are tied, we go from left to right. We calculate: $20 - 2 = 18$ to get

$$18 + 18 = 36$$

The key to arriving at the correct answer is to go slow and write down each step in the arithmetic.

Hidden Parentheses

You may think that since you always have a calculator or computer at hand, that you don't need to worry about order of operations. Unfortunately, the way that expressions are written is not the same as the way that they are entered into a computer or calculator. In particular, exponents need to be treated with care as do fractions bars.

Example 7.3.2

Evaluate 2.1^{6-2}

Solution

First, note that we use the symbol "^" to tell a computer or calculator to exponentiate. If you were to enter 2.1^6-2 into a computer, it would give you the answer of 83.766121 which is not correct, since the computer will first calculate 2.1^6 and then subtract 2. Since the subtraction is within the exponent, it must be performed first. To tell a calculator or computer to perform the subtraction first, we use parentheses:

$$2.1^{(6-2)} = 2.1^4 = 19.4481$$

Example 7.3.3

The "z-score" is defined by:

$$z = \frac{x - \mu}{\sigma}$$

Find the z-score rounded to one decimal place if:

$$x = 2.323, \mu = 1.297, \sigma = 0.241$$

Solution

Once again, if we put these numbers into the z-score formula and use a computer or calculator by entering $3.323 - 1.297 \div 0.241$ we will get -0.259 which is the wrong answer. Instead, we need to know that the fraction bar separates the numerator and the denominator, so the subtraction must be done first. We compute

$$\frac{2.323 - 1.297}{0.241} = (2.323 - 1.297) \div 0.241 = 4.25726141$$

Now round to one decimal place to get 4.3. Notice that if you rounded before you did the arithmetic, you would get exactly 5 which is very different. 4.3 is more accurate.

Try It

Suppose the equation of the regression line for the number of pairs of socks a person owns, \hat{y} , based on the number of pairs of shoes, x , the person owns is

$$\hat{y} = 6 + 2x$$

Use this regression line equation to predict the number of pairs of socks a person owns for a person who owns 4 pairs of shoes.

Answer

$$\text{When } x = 4, \hat{y} = 6 + 2(4) = 6 + 8 = 14.$$

A person who owns 4 pairs of shoes has 14 pairs of socks, according to the regression model.

Additional video resources:

- [Order of Operations - The Basics](#)
- [Order of Operations](#)

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7.4: Order of Operations in Expressions and Formulas

Learning Outcomes

- Use order of operations in statistics formulas.

We have already encountered the order of operations: Parentheses, Exponents, Multiplication and Division (from left to right), Addition and Subtraction (from left to right). In this section, we will give some additional examples where the order of operations must be used properly to evaluate statistics.

Example 7.4.1

The formula for the sample standard deviation asks us to add up the squared deviations from the mean, take the square root and divide by one less than the sample size. For example, suppose that there are three data values: 3, 5, 10. The mean of these values is 6. Then the standard deviation is:

$$s = \sqrt{\frac{(3-6)^2 + (5-6)^2 + (10-6)^2}{3-1}}$$

Evaluate this number rounded to the nearest hundredth.

Solution

The first thing in the order of operations is to do what is in the parentheses. We must subtract:

$$3 - 6 = -3, \quad 5 - 6 = -1, \quad 10 - 6 = 4$$

We can substitute the numbers in to get:

$$= \sqrt{\frac{(-3)^2 + (-1)^2 + (4)^2}{3-1}}$$

Next, we exponentiate:

$$(-3)^2 = 9, \quad (-1)^2 = 1, \quad 4^2 = 16$$

Substitute these in to get:

$$\sqrt{\frac{9+1+16}{3-1}}$$

We can now perform the addition inside the square root to get:

$$\sqrt{\frac{26}{3-1}}$$

Next, perform the subtraction of the denominator to get:

$$\sqrt{\frac{26}{2}}$$

We can divide to get:

$$\sqrt{13}$$

We don't want to do this by hand, so in a calculator or computer type in:

$$\sqrt{13} = 3.61$$

(rounded to the nearest hundredth)

Example 7.4.2

When calculating the probability that a value will be less than 4.6 if the value is taken randomly from a uniform distribution between 3 and 7, we have to calculate:

$$(4.6 - 3) \times \frac{1}{7 - 3}$$

Find this probability.

Solution

We can use a calculator or computer, but we must be very careful about the order of operations. Notice that there are implied parentheses due to the fraction bar. The answer is:

$$\frac{(4.6 - 3) \times 1}{7 - 3}$$

Using technology, we get:

$$(4.6 - 3) \times 1 / (7 - 3) = 0.4 \quad (7.4.1)$$

Try It

When finding the upper bound, U , of a confidence interval given the lower bound, L , and the margin of error, E , we use the formula

$$U = L + 2E$$

Find the upper bound of the confidence interval for the proportion of babies that are born preterm if the lower bound is 0.085 and the margin of error is 0.03.

Answer

Plug in $L = 0.085$ and $E = 0.03$:

$$U = L + 2E = 0.085 + 2(0.03) = 0.085 + 0.06 = 0.145$$

The upper bound of the confidence interval for the proportion of babies that are born preterm is 0.145 or 14.5%.

Additional video resources:

- [Ex: Evaluate an Expression Using the Order of Operations](#)
- [Order of Operations and Confidence Intervals](#)

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7.5: Perform Signed Number Arithmetic

Learning Outcomes

1. Add signed numbers.
2. Subtract signed numbers.
3. Multiply signed numbers.
4. Divide signed numbers.

Even though negative numbers seem not that common in the real world, they do come up often when doing comparisons. For example, a common question is how much bigger is one number than another, which involves subtraction. In statistics we don't know the means until we collect the data and do the calculations. This often results in subtracting a larger number from a smaller number which yields a negative number. Because of this and for many other reasons, we need to be able to perform arithmetic on both positive and negative numbers.

Adding Signed Numbers

We will assume that you are very familiar with adding positive numbers, but when there are negative numbers involved, there are some rules to follow:

1. When adding two negative numbers, ignore the negative signs, add the positive numbers and then make the result negative.
2. When adding two numbers such that one is positive and the other is negative, ignore the sign, subtract the smaller from the larger. If the larger of the positive numbers was originally negative, then make the result negative. Otherwise keep the result positive.

Example 7.5.1

Add:

$$-4 + (-3)$$

Solution

First we ignore the signs and add the positive numbers.

$$4 + 3 = 7$$

Next we make the result negative.

$$-4 + (-3) = -7$$

Example 7.5.2

Add:

$$-2 + 5$$

Solution

Since one of the numbers is positive and the other is negative, we subtract:

$$5 - 2 = 3$$

Of the two numbers, 2 and 5, 5 is the larger one and started positive. Hence we keep the result positive:

$$-2 + 5 = 3$$

Subtracting Numbers

Subtraction comes up often when we want to find the width of an interval in statistics. Here are the cases for subtracting: $a - b$:

1. If $a \geq b \geq 0$, then this is just ordinary subtraction.
2. If $b \geq a \geq 0$, then find $b - a$ and make the result negative.
3. If $a < 0$, $b \geq 0$, then make both positive, add the two positive numbers and make the result negative.
4. If $b < 0$ then you use the rule that subtracting a negative number is the same as adding the positive number.

Example 7.5.3

Evaluate $5 - 9$

Solution

Since 9 is bigger than 5, we subtract:

$$9 - 5 = 4$$

Next, we make the result negative to get:

$$5 - 9 = -4$$

Example 7.5.4

Evaluate $-9 - 4$

Solution

We are in the case $a < 0$, $b \geq 0$. Therefore, we first make both positive and add the positive numbers.

$$9 + 4 = 13$$

The final step is to make the answer negative to get

$$-9 - 4 = -13$$

Example 7.5.5: Uniform distributions

In statistics, we call a *distribution Uniform* if an event is just as likely to be in any given interval within the bounds as any other interval within the bounds as long as the intervals are both of the same width. Finding the width of a given interval is usually the first step in solving a question involving uniform distributions. Suppose that the temperature on a winter day has a Uniform distribution on $[-8, 4]$. Find the width of this interval

Solution

To find the width of an interval, we subtract the left endpoint from the right endpoint:

$$4 - (-8)$$

Since we are subtracting a negative number, the "-" signs become addition:

$$4 - (-8) = 4 + 8 = 12$$

Thus the width of the interval is 12.

Multiplying and Dividing Signed Numbers

When we have a multiplication or division problem, we just remember that two negatives make a positive. So if there are an even number of negative numbers that are multiplied or divided, the result is negative. If there are an odd number of negative numbers that are multiplied or divided, the result is positive.

Example 7.5.6

Perform the arithmetic:

$$\frac{(-6)(-10)}{(-4)(-5)}$$

Solution

First, just ignore all of the negative signs and multiply the numerator and denominator separately:

$$\frac{(6)(10)}{(4)(5)} = \frac{60}{20}$$

Now divide:

$$\frac{60}{20} = \frac{6}{2} = 3$$

Finally, notice that there are four negative numbers in the original multiplication and division problem. Four is an even number, so the answer is positive:

$$\frac{(-6)(-10)}{(-4)(-5)} = 3$$

Example 7.5.7

A confidence interval for the population mean difference in books read per year by men and women was found to be $[-4, 1]$. Find the midpoint of this interval.

Solution

First recall that to find the midpoint of two numbers, we add them and then divide by 2. Hence, our first step is to add -4 and 1. Since 1 is positive and -4 is negative, we first subtract the two numbers:

$$4 - 1 = 3$$

Of the two numbers, 4 and 1, 4 is the larger one and started negative. Hence we change the sign to negative::

$$-4 + 1 = -3$$

The final step in finding the midpoint is to divide by 2. First we divide them as positive numbers:

$$\frac{3}{2} = 1.5$$

Since the original quotient has a single negative number (an odd number of negative numbers), the answer is negative. Thus the midpoint of -4 and 1 is -1.5.

Exercise

The difference between the observed value and the expected value in linear regression is called the residual. Suppose that the three observed values are: -4, 2, and 5. The expected values are -3, 7, and -1. First find the residuals and then find the sum of the residuals.

- [Signed Number Operations \(L1.4\)](#)
- [signed arithmetic](#)

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7.6: Powers and Roots

Learning Outcomes

1. Raise a number to a power using technology.
2. Take the square root of a number using technology.
3. Apply the order of operations when there is root or a power.

It can be a challenge when we first try to use technology to raise a number to a power or take a square root of a number. In this section, we will go over some pointers on how to successfully take powers and roots of a number. We will also continue our practice with the order of operations, remembering that as long as there are no parentheses, exponents always come before all other operations. We will see that taking a power of a number comes up in probability and taking a root comes up in finding standard deviations.

Powers

Just about every calculator, computer, and smartphone can find powers of a number. We just need to remember that the symbol "^" is used to mean "to the power of." We also need to remember to use parentheses if we need to force other arithmetic operations to come before the exponentiation.

Example 7.6.1

Evaluate: 1.04^5 and round to two decimal places.

