8.E: Testing Hypotheses (Exercises)

These are homework exercises to accompany the Textmap created for "Introductory Statistics" by Shafer and Zhang.

8.1: The Elements of Hypothesis Testing

Q8.1.1

State the null and alternative hypotheses for each of the following situations. (That is, identify the correct number μ_0 and write $H_0 : \mu = \mu_0$ and the appropriate analogous expression for H_a.)

a. The average July temperature in a region historically has been $74.5^{\circ} F$. Perhaps it is higher now.
b. The average weight of a female airline passenger with luggage was 145 pounds ten years ago. The FAA believes it to be higher now.
c. The average stipend for doctoral students in a particular discipline at a state university is $\$14,756$. The department chairman believes that the national average is higher.
d. The average room rate in hotels in a certain region is $\$82.53$. A travel agent believes that the average in a particular resort area is different.
e. The average farm size in a predominately rural state was 69.4 acres. The secretary of agriculture of that state asserts that it is less today.

Q1.1.2

State the null and alternative hypotheses for each of the following situations. (That is, identify the correct number μ_0 and write $H_0 : \mu = \mu_0$ and the appropriate analogous expression for H_a.)

a. The average time workers spent commuting to work in Verona five years ago was 38.2 minutes. The Verona Chamber of Commerce asserts that the average is less now.
b. The mean salary for all men in a certain profession is $\$58,291$. A special interest group thinks that the mean salary for women in the same profession is different.
c. The accepted figure for the caffeine content of an 8-ounce cup of coffee is 133 mg. A dietitian believes that the average for coffee served in a local restaurants is higher.
d. The average yield per acre for all types of corn in a recent year was 161.9 bushels. An economist believes that the average yield per acre is different this year.
e. An industry association asserts that the average age of all self-described fly fishermen is 42.8 years. A sociologist suspects that it is higher.

Q1.1.3

Describe the two types of errors that can be made in a test of hypotheses.

Q1.1.4

Under what circumstance is a test of hypotheses certain to yield a correct decision?

Answers

1. a. $H_0 : \mu = 74.5 \ vs \ H_a : \mu > 74.5$
   b. $H_0 : \mu = 145 \ vs \ H_a : \mu > 145$
   c. $H_0 : \mu = 14756 \ vs \ H_a : \mu > 14756$
   d. $H_0 : \mu = 82.53 \ vs \ H_a : \mu \neq 82.53$
   e. $H_0 : \mu = 69.4 \ vs \ H_a : \mu < 69.4$
2.
3. A Type I error is made when a true H_0 is rejected. A Type II error is made when a false H_0 is not rejected.

8.2: Large Sample Tests for a Population Mean

Basic

1. Find the rejection region (for the standardized test statistic) for each hypothesis test.
   a. $H_0 : \mu = 27 \ vs \ H_a : \mu < 27 \ @ \ \alpha = 0.05$

b. $H_0 : \mu = 52\ vs\ H_a : \mu \neq 52\ @\ \alpha = 0.05$
c. $H_0 : \mu = -105\ vs\ H_a : \mu > -105\ @\ \alpha = 0.10$
d. $H_0 : \mu = 78.8\ vs\ H_a : \mu \neq 78.8\ @\ \alpha = 0.10$

2. Find the rejection region (for the standardized test statistic) for each hypothesis test.
   a. $H_0 : \mu = 17\ vs\ H_a : \mu < 17\ @\ \alpha = 0.01$
   b. $H_0 : \mu = 880\ vs\ H_a : \mu \neq 880\ @\ \alpha = 0.01$
   c. $H_0 : \mu = -12\ vs\ H_a : \mu > -12\ @\ \alpha = 0.05$
   d. $H_0 : \mu = 21.1\ vs\ H_a : \mu \neq 21.1\ @\ \alpha = 0.05$
3. Find the rejection region (for the standardized test statistic) for each hypothesis test. Identify the test as left-tailed, right-tailed, or two-tailed.
   a. $H_0 : \mu = 141\ vs\ H_a : \mu < 141\ @\ \alpha = 0.20$
   b. $H_0 : \mu = -54\ vs\ H_a : \mu < -54\ @\ \alpha = 0.05$
   c. $H_0 : \mu = 98.6\ vs\ H_a : \mu \neq 98.6\ @\ \alpha = 0.05$
   d. $H_0 : \mu = 3.8\ vs\ H_a : \mu > 3.8\ @\ \alpha = 0.001$
4. Find the rejection region (for the standardized test statistic) for each hypothesis test. Identify the test as left-tailed, right-tailed, or two-tailed.
   a. $H_0 : \mu = -62\ vs\ H_a : \mu \neq -62\ @\ \alpha = 0.005$
   b. $H_0 : \mu = 73\ vs\ H_a : \mu > 73\ @\ \alpha = 0.001$
   c. $H_0 : \mu = 1124\ vs\ H_a : \mu < 1124\ @\ \alpha = 0.001$
   d. $H_0 : \mu = 0.12\ vs\ H_a : \mu \neq 0.12\ @\ \alpha = 0.001$
5. Compute the value of the test statistic for the indicated test, based on the information given.
   a. Testing $H_0 : \mu = 72.2\ vs\ H_a : \mu > 72.2,\ \sigma$ unknown $n = 55,\ \bar{x} = 75.1,\ s = 9.25$
   b. Testing $H_0 : \mu = 58\ vs\ H_a : \mu > 58,\ \sigma = 1.22\ n = 40,\ \bar{x} = 58.5,\ s = 1.29$
   c. Testing $H_0 : \mu = -19.5\ vs\ H_a : \mu < -19.5,\ \sigma$ unknown $n = 30,\ \bar{x} = -23.2,\ s = 9.55$
   d. Testing $H_0 : \mu = 805\ vs\ H_a : \mu \neq 805,\ \sigma = 37.5\ n = 75,\ \bar{x} = 818,\ s = 36.2$
6. Compute the value of the test statistic for the indicated test, based on the information given.
   a. Testing $H_0 : \mu = 342\ vs\ H_a : \mu < 342,\ \sigma = 11.2\ n = 40,\ \bar{x} = 339,\ s = 10.3$
   b. Testing $H_0 : \mu = 105\ vs\ H_a : \mu > 105,\ \sigma = 5.3\ n = 80,\ \bar{x} = 107,\ s = 5.1$
   c. Testing $H_0 : \mu = -13.5\ vs\ H_a : \mu \neq -13.5,\ \sigma$ unknown $n = 32,\ \bar{x} = -13.8,\ s = 1.5$
   d. Testing $H_0 : \mu = 28\ vs\ H_a : \mu \neq 28,\ \sigma$ unknown $n = 68,\ \bar{x} = 27.8,\ s = 1.3$
7. Perform the indicated test of hypotheses, based on the information given.
   a. Test $H_0 : \mu = 212\ vs\ H_a : \mu < 212\ @\ \alpha = 0.10,\ \sigma$ unknown $n = 36,\ \bar{x} = 211.2,\ s = 2.2$
   b. Test $H_0 : \mu = -18\ vs\ H_a : \mu > -18\ @\ \alpha = 0.05,\ \sigma = 3.3\ n = 44,\ \bar{x} = -17.2,\ s = 3.1$
   c. Test $H_0 : \mu = 24\ vs\ H_a : \mu \neq 24\ @\ \alpha = 0.02,\ \sigma$ unknown $n = 50,\ \bar{x} = 22.8,\ s = 1.9$
8. Perform the indicated test of hypotheses, based on the information given.
   a. Test $H_0 : \mu = 105\ vs\ H_a : \mu > 105\ @\ \alpha = 0.05,\ \sigma$ unknown $n = 30,\ \bar{x} = 108,\ s = 7.2$
   b. Test $H_0 : \mu = 21.6\ vs\ H_a : \mu < 21.6\ @\ \alpha = 0.01,\ \sigma$ unknown $n = 78,\ \bar{x} = 20.5,\ s = 3.9$
   c. Test $H_0 : \mu = -375\ vs\ H_a : \mu \neq -375\ @\ \alpha = 0.01,\ \sigma = 18.5\ n = 31,\ \bar{x} = -388,\ s = 18.0$

