7.2.1: Large Sample Estimation of a Population Mean

Learning Objectives

- To become familiar with the concept of an interval estimate of the population mean.
- To understand how to apply formulas for a confidence interval for a population mean.

The Central Limit Theorem says that, for large samples (samples of size $n \geq 30$), when viewed as a random variable the sample mean $\overline{X}$ is normally distributed with mean $\mu_{\overline{X}} = \mu$ and standard deviation $\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}}$. The Empirical Rule says that we must go about two standard deviations from the mean to capture 95% of the values of $\overline{X}$ generated by sample after sample. A more precise distance based on the normality of $\overline{X}$ is 1.960 standard deviations, which is $E = \frac{1.960\sigma}{\sqrt{n}}$.

The key idea in the construction of the 95% confidence interval is this, as illustrated in Figure $7.2.1.1$, because in sample after sample 95% of the values of $\overline{X}$ lie in the interval $[\mu - E, \mu + E]$, if we adjoin to each side of the point estimate $x - a$ "wing" of length E, 95% of the intervals formed by the winged dots contain μ. The 95% confidence interval is thus $\bar{x} \pm 1.960 \frac{\sigma}{\sqrt{n}}$. For a different level of confidence, say 90% or 99%, the number 1.960 will change, but the idea is the same.

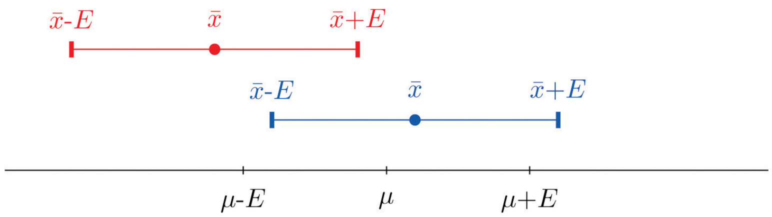


Figure 7.2.1.1: When Winged Dots Capture the Population Mean

Figure $7.2.1.2$ shows the intervals generated by a computer simulation of drawing 40 samples from a normally distributed population and constructing the 95% confidence interval for each one. We expect that about $(0.05)(40) = 2$ of the intervals so constructed would fail to contain the population mean μ, and in this simulation two of the intervals, shown in red, do.

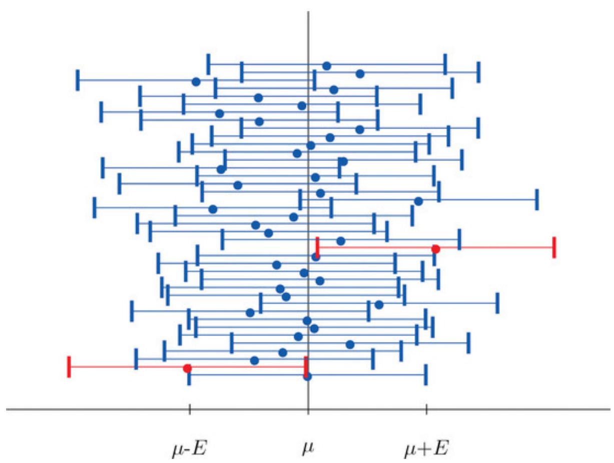


Figure 7.2.1.2: Computer Simulation of 40 95% Confidence Intervals for a Mean

It is standard practice to identify the level of confidence in terms of the area α in the two tails of the distribution of $\overline{X}$ when the middle part specified by the level of confidence is taken out. This is shown in Figure $7.2.1.3$, drawn for the general situation, and in Figure $7.2.1.4$, drawn for 95% confidence.

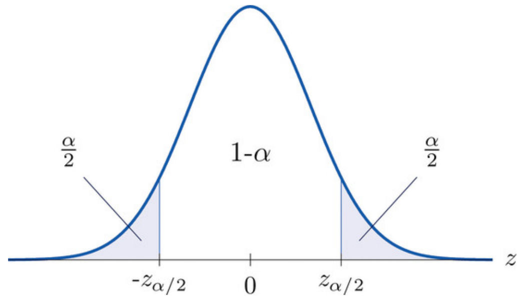


Figure 7.2.1.3: For $100(1-\alpha)\%$ confidence the area in each tail is $\alpha/2$.

Remember from Section 5.4 that the z-value that cuts off a right tail of area c is denoted z_c. Thus the number 1.960 in the example is $z_{.025}$, which is $z_{\frac{\alpha}{2}}$ for $\alpha = 1-0.95 = 0.05$.

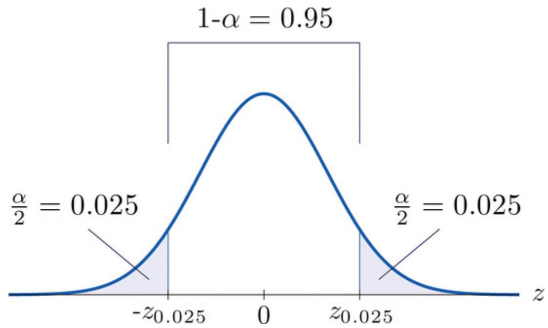


Figure 7.2.1.4: For 95% confidence the area in each tail is $\alpha/2 = 0.025$.

