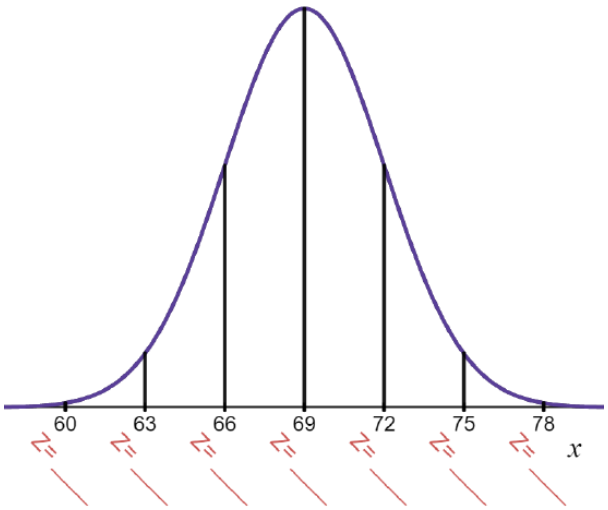
5.3: The Standard Normal Distribution

In the last lesson, we learned that all values from *any* normally distributed data can be *standardized* using Z-scores. In this lesson, we will be looking at the **standard normal distribution** which is the normal distribution that has mean $\mu = 0$ and standard deviation $\sigma = 1$. When using the standard normal distribution, we label the random variable with the letter z.

Accuracy

The empirical rule helps us to *approximate* probabilities of ranges of values that correspond to the eight regions given in the graph below.

1. Label the horizontal axis with the appropriate Z-scores and the appropriate areas for the eight regions given on the graph of the normal distribution of adult male heights below.



Images are created with the graphing calculator, used with permission from Desmos Studio PBC.

The benefit is that we can quickly approximate proportions from a normal distribution for a region between any integer value of z ($z = [-3, -2, -1, 0, 1, 2, 3]$). For example, using the empirical rule, I know that the proportion of adult male heights that are between 66 inches (one standard deviation below the mean) and 75 inches (two standard deviations above the mean) is about 68%+13.5% which sums to 81.5%. Using probability notation,

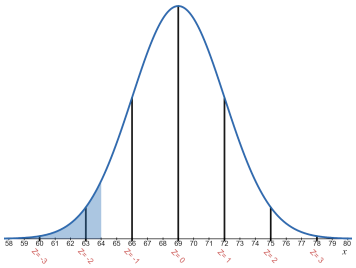
$$P(66 < x < 75) = P(-1 < z < 2) \approx 0.68 + 0.135 = 0.815 = 81.5\%.$$

We do not yet have enough information to find the proportion of adult men who are shorter than 64 inches tall because 64 inches does not correspond to an integer value of z.

2. Compute the Z-score (rounded to two decimal places) for an adult man who is 64 inches tall.

3. Below is a graph of the normal distribution of adult male heights. The proportion of adult males who are shorter than 64 inches has been shaded. In probability notation, this area is represented as $P(x < 64)$. Fill in the blank:

$$P(x < 64) = P(Z < \_\_\_\_\_)$$



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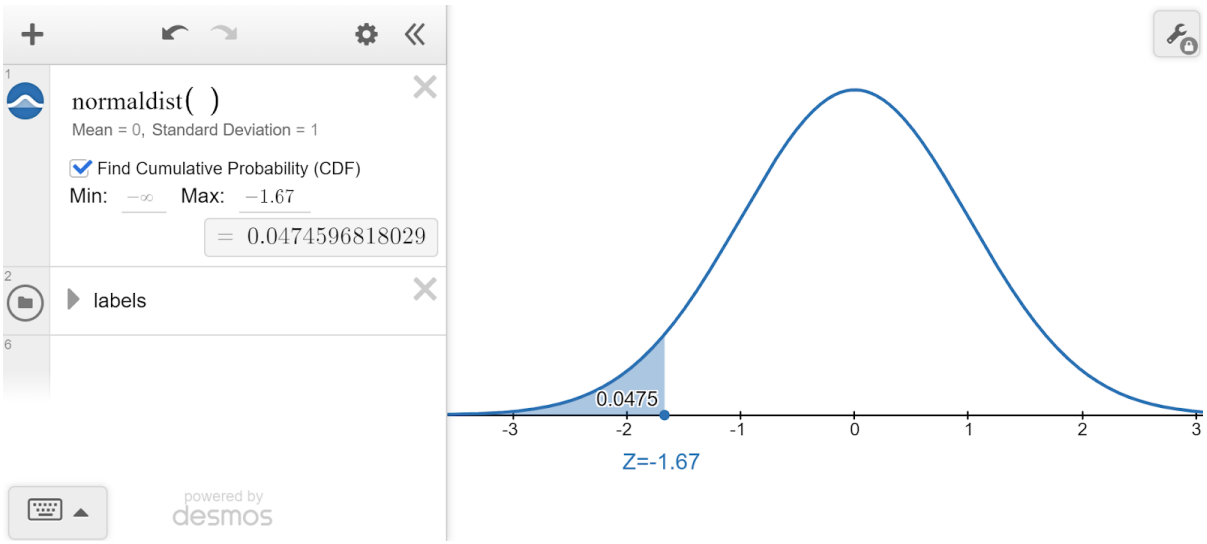
When a continuous random variable is approximately normal, we can use Z-scores to find probabilities for a desired range of values. This is done by translating between x-values and Z-scores and using Calculus to find the corresponding area under the curve. Luckily, mathematicians have found areas for us so that people can learn about statistics without knowing Calculus.