Solution

This definitely calls for the use of technology. Most calculators, whether hand calculators or computer calculators, use the symbol "^" (shift 6 on the keyboard) for exponentiation. We type in:

$$1.04^5 = 1.2166529$$

We are asked to round to two decimal places. Since the third decimal place is a 6 which is 5 or greater, we round up to get:

$$1.04^5 \approx 1.22$$

Example 7.6.2

Evaluate: $2.8^{5.3 \times 0.17}$ and round to two decimal places.

Solution

First note that on a computer we use "*" (shift 8) to represent multiplication. If we were to put in $2.8 \wedge 5.3 * 0.17$ into the calculator, we would get the wrong answer, since it will perform the exponentiation before the multiplication. Since the original question has the multiplication inside the exponent, we have to force the calculator to perform the multiplication first. We can ensure that multiplication occurs first by including parentheses:

$$2.8 \wedge (5.3 * 0.17) = 2.52865$$

Now round to decimal places to get:

$$2.8^{5.3 \times 0.17} \approx 2.53$$

Example 7.6.3

If we want to find the probability that if we toss a six-sided die five times that the first two rolls will each be a 1 or a 2 and the last three die rolls will be even, then the probability is:

$$\left(\frac{1}{3}\right)^2 \times \left(\frac{1}{2}\right)^3$$

What is this probability rounded to three decimal places?

Solution

We find:

$$[(1/3)^2] * [(1/2)^3] \approx 0.013888889$$

Now round to three decimal places to get

$$\left(\frac{1}{3}\right)^2 \times \left(\frac{1}{2}\right)^3 \approx 0.014$$

Square Roots

Square roots come up often in statistics, especially when we are looking at standard deviations. We need to be able to use a calculator or computer to compute a square root of a number. There are two approaches that usually work. The first approach is to use the $\sqrt{}$ symbol on the calculator if there is one. For a computer, using `sqrt()` usually works. For example if you put `10*sqrt(2)` in the Google search bar, it will show you 14.1421356, which is $10\sqrt{2}$. A second way that works for pretty much any calculator, whether it is a hand held calculator or a computer calculator, is to realize that the square root of a number is the same thing as the number raised to the $1/2$ power. In order to not have to wrap $1/2$ in parentheses, it is easier to type in the number to the 0.5 power.

Example 7.6.3

Evaluate $\sqrt{42}$ and round your answer to two decimal places.

Solution

Depending on the technology you are using you will either enter the square root symbol and then the number 42 and then close the parentheses if they are presented and then hit enter. If you are using a computer, you can use `sqrt(42)`. The third way that will work for both is to enter:

$$42^{0.5} \approx 6.4807407$$

You must then round to two decimal places. Since 0 is less than 5, we round down to get:

$$\sqrt{42} \approx 6.48$$

Example 7.6.4

The "z-score" is for the value of 28 for a sampling distribution with sample size 60 coming from a population with mean 28.3 and standard deviation 5 is defined by:

$$z = \frac{28 - 28.3}{\frac{5}{\sqrt{60}}}$$

Find the z-score rounded to two decimal places.

Solution

We have to be careful about the order of operations when putting it into the calculator. We enter:

$$(28 - 28.3) / (5 / 60^{0.5}) = -0.464758$$

Finally, we round to 2 decimal places. Since 4 is smaller than 5, we round down to get:

$$z = \frac{28 - 28.3}{\frac{5}{\sqrt{60}}} = -0.46$$

Try It

The standard error, which is an average of how far sample means are from the population mean is defined by:

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

where $\sigma_{\bar{x}}$ is the standard error, σ is the standard deviation, and n is the sample size. Find the standard error rounded to three decimal places if the population standard deviation, σ , is 14 and the sample size, n , is 11.

Answer

$$\sigma_{\bar{x}} = \frac{14}{\sqrt{11}} \approx 4.221$$

Additional video resources:

- [Square Root on the TI-83 plus and TI-84 family of Calculators](#)
- [Square Roots with a Computer](#)

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7.7: Using Summation Notation

Learning Outcomes

1. Evaluate an expression that includes summation notation.
2. Apply summation notation to calculate statistics.

This notation is called summation notation and appears as:

$$\sum_{i=1}^n a_i$$

In this notation, the a_i is an expression that contains the index i and you plug in 1 and then 2 and then 3 all the way to the last number n and then add up all of the results.

Example 7.7.1

Calculate

$$\sum_{i=1}^4 3i$$

Solution

First notice that $i = 1$, then 2, then 3 and finally 4. We are supposed to multiply each of these by 3 and add them up:

$$\begin{aligned}\sum_{i=1}^4 3i &= 3(1) + 3(2) + 3(3) + 3(4) \\ &= 3 + 6 + 9 + 12 = 30\end{aligned}$$

Example 7.7.2

The formula for the sample mean, sometimes called the average, is

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$$

A survey was conducted asking 8 older adults how many sexual partners they have had in their lifetime. Their answers were $\{4, 12, 1, 3, 4, 9, 24, 7\}$. Use the formula to find the sample mean.

Solution

Notice that the numerator of the formula just tells us to add the numbers up. Computing the numerator first gives:

$$\sum_{i=1}^8 x_i = 4 + 12 + 1 + 3 + 4 + 9 + 24 + 7 = 64$$

Now that we have the numerator calculated, the formula tells us to divide by n , which is just 8. We have:

$$\bar{x} = \frac{64}{8} = 8$$

Thus, the sample mean number of sexual partners this group had in their lifetimes is 8.

Example 7.7.3

The next most important statistic is the standard deviation. The formula for the sample standard deviation is:

$$s = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}}$$

Let's consider the data in the previous example. Find the standard deviation.

Solution

The formula is quite complicated, but if tackle it one piece at a time using the order of operations properly, we can succeed in finding the sample standard deviation for the data. Notice that there are parentheses, so based on the order of operations, we must do the subtraction within the parentheses first. Since this is all part of the sum, we have eight different subtractions to do. From our calculations in the previous example, the sample mean was $\bar{x} = 8$. We compute the 8 subtractions:

$$4 - 8 = -4, 12 - 8 = 4, 1 - 8 = -7, 3 - 8 = -5, \\ 4 - 8 = -4, 9 - 8 = 1, 24 - 8 = 16, 7 - 8 = -1$$

The next arithmetic to do is to square each of the differences to get:

$$(-4)^2 = 16, (4)^2 = 16, (-7)^2 = 49, (-5)^2 = 25, \\ (-4)^2 = 16, 1^2 = 1, 16^2 = 256, (-1)^2 = 1$$

Now we have all the entries in the summation, so we add them all up:

$$16 + 16 + 49 + 25 + 16 + 1 + 256 + 1 = 380$$

Now we can write

$$s = \sqrt{\frac{380}{8-1}} = \sqrt{\frac{380}{7}}$$

We can put this into the calculator or computer to get:

$$s = \sqrt{\frac{380}{7}} = 7.3679$$

Exercise: expected value

The expected value, EV, is defined by the formula

$$EV = \sum_{i=1}^n x_i P(x_i)$$

Where x_i are the possible outcomes and $P(x_i)$ are the probabilities of the outcomes occurring. Suppose the table below shows the number of eggs in a bald eagle clutch and the probabilities of that number occurring.

Probability Distribution Table with Outcomes, x, and probabilities, P(x)

x	1	2	3	4
P(x)	0.2	0.4	0.3	0.1

Find the expected value.

Ex 1: Find a Sum Written in Summation / Sigma Notation

Summation Notation and Expected Value

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7.8: Solving Equations Using the Subtraction and Addition Properties of Equality

Learning Objectives

By the end of this section, you will be able to:

- Determine whether a number is a solution of an equation
- Model the Subtraction Property of Equality
- Solve equations using the Subtraction Property of Equality
- Solve equations using the Addition Property of Equality
- Translate word phrases to algebraic equations
- Translate to an equation and solve

Be Prepared 2.7

Before you get started, take this readiness quiz.

Evaluate $x+8$ when $x=11$. Evaluate $x+8$ when $x=11$.

If you missed this problem, review Example 2.13.

Be Prepared 2.8

Evaluate $5x-3$ when $x=9$. Evaluate $5x-3$ when $x=9$.

If you missed this problem, review Example 2.14.

Be Prepared 2.9

Translate into algebra: the difference of xx and 8.8 .

If you missed this problem, review Example 2.24.

When some people hear the word *algebra*, they think of solving equations. The applications of solving equations are limitless and extend to all careers and fields. In this section, we will begin solving equations. We will start by solving basic equations, and then as we proceed through the course we will build up our skills to cover many different forms of equations.

Determine Whether a Number is a Solution of an Equation

Solving an equation is like discovering the answer to a puzzle. An algebraic equation states that two algebraic expressions are equal. To solve an equation is to determine the values of the variable that make the equation a true statement. Any number that makes the equation true is called a solution of the equation. It is the answer to the puzzle!

Solution of an Equation

A solution to an equation is a value of a variable that makes a true statement when substituted into the equation.

The process of finding the solution to an equation is called solving the equation.

To find the solution to an equation means to find the value of the variable that makes the equation true. Can you recognize the solution of $x+2=7$? $x+2=7$? If you said 5,5, you're right! We say 5 is a solution to the equation $x+2=7$ because when we substitute 5 for x the resulting statement is true.

$$x+2=5+2=? \quad 5+2=7 \quad 7=7 \quad \checkmark \quad x+2=5+2=? \quad 5+2=7 \quad 7=7 \quad \checkmark$$

Since $5+2=5+2=7$ is a true statement, we know that 5 is indeed a solution to the equation.

The symbol $=?$ asks whether the left side of the equation is equal to the right side. Once we know, we can change to an equal sign $(=)$ or not-equal sign (\neq) .

How To

Determine whether a number is a solution to an equation.

1. Step 1. Substitute the number for the variable in the equation.
2. Step 2. Simplify the expressions on both sides of the equation.
3. Step 3. Determine whether the resulting equation is true.
 - If it is true, the number is a solution.
 - If it is not true, the number is not a solution.

Example 2.28

Determine whether $x=5$ is a solution of $6x-17=16$. Determine whether $x=5$ is a solution of $6x-17=16$.

Answer

	$6x - 17 = 16$
Substitute 5 for x .	$6 \cdot 5 - 17 \stackrel{?}{=} 16$
Multiply.	$30 - 17 \stackrel{?}{=} 16$
Subtract.	$13 \neq 16$

So $x=5$ is not a solution to the equation $6x-17=16$.

Try It 2.55

Is $x=3$ a solution of $4x-7=16$? Is $x=3$ a solution of $4x-7=16$?

Try It 2.56

Is $x=2$ a solution of $6x-2=10$? Is $x=2$ a solution of $6x-2=10$?

Example 2.29

Determine whether $y=2$ is a solution of $6y-4=5y-2$. Determine whether $y=2$ is a solution of $6y-4=5y-2$.

Answer

Here, the variable appears on both sides of the equation. We must substitute 2 for each y .

	$6y - 4 = 5y - 2$
Substitute 2 for y .	$6(2) - 4 \stackrel{?}{=} 5(2) - 2$
Multiply.	$12 - 4 \stackrel{?}{=} 10 - 2$
Subtract.	$8 = 8 \checkmark$

Since $y=2$ results in a true equation, we know that 2 is a solution to the equation $6y-4=5y-2$.

