

7.5.1: Exercises

1. Jude was curious if the automated machine at his restaurant was filling drinks with the proper amount. He filled a sample of 20 drinks to test $H_0 : \mu = 530$ mL versus $H_a : \mu \neq 530$ where μ is the mean filling amount. The drinks in the sample contained a mean amount of 528 mL with a standard deviation of 4 mL. These results produced a test statistic of $T = -2.236$ and a P-value of approximately 0.038.

- a. If the significance level is 5%, what can we conclude about the null hypothesis?
- b. If the significance level is 5%, what can we conclude about the alternative hypothesis?
- c. Explain why you made the choices above.
- d. State the conclusion in context.
- e. What type of error might have occurred based on the above conclusion?
- f. Describe the error in context for this example.

2. A quality control engineer is testing the battery life of a new smartphone. The company is advertising that the battery lasts 24 hours on a full charge, but the engineer suspects that the battery life is actually less than that. They take a random sample of 30 of these phones to test $H_0 : \mu = 24$ versus $H_a : \mu < 24$ where μ is the mean battery life of these phones. The sample data had a mean of 21 hours and a standard deviation of 16 hours. These results produced a test statistic of $T \approx -1.03$ and a P-value of approximately 0.156.

a. If the significance level is 5%, state the conclusion in context.

b. What type of error might have occurred based on the conclusion above?

c. Describe the error in context for this example.

d. Suppose we increase the sample size to 300 and find the same sample mean and sample standard deviation.

i. Compute the new T-score.

ii. Is the sample mean statistically significant? Explain why or why not.

iii. Is the sample mean significant in a practical or real-world sense? Explain why or why not.

3. According to a report from the United States Environmental Protection Agency, burning one gallon of gasoline typically emits about 8.9 kg of CO_2 . A fuel company wants to test a new type of gasoline designed to have lower CO_2 emissions. Here are their hypotheses: $H_0 : \mu = 8.9 \text{ kg}$, $H_a : \mu < 8.9 \text{ kg}$ (where μ is the mean amount of CO_2 emitted by burning one gallon of this new gasoline).

a. Describe the type I error in context for this example.

b. Describe the type II error in context for this example.

c. Suppose Norton writes the following conclusion to the above hypothesis test:

The P-value is 0.04 which is greater than the level of significance of 1%. Therefore, we accept the null hypothesis and reject the alternative hypothesis. We have proved that the mean amount of CO_2 emitted by burning one gallon of this new gasoline is not less than 8.9 kg.

Improve Norton's conclusion.