## 7.4: Finite Population Correction Factor

We saw that the sample size has an important effect on the variance and thus the standard deviation of the sampling distribution. Also of interest is the proportion of the total population that has been sampled. We have assumed that the population is extremely large and that we have sampled a small part of the population. As the population becomes smaller and we sample a larger number of observations the sample observations are not independent of each other. To correct for the impact of this, the **Finite Population Correction Factor** can be used to adjust the variance of the sampling distribution. It is appropriate when more than 5% of the population is being sampled and the population has a known population size. There are cases when the population is known, and therefore the correction factor must be applied. The issue arises for both the sampling distribution of the means and the sampling distribution of proportions. The Finite Population Correction Factor for the variance of the means shown in the standardizing formula is:

$$Z=\frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}} \cdot \sqrt{\frac{N-n}{N-1}}}$$

and for the variance of proportions is:

$$\sigma_{\hat{p}}=\sqrt{\frac{p(1-p)}{n}} \times \sqrt{\frac{N-n}{N-1}}$$

The following examples show how to apply the factor. Sampling variances get adjusted using the above formula.

### Example 7.4.1

It is learned that the population of White German Shepherds in the USA is 4,000 dogs and the mean weight for German Shepherds is 75.45 pounds. It is also learned that the population standard deviation is 10.37 pounds. If the sample size is 100 dogs, then find the probability that a sample will have a mean that differs from the true population mean by less than 2 pounds.

**Answer**

We are given the following information:

$$N=4000, \quad n=100, \quad \sigma=10.37, \quad \mu=75.45, \quad \text{want } |\overline{X}-\mu|<\pm 2$$

Note that "differs by less" references the area on both sides of the mean within 2 pounds right or left. We apply the finite population correction factor as follows:

$$P(|\overline{X}-\mu|<2)=P\left(\frac{|\overline{X}-\mu|}{\frac{\sigma}{\sqrt{n}} \cdot \sqrt{\frac{N-n}{N-1}}}<\frac{2}{\frac{\sigma}{\sqrt{n}} \cdot \sqrt{\frac{N-n}{N-1}}}\right) \approx P\left(\frac{-2}{\frac{10.37}{\sqrt{100}} \cdot \sqrt{\frac{4000-100}{4000-1}}}<Z<\frac{2}{\frac{10.37}{\sqrt{100}} \cdot \sqrt{\frac{4000-100}{4000-1}}}\right)=P(-1.95<Z<1.95)$$

From the Z Table, we find that $P(0<Z<1.95)=0.4744$, which gives the following:

$$P(-1.95<Z<1.95)=0.4744 \cdot 2=0.9488$$

We have found that there is approximately a 94.88% chance that the mean weight of a sample of 100 dogs will differ from the true mean by less than 2 pounds.

### Example 7.4.2

When a customer places an order with Rudy's On-Line Office Supplies, a computerized accounting information system (AIS) automatically checks to see if the customer has exceeded his or her credit limit. Past records indicate that the probability of customers exceeding their credit limit is .06.

Suppose that on a given day, 3,000 orders are placed in total. If we randomly select 360 orders, what is the probability that between 10 and 20 customers will exceed their credit limit?

**Answer**

We are given the following information:

$$N=3000, \quad n=360, \quad p=0.06$$

We want to find $P(10<X<20)=P\left(\frac{10}{360}<\hat{p}<\frac{20}{360}\right)=P(0.0278<\hat{p}<0.0556)$ . We first find the standard deviation of $\hat{p}$ applying the finite population correction factor (because $n/N=360/3000=0.12>0.05$

$$\sigma_{\hat{p}}=\sqrt{\frac{p(1-p)}{n}} \times \sqrt{\frac{N-n}{N-1}}=\sqrt{\frac{0.06(1-0.06)}{360}} \times \sqrt{\frac{3000-360}{3000-1}}=0.0117$$

Note that we know the mean of $\hat{p}$ since we know the population proportion, $p=0.06$. Thus, the sampling distribution of $\hat{p}$ can be approximated by $N(0.06, 0.0117$.)So we compute:

$$P(0.0278 < \hat{p} < 0.0556) = P\left(\frac{0.0278 - 0.06}{0.0117} < \frac{\hat{p} - 0.06}{0.0117} < \frac{0.0556 - 0.06}{0.0117}\right) \approx P(-2.75 < Z < -0.38) = 0.4970 + 0.1480 = 0.6450.$$

There is approximately a 64.5% chance that between 10 and 20 customers in a sample of size 360 will exceed their credit limit.