Try It 2.57

Is $y=3$ a solution of $9y-2=8y+1$? Is $y=3$ a solution of $9y-2=8y+1$?

Try It 2.58

Is $y=4a$ a solution of $5y-3=3y+5$? Is $y=4a$ a solution of $5y-3=3y+5$?

Model the Subtraction Property of Equality

We will use a model to help you understand how the process of solving an equation is like solving a puzzle. An envelope represents the variable – since its contents are unknown – and each counter represents one.

Suppose a desk has an imaginary line dividing it in half. We place three counters and an envelope on the left side of desk, and eight counters on the right side of the desk as in Figure 2.3. Both sides of the desk have the same number of counters, but some counters are hidden in the envelope. Can you tell how many counters are in the envelope?

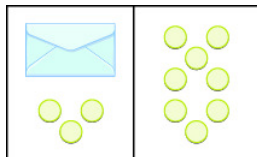


Figure 2.3

What steps are you taking in your mind to figure out how many counters are in the envelope? Perhaps you are thinking “I need to remove the 3” Figure 2.4 shows this process.

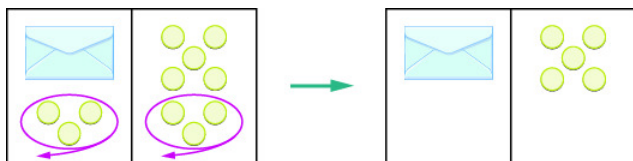


Figure 2.4

What algebraic equation is modeled by this situation? Each side of the desk represents an expression and the center line takes the place of the equal sign. We will call the contents of the envelope x , so the number of counters on the left side of the desk is $x+3$. On the right side of the desk are 8 counters. We are told that $x+3$ is equal to 8 so our equation is $x+3=8$.

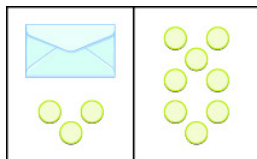


Figure 2.5

$$x+3=8$$

Let's write algebraically the steps we took to discover how many counters were in the envelope.

	$x + 3 = 8$
First, we took away three from each side.	$x + 3 - 3 = 8 - 3$
Then we were left with five.	$x = 5$

Now let's check our solution. We substitute 5 for x in the original equation and see if we get a true statement.

$$x + 3 = 8$$

$$5 + 3 \stackrel{?}{=} 8$$

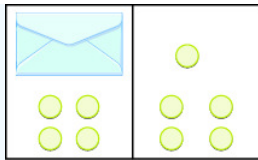
$$8 = 8 \checkmark$$

Our solution is correct. Five counters in the envelope plus three more equals eight.

Manipulative Mathematics

Example 2.30

Write an equation modeled by the envelopes and counters, and then solve the equation:



Answer

On the left, write xx for the contents of the envelope, add the 44 counters, so we have $x+4x+4$.

$$x+4x+4$$

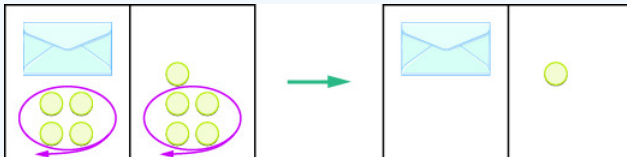
On the right, there are 55 counters.

$$55$$

The two sides are equal.

$$x+4=5x+4=5$$

Solve the equation by subtracting 44 counters from each side.



We can see that there is one counter in the envelope. This can be shown algebraically as:

$$\begin{aligned} x + 4 &= 5 \\ x + 4 - 4 &= 5 - 4 \\ x &= 1 \end{aligned}$$

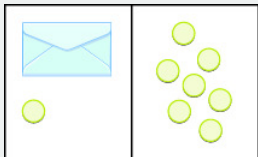
Substitute 11 for xx in the equation to check.

$$\begin{aligned} x + 4 &= 5 \\ 1 + 4 &\stackrel{?}{=} 5 \\ 5 &= 5 \checkmark \end{aligned}$$

Since $x=1x=1$ makes the statement true, we know that 11 is indeed a solution.

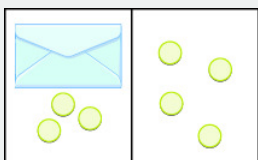
Try It 2.59

Write the equation modeled by the envelopes and counters, and then solve the equation:



Try It 2.60

Write the equation modeled by the envelopes and counters, and then solve the equation:



Solve Equations Using the Subtraction Property of Equality

Our puzzle has given us an idea of what we need to do to solve an equation. The goal is to isolate the variable by itself on one side of the equations. In the previous examples, we used the Subtraction Property of Equality, which states that when we subtract the same quantity from both sides of an equation, we still have equality.

Subtraction Property of Equality

For any numbers a, b, a, b , and c, c , if

$$a = b$$

then

$$a - c = b - c$$

Think about twin brothers Andy and Bobby. They are 17 years old. How old was Andy 3 years ago? He was 3 years less than 17, so his age was $17 - 3$, or 14. What about Bobby's age 3 years ago? Of course, he was 14 also. Their ages are equal now, and subtracting the same quantity from both of them resulted in equal ages 3 years ago.

$$a = b \quad a - 3 = b - 3$$

How To

Solve an equation using the Subtraction Property of Equality.

1. Step 1. Use the Subtraction Property of Equality to isolate the variable.
2. Step 2. Simplify the expressions on both sides of the equation.
3. Step 3. Check the solution.

Example 2.31

Solve: $x + 8 = 17$.

Answer

We will use the Subtraction Property of Equality to isolate x .

	$x + 8 = 17$
Subtract 8 from both sides.	$x + 8 - 8 = 17 - 8$
Simplify.	$x = 9$
	$x + 8 = 17$
	$9 + 8 = 17$
	$17 = 17 \checkmark$

Since $x = 9$ makes $x + 8 = 17$ a true statement, we know 9 is the solution to the equation.

Try It 2.61

Solve:

$$x + 6 = 19$$

Try It 2.62

Solve:

$$x+9=14 \quad x+9=14$$

Example 2.32

Solve: $100=y+74$.

Answer

To solve an equation, we must always isolate the variable—it doesn't matter which side it is on. To isolate y , we will subtract 74 from both sides.

	$100 = y + 74$
Subtract 74 from both sides.	$100 - 74 = y + 74 - 74$
Simplify.	$26 = y$
Substitute 26 for y to check. $100 = y + 74$ $100 \stackrel{?}{=} 26 + 74$ $100 = 100 \checkmark$	

Since $y=26$ makes $100=y+74$ a true statement, we have found the solution to this equation.

Try It 2.63

Solve:

$$95=y+67$$

Try It 2.64

Solve:

$$91=y+45$$

Solve Equations Using the Addition Property of Equality

In all the equations we have solved so far, a number was added to the variable on one side of the equation. We used subtraction to “undo” the addition in order to isolate the variable.

But suppose we have an equation with a number subtracted from the variable, such as $x-5=8$. We want to isolate the variable, so to “undo” the subtraction we will add the number to both sides.

We use the Addition Property of Equality, which says we can add the same number to both sides of the equation without changing the equality. Notice how it mirrors the Subtraction Property of Equality.

Addition Property of Equality

For any numbers a , b , and c , if

$$a=b$$

then

$$a+c=b+c$$

Remember the 17-year-old twins, Andy and Bobby? In ten years, Andy's age will still equal Bobby's age. They will both be 27.

$$a = b + 10 \Rightarrow b + 10 = b + 10$$

We can add the same number to both sides and still keep the equality.

How To

Solve an equation using the Addition Property of Equality.

1. Step 1. Use the Addition Property of Equality to isolate the variable.
2. Step 2. Simplify the expressions on both sides of the equation.
3. Step 3. Check the solution.

Example 2.33

Solve: $x - 5 = 8$.

Answer

We will use the Addition Property of Equality to isolate the variable.

	$x - 5 = 8$
Add 5 to both sides.	$x - 5 + 5 = 8 + 5$
Simplify.	$x = 13$
Now we can check. Let $x = 13$.	
$x - 5 = 8$	
$13 - 5 \stackrel{?}{=} 8$	
$8 = 8 \checkmark$	

Try It 2.65

Solve:

$$x - 9 = 13$$

Try It 2.66

Solve:

$$y - 1 = 3$$

Example 2.34

Solve: $27 = a - 16$.

Answer

We will add 16 to each side to isolate the variable.

	$27 = a - 16$
Add 16 to each side.	$27 + 16 = a - 16 + 16$
Simplify.	$43 = a$
Now we can check. Let $a = 43$.	$27 = a - 16$

$$27 \stackrel{?}{=} 43 - 16$$

$$27 = 27 \checkmark$$

The solution to $27 = a - 16$ is $a = 43$.

Try It 2.67

Solve:

$$19 = a - 18$$

Try It 2.68

Solve:

$$27 = n - 14$$

Translate Word Phrases to Algebraic Equations

Remember, an equation has an equal sign between two algebraic expressions. So if we have a sentence that tells us that two phrases are equal, we can translate it into an equation. We look for clue words that mean *equals*. Some words that translate to the equal sign are:

- is equal to
- is the same as
- is
- gives
- was
- will be

It may be helpful to put a box around the *equals* word(s) in the sentence to help you focus separately on each phrase. Then translate each phrase into an expression, and write them on each side of the equal sign.

We will practice translating word sentences into algebraic equations. Some of the sentences will be basic number facts with no variables to solve for. Some sentences will translate into equations with variables. The focus right now is just to translate the words into algebra.

Example 2.35

Translate the sentence into an algebraic equation: The sum of 66 and 99 is 15.15.

Answer

The word *is* tells us the equal sign goes between 9 and 15.

Locate the “equals” word(s).	The sum of 6 and 9 is 15.	
Write the = sign.	The sum of 6 and 9 = 15.	
Translate the words to the left of the <i>equals</i> word into an algebraic expression.	$6 + 9 = \underline{\quad}$	
Translate the words to the right of the <i>equals</i> word into an algebraic expression.	$6 + 9 = 15$	

Try It 2.69

Translate the sentence into an algebraic equation:

The sum of 77 and 66 gives 13.13.

Try It 2.70

Translate the sentence into an algebraic equation:

The sum of 88 and 66 is 14.14.

Example 2.36

Translate the sentence into an algebraic equation: The product of 88 and 77 is 56.56.

Answer

The location of the word *is* tells us that the equal sign goes between 7 and 56.

Locate the “equals” word(s).	The product of 8 and 7 <u>is</u> 56.	
Write the = sign.	The product of 8 and 7 = 56.	
Translate the words to the left of the <i>equals</i> word into an algebraic expression.	$8 \cdot 7 = \underline{\hspace{1cm}}$	
Translate the words to the right of the <i>equals</i> word into an algebraic expression.	$\underline{\hspace{1cm}} \cdot 7 = 56$	

Try It 2.71

Translate the sentence into an algebraic equation:

The product of 66 and 99 is 54.54.

Try It 2.72

Translate the sentence into an algebraic equation:

The product of 2121 and 33 gives 63.63.

Example 2.37

Translate the sentence into an algebraic equation: Twice the difference of x and 3 gives 18.18.

