

## 13.3: Linear Equations

Linear regression for two variables is based on a linear equation with one independent variable. The equation has the form:

$$y = a + bx$$

where  $a$  and  $b$  are constant numbers.

The variable  $x$  is the **independent variable**, and  $y$  is the **dependent variable**. Another way to think about this equation is a statement of cause and effect. The  $X$  variable is the cause and the  $Y$  variable is the hypothesized effect. Typically, you choose a value to substitute for the independent variable and then solve for the dependent variable.

### ? Example 13.3.1

The following examples are linear equations.

$$(y = 3 + 2x$$

$$y = -0.01 + 1.2x$$

The graph of a linear equation of the form  $y = a + bx$  is a **straight line**. Any line that is not vertical can be described by this equation

### ? Example 13.3.2

Graph the equation  $y = -1 + 2x$ .

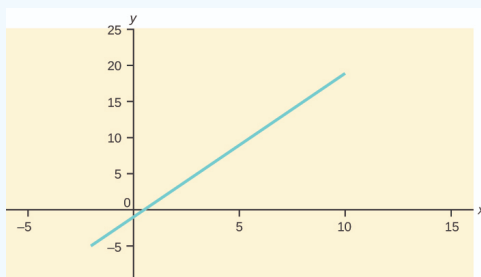


Figure 13.3.3

### ? Try It 13.3.1

Is the following an example of a linear equation? Why or why not?

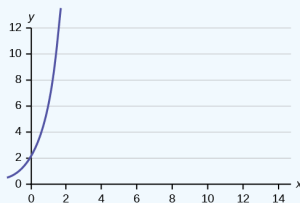


Figure 13.3.4

### ? Exercise 13.3.1

Aaron's Word Processing Service (AWPS) does word processing. The rate for services is \$32 per hour plus a \$31.50 one-time charge. The total cost to a customer depends on the number of hours it takes to complete the job.

Find the equation that expresses the **total cost** in terms of the **number of hours** required to complete the job.

**Answer**


Let  $x$  = the number of hours it takes to get the job done.

Let  $y$  = the total cost to the customer.

The \$31.50 is a fixed cost. If it takes  $x$  hours to complete the job, then  $(32)(x)$  is the cost of the word processing only. The total cost is:  $y = 31.50 + 32x$

## Slope and Y-Intercept of a Linear Equation

For the linear equation  $y = a + bx$ ,  $b$  = slope and  $a$  =  $y$ -intercept. From algebra recall that the slope is a number that describes the steepness of a line, and the  $y$ -intercept is the  $y$  coordinate of the point  $(0, a)$  where the line crosses the  $y$ -axis. From calculus the slope is the first derivative of the function. For a linear function the slope is  $dy/dx = b$  where we can read the mathematical expression as "the change in  $y$  ( $dy$ ) that results from a change in  $x$  ( $dx$ ) =  $b * dx$ ".

 Three possible graphs of the equation  $y = a + bx$ . For the first graph, (a),  $b$

0 and so the line slopes upward to the right. For the second,  $b = 0$  and the graph of the equation is a horizontal line. In the third graph, (c),  $b < 0$  and the line slopes downward to the right."

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Figure 13.3.1: Three possible graphs of  $y = a + bx$ . (a) If  $b > 0$ , the line slopes upward to the right. (b) If  $b = 0$ , the line is horizontal. (c) If  $b < 0$ , the line slopes downward to the right.

### ? Exercise 13.3.2

Svetlana tutors to make extra money for college. For each tutoring session, she charges a one-time fee of \$25 plus \$15 per hour of tutoring. A linear equation that expresses the total amount of money Svetlana earns for each session she tutors is  $y = 25 + 15x$ .

What are the independent and dependent variables? What is the  $y$ -intercept and what is the slope? Interpret them using complete sentences.

#### Answer

Solution 13.4

The independent variable ( $x$ ) is the number of hours Svetlana tutors each session. The dependent variable ( $y$ ) is the amount, in dollars, Svetlana earns for each session.

The  $y$ -intercept is 25 ( $a = 25$ ). At the start of the tutoring session, Svetlana charges a one-time fee of \$25 (this is when  $x = 0$ ). The slope is 15 ( $b = 15$ ). For each session, Svetlana earns \$15 for each hour she tutors.

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