4.2: The Geometric Distribution

There are many probability experiments where a trial has only two outcomes. For example, asking a group of individuals if they vote yes on a proposition, or randomly guessing on a multiple choice test. When we conduct a sequence of independent trials with only two outcomes per trial, we are conducting a binomial experiment.

1. Which of the following only have two possible outcomes:

   a. Rolling a 4 on a 6-sided die

   b. Examining the global temperature change over time.

   c. Measuring the height of adult in California

   d. Meeting a person that is infected with Covid-19

Characteristics of a Geometric Experiment

A geometric experiment is a probability experiment with the following characteristics:

- Each trial has exactly two possible outcomes which are labeled success and failure.
- The probability of success is the same for each trial. We denote the probability of success as p and the probability of failure as $q = 1 - p$.
- We look for when the first and only success occurs. There must be at least one trial, and in theory, we could repeat trials forever.

2. Go to https://www.random.org/dice/ and roll 1 die. Roll the die counting the number of trials it took to roll a 5. Keep track of your rolls in the table below.

Tally	On what attempt did you succeed in rolling a 5?

3. Assume we will roll a fair six-sided die.
   a. What is the probability of rolling a 5? We define rolling a 5 as success, and therefore, we are computing the probability of success.

   b. What is the probability that we will not roll a 5? Use the complement rule to compute the probability of failure.

4. Suppose we are rolling a fair six-sided die.
   a. What is the probability that we will roll a 5 (succeed) on the first attempt?

   b. What is the probability that we will roll a 5 (succeed) on the second attempt? In this case, we fail on the first try and succeed on the second try. Use the multiplication rule for independent events.

5. Suppose we are rolling a fair six-sided die.
   a. What is the probability that we will roll a die and succeed (roll a 5) on the third attempt? In this case, we fail on the first and second tries and succeed on the third try. Use the multiplication rule for independent events.

   b. What is the probability that we will roll a die and succeed (roll a 5) on the fourth attempt? In this case, we fail on the first and second and third tries and succeed on the fourth try. Use the multiplication rule for independent events.

6. What is the formula for computing geometric probability? So far, this is what we have come up with:

$$P(1) = \frac{1}{6}$$
$$P(2) = \frac{5}{6} \cdot \frac{1}{6}$$
$$P(3) = \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{1}{6} = \left(\frac{5}{6}\right)^2 \cdot \frac{1}{6}$$
$$P(4) = \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{1}{6} = \left(\frac{5}{6}\right)^3 \cdot \frac{1}{6}$$

   What patterns do you notice?

7. What is P(5)?

Geometric Probability

In general, the probability of succeeding only once on the xth attempt is

$$P(x) = q^{x-1}p$$

where p is the probability of success and $q = 1 - p$ is the probability of failure.

8. **You try!** You enter a darts tournament. The probability of hitting the bullseye is 17%. What is the probability that you hit the bullseye on the 7th attempt? You can upload an image to show your thinking.