Answer

Locate the “equals” word(s).	Twice the difference of x and 3 <u>gives</u> 18.
Recognize the key words: <i>twice</i> ; <i>difference of and</i>	<i>Twice</i> means two times.
Translate.	$\underbrace{2}_{\text{Twice}} \underbrace{(x - 3)}_{\text{difference of } x \text{ and } 3} \underbrace{=}_{\text{gives}} 18$

Try It 2.73

Translate the given sentence into an algebraic equation:

Twice the difference of xx and 55 gives 30.30.

Try It 2.74

Translate the given sentence into an algebraic equation:

Twice the difference of yy and 44 gives 16.16.

Translate to an Equation and Solve

Now let's practice translating sentences into algebraic equations and then solving them. We will solve the equations by using the Subtraction and Addition Properties of Equality.

Example 2.38

Translate and solve: Three more than xx is equal to 47.47.

Answer

		Three more than x is equal to 47.
Translate.		$x + 3 = 47$
Subtract 3 from both sides of the equation.		$x + 3 - 3 = 47 - 3$
Simplify.		$x = 44$
We can check. Let $x=44$.	$x + 3 = 47$	
	$44 + 3 \stackrel{?}{=} 47$	
	$47 = 47 \checkmark$	

So $x=44$ is the solution.

Try It 2.75

Translate and solve:

Seven more than xx is equal to 37.37.

Try It 2.76

Translate and solve:

Eleven more than yy is equal to 28.28.

Example 2.39

Translate and solve: The difference of yy and 14 is 18.18.

Answer

		The difference of y and 14 is 18.
Translate.		$y - 14 = 18$

Add 14 to both sides.

$$y - 14 + 14 = 18 + 14$$

Simplify.

$$y = 32$$

We can check. Let $y=32$.

$$y - 14 = 18$$

$$32 - 14 \stackrel{?}{=} 18$$

$$18 = 18 \checkmark$$

So $y=32$ is the solution.

Try It 2.77

Translate and solve:

The difference of z and 17 is equal to 37.

Try It 2.78

Translate and solve:

The difference of x and 19 is equal to 45.

Media

ACCESS ADDITIONAL ONLINE RESOURCES

- [Solving One Step Equations By Addition and Subtraction](#)

Section 2.3 Exercises

Practice Makes Perfect

Determine Whether a Number is a Solution of an Equation

In the following exercises, determine whether each given value is a solution to the equation.

147.

$$x + 13 = 21$$

1. $x = 8$
2. $x = 34$

148.

$$y + 18 = 25$$

1. $y = 7$
2. $y = 43$

149.

$$m - 4 = 13$$

1. $m = 9$
2. $m = 17$

150.

$$n - 9 = 6$$

1. $n = 3$
2. $n = 15$

151.

$$3p+6=15 \quad 3p+6=15$$

1. Ⓐ $p=3$
2. Ⓑ $p=7$

152.

$$8q+4=20 \quad 8q+4=20$$

1. Ⓐ $q=2$
2. Ⓑ $q=3$

153.

$$18d-9=27 \quad 18d-9=27$$

1. Ⓐ $d=1$
2. Ⓑ $d=2$

154.

$$24f-12=60 \quad 24f-12=60$$

1. Ⓐ $f=2$
2. Ⓑ $f=3$

155.

$$8u-4=40 \quad 8u-4=40$$

1. Ⓐ $u=3$
2. Ⓑ $u=11$

156.

$$7v-3=36 \quad 7v-3=36$$

1. Ⓐ $v=3$
2. Ⓑ $v=11$

157.

$$20h-5=15 \quad 20h-5=15$$

1. Ⓐ $h=6$
2. Ⓑ $h=8$

158.

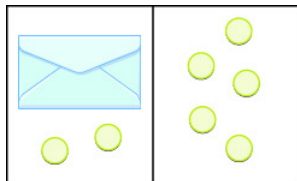
$$18k-3=12 \quad 18k-3=12$$

1. Ⓐ $k=1$
2. Ⓑ $k=6$

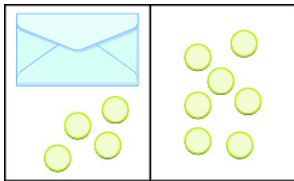
Model the Subtraction Property of Equality

In the following exercises, write the equation modeled by the envelopes and counters and then solve using the subtraction property of equality.

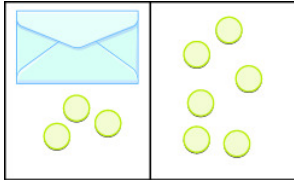
159.



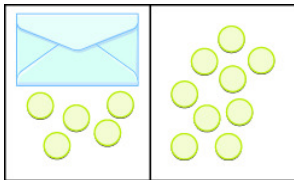
160.



161.



162.



Solve Equations using the Subtraction Property of Equality

In the following exercises, solve each equation using the subtraction property of equality.

163.

$$a + 2 = 18 \quad a + 2 = 18$$

164.

$$b + 5 = 13 \quad b + 5 = 13$$

165.

$$p + 18 = 23 \quad p + 18 = 23$$

166.

$$q + 14 = 31 \quad q + 14 = 31$$

167.

$$r + 76 = 100 \quad r + 76 = 100$$

168.

$$s + 62 = 95 \quad s + 62 = 95$$

169.

$$16 = x + 9 \quad 16 = x + 9$$

170.

$$17 = y + 6 \quad 17 = y + 6$$

171.

$$93 = p + 24 \quad 93 = p + 24$$

172.

$$116 = q + 79 \quad 116 = q + 79$$

173.

$$465 = d + 398 \quad 465 = d + 398$$

174.

$$932 = c + 641 \quad 932 = c + 641$$

Solve Equations using the Addition Property of Equality

In the following exercises, solve each equation using the addition property of equality.

175.

$$y - 3 = 19 \quad y - 3 = 19$$

176.

$$x - 4 = 12 \quad x - 4 = 12$$

177.

$$u - 6 = 24 \quad u - 6 = 24$$

178.

$$v - 7 = 35 \quad v - 7 = 35$$

179.

$$f - 55 = 123 \quad f - 55 = 123$$

180.

$$g - 39 = 117 \quad g - 39 = 117$$

181.

$$19 = n - 13 \quad 19 = n - 13$$

182.

$$18 = m - 15 \quad 18 = m - 15$$

183.

$$10 = p - 38 \quad 10 = p - 38$$

184.

$$18 = q - 72 \quad 18 = q - 72$$

185.

$$268 = y - 199 \quad 268 = y - 199$$

186.

$$204 = z - 149 \quad 204 = z - 149$$

Translate Word Phrase to Algebraic Equations

In the following exercises, translate the given sentence into an algebraic equation.

187.

The sum of 88 and 99 is equal to 17.17.

188.

The sum of 77 and 99 is equal to 16.16.

189.

The difference of 2323 and 1919 is equal to 4.4.

190.

The difference of 2929 and 1212 is equal to 17.17.

191.

The product of 33 and 99 is equal to 27.27.

192.

The product of 66 and 88 is equal to 48.48.

193.

The quotient of 5454 and 66 is equal to 9.9.

194.

The quotient of 4242 and 77 is equal to 6.6.

195.

Twice the difference of nn and 1010 gives 52.52.

196.

Twice the difference of mm and 1414 gives 64.64.

197.

The sum of three times yy and 1010 is 100.100.

198.

The sum of eight times xx and 44 is 68.68.

Translate to an Equation and Solve

In the following exercises, translate the given sentence into an algebraic equation and then solve it.

199.

Five more than pp is equal to 21.21.

200.

Nine more than qq is equal to 40.40.

201.

The sum of rr and 1818 is 73.73.

202.

The sum of ss and 1313 is 68.68.

203.

The difference of dd and 3030 is equal to 52.52.

204.

The difference of cc and 2525 is equal to 75.75.

205.

1212 less than uu is 89.89.

206.

1919 less than ww is 56.56.

207.

325325 less than cc gives 799.799.

208.

299299 less than dd gives 850.850.

Everyday Math

209.

Insurance Vince's car insurance has a \$500\$500 deductible. Find the amount the insurance company will pay, p,p, for an \$1800\$1800 claim by solving the equation $500+p=1800$. $500+p=1800$.

210.

Insurance Marta's homeowner's insurance policy has a \$750 deductible. The insurance company paid \$5800 to repair damages caused by a storm. Find the total cost of the storm damage, d , by solving the equation $d - 750 = 5800$.

211.

Sale purchase Arthur bought a suit that was on sale for \$120 off. He paid \$340 for the suit. Find the original price, p , of the suit by solving the equation $p - 120 = 340$.

212.

Sale purchase Rita bought a sofa that was on sale for \$1299. She paid a total of \$1409, including sales tax. Find the amount of the sales tax, t , by solving the equation $1299 + t = 1409$.

Writing Exercises

213.

Is $x = 1$ a solution to the equation $8x - 2 = 16 - 6x$? How do you know?

214.

Write the equation $y - 5 = 21$ in words. Then make up a word problem for this equation.

Self Check

a) After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
determine whether a number is a solution of an equation.			
model the subtraction property of equality.			
solve equations using the subtraction property of equality.			
solve equations using the addition property of equality.			
translate word phrases to algebraic equations.			
translate to an equation and solve.			

b) What does this checklist tell you about your mastery of this section? What steps will you take to improve?

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7.9: INTRODUCTION TO INEQUALITIES AND INTERVAL NOTATION

Learning Objectives

- Understand interval notation.
- Determine when intervals are open or closed.
- Graph the solutions of an inequality on a number line and express the solutions using interval notation.

UNBOUNDED INTERVALS

An algebraic inequality, such as $x \geq 2$, is read “ x is greater than or equal to 2.” This inequality has infinitely many solutions for x . Some of the solutions are 2, 3, 3.5, 5, 20, and 20.001. Since it is impossible to list all of the solutions, a system is needed that allows a clear communication of this infinite set. Two common ways of expressing solutions to an inequality are by graphing them on a number line and using interval notation.

To express the solution graphically, draw a number line and shade in all the values that are solutions to the inequality. Interval notation is textual and uses specific notation as follows:

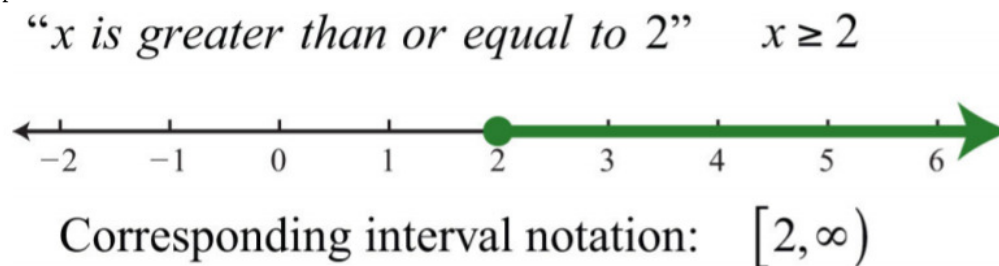


Figure 7.9.1

Determine the interval notation after graphing the solution set on a number line. The numbers in interval notation should be written in the same order as they appear on the number line, with smaller numbers in the set appearing first. In this example, there is an inclusive inequality, which means that the lower-bound 2 is included in the solution. Denote this with a closed dot on the number line and a square bracket in interval notation. The symbol (∞) is read as infinity and indicates that the set is unbounded to the right on a number line. Interval notation requires a parenthesis to enclose infinity. The square bracket indicates the boundary is included in the solution. The parentheses indicate the boundary is not included. **Infinity** is an upper bound to the real numbers, but is not itself a real number: it cannot be included in the solution set.