Applications

9. In the past the average length of an outgoing telephone call from a business office has been 143 seconds. A manager wishes to check whether that average has decreased after the introduction of policy changes. A sample of 100 telephone calls produced a mean of 133 seconds, with a standard deviation of 35 seconds. Perform the relevant test at the 1% level of significance.
10. The government of an impoverished country reports the mean age at death among those who have survived to adulthood as 66.2 years. A relief agency examines 30 randomly selected deaths and obtains a mean of 62.3 years with standard deviation 8.1 years. Test whether the agency's data support the alternative hypothesis, at the 1% level of significance, that the population mean is less than 66.2.
11. The average household size in a certain region several years ago was 3.14 persons. A sociologist wishes to test, at the 5% level of significance, whether it is different now. Perform the test using the information collected by the sociologist: in a random sample of 75 households, the average size was 2.98 persons, with sample standard deviation 0.82 person.

12. The recommended daily calorie intake for teenage girls is $2,200$ calories/day. A nutritionist at a state university believes the average daily caloric intake of girls in that state to be lower. Test that hypothesis, at the 5% level of significance, against the null hypothesis that the population average is $2,200$ calories/day using the following sample data: $n = 36,\ \bar{x} = 2,150,\ s = 203$
13. An automobile manufacturer recommends oil change intervals of $3,000$ miles. To compare actual intervals to the recommendation, the company randomly samples records of 50 oil changes at service facilities and obtains sample mean $3,752$ miles with sample standard deviation 638 miles. Determine whether the data provide sufficient evidence, at the 5% level of significance, that the population mean interval between oil changes exceeds $3,000$ miles.
14. A medical laboratory claims that the mean turn-around time for performance of a battery of tests on blood samples is 1.88 business days. The manager of a large medical practice believes that the actual mean is larger. A random sample of 45 blood samples yielded mean 2.09 and sample standard deviation 0.13 day. Perform the relevant test at the 10% level of significance, using these data.
15. A grocery store chain has as one standard of service that the mean time customers wait in line to begin checking out not exceed 2 minutes. To verify the performance of a store the company measures the waiting time in 30 instances, obtaining mean time 2.17 minutes with standard deviation 0.46 minute. Use these data to test the null hypothesis that the mean waiting time is 2 minutes versus the alternative that it exceeds 2 minutes, at the 10% level of significance.
16. A magazine publisher tells potential advertisers that the mean household income of its regular readership is $\$61,500$. An advertising agency wishes to test this claim against the alternative that the mean is smaller. A sample of 40 randomly selected regular readers yields mean income $\$59,800$ with standard deviation $\$5,850$. Perform the relevant test at the 1% level of significance.
17. Authors of a computer algebra system wish to compare the speed of a new computational algorithm to the currently implemented algorithm. They apply the new algorithm to 50 standard problems; it averages 8.16 seconds with standard deviation 0.17 second. The current algorithm averages 8.21 seconds on such problems. Test, at the 1% level of significance, the alternative hypothesis that the new algorithm has a lower average time than the current algorithm.
18. A random sample of the starting salaries of 35 randomly selected graduates with bachelor's degrees last year gave sample mean and standard deviation $\$41,202$ and $\$7,621$, respectively. Test whether the data provide sufficient evidence, at the 5% level of significance, to conclude that the mean starting salary of all graduates last year is less than the mean of all graduates two years before, $\$43,589$.

Additional Exercises

19. The mean household income in a region served by a chain of clothing stores is $\$48,750$. In a sample of 40 customers taken at various stores the mean income of the customers was $\$51,505$ with standard deviation $\$6,852$.
    a. Test at the 10% level of significance the null hypothesis that the mean household income of customers of the chain is $\$48,750$ against that alternative that it is different from $\$48,750$.
    b. The sample mean is greater than $\$48,750$, suggesting that the actual mean of people who patronize this store is greater than $\$48,750$. Perform this test, also at the 10% level of significance. (The computation of the test statistic done in part (a) still applies here.)
20. The labor charge for repairs at an automobile service center are based on a standard time specified for each type of repair. The time specified for replacement of universal joint in a drive shaft is one hour. The manager reviews a sample of 30 such repairs. The average of the actual repair times is 0.86 hour with standard deviation 0.32 hour.
    a. Test at the 1% level of significance the null hypothesis that the actual mean time for this repair differs from one hour.
    b. The sample mean is less than one hour, suggesting that the mean actual time for this repair is less than one hour. Perform this test, also at the 1% level of significance. (The computation of the test statistic done in part (a) still applies here.)