For 95% confidence the area in each tail is $\alpha/2 = 0.025$.

The level of confidence can be any number between 0 and 100%, but the most common values are probably 90% $(\alpha = 0.10)$, 95% $(\alpha = 0.05)$, and 99% $(\alpha = 0.01)$.

Thus in general for a $100(1-\alpha)\%$ confidence interval, $E = z_{\alpha/2}(\sigma/\sqrt{n})$, so the formula for the confidence interval is $\bar{x} \pm z_{\alpha/2}(\sigma/\sqrt{n})$. While sometimes the population standard deviation σ is known, typically it is not. If not, for $n \geq 30$ it is generally safe to approximate σ by the sample standard deviation s.

Large Sample $100(1-\alpha)\%$ Confidence Interval for a Population Mean

- If σ is known:

$$\bar{x} \pm z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

- If σ is unknown:

$$\bar{x} \pm z_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$$

A sample is considered large when $n \geq 30$.

As mentioned earlier, the number

$$E = z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

or

$$E = z_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$$

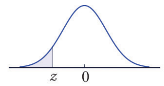
is called the *margin of error of the estimate*.

Example 7.2.1.1

Find the number $z_{\alpha/2}$ needed in construction of a confidence interval:

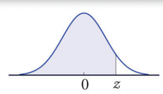
1. when the level of confidence is 90%;
2. when the level of confidence is 99%.

using the tables in Figure $7.2.1.5$below.



Cumulative Probability $P(Z \le z)$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−3.9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
−3.8	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
−3.7	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
−3.6	0.0002	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
−3.5	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641



Cumulative Probability $P(Z \le z)$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8304	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981

2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Figure $7.2.1.5$: Cumulative Normal Probability

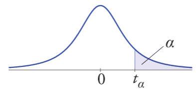
Solution:

1. For confidence level 90%, $\alpha = 1 - 0.90 = 0.10$, so $z_{\alpha/2} = z_{0.05}$. Since the area under the standard normal curve to the right of $z_{0.05}$ is 0.05, the area to the left of $z_{0.05}$ is 0.95. We search for the area 0.9500 in Figure $7.2.1.5$. The closest entries in the table are 0.9495 and 0.9505, corresponding to z-values 1.64 and 1.65. Since 0.95 is halfway between 0.9495 and 0.9505 we use the average 1.645 of the z-values for $z_{0.05}$.
2. For confidence level 99%, $\alpha = 1 - 0.99 = 0.01$, so $z_{\alpha/2} = z_{0.005}$. Since the area under the standard normal curve to the right of $z_{0.005}$ is 0.005, the area to the left of $z_{0.005}$ is 0.9950. We search for the area 0.9950 in Figure $7.2.1.5$. The closest entries in the table are 0.9949 and 0.9951, corresponding to z-values 2.57 and 2.58. Since 0.995 is halfway between 0.9949 and 0.9951 we use the average 2.575 of the z-values for $z_{0.005}$.

✔ Example $7.2.1.2$

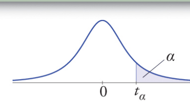
Use Figure $7.2.1.6$ below to find the number $z_{\alpha/2}$ needed in construction of a confidence interval:

1. when the level of confidence is 90%;
2. when the level of confidence is 99%.



Critical Values of t

df	$t_{0.200}$	$t_{0.100}$	$t_{0.050}$	$t_{0.025}$	$t_{0.010}$	$t_{0.005}$	$t_{0.0025}$	$t_{0.001}$	$t_{0.0005}$
1	1.376	3.078	6.314	12.706	31.821	63.657	127.321	318.309	636.619
2	1.061	1.886	2.920	4.303	6.965	9.925	14.089	22.327	31.599
3	0.978	1.638	2.353	3.182	4.541	5.841	7.453	10.215	12.924
4	0.941	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.920	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.906	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.896	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.889	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.883	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.879	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.876	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.873	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.870	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.868	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.866	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.865	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.863	1.333	1.740	2.110	2.576	2.898	3.222	3.646	3.965
18	0.862	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.861	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.860	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.859	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.858	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.858	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.768
24	0.857	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.856	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.856	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.855	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	0.855	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.854	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	0.854	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
31	0.853	1.309	1.696	2.040	2.453	2.744	3.022	3.375	3.633
32	0.853	1.309	1.694	2.037	2.449	2.738	3.015	3.365	3.622
33	0.853	1.308	1.692	2.035	2.445	2.733	3.008	3.356	3.611
34	0.852	1.307	1.691	2.032	2.441	2.728	3.002	3.348	3.601
35	0.852	1.306	1.690	2.030	2.438	2.724	2.996	3.340	3.591
36	0.852	1.306	1.688	2.028	2.434	2.719	2.990	3.333	3.582
37	0.851	1.305	1.687	2.026	2.431	2.715	2.985	3.326	3.574
38	0.851	1.304	1.686	2.024	2.429	2.712	2.980	3.319	3.566
39	0.851	1.304	1.685	2.023	2.426	2.708	2.976	3.313	3.558
40	0.851	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
41	0.851	1.303	1.683	2.020	2.421	2.701	2.967	3.301	3.544
42	0.851	1.302	1.682	2.018	2.418	2.698	2.963	3.296	3.538
43	0.851	1.302	1.681	2.017	2.416	2.695	2.959	3.291	3.532
44	8.850	1.301	1.680	2.015	2.414	2.692	2.956	3.286	3.526
45	0.850	1.301	1.679	2.014	2.412	2.690	2.952	3.281	3.520
46	0.850	1.300	1.679	2.013	2.410	2.687	2.949	3.277	3.515
47	0.849	1.300	1.678	2.012	2.408	2.685	2.946	3.273	3.510
48	0.849	1.299	1.677	2.011	2.407	2.682	2.943	3.269	3.505
49	0.849	1.299	1.677	2.010	2.405	2.680	2.940	3.265	3.500
50	0.849	1.299	1.676	2.009	2.403	2.678	2.937	3.261	3.496