Now compare the interval notation in the previous example to that of the strict, or noninclusive, inequality that follows:

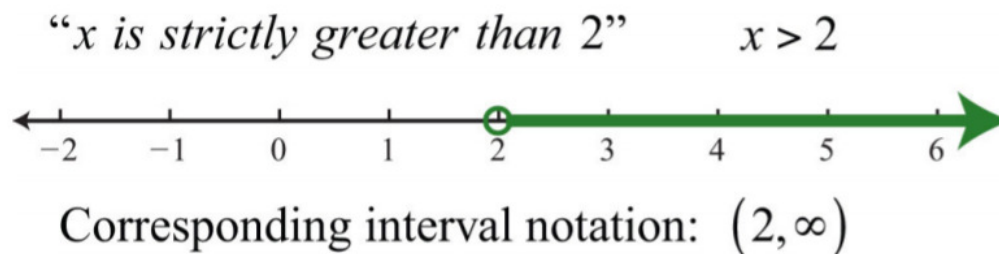


Figure 7.9.2

Strict inequalities imply that solutions may get very close to the boundary point, in this case 2, but not actually include it. Denote this idea with an open dot on the number line and a round parenthesis in interval notation.

Example 7.9.1

Graph and give the interval notation equivalent:

$$x < 3.$$

Solution:

Use an open dot at 3 and shade all real numbers strictly less than 3. Use negative infinity $(-\infty)$ to indicate that the solution set is unbounded to the left on a number line.



Figure 7.9.3

Answer:

Interval notation: $(-\infty, 3)$

Example 7.9.2

Graph and give the interval notation equivalent:

$$x \leq 5.$$

Solution:

Use a closed dot and shade all numbers less than and including 5.



Figure 7.9.4

Answer:

Interval notation: $(-\infty, 5]$

It is important to see that $5 \geq x$ is the same as $x \leq 5$. Both require values of x to be smaller than or equal to 5. To avoid confusion, it is good practice to rewrite all inequalities with the variable on the left.

In summary,

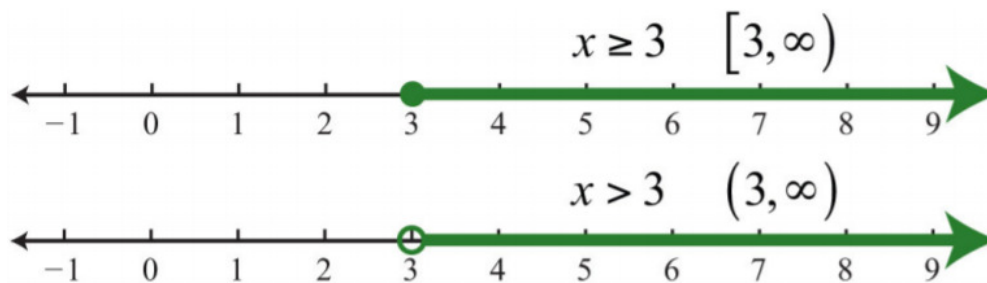


Figure 7.9.5

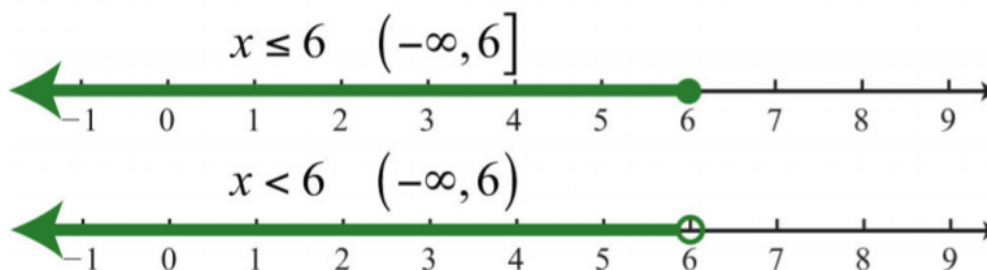


Figure 7.9.6

BOUNDED INTERVALS

An inequality such as

$$-1 \leq x < 3$$

reads “negative one is less than or equal to x and x is less than three.” This is a compound inequality because it can be decomposed as follows:

$$-1 \leq x \text{ and } x < 3$$

The logical “and” requires that both conditions must be true. Both inequalities are satisfied by all the elements in the intersection, denoted \cap , of the solution sets of each.

Alternatively, we may interpret $-1 \leq x < 3$ as all possible values for x between or bounded by -1 and 3 on a number line. For example, one such solution is $x = 1$. Notice that 1 is between -1 and 3 on a number line, or that $-1 < 1 < 3$. Similarly, we can see that other possible solutions are $-1, -0.99, 0, 0.0056, 1.8$, and 2.99 . Since there are infinitely many real numbers between -1 and 3 , we must express the solution graphically and/or with interval notation, in this case $[-1, 3)$.

Example 7.9.3

Graph and give the interval notation equivalent:

$$-\frac{3}{2} < x < 2.$$

Solution:

Shade all real numbers bounded by, or strictly between, $-\frac{3}{2} = -1\frac{1}{2}$ and 2 .



Figure 7.9.7

Answer:

Interval notation: $(-\frac{3}{2}, 2)$

Example 7.9.7

Graph and give the interval notation equivalent:

$$-5 < x \leq 15$$

Solution:

Shade all real numbers between -5 and 15 , and indicate that the upper bound, 15 , is included in the solution set by using a closed dot.

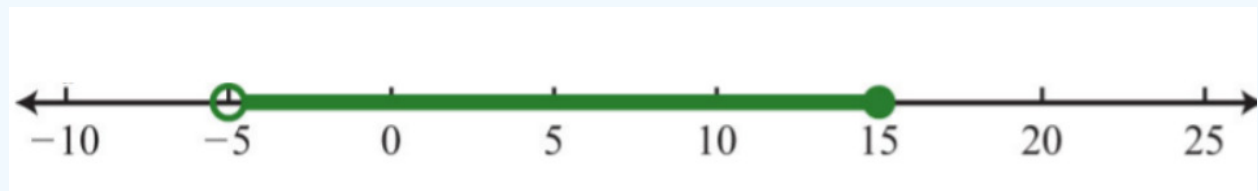


Figure 7.9.8

Answer:

Interval notation: $(-5, 15]$

In the previous two examples, we did not decompose the inequalities; instead we chose to think of all real numbers between the two given bounds.

In summary,

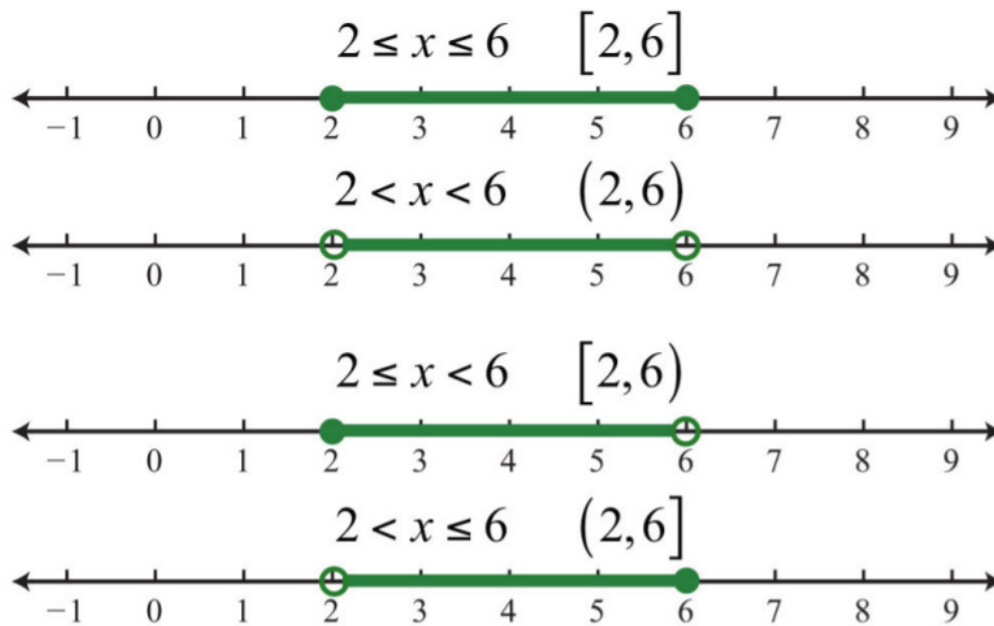


Figure 7.9.9

KEY TAKEAWAYS

- Inequalities usually have infinitely many solutions, so rather than presenting an impossibly large list, we present such solutions sets either graphically on a number line or textually using interval notation.
- Inclusive inequalities with the “or equal to” component are indicated with a closed dot on the number line and with a square bracket using interval notation.
- Strict inequalities without the “or equal to” component are indicated with an open dot on the number line and a parenthesis using interval notation.
- Compound inequalities of the form $m < x < n$ can be decomposed into two inequalities using the logical “and.” However, it is just as valid to consider the argument x to be bounded between the values m and n .

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7.10: Translating English to Math

Learning Objectives

- Translate English phrases to mathematical expressions

Words Are Important!

When working with probability and statistics, words such as “more than” or “less than” can drastically change the answer. Table 7.10.1 shows some of the common phrases you may run into while reading a problem. It will be essential later in the course that you can correctly match these phrases with their correct symbol.

=		\leq	\geq
is the same as		is less than or equal to	is greater than or equal to
is equal to		is at most	is at least
is exactly the same as		is not greater than	is not less than
has not changed from		within	

\neq	$>$	$<$
is not	is more than	is less than
is not equal to	is greater than	is below
is different from	is above	is lower than
has changed from	is higher than	is shorter than
is not the same as	is longer than	is smaller than
	is bigger than	has decreased
	has increased	is reduced

Table 7.10.1

✓ Example 7.10.1

Translate the following to mathematical expressions when rolling 2 dice:

- Rolling a sum of less than 5.
- Rolling a sum that is above 5.
- Rolling a sum of no less than 8.
- Rolling a sum of at most 8.

Solution

Let x be the sum of the dice.

- The mathematical expression for rolling a sum of less than 5 would be:

$$x < 5$$

- The mathematical expression for rolling a sum that is above 5 would be:

$$x > 5$$

c. The mathematical expression for rolling a sum of no less than 8 would be:

$$x \geq 8$$

d. The mathematical expression for rolling a sum of at most 8 would be:

$$x \leq 8$$

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CHAPTER OVERVIEW

8: Graphing Points and Lines in Two Dimensions

This chapter is from the [Support Course for Elementary Statistics](#) by Larry Green from Lake Tahoe Community College.

