

5.2: The Uniform Distribution

The uniform distribution is a continuous probability distribution and is concerned with events that are equally likely to occur. When working out problems that have a uniform distribution, be careful to note if the data is inclusive or exclusive of endpoints.

The mathematical statement of the uniform distribution is

$$f(x) = \frac{1}{b-a} \text{ for } a \leq x \leq b$$

where a = the lowest value of x and b = the highest value of x .

Formulas for the theoretical mean and standard deviation are

$$\mu = \frac{a+b}{2} \text{ and } \sigma = \sqrt{\frac{(b-a)^2}{12}}$$

? Exercise 5.2.1

The data that follow are the number of passengers on 35 different charter fishing boats. The sample mean = 7.9 and the sample standard deviation = 4.33. The data follow a uniform distribution where all values between and including zero and 14 are equally likely. State the values of a and b . Write the distribution in proper notation, and calculate the theoretical mean and standard deviation.

1	12	4	10	4	14	11
7	11	4	13	2	4	6
3	10	0	12	6	9	10
5	13	4	10	14	12	11
6	10	11	0	11	13	2

Table 5.2.1

? Example 5.2.1

The amount of time, in minutes, that a person must wait for a bus is uniformly distributed between zero and 15 minutes, inclusive.

a. What is the probability that a person waits fewer than 12.5 minutes?

Answer

a. Let X = the number of minutes a person must wait for a bus. $a = 0$ and $b = 15$. $X \sim U(0, 15)$. Write the probability density function. $f(x) = \frac{1}{15-0} = \frac{1}{15}$ for $0 \leq x \leq 15$.

Find $P(x < 12.5)$. Draw a graph.

$$P(x < k) = (\text{base})(\text{height}) = (12.5 - 0) \left(\frac{1}{15} \right) = 0.8333$$

The probability a person waits less than 12.5 minutes is 0.8333.

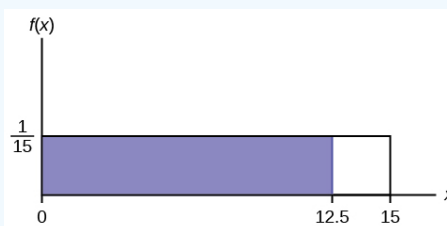


Figure 5.2.1

b. On the average, how long must a person wait? Find the mean, μ , and the standard deviation, σ .

Answer

b. $\mu = \frac{a+b}{2} = \frac{15+0}{2} = 7.5$. On the average, a person must wait 7.5 minutes.

$\sigma = \sqrt{\frac{(b-a)^2}{12}} = \sqrt{\frac{(15-0)^2}{12}} = 4.3$. The Standard deviation is 4.3 minutes.

c. Ninety percent of the time, the time a person must wait falls below what value?

Note

This asks for the 90th percentile.

Answer

c. Find the 90th percentile. Draw a graph. Let k = the 90th percentile.

$$P(x < k) = 0.90 = (k) \left(\frac{1}{15} \right)$$

$$k = (0.90)(15) = 13.5$$

The 90th percentile is 13.5 minutes. Ninety percent of the time, a person must wait at most 13.5 minutes.

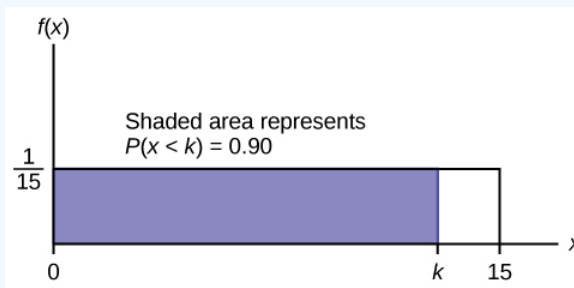


Figure 5.2.2

? Exercise 5.2.2

The total duration of baseball games in the major league in the 2011 season is uniformly distributed between 447 hours and 521 hours inclusive.

1. Find a and b and describe what they represent.
2. Write the distribution.
3. Find the mean and the standard deviation.
4. What is the probability that the duration of games for a team for the 2011 season is between 480 and 500 hours?

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