Using Summation Notation

Learning Outcomes

1. Evaluate an expression that includes summation notation.
2. Apply summation notation to calculate statistics.

This notation is called summation notation and appears as:

$$\sum_{i=1}^{n} a_i$$

In this notation, the a_i is an expression that contains the index i and you plug in 1 and then 2 and then 3 all the way to the last number n and then add up all of the results.

Example 1

Calculate

$$\sum_{i=1}^{4} 3i$$

Solution

First notice that i = 1, then 2, then 3 and finally 4. We are supposed to multiply each of these by 3 and add them up:

$$\begin{aligned}\sum_{i=1}^{4} 3i \ &= 3\,(1) + 3\,(2) + 3\,(3) + 3\,(4) \\ &= 3 + 6 + 9 + 12 \ = \ 30\end{aligned}$$

Example 2

The formula for the sample mean, sometimes called the average, is

$$\bar{x} \ = \ \frac{\sum_{i=1}^{n} x_i}{n}$$

A survey was conducted asking 8 older adults how many sexual partners they have had in their lifetime. Their answers were {4,12,1,3,4,9,24,7}. Use the formula to find the sample mean.

Solution

Notice that the numerator of the formula just tells us to add the numbers up. Computing the numerator first gives:

$$\sum_{i=1}^{8} x_i = 4 + 12 + 1 + 3 + 4 + 9 + 24 + 7 \ = 64$$

Now that we have the numerator calculated, the formula tells us to divide by n, which is just 8. We have:

$$\bar{x} \ = \frac{64}{8} = 8$$

Thus, the sample mean number of sexual partners this group had in their lifetimes is 8.

Example 3

The next most important statistic is the standard deviation. The formula for the sample standard deviation is:

$$s=\sqrt{\frac{\sum_{i=1}^{n}\left(x_i-\bar{x}\right)^2}{n-1}}$$

Let's consider the data in the previous example. Find the standard deviation.

Solution

The formula is quite complicated, but if tackle it one piece at a time using the order of operations properly, we can succeed in finding the sample standard deviation for the data. Notice that there are parentheses, so based on the order of operations, we must do the subtraction within the parentheses first. Since this is all part of the sum, we have eight different subtractions to do. From our calculations in the previous example, the sample mean was $\bar{x}=8$. We compute the 8 subtractions:

$$4-\ 8\ =\ -4,\ \ 12-8=4,\ 1-8=-7,\ 3-8=-5,$$

$$4-8=-4,\ 9-8=1,\ 24-8=16,\ 7-8=-1$$

The next arithmetic to do is to square each of the differences to get:

$$(-4)^2=16,\ \ (4)^2=16, (-7)^2=49,\ (-5)^2=25,$$

$$(-4)^2=16,\ 1^2=1,\ 16^2=256,\ (-1)^2=1$$

Now we have all the entries in the summation, so we add them all up:

$$16+16+49+25+16+1+256+1=380$$

Now we can write

$$s=\sqrt{\frac{380}{8-1}}=\sqrt{\frac{380}{7}}$$

We can put this into the calculator or computer to get:

$$s=\sqrt{\frac{380}{7}}=7.3679$$

Exercise: expected value

The expected value, EV, is defined by the formula

$$EV=\sum_{i=1}^{n} x_i\, P\left(x_i\right)$$

Where x_i are the possible outcomes and $P\left(x_i\right)$ are the probabilities of the outcomes occurring. Suppose the table below shows the number of eggs in a bald eagle clutch and the probabilities of that number occurring.

Probability Distribution Table with Outcomes, x, and probabilities, P(x)

x	1	2	3	4
P(x)	0.2	0.4	0.3	0.1

Find the expected value.

Ex 1: Find a Sum Written in Summation / Sigma Notation

Summation Notation and Expected Value