- [8.1: Plot an Ordered Pair](#)
- [8.2: Find y given x and the Equation of a Line](#)
- [8.3: Graph a Line Given its Equation](#)
- [8.4: Find the Equation of a Line Given its Graph](#)
- [8.5: Interpreting the Slope of a Line](#)
- [8.6: Interpreting the y-Intercept of a Line](#)
- [8.7: Finding Residuals](#)

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8.1: Plot an Ordered Pair

Learning Outcomes

1. Draw x and y axes.
2. Plot a point in the xy -plane

We have already gone into detail about how to plot points on a number line, and that is very useful for single variable presentations. Now we will move to questions that involve comparing two variables. Working with two variables is frequently encountered in statistical studies and we would like to be able to display the results graphically. This is best done by plotting points in the xy -plane, also called the Cartesian plane.

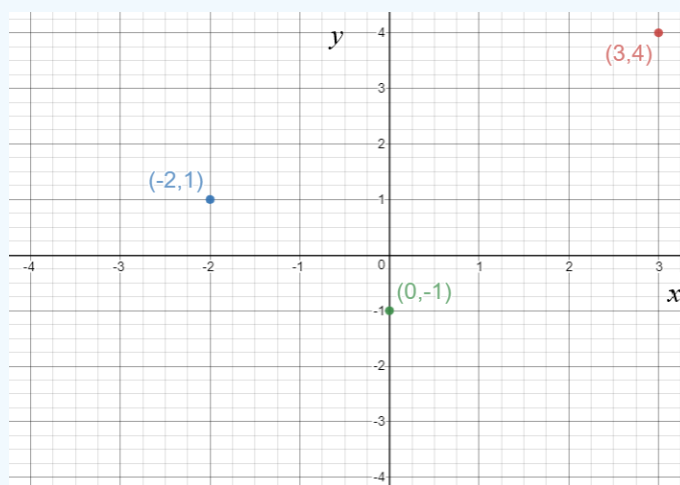
The Cartesian plane is defined by two perpendicular lines. The horizontal line is the x -axis, and the vertical line is the y -axis. The two axes meet at a point called the origin. The location of a point is given by an ordered pair (x, y) . The x -coordinate gives the point's horizontal location from the origin, and the y -coordinate gives the point's vertical location from the origin. The coordinates of the origin are $(0,0)$.

Example 8.1.1

Plot the points: $(3, 4)$, $(-2, 1)$, and $(0, -1)$

Solution

The first thing to do when plotting points is to sketch the x -axis and y -axis and decide on the tick marks. Here the numbers are all less than 5, so it is reasonable to count by 1's. Next, we plot the first point, $(3, 4)$. This means to start at the origin, where the axes intersect. Then move 3 units to the right and 4 units up. After arriving there, we just draw a dot. For the next point, $(-2, 1)$, we start at the origin, move 2 units to the left and 1 unit up and draw the dot. For the third point, $(0, -1)$, we don't move left or right at all since the x -coordinate is 0, but we do move 1 unit down and draw the dot. The plot is shown below.



Example 8.1.2

A survey was done to look at the relationship between a person's age and their annual income. The first three answers are shown in the table below:

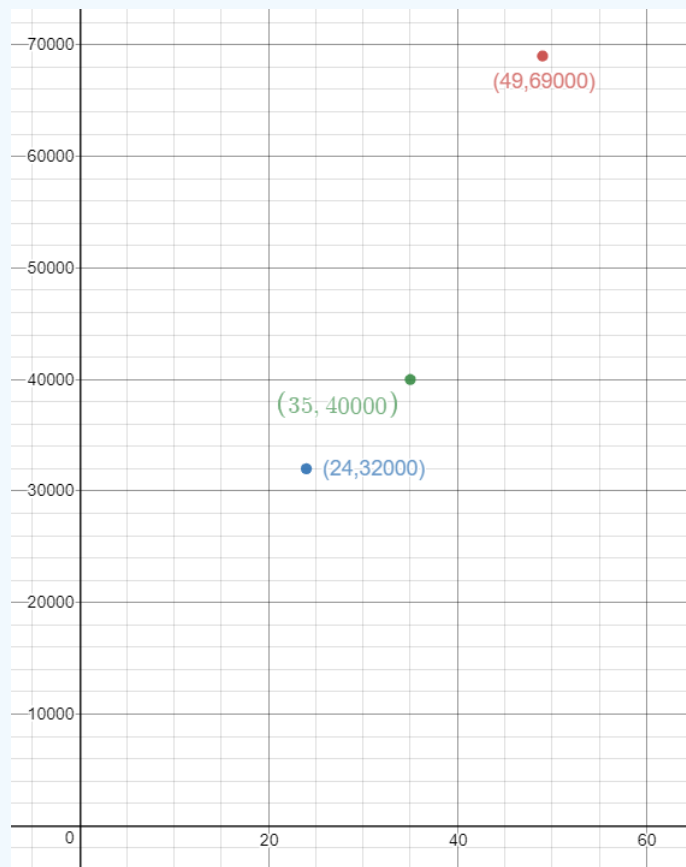
Table of ages and income

Age	49	24	35
Income	69,000	32,000	40,000

Graph the three points on the xy -plane.

Solution

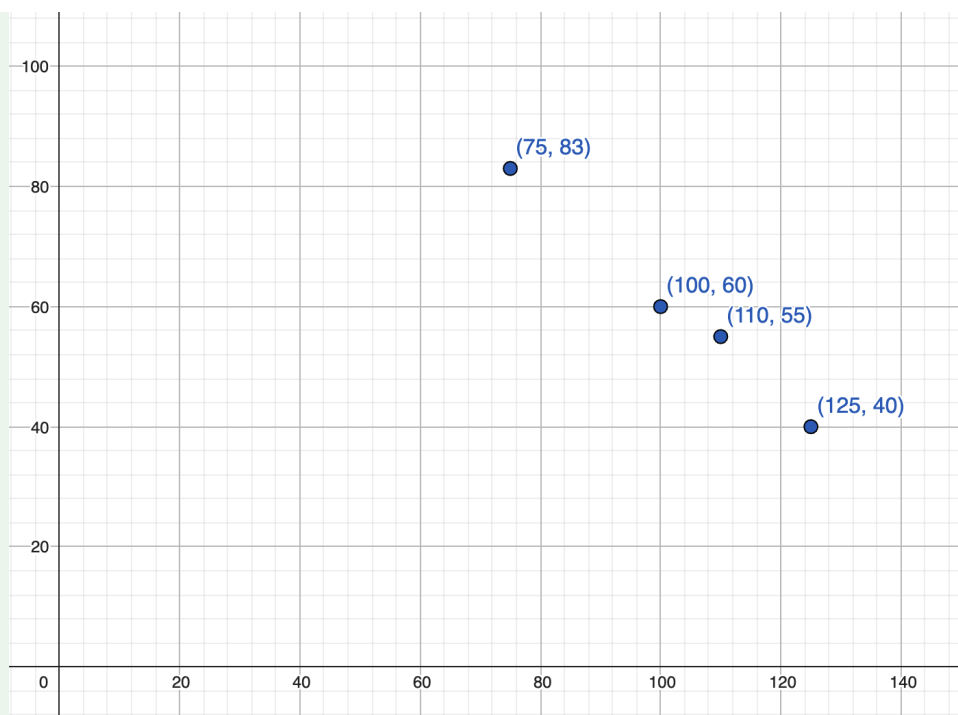
Notice that the numbers are all relatively large. Therefore counting by 1's would not make sense. Instead, it makes better sense to count the Age axis, x , by 10's and the Income axis, y , by 1000's. The points are plotted below.



Exercise

A hotel manager was interested in seeing the relationship between the price per night, x , that the hotel charged and the number of occupied rooms, y . The results were (75,83), (100,60), (110,55), and (125,40). Plot these points in the xy -plane.

Answer



Additional video resources:

[Ex: Plotting Points on the Coordinate Plane](#)

[Plotting Points](#)

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8.2: Find y given x and the Equation of a Line

Learning Outcomes

1. Find the value of y given x and the equation of a line.
2. Use a line to make predictions.

A line can be thought of as a function, which means that if a value of x is given, the equation of the line produces exactly one value of y ; This is particularly useful in regression analysis where the line is used to make a prediction of one variable given the value of the other variable.

Example 8.2.1

Consider the line with equation:

$$y = 3x - 4$$

Find the value of y when x is 5.

Solution

Just replace the variable x with the number 5 in the equation and perform the arithmetic:

$$y = 3(5) - 4 = 15 - 4 = 11$$

Example 8.2.2

A survey was done to look at the relationship between a woman's height, x and the woman's weight, y . The equation of the regression line was found to be:

$$y = -220 + 5.5x$$

Use this equation to estimate the weight in pounds of a woman who is 5' 2" (62 inches) tall.

Solution

Just replace the variable x with the number 62 in the equation and perform the arithmetic:

$$y = -220 + 5.5(62)$$

We can put this into a calculator or computer to get:

$$y = 121$$

Therefore, our best prediction for the weight of a woman who is 5' 2" tall is that she is 121 lbs.

Exercise

A biologist has collected data on the girth (how far around) of pine trees and the pine tree's height. She found the equation of the regression line to be:

$$y = 1.3 + 2.7x$$

Where the girth, x , is measured in inches and the height, y , is measured in feet. Use the regression line to predict the height of a tree with girth 28 inches.



<https://youtu.be/cS95PIUKZ6I>

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8.3: Graph a Line Given its Equation

Learning Outcomes

1. Identify the slope and y-intercept from the equation of a line.
2. Plot the y-intercept of a line given its equation.
3. Plot a second point on a line given the y-intercept and the slope.
4. Graph a line given its equation in slope y-intercept form.

Often we are given an equation of a line and we want to visualize it. For this reason, it is important to be able to graph a line given its equation. We will look at lines that are in slope intercept form: $y = a + bx$ where a is the y-intercept of the line and b is the slope of the line. The y-intercept is the value of y where the line crosses the y-axis. The slope is the rise over run. If we write the slope as a fraction, then the numerator tells us how far to move up (or down if it is negative) and the denominator tells us how far to the right we need to go. The main application to statistics is in regression analysis which is the study of how to use a line to make a prediction about one variable based on the value of the other variable.

