7.3: Estimating a Population Mean

In this section, we will be constructing confidence intervals for population means. You will find that the process is similar to estimating a population proportion, however, we have different criteria, formulae, and frequently use different distributions.

Confidence Interval for a Population Mean

We will estimate the mean number of landfills in states with high poverty rates. We will use the T-distribution to compute margins of error and confidence intervals for a population mean.

Recall the five step process of building a confidence interval:

Step 1. Verify that the sampling distribution is approximately normal.

Step 2. Compute the critical value.

Step 3. Compute the margin of error $E = \text{ critical value } \cdot \text{ standard error }$.

Step 4. Compute the bounds of the interval and write the interval in interval notation.

Step 5. Interpret the interval in context/state a conclusion in context.

Step 1: Verify that the Sampling Distribution of Sample Means is Approximately Normal

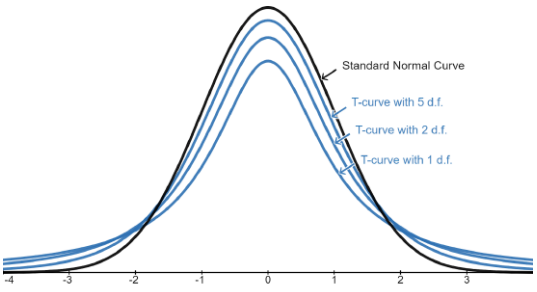
Recall, the normality conditions for the sampling distribution of sample means are **either:**

- The sample size is greater than 30 **OR**
- The sample is from a normally distributed population.

Step 2: Find the Critical Value

If the population standard deviation (σ) is *known*, the critical value comes from the normal distribution. If the population standard deviation (σ)

is *unknown*, the critical value comes from the student's T distribution which varies based on sample size. This is usually the case.



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Step 3: Compute the Margin of Error

The margin of error is computed using the equation

$$E = \text{ critical value } \cdot \text{ standard error}$$

Since we often use the sample standard deviation, s, to estimate the unknown population standard deviation, , the estimated margin of error is found using the equation

$$E \approx T_c \cdot \frac{s}{\sqrt{n}}$$

Step 4: Compute the Confidence Interval

Now, we are estimating a population *mean* rather than a population *proportion*. Therefore, the point estimate we use is the sample mean, $\bar{x}$. The confidence interval is

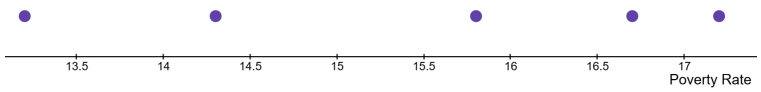
$$(\bar{x} - E, \bar{x} + E)$$

Step 5: Interpret the Interval and State a Conclusion in Context

Arguably the most important step, the five step process always ends with writing the conclusion in plain language. The conclusion should include the confidence level, the parameter that is being estimated, and the bounds of the interval with appropriate units.

Example

A researcher wants to know the average poverty rate in California. They randomly select 5 counties and find that the average poverty rate is 15.44% with a standard deviation of 1.668%. Researchers have determined that the population of poverty rates by county in California is approximately normal. The dotplot of the sample of 5 counties' poverty rates is given below.



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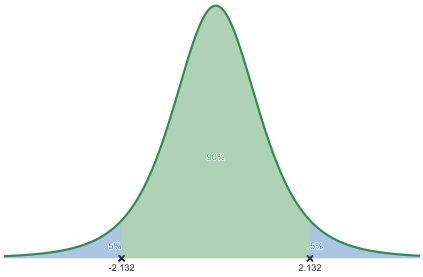
We will construct a 90% confidence interval for the true mean number of landfills in poor counties of California.

Step 1. Although the sample size is less than ____, the problem states that the population of poverty rates by county in California is approximately normal. Therefore, the sample is from a normal population. It follows that the sampling distribution of sample _____ is approximately normal.

Step 2. Since the population standard deviation is unknown, we will find the critical value from the T-distribution. The T-distribution depends on the degrees of freedom. Here, the sample size is 5, so there are _____ degrees of freedom. We will need to know the area left of the critical value. Since the confidence level is 90%=_____, there is 5% or 0.05 in each tail. Therefore, the area left of the critical value is _____.

- In https://www.desmos.com/calculator, we enter tdist(4).inversecdf(0.95) to find the critical value. So the critical value $T_c = 2.132$.



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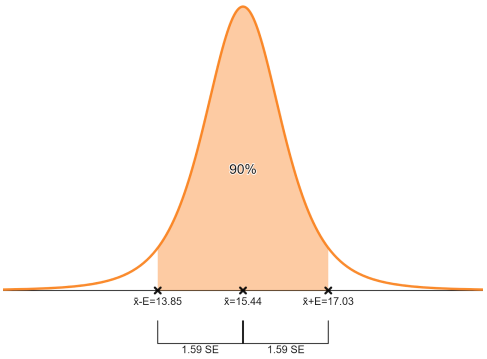
Step 3. To compute the margin of error, we use $E \approx T_c \cdot \frac{s}{\sqrt{n}} = 2.132 \cdot \frac{}{\sqrt{5}} \approx 1.59\%$.

Step 4. The interval is

$$(\bar{x} - E, \bar{x} + E) = (15.44\% - 1.59\%, 15.44\% + 1.59\%) = (13.85\%, 17.03\%)$$



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Step 5. We are _____ confident that the _____ poverty rate for all counties in California is between _____% and _____%.

How the Margin of Error Changes

The following questions and animations will help you think about the relationship between sample size, the confidence level, the standard deviation and the margin of error.

1. Fill in the blanks with one of the following: *increases*, *decreases*, or *stays the same* where $E = (\text{ critical value }) \cdot \frac{s}{\sqrt{n}}$. (If you are accessing on a hard copy, go to https://www.desmos.com/calculator/rxiaadmqrm and adjust the indicated variable).

   a. As the sample size (n) increases, the margin of error (E) _____

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   b. As the confidence level increases, the margin of error (E) _____

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c. As the sample standard deviation (s) increases, the margin of error (E) _____

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You try!

2. You work for a consumer advocate agency and want to find the mean repair cost of a washing machine. As part of your study, you randomly select 40 repair costs and find the mean to be $100.00 and the standard deviation $17.50. Construct a 95% confidence interval for the mean repair cost of a washing machine.

Step 1.

Step 2.

Step 3.

Step 4.

Step 5.