7.4: Hypothesis Tests for a Single Population Mean

In previous lessons, we have learned that there are two fundamental forms of statistical inference: confidence intervals and hypothesis tests. In the last section, we used confidence intervals to estimate a single population mean. We will now apply the four step hypothesis testing process to test hypotheses about the value of a single population mean. To explore this concept, we will examine examples of water contamination across the US.

The Flint Water Crisis

The water crisis in Flint, Michigan, is an example of environmental injustice that took place beginning in 2014. The city decided to switch its drinking water supply from Detroit's system to the water from the Flint River to save money. This switch was made despite inadequate treatment and testing of the water from the river. As a result, the community was deeply impacted. Their water supply turned yellow and began to smell and taste of sewage. Members of the community suffered from skin rashes and hair loss. Later, independent research would reveal that the contaminated water was contributing to dangerous increases of blood lead levels in the city's youth, and the lead level in the water greatly exceeded the EPAs standard.

Eventually, in 2016, with the efforts of community activists and scientists, a federal judge sided with Flint residents and ordered clean bottled water to be delivered to citizens. The following year, the city was ordered to replace the city's lead pipes with funding from the state and the community was provided with resources to support their health and well-being. However, the fight to bring safe access to the water supply is ongoing.

Read more about the water crisis in Flint, Michigan: https://www.nrdc.org/stories/flint-water-crisis-everything-you-need-know

Use Statistics to Defend Environmental Justice

To test whether the water was safe, the scientists likely conducted hypothesis tests. Now you will conduct a hypothesis test, like those scientists who fought for the citizens in Flint.

1. A mean lead level of 15 parts per billion (ppb) is considered safe, while anything above 15 ppb is considered dangerous. You believe the Flint water is unsafe, so you take a random sample of 200 households. You find that the average lead level of the sample is 16.5 ppb, with a standard deviation of 8 ppb. Do we have reason to believe that the true mean lead level for the entire water supply is dangerous, at a significance level of 1%?

Step 1 Determine the hypotheses

Let μ represent the _____________ lead level for the entire water supply.

$H_0 : \mu =$ _____ ppb

Select the appropriate alternative hypothesis:

a. $H_a : \mu < 15$ ppb

b. $H_a : \mu > 15$ ppb

c. $H_a : \mu \neq 15$ ppb

Select the appropriate test, and explain why:

a. Left-tailed

b. Right-tailed

c. Two-tailed

Step 2 Collect sample data

The sample mean, $\bar{x}$, is __________ ppb.

The sample standard deviation, s, is __________ ppb.

The sample size, n, is _________ households.

There are __________ degrees of freedom.

Explain why the conditions of the Central Limit Theorem are met:

Step 3 Assess the evidence

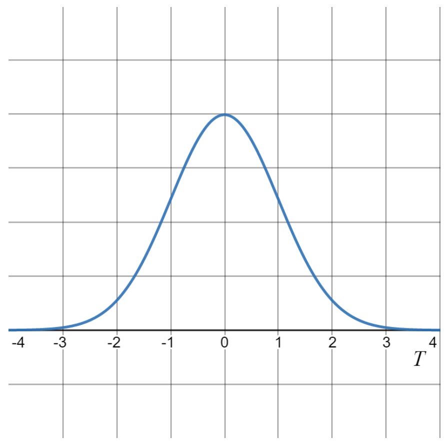
Which distribution will we use to find the P-value?

a. The normal distribution because the population standard deviation is given.
b. The student's T-distribution because the population standard deviation is unknown.

The test statistic, rounded to three decimal places, is

$$T = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{\_\_\_\_\_ - \_\_\_\_\_}{\frac{\_\_\_\_\_}{\sqrt{\_\_\_\_\_}}} = \_\_\_\_\_$$

Below is the graph of the T-distribution with _______ degrees of freedom. Label the T-statistic on the horizontal axis. Then shade the region that represents the P-value.



Images are created with the graphing calculator, used with permission from Desmos Studio PBC.

Use https://www.desmos.com/calculator to find the P-value:

1. Enter tdist(199) in the first line.
2. Check the box for Find Cumulative Probability (CDF)
3. The minimum and maximum will default to $-\infty$ and ∞ respectively. Enter the T-score in the min or max so that the graph matches the graph above.

Step 4 State a conclusion in context

The level of significance is $\alpha =$ _____. The P-value (rounded to three decimal places) is 0.004. Fill in the blank with $\leq$ or $>$:

$$0.004 \ \_\_\_\_\_ \ 0.01$$

Defend the citizens of Flint:

The evidence supports the claim that the true _____________ lead level for the entire water supply in Flint is ___________________ _______________________ 15 ppb. Therefore, by the current standards, the lead level in Flint's water supply is dangerously high.

Summary of Hypothesis Testing Process for a Single Population Mean

Step 1: Determine the Hypotheses

In order to test a claim about a population parameter, we create two opposing hypotheses. We call these the null hypothesis, H_0, and the alternative hypothesis, H_a. Let μ represent a given population mean.

The Null Hypothesis

In every hypothesis test, we assume that the null hypothesis is true. The null hypothesis is always a statement of equality and therefore, should always contain an equal symbol (=). When a test involves a single population mean, the null hypothesis will be

$$H_0 : \mu = value$$

The Alternative Hypothesis

The alternative hypothesis is a claim implied by the research question and is an inequality. The alternative hypothesis states that population mean is greater than (>), less than (<), or not equal (≠) to the assumed value in the null hypothesis.

