5.4: Finding Critical Values from the Normal Distribution

Sometimes we know the proportion or area that is defined by a range of normally distributed values. But we may not know the cutoff values (also called **critical values**) for the range.

Suppose we want to find the critical value that separates the top 15% of men's heights from the lower 85%. We can use technology to calculate this value from the standard normal distribution. This value is a Z-score.

Using desmos to Find a Critical Value from The Standard Normal Distribution

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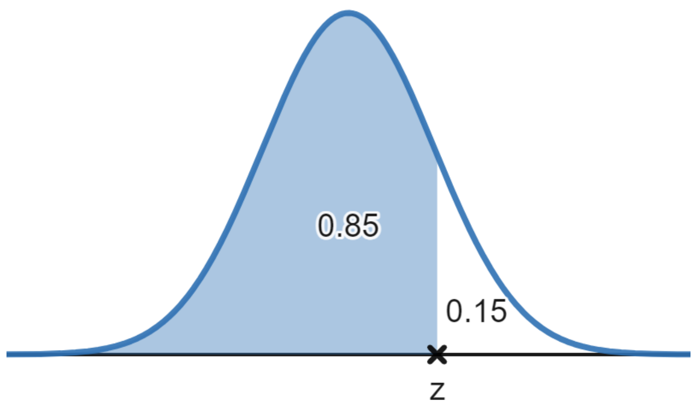

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To calculate a value of z from a given probability, use the following steps:

1. Go to https://www.desmos.com/calculator.
2. The function we will use takes the area or probability that is less than the critical value. Let's call this area A. It returns a value of z. In the first line, type normaldist().inversecdf(A).

In our example above, we learn that $z = \text{normaldist}().\text{inversecdf}(0.85) \approx 1.036$ is the Z-score that separates the lower 85% from the upper 15%. This is useful information, but we haven't yet found the adult male height that separates the lower 85% from the upper 15%. We need to find the adult male height that corresponds to the Z-score 1.036.

Recall, that a Z-score represents the distance (in standard deviations) a value is above or below the mean. Therefore, the male height that corresponds to this Z-score is 1.036 standard deviations above the mean (since it is positive). x represents random adult male heights, the mean of the distribution is 69 inches, and the standard deviation of the distribution is 3 inches. Let's translate this into a mathematical sentence:

$$\underbrace{\text{This adult male height}}_{x} \text{ is } \underbrace{1.036}_{=1.036} \underbrace{\text{standard deviations}}_{\sigma} \underbrace{\text{above}}_{+} \underbrace{\text{the mean}}_{\mu}$$

$$x = 1.036 \cdot 3 + 69 = 72.108 \text{ inches}$$

Therefore, the adult male height that separates the lower 85% from the top 15% is 72.108 inches. Furthermore, we can say that this adult male height is in the 85th **percentile** because said adult man is taller than 85% of all adult men.

In general, to convert a Z-score to an x-value that is from a normal population that has mean and standard deviation , we use the formula

$$x = z \cdot \sigma + \mu$$

You try!

1. The Welcher Adult Intelligence Test Scale is composed of a number of subtests. On one subtest, the raw scores have a mean of 35 and a standard deviation of 6. Assuming these raw scores form a normal distribution:
   a. What number represents the 65^{th} percentile (what number separates the lower 65% of the distribution)?

   b. What number represents the 90th percentile?

   c. What numbers separate the middle 95% of scores?

2. Kelly’s score on the SAT was in the 92nd percentile. Explain what this means about her score relative to all students who took the SAT.