9.5: Fisher exact test

Introduction

We mentioned that chi-square tests for contingency tables are fine as long as two conditions are met. These are the assumptions of a χ^2 test:

1. No cell should have expected values less than 5%.
2. The test performs poorly at DF = 1 because we are approximating an infinite distribution with an exact test.

You will note that any time you have a 2×2 table, the second condition is always an issue because 2×2 tables have $DF = 1$. Thus, in biomedical research, it is common to have an experiment that may be appropriate for a contingency analysis but the data may suffer from one or both of these limitations. **Fisher's exact test** is always an option for these types of problems, but with the advantage that it always returns the exact p-value.

As a reminder, the 2×2 table looks like:

Table $9.5.1$. 2×2 table reporting numbers of subjects who have (Yes) or do not have (No) the event.

		Column 1	Column 2
	Subjects	Yes	No
Row 1	Treatment 1	**a**	**b**
Row 2	Treatment 2	**c**	**d**

where **a** is the count of Treatment 1-treated subjects who have the event, **b** is the count of Treatment 1-treated subjects who do not have the event, **c** is the count of Treatment 2-treated subjects who have the event, and **d** is the count of Treatment 2-treated subjects who do not have the event. Note the row and column totals:

$$\begin{aligned} \text{Row 1 } &= a + b \\ \text{Row 2 } &= c + d \\ \text{Column 1 } &= a + c \\ \text{Column 2 } &= b + d \end{aligned}$$

For example, a fairly common "Gee, that's curious" fact is that the seven left-handed US Presidents since 1901 (out of a total of 21 presidents) exceed the proportion of left-handers in the general population (about 10%). For comparison, we could ask the same question about Vice Presidents.†

Table $9.5.2$. Left-handedness of US presidents and vice presidents since 1901.

Subjects	Yes	No
Presidents	7	14
Vice presidents	5	20

†Seven Vice-Presidents went on to become President, four right-handers, 3 left-handers.

Ronald A. Fisher came up with a test that is now called "Fisher's Exact test" that circumvents this problem. It is an extremely useful test to know about because it provides a way to get an exact probability of the outcome compared to all other possible outcomes. Thus, when asked for a possible alternate to the chi-square contingency test for a 2×2 table, you can respond "Fisher's Exact test."

Although tedious to calculate by hand and resource demanding when done by computer because of the multiple factorial expressions, the major advantage of the test is that it does not rely on the assumption that an underlying distribution applies. The Fisher Exact test can be used to calculate the exact probability of the observed outcome (P).

The equation for the Fisher Exact test can be written as

$$P = \frac{R_1! \cdot R_2! \cdot C_1! \cdot C_2!}{a! \cdot b! \cdot c! \cdot d! \cdot n!}$$

where R stands for row total, C stands for column total, n is the sample size, ! is the **factorial**, and a, b, c, and d are defined as in Table 9.5.1.

How does Fisher's Exact test work? The data are set up in the usual way for a contingency problem, but now, we calculate the probability for all possible outcomes that we COULD have seen from our experiment, and ask if the actual outcome is unusual (low p-value). The trick is recognizing that you have to keep the totals constrained (note row and column totals stay the same).

Table 9.5.3. Original 2×2 contingency table (bold), with the next two more extreme outcomes

original data	————>	more extreme	————>	next more extreme still	————>
Yes	**No**	Yes	No	Yes	No
10	**5**	11	4	12	3
4	**12**	3	13	2	14
	p-value=0.0206		p-value=0.0029		p-value=0.0002

I've shown just the one-tailed outcomes, so the p-values are for one-tailed tests of hypothesis. The essence of the test is to find all outcomes MORE extreme than the original, in one direction. The one-tailed P-value then is the sum of all probabilities from those more extreme tables of outcomes.

To get the two-tailed probability, remember that you multiply the one-tailed probability by two. More accurate methods are also available (Agresti 1992).

Calculation of Fisher's test involves using all possible combinations and factorials. Rcmdr has Fisher's 2×2 built in via the Contingency table and as part of some `Rcmdr` plugins (e.g., `RcmdrPlugin.EBM` , the Evidence Based Medicine plugin). Here we illustrate Fisher Exact test from the context menu in the main Statistics menu.

Alternatively, there are many web sites out there that provide an online calculator for Fisher's Exact test. Here's a link to one such calculator on GraphPad's web site, cookies must be enabled to run this calculator).

To get the Fisher Exact test, your data must already be summarized into a 2×2 table, in which case you can use

Rcmdr: Statistics → Contingency tables… → Enter and analyze two way table (then select Fisher's Exact test option).

	Smoker: No	Smoker: Yes
Vitamin use: No	14	26
Vitamin use: Yes	19	15

If the original data are available, do not tally the counts, let R do the work for you. The worksheet would be stacked like so. The image of the R worksheet below contains 4 columns: Sex (M/F), Smoker (Never, Former, Current), Smoke (Y/N), and Vitamin User (No, Regular).

Stacked worksheet for Continency table or Fisher exact test.

R code

To carry out contingency table analysis or Fisher Exact test,

Rcmdr: Statistics → Contingency tables… → Two way table …

Check the box next to the Fisher's exact test.

Select `Vitamin.Use` for Row variable and `Smoke` for Column variable. Click OK, and here is the R output.

```
> fisher.test(.Table)
Fisher's Exact Test for Count Data
data: .Table
p-value = 0.1008
```

```
alternative hypothesis: true odds ratio is not equal to 1
95 percent confidence interval:
0.1496417 1.1985775
sample estimates:
odds ratio
0.4302094
```

We accepted the defaults. Is this a one- or two-tailed test of hypothesis?

What can we conclude about the null hypothesis? Do we accept or reject?

Want to know what the "odds ratio" is? Follow the link to the next subchapter.

When to use the Fisher Exact Test?

Here's the take-home message: the Fisher exact test is an alternate and better choice over the contingency table chi-square for 2×2 tables if one or more of the cells has expected values less than 5%. It is also appropriate for cases in which you have only 1 degree of freedom (as do all 2×2 tables!), but it doesn't make sense if each cell has more than 5% expected values (the calculation is too tedious), but rather, apply the Yate's correction. As the sample sizes get larger, the different methods converge to virtually identical answers.

Some examples.

Is there an association between final grades and attendance on a randomly selected day?

Table $9.5.4$. First scenario

cc	Yes	No
Letter grade A	2	3
Other letter grade	1	6

Table $9.5.5$. Second scenario.

cc	Yes	No
Letter grade A	5	6
Other letter grade	2	12

Table $9.5.6$. Third scenario

cc	Yes	No
Letter grade A	10	12
Other letter grade	4	24

Code for tests are as follows for Table $9.5.4$ as an example:

Data table:

```
grades.Table <- matrix(c(2,3,1,6), 2, 2, byrow=TRUE)
```

Chi-square test of independence:

```
.Test <- chisq.test(grades.Table, correct=TRUE)
```

Fisher Exact test:

```
fisher.test(grades.Table, alternative = “greater”)
```

Questions

1. Apply the Fisher exact test on the four contingency tables (a – d) introduced in Section 9.3, question 2. Make note of the p-value from Fisher exact test and from analyses used to complete question 2 in Section 9.3. Note any trends. (Hint: make sure you are testing the same null hypothesis.)

(a)

	Yes	No
A	18	6
B	3	8

(b)

	Yes	No
A	10	12
B	3	14

(c)

	Yes	No
A	5	12
B	12	18

(d)

	Yes	No
A	8	12
B	3	3