Using desmos to Find Probabilities from The Standard Normal Distribution

We can use desmos to find probabilities from the normal distribution. To find the probability of an adult male being less than 64 inches tall, go to https://www.desmos.com/calculator.

1. In the first line type normaldist(). This function graphs the standard normal distribution whose horizontal axis is labeled with values of z.
2. Click the Zoom Fit button which is represented as a magnifying glass icon.
3. Click the checkbox that says Find Cumulative Probability (CDF)
4. The minimum and maximum values will default to $-\infty$ and ∞ respectively. The bounds of the region we are finding the area for are $-\infty$ and -1.67, therefore, enter -1.67 for the maximum.



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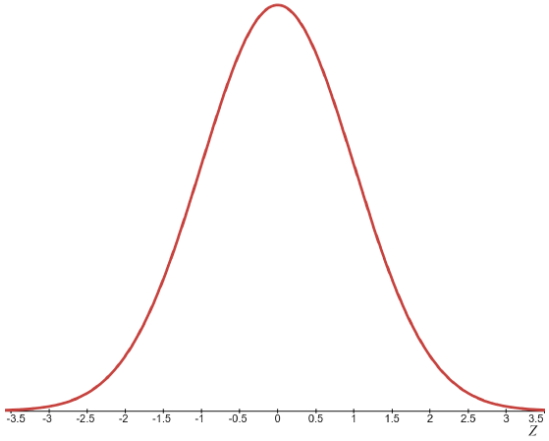
$$P(x < 64) = P(z < -1.67) = 0.0475$$

Therefore, 4.75% of adult men are shorter than 64 inches tall.

You try!

4. Fill in the probability notation with the missing Z-score (rounded to two decimal places) for the x-value 73. Shade the region of the standard normal distribution that represents the proportion of adult males who are at least 73 inches tall. Then use desmos to find the proportion (rounded to four decimal places) of adult males who are at least 73 inches tall.



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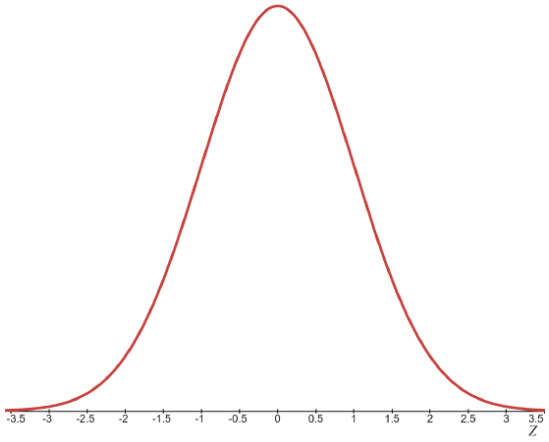
$$P(x \geq 73) = P(Z \geq \_\_\_\_\_) = \_\_\_\_\_$$

5. Fill in the probability notation with the missing Z-scores (rounded to two decimal places) for the x-values 67.5 and 71.8. Shade the region of the standard normal distribution that represents the proportion of adult males who are between 67.5 and 71.8 inches tall. Then use desmos to find the proportion (rounded to four decimal places) of adult males who are between 67.5 and 71.8 inches tall.



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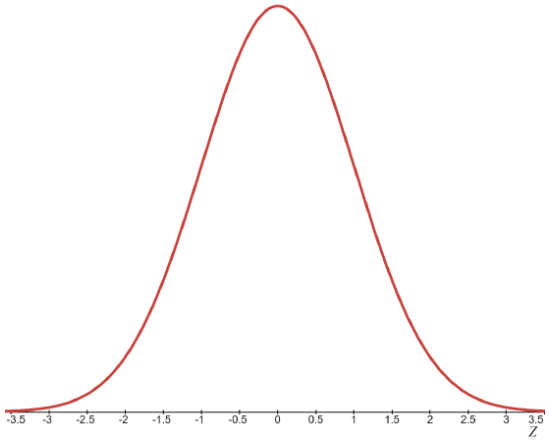
$$P(67.5 \leq x \leq 71.8) = P(\_\_\_\_\_ \leq Z \leq \_\_\_\_\_) = \_\_\_\_\_$$

6. Find the probability that a randomly selected adult male is at most 75.7 inches tall. Round Z-scores to two decimal places, and probability to four decimal places. Include probability notation in your answer and shade the appropriate region on the graph of the standard normal distribution.



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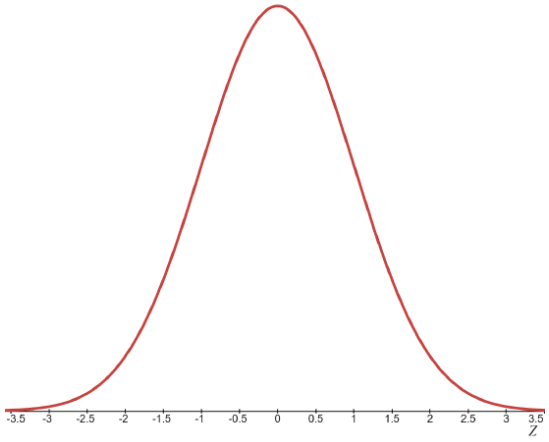
7. What proportion of adult men are between 70.5 inches tall and 71.1 inches tall? Round Z-scores to two decimal places, and probability to four decimal places. Include probability notation in your answer and shade the appropriate region on the graph of the standard normal distribution.



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