7.5: Conclusions (2)

When we estimate a population parameter or conduct a hypothesis test, our last step is to state a conclusion in context. In this section, we will focus on conclusions of a hypothesis test for a population mean. These concepts can be extended to other estimations and hypothesis tests.

Example 1

Economists use average household credit card debt to gauge the financial well-being of families in the U.S. Excessive credit card debt can lead to financial challenges and prevent individuals from saving money or investing money for future expenses. In 2019, U.S. credit card debt hit a record high of $930 billion. On average, Americans carried $6194 in credit card debt in 2019[13].

Suppose we are interested in comparing the mean credit card debt of households from the Baby Boomer generation against the national average. In a random sample of 32 baby boomer households, the mean credit card debt per household was $6043 with a sample standard deviation of $440. At the 1% level of significance, can we conclude that the mean credit card debt of baby boomer households is less than the national average?

Solution:

Step 1. Let μ represent the _____ credit card debt of all baby boomer households.

$H_0 : \mu = \$6194$

$H_a :$ _____

We will perform a _____-tailed test because:

Step 2. Are the criteria for the approximate normality of the sampling distribution of sample means satisfied? Explain.

$n =$ _____

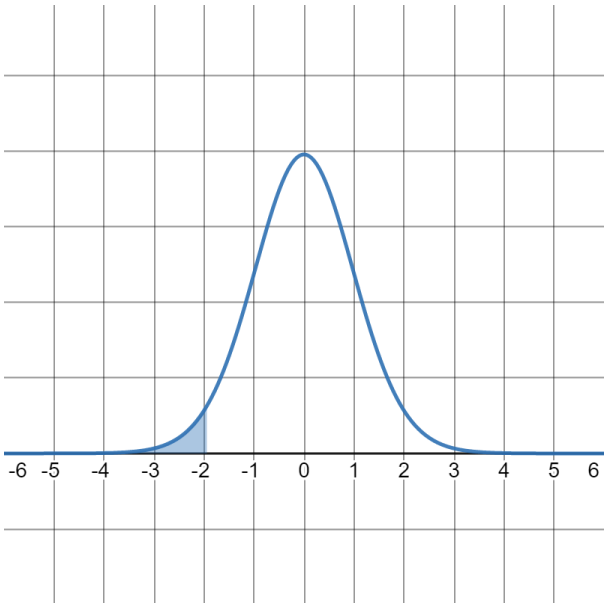
$df =$ _____

$\bar{x} =$ _____

$s =$ _____

Step 3. Calculate the test statistic (rounded to two decimal places) and label it on the graph of the T-distribution below.

$$T = \frac{\_\_\_\_\_}{\frac{\_\_\_\_\_}{\sqrt{\_\_\_\_\_}}} =$$



Images are created with the graphing calculator, used with permission from Desmos Studio PBC.

The P-value is 0.031.

Step 4. The P-value $= 0.031 > 0.01 = 1\% = \alpha$ the level of significance, so we fail to reject the _____ hypothesis and cannot support the _____ hypothesis. Is the sample mean statistically significant? State a conclusion in the context of the problem.

Errors in Hypothesis Tests

In the above example, we failed to reject the null hypothesis, meaning we were not convinced that the null hypothesis was false. This decision is based on sample data and sometimes, there is a small possibility that we made the wrong decision.

Recall, there are two possible conclusions to a hypothesis test, which means there are two possible errors:

Type I. We reject the null hypothesis in support of the alternative hypothesis.

- If we made the wrong decision here, what is actually true?

  We decide that the null hypothesis is _____, when actually, it is _____.

Type II. We fail to reject the null hypothesis in support of the alternative hypothesis.

- If we made the wrong decision here, what is actually true?

  We decide that the null hypothesis is _____, when actually, it is _____.

Notice that Type I and II errors are exclusively about the null hypothesis.

You try!

1. In example 1, μ represented the mean credit card debt for all baby boomer households in 2019.

   The null hypothesis was that the mean credit card debt for all baby boomer households in 2019 was \$6194 $(H_0 : \mu = \$6194)$.

   a. Based on the conclusion of example 1, what type of error might have occurred? Explain.

   b. Describe Type I error in context.

   c. Describe Type II error in context.

2. In example 1, we used data from 32 randomly selected baby boomer households. The mean credit card debt was $6043 with standard deviation $440. We concluded that the sample results based on this small sample were not statistically significant. Can we generalize the results of the hypothesis test to other generations in the U.S.? Explain.

3. Under what conditions would it be appropriate to generalize the results of the hypothesis test?

Statistical and Practical Significance

In example 1, we concluded that the sample mean of $6043 was not statistically significant. The sample did not provide sufficient evidence to conclude that the mean debt of all baby boomer households was less than the national average.

4. The sample mean of $6043 differed from the national average of $6194 by _____ dollars. Do you think the difference between the observed sample mean and the national average is significant in a real-world or practical sense? Explain.

5. How would our assessment of statistical significance change if the sample mean of $6043 was observed for a sample of 100 baby boomer households? Assuming the sample standard deviation is still $440, determine whether the sample mean of $6043 is now statistically significant. Would the conclusion of the hypothesis test change? Explain your answer.

Reference

[13] "Alaskans carry the highest credit card balance–here's the average credit card balance in every state", Alexandria White, May 10 2022, accessed June 21 2022, https://www.cnbc.com/select/average-credit-card-balance-by-state/#:~:text=If%20you%20have%20credit%20card,card%20balance%2C%20on%20average%20%248%2C026.