Large Data Set Exercises

Large Data Set missing from the original

21. Large Data Set 1 records the SAT scores of $1,000$ students. Regarding it as a random sample of all high school students, use it to test the hypothesis that the population mean exceeds $1,510$, at the 1% level of significance. (The null hypothesis is that $\mu = 1510$).
22. Large Data Set 1 records the GPAs of $1,000$ college students. Regarding it as a random sample of all college students, use it to test the hypothesis that the population mean is less than 2.50, at the 10% level of significance. (The null hypothesis is that $\mu = 2.50$).

23. Large Data Set 1 lists the SAT scores of $1,000$ students.
    a. Regard the data as arising from a census of all students at a high school, in which the SAT score of every student was measured. Compute the population mean μ.
    b. Regard the first 50 students in the data set as a random sample drawn from the population of part (a) and use it to test the hypothesis that the population mean exceeds $1,510$, at the 10% level of significance. (The null hypothesis is that $\mu = 1510$).
    c. Is your conclusion in part (b) in agreement with the true state of nature (which by part (a) you know), or is your decision in error? If your decision is in error, is it a Type I error or a Type II error?
24. Large Data Set 1 lists the GPAs of $1,000$ students.
    a. Regard the data as arising from a census of all freshman at a small college at the end of their first academic year of college study, in which the GPA of every such person was measured. Compute the population mean μ.
    b. Regard the first 50 students in the data set as a random sample drawn from the population of part (a) and use it to test the hypothesis that the population mean is less than 2.50, at the 10% level of significance. (The null hypothesis is that $\mu = 2.50$).
    c. Is your conclusion in part (b) in agreement with the true state of nature (which by part (a) you know), or is your decision in error? If your decision is in error, is it a Type I error or a Type II error?

Answers

1. a. $Z \le -1.645$
   b. $Z \le -1.645 \text{ or } Z \ge 1.96$
   c. $Z \ge 1.28$
   d. $Z \le -1.645 \text{ or } Z \ge 1.645$
2.
3. a. $Z \le -0.84$
   b. $Z \le -1.645$
   c. $Z \le -1.96 \text{ or } Z \ge 1.96$
   d. $Z \ge 3.1$
4.
5. a. $Z = 2.235$
   b. $Z = 2.592$
   c. $Z = -2.122$
   d. $Z = 3.002$
6.
7. a. $Z = -2.18,\ -z_{0.10} = -1.28,\ \text{reject } H_0$
   b. $Z = 1.61,\ z_{0.05} = 1.645,\ \text{do not reject } H_0$
   c. $Z = -4.47,\ -z_{0.01} = -2.33,\ \text{reject } H_0$
8.
9. $Z = -2.86,\ -z_{0.01} = -2.33,\ \text{reject } H_0$
10.
11. $Z = -1.69,\ -z_{0.025} = -1.96,\ \text{do not reject } H_0$
12.
13. $Z = 8.33,\ z_{0.05} = 1.645,\ \text{reject } H_0$
14.
15. $Z = 2.02,\ z_{0.10} = 1.28,\ \text{reject } H_0$
16.
17. $Z = -2.08,\ -z_{0.01} = -2.33,\ \text{do not reject } H_0$
18.
19. a. $Z = 2.54,\ z_{0.05} = 1.645,\ \text{reject } H_0$
    b. $Z = 2.54,\ z_{0.10} = 1.28,\ \text{reject } H_0$
20.

21. $H_0 : \mu = 1510 \ vs \ H_a : \mu > 1510$. Test Statistic: $Z = 2.7882$. Rejection Region: $[2.33, \infty)$. Decision: Reject H_0.
22.
23. a. $\mu_0 = 1528.74$
    b. $H_0 : \mu = 1510 \ vs \ H_a : \mu > 1510$. Test Statistic: $Z = -1.41$. Rejection Region: $[1.28, \infty)$. Decision: Fail to reject H_0.
    c. No, it is a Type II error.

8.3: The Observed Significance of a Test

Basic

1. Compute the observed significance of each test.
   a. Testing $H_0 : \mu = 54.7 \ vs \ H_a : \mu < 54.7, \text{ test statistic } z = -1.72$
   b. Testing $H_0 : \mu = 195 \ vs \ H_a : \mu \neq 195, \text{ test statistic } z = -2.07$
   c. Testing $H_0 : \mu = -45 \ vs \ H_a : \mu > -45, \text{ test statistic } z = 2.54$
2. Compute the observed significance of each test.
   a. Testing $H_0 : \mu = 0 \ vs \ H_a : \mu \neq 0, \text{ test statistic } z = 2.82$
   b. Testing $H_0 : \mu = 18.4 \ vs \ H_a : \mu < 18.4, \text{ test statistic } z = -1.74$
   c. Testing $H_0 : \mu = 63.85 \ vs \ H_a : \mu > 63.85, \text{ test statistic } z = 1.93$
3. Compute the observed significance of each test. (Some of the information given might not be needed.)
   a. Testing $H_0 : \mu = 27.5 \ vs \ H_a : \mu > 27.5, \ n = 49, \ \bar{x} = 28.9, \ s = 3.14, \text{ test statistic } z = 3.12$
   b. Testing $H_0 : \mu = 581 \ vs \ H_a : \mu < 581, \ n = 32, \ \bar{x} = 560, \ s = 47.8, \text{ test statistic } z = -2.49$
   c. Testing $H_0 : \mu = 138.5 \ vs \ H_a : \mu \neq 138.5, \ n = 44, \ \bar{x} = 137.6, \ s = 2.45, \text{ test statistic } z = -2.44$
4. Compute the observed significance of each test. (Some of the information given might not be needed.)
   a. Testing $H_0 : \mu = -17.9 \ vs \ H_a : \mu < -17.9, \ n = 34, \ \bar{x} = -18.2, \ s = 0.87, \text{ test statistic } z = -2.01$
   b. Testing $H_0 : \mu = 5.5 \ vs \ H_a : \mu \neq 5.5, \ n = 56, \ \bar{x} = 7.4, \ s = 4.82, \text{ test statistic } z = 2.95$
   c. Testing $H_0 : \mu = 1255 \ vs \ H_a : \mu > 1255, \ n = 152, \ \bar{x} = 1257, \ s = 7.5, \text{ test statistic } z = 3.29$
5. Make the decision in each test, based on the information provided.
   a. Testing $H_0 : \mu = 82.9 \ vs \ H_a : \mu < 82.9$ @ $\alpha = 0.05$, observed significance $p = 0.038$
   b. Testing $H_0 : \mu = 213.5 \ vs \ H_a : \mu \neq 213.5$ @ $\alpha = 0.01$, observed significance $p = 0.038$
6. Make the decision in each test, based on the information provided.
   a. Testing $H_0 : \mu = 31.4 \ vs \ H_a : \mu > 31.4$ @ $\alpha = 0.10$, observed significance $p = 0.062$
   b. Testing $H_0 : \mu = -75.5 \ vs \ H_a : \mu < -75.5$ @ $\alpha = 0.05$, observed significance $p = 0.062$