Example 8.3.1

Graph the line given by the equation:

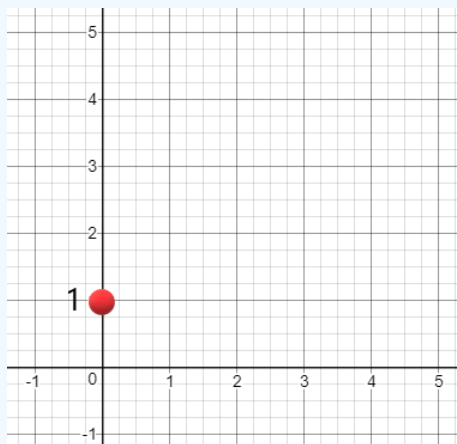
$$y = 1 + \frac{3}{2}x$$

Solution

We follow the three step process:

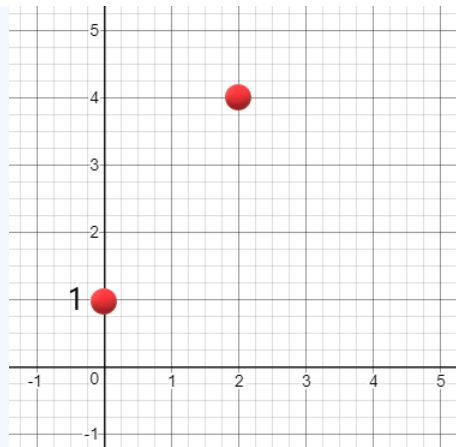
Step 1: Plot the y-intercept

The y-intercept is the number that is not associated with the x . For this example, it is 1. The x-coordinate of the y-intercept is always 0. So the coordinates of the y-intercept are (0, 1). Thus start at the origin and move up 1:



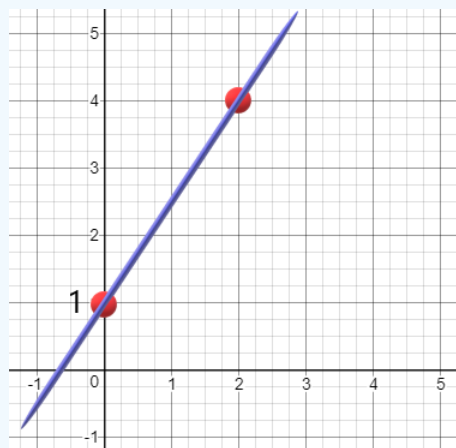
Step 2: Plot the Slope.

The slope of a line is the coefficient of the x term. Here it is $\frac{3}{2}$. What this means is that we rise 3 and run to the right 2. Rising 3 from an original y-coordinate of 1 gives a new y-coordinate of 4. Running 2 to the right from an initial x-coordinate of 0 gives a new x-coordinate of 2. Thus we next plot the point (2, 4).



Step 3: Connect the Dots

The last thing we need to do is connect the dots with a line:



Example 8.3.2

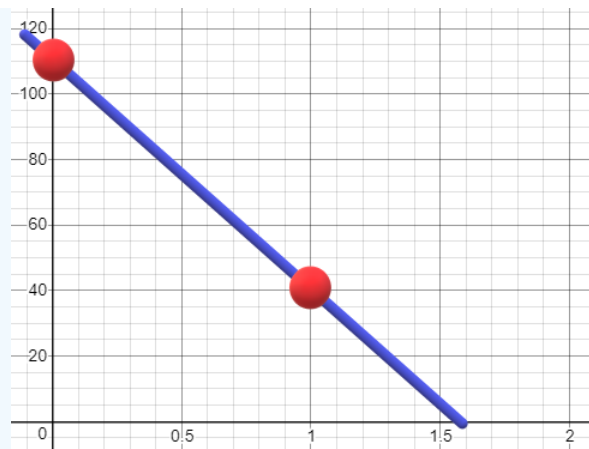
A study was done to look at the relationship between the weight of a car, x , in tons and its gas mileage in mpg, y . The equation of the regression line was found to be:

$$y = 110 - 70x$$

Graph this line.

Solution

The first step is to note that the y-intercept is 110, hence the graph goes through the point (0, 110). The next step is to see that the slope is -70. We can always put a number over 1 in order to make it a fraction. The slope of $-\frac{70}{1}$ tells us that y goes down by 70 if x goes up by 1. We use this to find the second point. The y-coordinate is: $110 - 70 = 40$. The x-coordinate is 1. Thus, a second point is (1, 40). We can now plot the two points and connect the dots with a line.



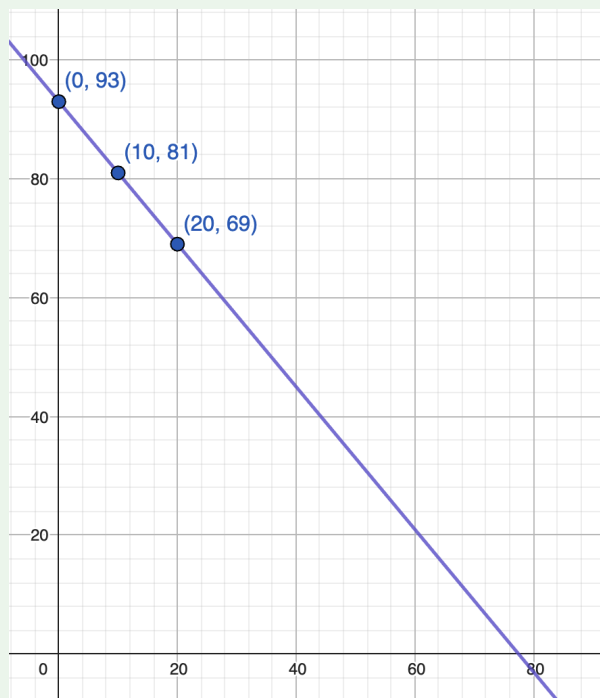
Exercise

The regression line that relates the ounces of beer consumed just before a test, x , and the score on the test, y , is given by

$$y = 93 - 1.2x$$

Graph this line.

Answer



Additional video resources:

[Graphing a Line in Slope-Intercept Form](#)

[Graphing a Line](#)

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8.4: Find the Equation of a Line Given its Graph

Learning Outcomes

1. Find the slope of a line given its graph.
2. Find the y-intercept of a line given its graph.
3. Find the equation of a line given its graph.

There are two main ways of representing a line: the first is with its graph, and the second is with its equation. In this section, we will practice how to find the equation of the line if we are given the graph of the line. The two key numbers in the equation of a line are the slope and the y -intercept. Thus the main steps in finding the equation of a line are finding the slope and finding the y -intercept. In statistics we are often presented with a **scatter plot** where we can eyeball the line. Once we have the graph of the line, getting the equation is helpful for making predictions based on the line.

Finding the Slope of a Line Given Its Graph

The steps to follow to find the slope of the line given its graph are the following.

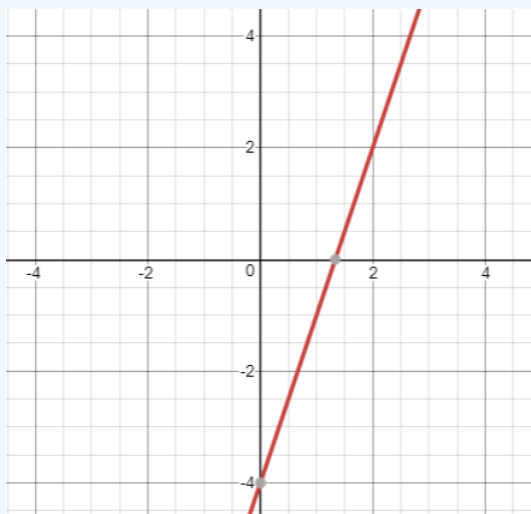
Step 1: Identify two points on the line. Any two points will do, but it is recommended to find points with nice x and y coordinates.

Step 2: The slope is the rise over the run. Thus if the points have coordinates (x_1, y_1) and (x_2, y_2) , then the slope is:

$$\text{Slope} = \frac{\text{Rise}}{\text{Run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

Example 8.4.1

Find the slope of the line shown below.



Solution

First, we locate points on the line that are as easy as possible to work with. The points with integer coordinates are $(0, -4)$ and $(2, 2)$.

Next, we use the rise over run formula to find the slope of the line.

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - (-4)}{2 - 0} = \frac{6}{2} = 3$$

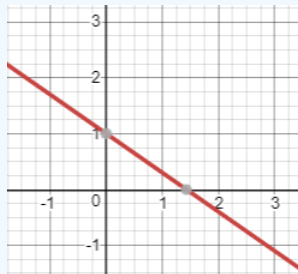
Finding the y-intercept from the graph

If the portion of the graph that is in view includes the y -axis, then the y -intercept is very easy to spot. You just see where it crosses the y -axis. On the other hand, if the portion of the graph in view does not contain the y -axis, then it is best to first find the equation

of the line and then use the equation to find the y -intercept.

Example 8.4.2

Find the y -intercept of the line shown below.



Solution

We just look at the line and notice that it crosses the y -axis at $y = 1$. Therefore, the y -intercept is 1 or $(0, 1)$.

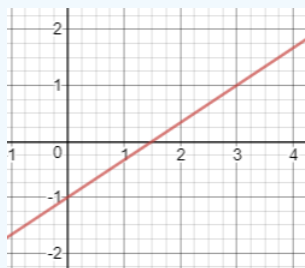
Finding the equation of the line given its graph

If you are given the graph of a line and want to find its equation, then you first find the slope as in Example 8.4.1. Then you use y -intercept, which is b , and the slope, m , and put it into the **slope-intercept equation**:

$$y = mx + b$$

Example 8.4.3

Find the equation of the line shown below.



Solution

First we find the slope by identifying two nice points. Notice that the line passes through $(0, -1)$ and $(3, 1)$. Now compute the slope using the rise over run formula:

$$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{1 - (-1)}{3 - 0} = \frac{2}{3}$$

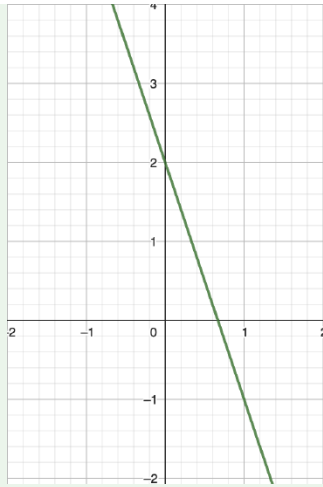
We can easily see that the y -intercept is $(0, -1)$. Thus, $b = -1$.

Next use the slope-intercept equation with $m = \frac{2}{3}$ and $b = -1$.

$$y = \frac{2}{3}x - 1$$

Try It

Find the equation of this line in slope-intercept form.



Answer

$$y = -3x + 2$$

Additional video resources:

[Ex 1: Find the Equation of a Line in Slope Intercept Form Given the Graph of a Line](#)

[Finding the Equation of a Line Given Its Graph](#)

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8.5: Interpreting the Slope of a Line

Learning Outcomes

1. Interpret the slope of a line as the change in y when x changes by 1.

Interpreting the Slope of a Line

For every increase in the x -variable by 1, the y -variable tends to change by the amount of the slope.

A common issue when we learn about the equation of a line in algebra is to state the slope as a number, but have no idea what it represents in the real world. The slope of a line is the rise over the run. If the slope is given by an integer or decimal value we can always put it over the number 1. In this case, the line rises by the slope when it runs 1. "Runs 1" means that the x -value increases by 1 unit. Therefore the slope represents how much the y -value changes when the x -value changes by 1 unit. In statistics, especially regression analysis, the x -value has real life meaning and so does the y -value.

Example 8.5.1

A study was done to see the relationship between the time it takes, x , to complete a college degree and the student loan debt incurred, y . The equation of the regression line was found to be:

$$y = 25,142 + 14,329x$$

Interpret the slope of the regression line in the context of the study.

Solution

First, note that the slope is the coefficient in front of the x . Thus, the slope is 14,329. Next, the slope is the rise over the run, so it helps to write the slope as a fraction:

$$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{14,329}{1}$$

The rise is the change in y and y represents student loan debt. Thus, the numerator represents an increase of \$14,329 of student loan debt. The run is the change in x and x represents the time it takes to complete a college degree. Thus, the denominator represents an increase of 1 year to complete a college degree. We can put this all together and interpret the slope as telling us that:

For every additional year it takes to complete a college degree, on average the student loan debt tends to increase by \$14,329.

