Perform Signed Number Arithmetic

Learning Outcomes

1. Add signed numbers.
2. Subtract signed numbers.
3. Multiply signed numbers.
4. Divide signed numbers.

Even though negative numbers seem not that common in the real world, they do come up often when doing comparisons. For example, a common question is how much bigger is one number than another, which involves subtraction. In statistics we don't know the means until we collect the data and do the calculations. This often results in subtracting a larger number from a smaller number which yields a negative number. Because of this and for many other reasons, we need to be able to perform arithmetic on both positive and negative numbers.

Adding Signed Numbers

We will assume that you are very familiar with adding positive numbers, but when there are negative numbers involved, there are some rules to follow:

1. When adding two negative numbers, ignore the negative signs, add the positive numbers and then make the result negative.
2. When adding two numbers such that one is positive and the other is negative, ignore the sign, subtract the smaller from the larger. If the larger of the positive numbers was originally negative, then make the result negative. Otherwise keep the result positive.

Example 1

Add:

$$-4+(-3)$$

Solution

First we ignore the signs and add the positive numbers.

$$4+3=7$$

Next we make the result negative.

$$-4+(-3)=-7$$

Example 2

Add:

$$-2+5$$

Solution

Since one of the numbers is positive and the other is negative, we subtract:

$$5-2=3$$

Of the two numbers, 2 and 5, 5 is the larger one and started positive. Hence we keep the result positive:

$$-2+5=3$$

Subtracting Numbers

Subtraction comes up often when we want to find the width of an interval in statistics. Here are the cases for subtracting: $a-b$:

1. If $a \geq b \geq 0$, then this is just ordinary subtraction.
2. If $b \geq a \geq 0$, then find $b-a$ and make the result negative.
3. If $a < 0, \ b \geq 0$, then make both positive, add the two positive numbers and make the result negative.
4. If $b < 0$ then you use the rule that subtracting a negative number is the same as adding the positive number.

Example 3

Evaluate $5-9$

Solution

Since 9 is bigger than 5, we subtract:

$$9-5 \ = \ 4$$

Next, we make the result negative to get:

$$5-9=-4$$

Example 4

Evaluate $-9-4$

Solution

We are in the case $a < 0, \ b \geq 0$. Therefore, we first make both positive and add the positive numbers.

$$9+4 \ = \ 13$$

The final step is to make the answer negative to get

$$-9-4=-13$$

Example 5: Uniform distributions

In statistics, we call a *distribution Uniform* if an event is just as likely to be in any given interval within the bounds as any other interval within the bounds as long as the intervals are both of the same width. Finding the width of a given interval is usually the first step in solving a question involving uniform distributions. Suppose that the temperature on a winter day has a Uniform distribution on [-8,4]. Find the width of this interval

Solution

To find the width of an interval, we subtract the left endpoint from the right endpoint:

$$4 \ - \ (-8)$$

Since we are subtracting a negative number, the "-" signs become addition:

$$4-(-8) \ = \ 4+8=12$$

Thus the width of the interval is 12.

Multiplying and Dividing Signed Numbers

When we have a multiplication or division problem, we just remember that two negatives make a positive. So if there are an even number of negative numbers that are multiplied or divided, the result is negative. If there are an odd number of negative numbers that are multiplied or divided, the result is positive.

Example 6

Perform the arithmetic:

$$\frac{(-6)(-10)}{(-4)(-5)}$$

Solution

First, just ignore all of the negative signs and multiply the numerator and denominator separately:

$$\frac{(6)(10)}{(4)(5)} = \frac{60}{20}$$

Now divide:

$$\frac{60}{20} = \frac{6}{2} = 3$$

Finally, notice that there are four negative numbers in the original multiplication and division problem. Four is an even number, so the answer is positive:

$$\frac{(-6)(-10)}{(-4)(-5)} = 3$$

Example 7

A confidence interval for the population mean difference in books read per year by men and women was was found to be [-4,1]. Find the midpoint of this interval.

Solution

First recall that to find the midpoint of two numbers, we add then and then divide by 2. Hence, our first step is to add -4 and 1. Since 1 is positive and -4 is negative, we first subtract the two numbers:

$$4 - 1 = 3$$

Of the two numbers, 4 and 1, 4 is the larger one and started negative. Hence we change the sign to negative::

$$-4 + 1 = -3$$

The final step in finding the midpoint is to divide by 2. First we divide them as positive numbers:

$$\frac{3}{2} = 1.5$$

Since the original quotient has a single negative number (an odd number of negative numbers), the answer is negative. Thus the midpoint of -4 and 1 is -1.5.

Exercise

The difference between the observed value and the expected value in linear regression is called the residual. Suppose that the three observed values are: -4, 2, and 5. The expected values are -3, 7, and -1. First find the residuals and then find the sum of the residuals.

- Signed Number Operations (L1.4)
- signed arithmetic

This page titled Perform Signed Number Arithmetic is shared under a CC BY 4.0 license and was authored, remixed, and/or curated by Larry Green.