Applications

7. A lawyer believes that a certain judge imposes prison sentences for property crimes that are longer than the state average 11.7 months. He randomly selects 36 of the judge's sentences and obtains mean 13.8 and standard deviation 3.9 months.
   a. Perform the test at the 1% level of significance using the critical value approach.
   b. Compute the observed significance of the test.
   c. Perform the test at the 1% level of significance using the p-value approach. You need not repeat the first three steps, already done in part (a).
8. In a recent year the fuel economy of all passenger vehicles was 19.8 mpg. A trade organization sampled 50 passenger vehicles for fuel economy and obtained a sample mean of 20.1 mpg with standard deviation 2.45 mpg. The sample mean 20.1 exceeds 19.8, but perhaps the increase is only a result of sampling error.
   a. Perform the relevant test of hypotheses at the 20% level of significance using the critical value approach.
   b. Compute the observed significance of the test.
   c. Perform the test at the 20% level of significance using the p-value approach. You need not repeat the first three steps, already done in part (a).
9. The mean score on a 25-point placement exam in mathematics used for the past two years at a large state university is 14.3. The placement coordinator wishes to test whether the mean score on a revised version of the exam differs from 14.3. She gives the revised exam to 30 entering freshmen early in the summer; the mean score is 14.6 with standard deviation 2.4.
   a. Perform the test at the 10% level of significance using the critical value approach.

b. Compute the observed significance of the test.
c. Perform the test at the 10% level of significance using the p-value approach. You need not repeat the first three steps, already done in part (a).

10. The mean increase in word family vocabulary among students in a one-year foreign language course is 576 word families. In order to estimate the effect of a new type of class scheduling, an instructor monitors the progress of 60 students; the sample mean increase in word family vocabulary of these students is 542 word families with sample standard deviation 18 word families.
    a. Test at the 5% level of significance whether the mean increase with the new class scheduling is different from 576 word families, using the critical value approach.
    b. Compute the observed significance of the test.
    c. Perform the test at the 5% level of significance using the p-value approach. You need not repeat the first three steps, already done in part (a).
11. The mean yield for hard red winter wheat in a certain state is 44.8 bu/acre. In a pilot program a modified growing scheme was introduced on 35 independent plots. The result was a sample mean yield of 45.4 bu/acre with sample standard deviation 1.6 bu/acre, an apparent increase in yield.
    a. Test at the 5% level of significance whether the mean yield under the new scheme is greater than 44.8 bu/acre, using the critical value approach.
    b. Compute the observed significance of the test.
    c. Perform the test at the 5% level of significance using the p-value approach. You need not repeat the first three steps, already done in part (a).
12. The average amount of time that visitors spent looking at a retail company's old home page on the world wide web was 23.6 seconds. The company commissions a new home page. On its first day in place the mean time spent at the new page by $7,628$ visitors was 23.5 seconds with standard deviation 5.1 seconds.
    a. Test at the 5% level of significance whether the mean visit time for the new page is less than the former mean of 23.6 seconds, using the critical value approach.
    b. Compute the observed significance of the test.
    c. Perform the test at the 5% level of significance using the p-value approach. You need not repeat the first three steps, already done in part (a).

Answers

1. a. $p\text{-value} = 0.0427$
   b. $p\text{-value} = 0.0384$
   c. $p\text{-value} = 0.0055$
2.
3. a. $p\text{-value} = 0.0009$
   b. $p\text{-value} = 0.0064$
   c. $p\text{-value} = 0.0146$
4.
5. a. reject H_0
   b. do not reject H_0
6.
7. a. $Z = 3.23,\ z_{0.01} = 2.33$, reject H_0
   b. $p\text{-value} = 0.0006$
   c. reject H_0
8.
9. a. $Z = 0.68,\ z_{0.05} = 1.645$, do not reject H_0
   b. $p\text{-value} = 0.4966$
   c. do not reject H_0
10.
11. a. $Z = 2.22,\ z_{0.05} = 1.645$, reject H_0