Example 8.5.2

Suppose that a research group tested the cholesterol level of a sample of 40-year-old women and then waited many years to see the relationship between a woman's HDL cholesterol level in mg/dl, x , and her age of death, y . The equation of the regression line was found to be:

$$y = 103 - 0.3x$$

Interpret the slope of the regression line in the context of the study.

Solution

The slope of the regression line is -0.3. The slope as a fraction is:

$$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{-0.3}{1}$$

The rise is the change in y and y represents age of death. Since the slope is negative, the numerator indicates a decrease in lifespan in terms of a lower age of death. Thus, the numerator represents a decrease in lifespan of 0.3 years. The run is the change in x and x represents the HDL cholesterol level. Thus, the denominator represents an HDL cholesterol level increase of 1 mg/dl. Now, put this all together and interpret the slope as telling us that:

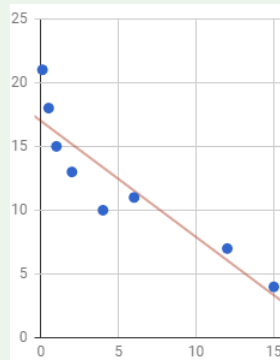
For every additional 1 mg/dl of HDL cholesterol level, on average women are predicted to die 0.3 years younger.

Try It

The scatterplot and regression line below are from a study that collected data on the population (in hundred thousands) of cities, x , and the average number of hours per week the city's residents spend outdoors, y . The equation of the regression line was found to be:

$$y = 17 - 0.93x$$

Interpret the slope of this regression line in the context of the study.



Answer

For every additional 100,000 people in the population of a city, on average the number of hours per week the city's residents spend outdoors decreases by 0.93 hours.

Additional video resources:

[Interpret the Meaning of the Slope of a Linear Equation - Smokers](#)

[Interpreting the Slope of a Regression Line](#)

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8.6: Interpreting the y-Intercept of a Line

Learning Outcomes

1. Interpret the y -intercept of a line as the value of y when x equals 0.
2. Determine whether the y -intercept is useful for interpreting the relationship between x and y .

Just like the slope of a line, many algebra classes go over the y -intercept of a line without explaining how to use it in the real world. The y -intercept of a line is the value of y where the line crosses the y -axis. In other words, it is the value of y when the value of x is equal to 0. Sometimes this has true meaning for the model that the line provides, but other times it is meaningless. We will encounter examples of both types in this section.

Interpreting the y -Intercept of a Line

When the value for the x -variable is 0, the best prediction for the value of the y -variable is the value of the y -intercept.

Example 8.6.1

A study was done to see the relationship between the ounces of meat, x , that people eat each day on average and the hours per week, y they watch sports. The equation of the regression line was found to be:

$$y = 1.3 + 0.4x$$

Interpret the y -intercept of the regression line in the context of the study or explain why it has no practical meaning.

Solution

First, note that the y -intercept is the number that is not in front of the x . Thus, the y -intercept is 1.3. Next, the y -intercept is the value of y when x equals zero. For this example, x represents the ounces of meat consumed each day.

When the consumption of meat is 0 ounces, the best prediction for the value of the hours of sports watched each week is 1.3.

If x is equal to 0, this means the person does not consume any meat. Since there are people, namely vegetarians or vegans, who consume no meat, it is meaningful to have an x -value of 0. The y -value of 1.3 represents the hours of sports the person watches per week. Putting this all together we can state:

A person who does not eat meat (vegetarian/vegan) is predicted to watch 1.3 hours of sports each week.

Example 8.6.2

A neonatal nurse at Children's Hospital has collected data on the birth weight, x , in pounds and the number of days, y , that the newborns stay in the hospital. The equation of the regression line was found to be

$$y = 45 - 3.9x$$

Interpret the y -intercept of the regression line in the context of the study or explain why it has no practical meaning.

Solution

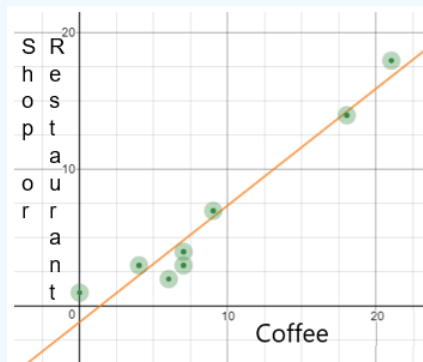
Again, we note that the y -intercept is the number that is not in front of the x . Thus, the y -intercept is 45. Next, the y -intercept is the value of y when x equals zero.

When the birth weight in pounds is 0, the best prediction for the value of the number of days the newborn is predicted to stay in the hospital is 45 days.

For this example, x represents the new born baby's birth weight in pounds. If x is equal to 0, this means the baby was born with a weight of 0 pounds. Since it makes no sense for a baby to weigh 0 pounds, we can say that the y -intercept of this regression line has no practical meaning.

Example 8.6.3

A researcher asked several people "How many cups of coffee did you drink last week?" and "How many times did you go to a shop or restaurant for a meal or a drink last week?" The scatterplot and the regression line from this study are shown below.



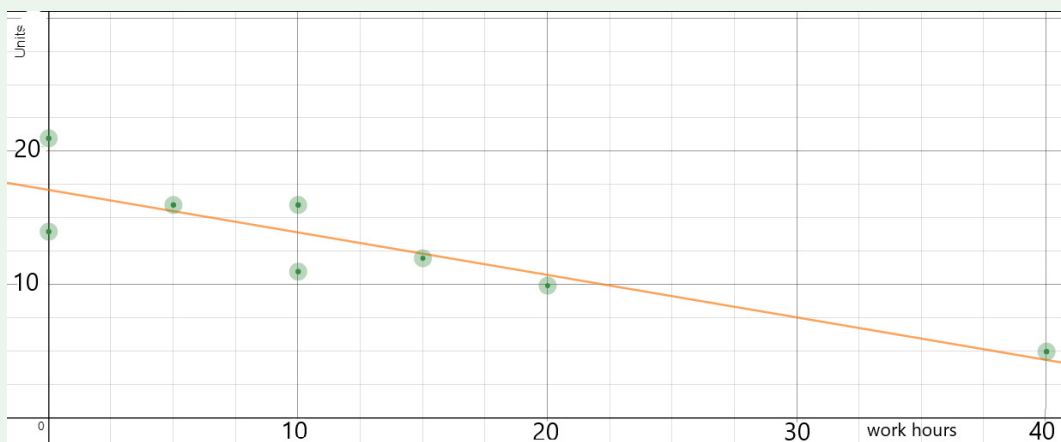
Interpret the y -intercept of the regression line in the context of the study or explain why it has no practical meaning.

Solution

The y -intercept of a line is where it crosses the y -axis. In this case, the line crosses at around $y = -1$. The value of x , by definition is 0 and the x -axis represents the number of cups of coffee a person drank last week. Since there are people who don't drink coffee, it does make sense to have an x -value of 0. The y -axis represents the number of times the person went to a shop or restaurant last week to purchase a meal or a drink. It makes no sense to say that a person went -1 times to a shop or restaurant last week to purchase a meal or a drink. Therefore the y -intercept of this regression line has no practical meaning.

Try It

The scatterplot and regression line below are from a study that collected data from a group of college students on the number of hours per week during the school year they work at a paid job, x , and the number of units they are taking, y . Interpret the y -intercept of the regression line or explain why it has no practical meaning.



Answer

A student who works 0 hours at a paid job (does not work) is predicted to be taking about 17 units.

Additional video resources:

- [Interpret the Meaning of the y-intercept Given a Linear Equation](#)
- [Interpreting the y-Intercept](#)

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8.7: Finding Residuals

Learning Outcomes

- Given a Regression line and a data point, find the residual

In the linear regression part of statistics we are often asked to find the residuals. Given a data point and the regression line, the residual is defined by the vertical difference between the observed value of y and the computed value of \hat{y} based on the equation of the regression line:

$$\text{Residual} = y - \hat{y}$$

Example 8.7.1

A study was conducted asking female college students how tall they are and how tall their mother is. The results are shown in the table below:

Table of Mother and Daughter Heights

Mother's Height	63	67	64	60	65	67	59	60
Daughter's Height	58	64	65	61	65	67	61	64

The equation of the regression line is

$$\hat{y} = 30.28 + 0.52x$$

Find the residual for the mother who is 59 inches tall.

Solution

First note that the Daughter's Height associated with the mother who is 59 inches tall is 61 inches. This is y . Next we use the equation of the regression line to find \hat{y} . Since $x = 59$, we have

$$\hat{y} = 30.28 + 0.52(59)$$

We can use a calculator to get:

$$\hat{y} = 60.96$$

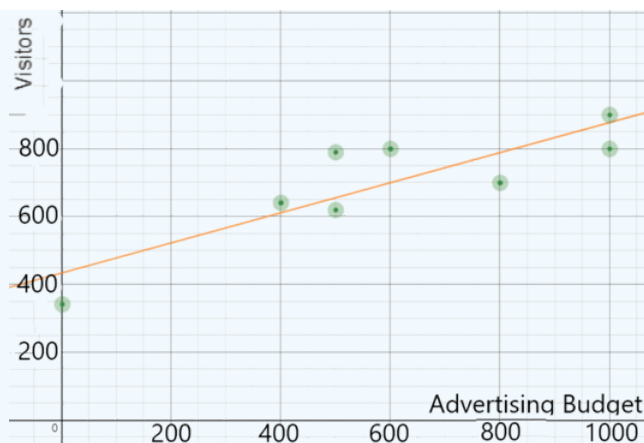
Now we are ready to put the values into the residual formula:

$$\text{Residual} = y - \hat{y} = 61 - 60.96 = 0.04$$

Therefore the residual for the 59 inch tall mother is 0.04. Since this residual is very close to 0, this means that the regression line was an accurate predictor of the daughter's height.

Example 8.7.2

An online retailer wanted to see how much bang for the buck was obtained from online advertising. The retailer experimented with different weekly advertising budgets and logged the number of visitors who came to the retailer's online site. The regression line for this is shown below.



Find the residual for the week when the retailer spent \$600 on advertising.

Solution

First notice that the point of the scatterplot with x-coordinate of 600 has y-coordinate 800. Thus $y = 800$. Next note that the point on the line with x-coordinate 600 has y-coordinate 700. Thus $\hat{y} = 700$. Now we are ready to put the values into the residual formula:

$$\text{Residual} = y - \hat{y} = 800 - 700 = 100$$

Therefore the residual for the \$600 advertising budget is 100.

Exercise

Data was taken from the recent Olympics on the GDP in trillions of dollars of 8 of the countries that competed and the number of gold medals that they won. The equation of the regression line is:

$$\hat{y} = 7.55 + 1.57x$$

The table below shows the data:

GDP	21	1.6	16	1.8	4	5.4	3.1	2.3
Medals	46	8	26	19	17	12	10	9

Find the residual for the country with a GDP of 4 trillion dollars.

- [Calculating residual example | Exploring bivariate numerical data | AP Statistics | Khan Academy](#)
- [Finding a Residual](#)

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