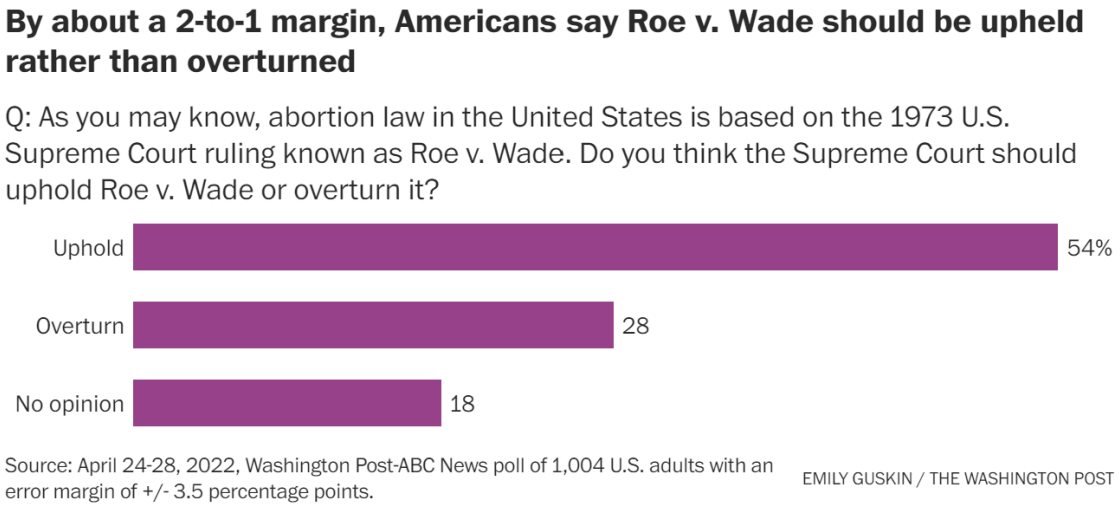
6.2: Estimating a Population Proportion

On May 3rd, 2022, The Washington Post published an article[4] that included results of a poll that asked US adults whether they think the supreme court should uphold Roe v. Wade or overturn it, after reports of the Supreme Court's plans to overturn the right to abortion were leaked to the public.

Findings from a survey of 1,004 US adults suggest that 54% believe the ruling should be upheld, 28% believe the ruling should be overturned, while 18% have no opinion, with an error margin of +/- 3.5 percentage points.



Identifying Important Information

1. Washington Post-ABC News used sample data to estimate the true population proportion of those adults in the US who believe Roe v. Wade should be upheld.

   a. What is the sample size?

   b. Is 54% a parameter or a statistic? Justify your answer.

c. Do you think the population proportion of adults in the US who believe Roe v. Wade should be upheld is *exactly* 54%? Why or why not?

d. What do you think the “error margin of +/- 3.5 percentage points” tells you about the population proportion?

The sample proportion $\hat{p}$ is a single estimate of the population proportion, and is often referred to as a **point estimate**. An **interval estimate** is a range of values used to estimate a population proportion. Interval estimates are centered at a point estimate. The interval is formed as

$$\text{Point Estimate} \pm \text{Margin of Error}$$

e. Identify the point estimate and the margin of error.

$\hat{p} =$

$E =$

f. Construct the endpoints of the interval given in the results of the poll.

Interval estimates like you found above, give a range of values that *usually* contain the true population proportion, which is the proportion of *all* adults in the US who believe Roe v. Wade should be upheld in the example above.

g. Do you think that it is likely that the true proportion of adults in the US who believe Roe v. Wade should be upheld is as high as 60%? Explain.

h. Do you think it's likely that the population proportion of adults in the US who believe Roe v. Wade should be upheld is as low as 45%? Explain.

We call the interval estimate for a population parameter a **confidence interval**. To construct a confidence interval, we need

- A point estimate (a statistic is a point estimate)
- A margin of error

The interval includes values between the point estimate minus the margin of error and the point estimate plus the margin of error. We write the interval in interval notation as

$$(\text{point estimate - margin of error}, \text{point estimate} + \text{margin of error})$$

When we estimate a single population proportion, we can translate this interval as

$$(\hat{p} - E, \hat{p} + E)$$

The Sampling Distribution of Sample Proportions and the Margin of Error

To learn about margins of error, we will need to think about the sampling distribution of sample proportions. Recall,

- A sampling distribution of sample proportions is approximately normal if there are at least 10 expected successes and failures in the random sample.
- The mean of the sampling distribution of sample proportions is $\mu_{\hat{p}} = p$.
- The standard error (standard deviation of the sampling distribution of sample proportions) is $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$.

When p is Unknown

When we construct a confidence interval, we *estimate* the true population proportion and we therefore, do not know what the true population proportion is. When the population proportion p is unknown, we use a sample proportion $\hat{p}$ in its place.