b. p-value $= 0.0132$
c. reject H_0

8.4: Small Sample Tests for a Population Mean

Basic

1. Find the rejection region (for the standardized test statistic) for each hypothesis test based on the information given. The population is normally distributed.
   a. $H_0 : \mu = 27\ vs\ H_a : \mu < 27\ @\ \alpha = 0.05,\ n = 12,\ \sigma = 2.2$
   b. $H_0 : \mu = 52\ vs\ H_a : \mu \neq 52\ @\ \alpha = 0.05,\ n = 6,\ \sigma$ unknown
   c. $H_0 : \mu = -105\ vs\ H_a : \mu > -105\ @\ \alpha = 0.10,\ n = 24,\ \sigma$ unknown
   d. $H_0 : \mu = 78.8\ vs\ H_a : \mu \neq 78.8\ @\ \alpha = 0.10,\ n = 8,\ \sigma = 1.7$
2. Find the rejection region (for the standardized test statistic) for each hypothesis test based on the information given. The population is normally distributed.
   a. $H_0 : \mu = 17\ vs\ H_a : \mu < 17\ @\ \alpha = 0.01,\ n = 26,\ \sigma = 0.94$
   b. $H_0 : \mu = 880\ vs\ H_a : \mu \neq 880\ @\ \alpha = 0.01,\ n = 4,\ \sigma$ unknown
   c. $H_0 : \mu = -12\ vs\ H_a : \mu > -12\ @\ \alpha = 0.05,\ n = 18,\ \sigma = 1.1$
   d. $H_0 : \mu = 21.1\ vs\ H_a : \mu \neq 21.1\ @\ \alpha = 0.05,\ n = 23,\ \sigma$ unknown
3. Find the rejection region (for the standardized test statistic) for each hypothesis test based on the information given. The population is normally distributed. Identify the test as left-tailed, right-tailed, or two-tailed.
   a. $H_0 : \mu = 141\ vs\ H_a : \mu < 141\ @\ \alpha = 0.20,\ n = 29,\ \sigma$ unknown
   b. $H_0 : \mu = -54\ vs\ H_a : \mu < -54\ @\ \alpha = 0.05,\ n = 15,\ \sigma = 1.9$
   c. $H_0 : \mu = 98.6\ vs\ H_a : \mu \neq 98.6\ @\ \alpha = 0.05,\ n = 12,\ \sigma$ unknown
   d. $H_0 : \mu = 3.8\ vs\ H_a : \mu > 3.8\ @\ \alpha = 0.001,\ n = 27,\ \sigma$ unknown
4. Find the rejection region (for the standardized test statistic) for each hypothesis test based on the information given. The population is normally distributed. Identify the test as left-tailed, right-tailed, or two-tailed.
   a. $H_0 : \mu = -62\ vs\ H_a : \mu \neq -62\ @\ \alpha = 0.005,\ n = 8,\ \sigma$ unknown
   b. $H_0 : \mu = 73\ vs\ H_a : \mu > 73\ @\ \alpha = 0.001,\ n = 22,\ \sigma$ unknown
   c. $H_0 : \mu = 1124\ vs\ H_a : \mu < 1124\ @\ \alpha = 0.001,\ n = 21,\ \sigma$ unknown
   d. $H_0 : \mu = 0.12\ vs\ H_a : \mu \neq 0.12\ @\ \alpha = 0.001,\ n = 14,\ \sigma = 0.026$
5. A random sample of size 20 drawn from a normal population yielded the following results: $\bar{x} = 49.2,\ s = 1.33$
   a. Test $H_0 : \mu = 50\ vs\ H_a : \mu \neq 50\ @\ \alpha = 0.01$.
   b. Estimate the observed significance of the test in part (a) and state a decision based on the p-value approach to hypothesis testing.
6. A random sample of size 16 drawn from a normal population yielded the following results: $\bar{x} = -0.96,\ s = 1.07$
   a. Test $H_0 : \mu = 0\ vs\ H_a : \mu < 0\ @\ \alpha = 0.001$.
   b. Estimate the observed significance of the test in part (a) and state a decision based on the p-value approach to hypothesis testing.
7. A random sample of size 8 drawn from a normal population yielded the following results: $\bar{x} = 289,\ s = 46$
   a. Test $H_0 : \mu = 250\ vs\ H_a : \mu > 250\ @\ \alpha = 0.05$.
   b. Estimate the observed significance of the test in part (a) and state a decision based on the p-value approach to hypothesis testing.
8. A random sample of size 12 drawn from a normal population yielded the following results: $\bar{x} = 86.2,\ s = 0.63$
   a. Test $H_0 : \mu = 85.5\ vs\ H_a : \mu \neq 85.5\ @\ \alpha = 0.01$.
   b. Estimate the observed significance of the test in part (a) and state a decision based on the p-value approach to hypothesis testing.

Applications

9. Researchers wish to test the efficacy of a program intended to reduce the length of labor in childbirth. The accepted mean labor time in the birth of a first child is 15.3 hours. The mean length of the labors of 13 first-time mothers in a pilot program was 8.8

hours with standard deviation 3.1 hours. Assuming a normal distribution of times of labor, test at the 10% level of significance test whether the mean labor time for all women following this program is less than 15.3 hours.