When a test involves a single population mean, alternative hypothesis will be one of the following:

$$\begin{aligned} H_a &: \mu > \text{ value} \\ H_a &: \mu < \text{ value} \\ H_a &: \mu \neq \text{ value} \end{aligned}$$

Step 2: Collect Sample Data

During a hypothesis test, we work to know if a sample statistic is unusual or not. Therefore, we must think about probabilities from a sampling distribution.

In a previous lesson, we learned about the sampling distribution of sample means. The Central Limit Theorem says that a sampling distribution of sample means is approximately normal if either the sample size, n, is greater than 30 or sampling was performed from a normally distributed population. In the second step of a hypothesis test, we verify that the sampling distribution is approximately normal and we identify or compute any sample statistics.

Step 3: Assess the Evidence

This step is all about probability. Since the sampling distribution is approximately normal (as determined in step 2), and the population standard deviation is likely unknown, we can compute a T-score and use the student's T-distribution to find probabilities. The sampling distribution of sample means has mean

$$\mu_{\bar{x}} = \mu$$

and standard error

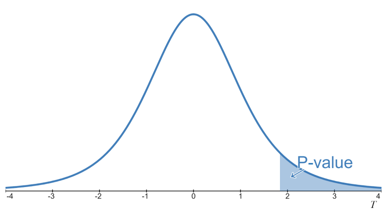
$$\sigma_{\bar{x}} = \frac{s}{\sqrt{n}}$$

where μ is the assumed population mean, s is the sample standard deviation, and n is the sample size. The test statistic is

$$T = \frac{x-\mu}{\sigma} \text{ which translates to } T = \frac{\bar{x}-\mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}$$

when looking at the sampling distribution of sample means.

- When the alternative hypothesis is $H_a : \mu > \text{ value }$, we are conducting a right-tailed test. The P-value is the probability of observing a sample mean at least as extreme as the one we observed. In this case at least as extreme means "as high or higher". The P-value is the area to the right of the test statistic (T.S.).



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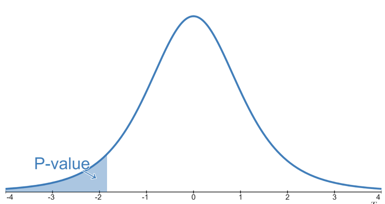
- When the alternative hypothesis is $H_a : \mu < \text{ value }$, we are conducting a left-tailed test. The P-value is the probability of observing a sample mean at least as extreme as the one we observed. In this case at least as extreme means "as low or lower". The P-value is the area to the left of the test statistic (T.S.).



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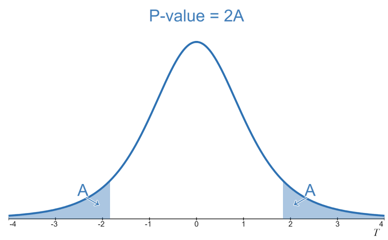
- When the alternative hypothesis is $H_a: \mu \neq \text{value}$, we are conducting a two-tailed test, and the P-value is twice the area of either the tail to the right of a positive test statistic (T.S.), or the tail to the left of a negative test statistic (T.S.).



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Step 4: State a Conclusion

Hypothesis tests are all about making decisions. We use the P-value to make a decision about the null and alternative hypotheses.

We compare our P-value to a level of significance. The level of significance, denoted α (the greek letter "alpha"), is how unlikely a sample statistic needs to be to convince us about a claim. It is also the level of risk we accept in being wrong.

We have only two possible conclusions:

- If the P-value $\leq \alpha$, we reject the null hypothesis and support the alternative hypothesis.
- If the P-value $> \alpha$, we fail to reject the null hypothesis and cannot support the alternative hypothesis.
    - This does not make the null hypothesis true—we cannot prove the null hypothesis because sample data cannot reveal the true value of the population mean.

You try!

2. A machine is designed to fill jars with 16 ounces of coffee. A consumer suspects that the machine is not filling the jars completely. They randomly sampled 12 jars shown below. Is there enough evidence to support the consumer's claim at a 10% significance level? Assume that the population of volumes in each jar fill is approximately normal.

15	15.4	16.2	16.1	15.8	16.2
15.7	15.6	16	16.3	15.3	15.9

Step 1.

Let μ represent:

H_0:

H_a:

Which type of test and why?

Step 2.

The sampling distribution is approximately normal because:

Go to https://www.desmos.com/calculator. Type $C =$ into the first line and copy and paste the data.

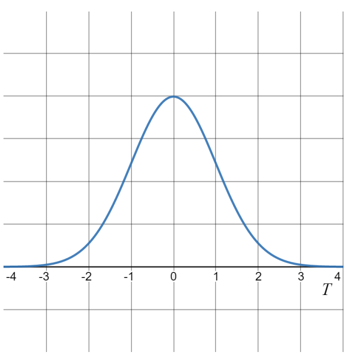
$\bar{x} = \text{mean}(C) = \_\_\_\_\_$ ounces

$s = \text{stdev}(C) = \_\_\_\_\_$ ounces

$n = \_\_\_\_\_$ and $df = \_\_\_\_\_$

Step 3.

$T =$



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P-value is __________________________

Step 4.

Compare the P-value and the significance level. Make a decision, and state a conclusion in context: