

## 8.3: Inference for a Difference in Two Population Means

### Constructing a Confidence Interval for the Difference in Two Population Means

An experiment conducted by a leadership consulting firm, Nextion, was used to investigate the unconscious biases in the workplace at a law firm.<sup>18</sup> Researchers wrote a memo with 22 errors. The errors consisted of minor spelling and grammar errors, major technical writing errors, errors in fact, and errors in analysis of the facts. Fifty-three partners from 22 law firms received copies of the memo. 24 were told the memo was written by an African-American man named Thomas Meyer, and the remaining 29 were told the writer was a Caucasian man named Thomas Meyer. The partners gave the memo from the white Thomas Meyer an average rating of 4.1 out of 5 with a standard deviation of 0.8, while they gave the memo from the black Thomas Meyer an average rating of 3.2 out of 5 and a standard deviation of 1.12. Let's assume that scores for each memo are approximately normal.

1. State  $n_1$ ,  $n_2$ ,  $\bar{x}_1$ ,  $\bar{x}_2$ ,  $s_1$ ,  $s_2$ , and  $\bar{x}_1 - \bar{x}_2$ .

2. If the partners do have unconscious bias, which sample mean do you expect to be larger?

- One-sample situations: you compare a statistic in \_\_\_\_\_ population against a \_\_\_\_\_ about that population.
  - Example: Is the mean lead level in the Flint, Michigan water supply higher than the acceptable safe level of 15 parts per billion (ppb)?
- Two-sample situations: you measure the \_\_\_\_\_ in \_\_\_\_\_ and see if they are significantly different.
  - Example: Are black men assessed more harshly than white men, on average, by partners at law firms?

Let  $\mu_1$  represent the mean score given to \_\_\_\_\_ white Thomas Meyer memos by partners at law firms. Let  $\mu_2$  represent the mean score given to \_\_\_\_\_ black Thomas Meyer memos by partners at law firms.

## Five Step Process for Constructing a Confidence Interval for the Difference in Two Population Means

The process we use to build confidence intervals has not changed. The following is a list of familiar steps with the appropriate formulas to use for this situation.

1. Verify that the sampling distribution is approximately normal by checking that *each* sample size is greater than 30 OR *each* sample came from a normal population.
2. Find the critical value from the Student's T-distribution that corresponds to the provided confidence level.
3. Compute the margin of error  $E = T_c \cdot \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$
4. Compute the lower and upper limits of the interval, and write the interval in interval notation.  $(\bar{x}_1 - \bar{x}_2 - E, \bar{x}_1 - \bar{x}_2 + E)$
5. Write a conclusion in context. Interpret the interval.

Apply this process to the following example: Researchers wrote a memo with 22 errors. The errors consisted of minor spelling and grammar errors, major technical writing errors, errors in fact, and errors in analysis of the facts. Fifty-three partners from 22 law firms received copies of the memo. 24 were told the memo was written by an African-American man named Thomas Meyer, and the remaining 29 were told the writer was a Caucasian man named Thomas Meyer. The partners gave the memo from the white Thomas Meyer an average rating of 4.1 out of 5 with a standard deviation of 0.8, while they gave the memo from the black Thomas Meyer an average rating of 3.2 out of 5 and a standard deviation of 1.12. Let's assume that scores for each memo are approximately normal. We will construct a 95% confidence interval for the true mean difference in memo scores for memos written by white and black men at law firms. Use 51 degrees of freedom.

1. Explain why the sampling distribution of differences in sample means is approximately normal.

2. The confidence level is \_\_\_\_\_% so the critical value (rounded to three decimal places) is

$$T_c = \text{tdist}(\text{_____}) \cdot \text{inversecdf}(\text{_____}) = \text{_____} \quad (8.3.1)$$

$$3. E = T_c \cdot \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = \text{_____} \cdot \sqrt{\frac{(\text{_____})^2}{\text{_____}} + \frac{(\text{_____})^2}{\text{_____}}} = \text{_____}$$

$$4. (\bar{x}_1 - \bar{x}_2 - E, \bar{x}_1 - \bar{x}_2 + E) =$$

5. We are \_\_\_\_\_% confident that the true \_\_\_\_\_ in memo scores for memos written by white and black men at law firms is between \_\_\_\_\_ points out of 5 and \_\_\_\_\_ points out of 5.

Does the interval suggest there is a difference? Why or why not?

## Testing a Claim about the Difference in Two Population Means

### Four Step Process for Testing a Claim about the Difference between Two Population Means

The process we use to test a claim about a population parameter has not changed. The following is a list of familiar steps with the appropriate formulas to use for this situation.

1. State the hypotheses. Define  $\mu_1$  and  $\mu_2$ . The null hypothesis will always be  $H_0 : \mu_1 = \mu_2$ . The alternative will be one of the following  $H_a : \mu_1 < \mu_2$ ,  $H_a : \mu_1 > \mu_2$ , or  $H_a : \mu_1 \neq \mu_2$  based on the statement of the claim in the problem.
2. Verify that the sampling distribution is approximately normal by checking that *each* sample size is greater than 30 OR *each* sample came from a normal population.
3. Compute the T-score:  $T = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$ . The degrees of freedom will be provided. Use the Student's T-distribution to find the P-value (the probability of observing a sample difference as extreme or more extreme than the calculated sample difference just by chance).
4. Make a decision about the null and alternative hypotheses and state a conclusion in context.

Apply this process to the following example: Researchers wrote a memo with 22 errors. The errors consisted of minor spelling and grammar errors, major technical writing errors, errors in fact, and errors in analysis of the facts. Fifty-three partners from 22 law firms received copies of the memo. 24 were told the memo was written by an African-American man named Thomas Meyer, and the remaining 29 were told the writer was a Caucasian man named Thomas Meyer. The partners gave the memo from the white Thomas Meyer an average rating of 4.1 out of 5 with a standard deviation of 0.8, while they gave the memo from the black Thomas Meyer an average rating of 3.2 out of 5 and a standard deviation of 1.12. Let's assume that scores for each memo are approximately normal. Do the results give convincing statistical evidence that partners at law firms assess black men more harshly than white men on average? Use a 5% level of significance and 51 degrees of freedom.

1. Let  $\mu_1$  represent:

Let  $\mu_2$  represent:

$H_0$  :

$H_a$  :

Use a \_\_\_\_\_-tailed test because:

2. Explain why the sampling distribution of differences in sample means is approximately normal.

$$3. T = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{\text{_____} - \text{_____}}{\sqrt{\frac{(\text{_____})^2}{\text{_____}} + \frac{(\text{_____})^2}{\text{_____}}}} =$$

P-value is \_\_\_\_\_.

4. We \_\_\_\_\_ the null hypothesis, we \_\_\_\_\_ the alternative hypothesis.

Conclusion in context:

## Reference

<sup>18</sup>Debra Cassens Weiss. "Partners in study gave legal memo a lower rating when told author wasn't white" *ABA Journal*, April 21, 2014. Accessed \_\_\_\_\_ July \_\_\_\_\_ 5, \_\_\_\_\_ 2022.  
[https://www.abajournal.com/news/article/hypothetical\\_legal\\_memo\\_demonstrates\\_unconscious\\_biases](https://www.abajournal.com/news/article/hypothetical_legal_memo_demonstrates_unconscious_biases)

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