10. A dairy farm uses the somatic cell count (SCC) report on the milk it provides to a processor as one way to monitor the health of its herd. The mean SCC from five samples of raw milk was $250,000$ cells per milliliter with standard deviation $37,500$ cell/ml. Test whether these data provide sufficient evidence, at the 10% level of significance, to conclude that the mean SCC of all milk produced at the dairy exceeds that in the previous report, $210,250$ cell/ml. Assume a normal distribution of SCC.
11. Six coins of the same type are discovered at an archaeological site. If their weights on average are significantly different from 5.25 grams then it can be assumed that their provenance is not the site itself. The coins are weighed and have mean 4.73 g with sample standard deviation 0.18 g. Perform the relevant test at the 0.1% ($1/10\text{th}$ of 1%) level of significance, assuming a normal distribution of weights of all such coins.
12. An economist wishes to determine whether people are driving less than in the past. In one region of the country the number of miles driven per household per year in the past was 18.59 thousand miles. A sample of 15 households produced a sample mean of 16.23 thousand miles for the last year, with sample standard deviation 4.06 thousand miles. Assuming a normal distribution of household driving distances per year, perform the relevant test at the 5% level of significance.
13. The recommended daily allowance of iron for females aged $19-50$ is 18 mg/day. A careful measurement of the daily iron intake of 15 women yielded a mean daily intake of 16.2 mg with sample standard deviation 4.7 mg.
    a. Assuming that daily iron intake in women is normally distributed, perform the test that the actual mean daily intake for all women is different from 18 mg/day, at the 10% level of significance.
    b. The sample mean is less than 18, suggesting that the actual population mean is less than 18 mg/day. Perform this test, also at the 10% level of significance. (The computation of the test statistic done in part (a) still applies here.)
14. The target temperature for a hot beverage the moment it is dispensed from a vending machine is $170^{\circ}F$. A sample of ten randomly selected servings from a new machine undergoing a pre-shipment inspection gave mean temperature $173^{\circ}F$ with sample standard deviation $6.3^{\circ}F$.
    a. Assuming that temperature is normally distributed, perform the test that the mean temperature of dispensed beverages is different from $170^{\circ}F$, at the 10% level of significance.
    b. The sample mean is greater than 170, suggesting that the actual population mean is greater than $170^{\circ}F$. Perform this test, also at the 10% level of significance. (The computation of the test statistic done in part (a) still applies here.)
15. The average number of days to complete recovery from a particular type of knee operation is 123.7 days. From his experience a physician suspects that use of a topical pain medication might be lengthening the recovery time. He randomly selects the records of seven knee surgery patients who used the topical medication. The times to total recovery were:
$$128 \quad 135 \quad 121 \quad 142 \quad 126 \quad 151 \quad 123 \tag{8.E.1}$$
    a. Assuming a normal distribution of recovery times, perform the relevant test of hypotheses at the 10% level of significance.
    b. Would the decision be the same at the 5% level of significance? Answer either by constructing a new rejection region (critical value approach) or by estimating the p-value of the test in part (a) and comparing it to α.
16. A 24-hour advance prediction of a day's high temperature is "unbiased" if the long-term average of the error in prediction (true high temperature minus predicted high temperature) is zero. The errors in predictions made by one meteorological station for 20 randomly selected days were:
$$\begin{array}{ccccc} 2 & 0 & -3 & 1 & -2 \\ 1 & 0 & -1 & 1 & -1 \\ -4 & 1 & 1 & -4 & 0 \\ -4 & -3 & -4 & 2 & 2 \end{array} \tag{8.E.2}$$
    a. Assuming a normal distribution of errors, test the null hypothesis that the predictions are unbiased (the mean of the population of all errors is 0) versus the alternative that it is biased (the population mean is not 0), at the 1% level of significance.
    b. Would the decision be the same at the 5% level of significance? The 10% level of significance? Answer either by constructing new rejection regions (critical value approach) or by estimating the p-value of the test in part (a) and comparing it to α.
17. Pasteurized milk may not have a standardized plate count (SPC) above $20,000$ colony-forming bacteria per milliliter (cfu/ml). The mean SPC for five samples was $21,500$ cfu/ml with sample standard deviation 750 cfu/ml. Test the null hypothesis that the

mean SPC for this milk is $20,000$ versus the alternative that it is greater than $20,000$, at the 10% level of significance. Assume that the SPC follows a normal distribution.

18. One water quality standard for water that is discharged into a particular type of stream or pond is that the average daily water temperature be at most $18^{\circ}F$. Six samples taken throughout the day gave the data:

$$16.8 \quad 21.5 \quad 19.1 \quad 12.8 \quad 18.0 \quad 20.7 \tag{8.E.3}$$

The sample mean exceeds $\bar{x} = 18.15$, but perhaps this is only sampling error. Determine whether the data provide sufficient evidence, at the 10% level of significance, to conclude that the mean temperature for the entire day exceeds $18^{\circ}F$.

Additional Exercises

19. A calculator has a built-in algorithm for generating a random number according to the standard normal distribution. Twenty-five numbers thus generated have mean 0.15 and sample standard deviation 0.94. Test the null hypothesis that the mean of all numbers so generated is 0 versus the alternative that it is different from 0, at the 20% level of significance. Assume that the numbers do follow a normal distribution.
20. At every setting a high-speed packing machine delivers a product in amounts that vary from container to container with a normal distribution of standard deviation 0.12 ounce. To compare the amount delivered at the current setting to the desired amount 64.1 ounce, a quality inspector randomly selects five containers and measures the contents of each, obtaining sample mean 63.9 ounces and sample standard deviation 0.10 ounce. Test whether the data provide sufficient evidence, at the 5% level of significance, to conclude that the mean of all containers at the current setting is less than 64.1 ounces.
21. A manufacturing company receives a shipment of $1,000$ bolts of nominal shear strength $4,350$ lb. A quality control inspector selects five bolts at random and measures the shear strength of each. The data are:

$$4,320 \quad 4,290 \quad 4,360 \quad 4,350 \quad 4,320 \tag{8.E.4}$$

   a. Assuming a normal distribution of shear strengths, test the null hypothesis that the mean shear strength of all bolts in the shipment is $4,350$ lb versus the alternative that it is less than $4,350$ lb, at the 10% level of significance.
   b. Estimate the p-value (observed significance) of the test of part (a).
   c. Compare the p-value found in part (b) to $\alpha = 0.10$ andmake a decision based on the p-value approach. Explain fully.

22. A literary historian examines a newly discovered document possibly written by Oberon Theseus. The mean average sentence length of the surviving undisputed works of Oberon Theseus is 48.72 words. The historian counts words in sentences between five successive 101 periods in the document in question to obtain a mean average sentence length of 39.46 words with standard deviation 7.45 words. (Thus the sample size is five.)
   a. Determine if these data provide sufficient evidence, at the 1% level of significance, to conclude that the mean average sentence length in the document is less than 48.72.
   b. Estimate the p-value of the test.
   c. Based on the answers to parts (a) and (b), state whether or not it is likely that the document was written by Oberon Theseus.

