3.8: Discrete Distribution Experiment (Worksheet) **

Work in groups on these problems. You should try to answer the questions without referring to your textbook. If you get stuck, try asking another group for help.

Student Learning Outcomes

- The student will compare empirical data and a theoretical distribution to determine if an everyday experiment fits a discrete distribution.
- The student will demonstrate an understanding of long-term probabilities.

The Experiment and Random Variable

The experimental procedure is to select a playing card from a full set of denominations, i.e., Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, and King. The procedure is repeated three times and the number of Kings picked is recorded.

1. Shuffle the set of cards.
2. Pick one card from the set at random.
3. Record whether it was a King or not a King.
4. Put the card back and reshuffle the set.
5. Do this a total of three times.
6. Record the number of Kings picked.

We consider the **random variable *X* = number of Kings picked.**

Empirical Distribution of *X*

1. Record the number of Kings picked by the class in a table like the one below and calculate the relative frequencies.
2. Using the table as the empirical probability distribution of *X*, calculate the **mean** and **standard deviation** for *X*.
3. Construct a **histogram** for *X* using the empirical distribution.
4. Calculate the following **probabilities** using the empirical distribution.
   a. the probability of picking exactly 3 Kings: $P(X = 3)$
   b. the probability of picking between 0 and 3 Kings, not inclusive: $P(0 < X < 3)$
   c. the probability of picking at least 2 Kings: $P(X \geq 2)$

Empirical Distribution of *X*

x	Frequency	Relative Frequency $\approx p(x)$
0		
1		
2		
3		

Theoretical Distribution of *X*

The distribution of *X* can be modeled theoretically with the **binomial distribution**. The binomial distribution applies when

- we have n identical trials - each with two possible outcomes (called success or failure),
- the probability of success, p, is the same for each trial, and
- the trials are independent.

The pdf of a binomial distribution is given by the following formula:

$$p(x) = \binom{n}{x} p^x (1-p)^{n-x},$$

where

$$\binom{n}{x} = \frac{n!}{x!(n-x)!}. \quad (3.8.1)$$

Remember $n! = n(n-1)\cdots 2\cdot 1$. For example, $3! = 3\cdot 2\cdot 1 = 6$. Note that $0! = 1$.

1. **Explain** why the random variable X = the number of Kings picked can be modeled using a binomial distribution. Identify the values of n, the number of trials, and p, the probability of "success" for X.
2. Present the theoretical PDF in a table like the one below.
3. Using the theoretical distribution of X, calculate the **mean** and **standard deviation** for X.
4. Construct a **histogram** for X using the theoretical distribution.
5. Calculate the following **probabilities** using the theoretical distribution.
   a. the probability of picking exactly 3 Kings: $P(X = 3)$
   b. the probability of picking between 0 and 3 Kings, not inclusive: $P(0 < X < 3)$
   c. the probability of picking at least 2 Kings: $P(X \geq 2)$

Theoretical Distribution of X

x	$p(x)$
0	
1	
2	
3	

Reflection Questions

For questions 1 and 2 below, think about the shapes of the two graphs, the probabilities you calculated, the relative frequencies, the means, and the standard deviations.

1. Knowing that data vary, describe three similarities between the graphs and distributions of the theoretical and empirical distributions. Use complete sentences.
2. Describe the three most significant differences between the graphs or distributions of the theoretical and empirical distributions.
3. Using your answers from questions 1 and 2, does it appear that the data fit the theoretical distribution? In complete sentences, explain why or why not.
4. Suppose that the experiment had been repeated 500 times. Would you expect the Empirical Distribution or the Theoretical Distribution to change, and how would it change? Why? Why wouldn't the other distribution change?