- A sampling distribution of sample proportions is approximately normal if there are at least 10 **observed** successes and failures in the random sample. This is because we don't know the population proportion and cannot compute np and $n(1-p)$.
- The mean of the sampling distribution of sample proportions is $\mu_{\hat{p}} \approx \hat{p}$ since we do not know the value of p.
- The standard error (standard deviation of the sampling distribution of sample proportions) is $\sigma_{\hat{p}} \approx \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$.

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2. Suppose Garrett wants to estimate the proportion of adults in Texas who support upholding Roe v. Wade. He randomly surveys 200 adults in Texas. He finds that 114 surveyed adults say they support upholding Roe v. Wade.

Step 1: Verify normality criteria

a. Are the criteria for the approximate normality of the sampling distribution met?

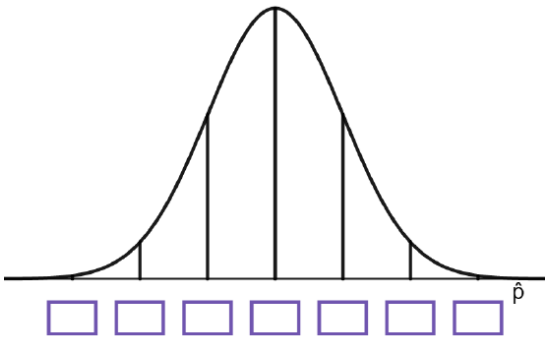
b. Compute the sample proportion $\hat{p}$.

Step 2: Compute the critical value

a. Compute the mean of the sampling distribution of sample proportions, $\mu_{\hat{p}}$.

b. Compute the standard error, $\sigma_{\hat{p}}$, rounded to three decimal places.

c. Label the horizontal axis of the normal sampling distribution of sample proportions below.



Images are created with the graphing calculator, used with permission from Desmos Studio PBC.

d. Garrett would like to be 97% sure or *confident* that the true proportion of all Texans who support upholding Roe v. Wade is in his interval estimate. Shade the middle 97% of the normal sampling distribution above.

e. We want to work backwards from the standard normal distribution to find the lower and upper values of $\hat{p}$ that are limits of the middle 97% of data (all sample proportions). Recall, we find critical values to achieve this. Compute the critical value Z_c that corresponds to the upper limit of the 97% confidence level. Draw a picture to show you thinking.

f. Usually in confidence interval problems, we use three main confidence levels: 90%, 95%, and 99%. Access this desmos graph or the QR code below to compute the critical values Zc that correspond to these three confidence levels.

Step 3: Compute the margin of error E.

a. The margin of error is $E = Z_c \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$. Use the information above to compute the margin of error rounded to three decimal places.

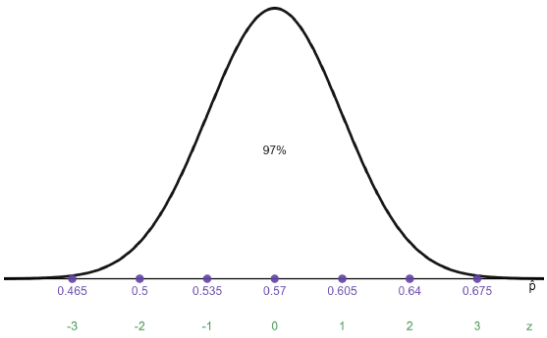
Step 4: Compute the lower and upper limits of the interval

a. Compute the proportion that is the lower limit of the interval $\hat{p} - E$.

b. Compute the proportion that is the upper limit of the interval $\hat{p} + E$.

c. Write the interval in interval notation as $(\hat{p} - E, \hat{p} + E)$.

d. Locate these proportions on the graph below and shade the area in the middle.



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Step 5: Interpret the interval in context.

a. Fill in the blanks: We are _____ % confident that the true proportion of all Texans that _____ is between _____% and _____%.

b. Is it likely that the majority of Texans believe Roe v. Wade should be upheld? Use the interval to support your answer.

The Five Step Process for Building a Confidence Interval

Use the following steps when constructing a confidence interval:

1. Verify that the sampling distribution is approximately normal.
   - When estimating with a single population proportion, we check that there are at least 10 observed successes $(n\hat{p})$ and failures $(n(1-\hat{p}))$ in the sample.
2. Compute the critical value.
   - When estimating with a single population proportion, we find the critical value, Z_c, from the standard normal distribution. In desmos, we enter normaldist().inversecdf(A) where A is the area left of the critical value.
3. Compute the margin of error.
   - When estimating with a single population proportion, $E = Z_c \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$.
4. Compute the limits of the interval and write the interval in interval notation.
   - When estimating with a single population proportion, the interval is $(\hat{p} - E, \hat{p} + E)$.
5. Write a conclusion and interpret the interval in context.
   - Include the confidence level, the parameter that you are estimating, and the bounds (with units) in your conclusion.

Reference

[4] "Majority of Americans say Supreme Court should uphold Roe, Post-ABC poll finds," Emily Guskin and Scott Clement, May 3, 2022, accessed May 13, 2022, https://www.washingtonpost.com/politics/2022/05/03/most-americans-say-supreme-court-should-uphold-roe-post-abc-poll-finds/