Answers

1. a. $Z \leq -1.645$
   b. $T \leq -2.571 \text{ or } T \geq 2.571$
   c. $T \geq 1.319$
   d. $Z \leq -1645 \text{ or } Z \geq 1.645$
2.
3. a. $T \leq -0.855$
   b. $Z \leq -1.645$
   c. $T \leq -2.201 \text{ or } T \geq 2.201$
   d. $T \geq 3.435$
4.
5. a. $T = -2.690,\ df = 19,\ -t_{0.005} = -2.861,\ \text{do not reject } H_0$
   b. $0.01 < p-value < 0.02,\ \alpha = 0.01,\ \text{do not reject } H_0$
6.
7. a. $T = 2.398,\ df = 7,\ t_{0.05} = 1.895,\ \text{reject } H_0$

b. $0.01 < p-value < 0.025,\ \alpha=0.05,\ \text{reject } H_0$

8.
9. $T=-7.560,\ df=12,\ -t_{0.10}=-1.356,\ \text{reject } H_0$
10.
11. $T=-7.076,\ df=5,\ -t_{0.0005}=-6.869,\ \text{reject } H_0$
12.
13. a. $T=-1.483,\ df=14,\ -t_{0.05}=-1.761,\ \text{do not reject } H_0$
    b. $T=-1.483,\ df=14,\ -t_{0.10}=-1.345,\ \text{reject } H_0$
14.
15. a. $T=2.069,\ df=6,\ t_{0.10}=1.44,\ \text{reject } H_0$
    b. $T=2.069,\ df=6,\ t_{0.05}=1.943,\ \text{reject } H_0$
16.
17. $T=4.472,\ df=4,\ t_{0.10}=1.533,\ \text{reject } H_0$
18.
19. $T=0.798,\ df=24,\ t_{0.10}=1.318,\ \text{do not reject } H_0$
20.
21. a. $T=-1.773,\ df=4,\ -t_{0.05}=-2.132,\ \text{do not reject } H_0$
    b. $0.05 < p-value < 0.10$
    c. $\alpha=0.05,\ \text{do not reject } H_0$

8.5: Large Sample Tests for a Population Proportion

Basic

On all exercises for this section you may assume that the sample is sufficiently large for the relevant test to be validly performed.

1. Compute the value of the test statistic for each test using the information given.
   a. Testing $H_0: p=0.50\ vs\ H_a: p>0.50,\ n=360,\ \hat{p}=0.56$.
   b. Testing $H_0: p=0.50\ vs\ H_a: p\neq 0.50,\ n=360,\ \hat{p}=0.56$.
   c. Testing $H_0: p=0.37\ vs\ H_a: p<0.37,\ n=1200,\ \hat{p}=0.35$.
2. Compute the value of the test statistic for each test using the information given.
   a. Testing $H_0: p=0.72\ vs\ H_a: p<0.72,\ n=2100,\ \hat{p}=0.71$.
   b. Testing $H_0: p=0.83\ vs\ H_a: p\neq 0.83,\ n=500,\ \hat{p}=0.86$.
   c. Testing $H_0: p=0.22\ vs\ H_a: p<0.22,\ n=750,\ \hat{p}=0.18$.
3. For each part of Exercise 1 construct the rejection region for the test for $\alpha=0.05$ and make the decision based on your answer to that part of the exercise.
4. For each part of Exercise 2 construct the rejection region for the test for $\alpha=0.05$ and make the decision based on your answer to that part of the exercise.
5. For each part of Exercise 1 compute the observed significance (p-value) of the test and compare it to $\alpha=0.05$ in order to make the decision by the p-value approach to hypothesis testing.
6. For each part of Exercise 2 compute the observed significance (p-value) of the test and compare it to $\alpha=0.05$ in order to make the decision by the p-value approach to hypothesis testing.
7. Perform the indicated test of hypotheses using the critical value approach.
   a. Testing $H_0: p=0.55\ vs\ H_a: p>0.55\ @\ \alpha=0.05,\ n=300,\ \hat{p}=0.60$.
   b. Testing $H_0: p=0.47\ vs\ H_a: p\neq 0.47\ @\ \alpha=0.01,\ n=9750,\ \hat{p}=0.46$.
8. Perform the indicated test of hypotheses using the critical value approach.
   a. Testing $H_0: p=0.15\ vs\ H_a: p\neq 0.15\ @\ \alpha=0.001,\ n=1600,\ \hat{p}=0.18$.
   b. Testing $H_0: p=0.90\ vs\ H_a: p>0.90\ @\ \alpha=0.01,\ n=1100,\ \hat{p}=0.91$.
9. Perform the indicated test of hypotheses using the p-value approach.
   a. Testing $H_0: p=0.37\ vs\ H_a: p\neq 0.37\ @\ \alpha=0.005,\ n=1300,\ \hat{p}=0.40$.
   b. Testing $H_0: p=0.94\ vs\ H_a: p>0.94\ @\ \alpha=0.05,\ n=1200,\ \hat{p}=0.96$.

10. Perform the indicated test of hypotheses using the p-value approach.
    a. Testing $H_0 : p = 0.25\ vs\ H_a : p < 0.25\ @\ \alpha = 0.10,\ n = 850,\ \hat{p} = 0.23$.
    b. Testing $H_0 : p = 0.33\ vs\ H_a : p \neq 0.33\ @\ \alpha = 0.05,\ n = 1100,\ \hat{p} = 0.30$.

Applications

11. Five years ago 3.9% of children in a certain region lived with someone other than a parent. A sociologist wishes to test whether the current proportion is different. Perform the relevant test at the 5% level of significance using the following data: in a random sample of $2,759$ children, 119 lived with someone other than a parent.
12. The government of a particular country reports its literacy rate as 52%. A nongovernmental organization believes it to be less. The organization takes a random sample of 600 inhabitants and obtains a literacy rate of 42%. Perform the relevant test at the 0.5% (one-half of 1%) level of significance.
13. Two years ago 72% of household in a certain county regularly participated in recycling household waste. The county government wishes to investigate whether that proportion has increased after an intensive campaign promoting recycling. In a survey of 900 households, 674 regularly participate in recycling. Perform the relevant test at the 10% level of significance.
14. Prior to a special advertising campaign, 23% of all adults recognized a particular company's logo. At the close of the campaign the marketing department commissioned a survey in which 311 of $1,200$ randomly selected adults recognized the logo. Determine, at the 1% level of significance, whether the data provide sufficient evidence to conclude that more than 23% of all adults now recognize the company's logo.
15. A report five years ago stated that 35.5% of all state-owned bridges in a particular state were "deficient." An advocacy group took a random sample of 100 state-owned bridges in the state and found 33 to be currently rated as being "deficient." Test whether the current proportion of bridges in such condition is 35.5% versus the alternative that it is different from 35.5%, at the 10% level of significance.
16. In the previous year the proportion of deposits in checking accounts at a certain bank that were made electronically was 45%. The bank wishes to determine if the proportion is higher this year. It examined $20,000$ deposit records and found that $9,217$ were electronic. Determine, at the 1% level of significance, whether the data provide sufficient evidence to conclude that more than 45% of all deposits to checking accounts are now being made electronically.
17. According to the Federal Poverty Measure 12% of the U.S. population lives in poverty. The governor of a certain state believes that the proportion there is lower. In a sample of size $1,550, 163$were impoverished according to the federal measure.
    a. Test whether the true proportion of the state's population that is impoverished is less than 12%, at the 5% level of significance.
    b. Compute the observed significance of the test.
18. An insurance company states that it settles 85% of all life insurance claims within 30 days. A consumer group asks the state insurance commission to investigate. In a sample of 250 life insurance claims, 203 were settled within 30 days.
    a. Test whether the true proportion of all life insurance claims made to this company that are settled within 30 days is less than 85%, at the 5% level of significance.
    b. Compute the observed significance of the test.
19. A special interest group asserts that 90% of all smokers began smoking before age 18. In a sample of 850 smokers, 687 began smoking before age 18.
    a. Test whether the true proportion of all smokers who began smoking before age 18 is less than 90%, at the 1% level of significance.
    b. Compute the observed significance of the test.
20. In the past, 68% of a garage's business was with former patrons. The owner of the garage samples 200 repair invoices and finds that for only 114 of them the patron was a repeat customer.
    a. Test whether the true proportion of all current business that is with repeat customers is less than 68%, at the 1% level of significance.
    b. Compute the observed significance of the test.

Additional Exercises

21. A rule of thumb is that for working individuals one-quarter of household income should be spent on housing. A financial advisor believes that the average proportion of income spent on housing is more than 0.25. In a sample of 30 households, the

mean proportion of household income spent on housing was 0.285 with a standard deviation of 0.063. Perform the relevant test of hypotheses at the 1% level of significance. Hint: This exercise could have been presented in an earlier section.

22. Ice cream is legally required to contain at least 10% milk fat by weight. The manufacturer of an economy ice cream wishes to be close to the legal limit, hence produces its ice cream with a target proportion of 0.106 milk fat. A sample of five containers yielded a mean proportion of 0.094 milk fat with standard deviation 0.002. Test the null hypothesis that the mean proportion of milk fat in all containers is 0.106 against the alternative that it is less than 0.106, at the 10% level of significance. Assume that the proportion of milk fat in containers is normally distributed. Hint: This exercise could have been presented in an earlier section.

Large Data Set Exercises

Large Data Sets missing

23. Large $\text{Data Sets 4 and 4A}$ list the results of 500 tosses of a die. Let p denote the proportion of all tosses of this die that would result in a five. Use the sample data to test the hypothesis that p is different from $1/6$, at the 20% level of significance.
24. Large Data Set 6 records results of a random survey of 200 voters in each of two regions, in which they were asked to express whether they prefer Candidate A for a U.S. Senate seat or prefer some other candidate. Use the full data set (400 observations) to test the hypothesis that the proportion p of all voters who prefer Candidate A exceeds 0.35. Test at the 10% level of significance.
25. Lines 2 through 536 in Large Data Set 11 is a sample of 535 real estate sales in a certain region in 2008. Those that were foreclosure sales are identified with a 1 in the second column. Use these data to test, at the 10% level of significance, the hypothesis that the proportion p of all real estate sales in this region in 2008 that were foreclosure sales was less than 25%. (The null hypothesis is $H_0 : p = 0.25$).
26. Lines 537 through 1106 in Large Data Set 11 is a sample of 570 real estate sales in a certain region in 2010. Those that were foreclosure sales are identified with a 1 in the second column. Use these data to test, at the 5% level of significance, the hypothesis that the proportion p of all real estate sales in this region in 2010 that were foreclosure sales was greater than 23%. (The null hypothesis is $H_0 : p = 0.25$).

Answers

1. a. $Z = 2.277$
   b. $Z = 2.277$
   c. $Z = -1.435$
2.
3. a. $Z \geq 1.645$; reject H_0
   b. $Z \leq -1.96 \text{ or } Z \geq 1.96$; reject H_0
   c. $Z \leq -1.645$; do not reject H_0
4.
5. a. $p-value = 0.0116, \ \alpha = 0.05$; reject H_0
   b. $p-value = 0.0232, \ \alpha = 0.05$; reject H_0
   c. $p-value = 0.0749, \ \alpha = 0.05$; do not reject H_0
6.
7. a. $Z = 1.74, \ z_{0.05} = 1.645$; reject H_0
   b. $Z = -1.98, \ -z_{0.005} = -2.576$; do not reject H_0
8.
9. a. $Z = 2.24, \ p-value = 0.025, \alpha = 0.005$; do not reject H_0
   b. $Z = 2.92, \ p-value = 0.0018, \alpha = 0.05$; reject H_0
10.
11. $Z = 1.11, \ z_{0.025} = 1.96$; do not reject H_0
12.
13. $Z = 1.93, \ z_{0.10} = 1.28$; reject H_0
14.
15. $Z = -0.523, \ \pm z_{0.05} = \pm 1.645$; do not reject H_0
16.

17. a. $Z = -1.798,\ -z_{0.05} = -1.645$; reject H_0
    b. $p - value = 0.0359$
18.
19. a. $Z = -8.92,\ -z_{0.01} = -2.33$; reject H_0
    b. $p - value \approx 0$
20.
21. $Z = 3.04,\ z_{0.01} = 2.33$; reject H_0
22.
23. $H_0 : p = 1/6\ vs\ H_a : p \neq 1/6$. Test Statistic: $Z = -0.76$. Rejection Region: $(-\infty, -1.28] \cup [1.28, \infty)$. Decision: Fail to reject H_0.
24.
25. $H_0 : p = 0.25\ vs\ H_a : p < 0.25$. Test Statistic: $Z = -1.17$. Rejection Region: $(-\infty, -1.28]$. Decision: Fail to reject H_0.

Contributor

- Anonymous