Critical Values of t

df	$t_{0.200}$	$t_{0.100}$	$t_{0.050}$	$t_{0.025}$	$t_{0.010}$	$t_{0.005}$	$t_{0.0025}$	$t_{0.001}$	$t_{0.0005}$
51	0.849	1.298	1.675	2.008	2.402	2.676	2.934	3.258	3.492
52	0.849	1.298	1.675	2.007	2.400	2.674	2.932	3.255	3.488
53	0.849	1.298	1.674	2.006	2.399	2.672	2.929	3.251	3.484
54	0.848	1.297	1.674	2.005	2.397	2.670	2.927	3.248	3.480
55	0.848	1.297	1.673	2.004	2.396	2.668	2.925	3.245	3.476
56	0.848	1.297	1.673	2.003	2.395	2.667	2.923	3.242	3.473
57	0.848	1.297	1.672	2.002	2.394	2.665	2.920	3.239	3.470
58	0.848	1.296	1.672	2.002	2.392	2.663	2.918	3.237	3.466
59	0.848	1.296	1.671	2.001	2.391	2.662	2.916	3.234	3.463
60	0.848	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
61	0.848	1.296	1.670	2.000	2.389	2.659	2.913	3.229	3.457
62	0.848	1.295	1.670	1.999	2.388	2.657	2.911	3.227	3.454

62	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
63	0.847	1.295	1.669	1.998	2.387	2.656	2.909	3.225	3.452
64	0.847	1.295	1.669	1.998	2.386	2.655	2.908	3.223	3.449
65	0.847	1.295	1.669	1.997	2.385	2.654	2.906	3.220	3.447
66	0.847	1.295	1.668	1.997	2.384	2.652	2.904	3.218	3.444
67	0.847	1.294	1.668	1.996	2.383	2.651	2.903	3.216	3.442
68	0.847	1.294	1.668	1.995	2.382	2.650	2.902	3.214	3.439
69	0.847	1.294	1.667	1.995	2.382	2.649	2.900	3.213	3.437
70	0.847	1.294	1.667	1.994	2.381	2.648	2.899	3.211	3.435
71	0.847	1.294	1.667	1.994	2.380	2.647	2.897	3.209	3.433
72	0.847	1.293	1.666	1.993	2.379	2.646	2.896	3.207	3.431
73	0.847	1.293	1.666	1.993	2.379	2.645	2.895	3.206	3.429
74	0.847	1.293	1.666	1.993	2.378	2.644	2.894	3.204	3.427
75	0.846	1.293	1.665	1.992	2.377	2.643	2.892	3.202	3.425
76	0.846	1.293	1.665	1.992	2.376	2.642	2.891	3.201	3.423
77	0.846	1.293	1.665	1.991	2.376	2.641	2.890	3.199	3.421
78	0.846	1.292	1.665	1.991	2.375	2.640	2.889	3.198	3.420
79	0.846	1.292	1.664	1.990	2.374	2.640	2.888	3.197	3.418
80	0.846	1.292	1.664	1.990	2.374	2.639	2.887	3.195	3.416
81	0.846	1.292	1.664	1.990	2.373	2.638	2.886	3.194	3.415
82	0.846	1.292	1.664	1.989	2.373	2.637	2.885	3.193	3.413
83	0.846	1.292	1.663	1.989	2.372	2.636	2.884	3.191	3.412
84	0.846	1.292	1.663	1.989	2.372	2.636	2.883	3.190	3.410
85	0.846	1.292	1.663	1.988	2.371	2.635	2.882	3.189	3.409
86	0.846	1.291	1.663	1.988	2.370	2.634	2.881	3.188	3.407
87	0.846	1.291	1.663	1.988	2.370	2.634	2.880	3.187	3.406
88	0.846	1.291	1.662	1.987	2.369	2.633	2.880	3.185	3.405
89	0.846	1.291	1.662	1.987	2.369	2.632	2.879	3.184	3.403
90	0.846	1.291	1.662	1.987	2.368	2.632	2.878	3.183	3.402
91	0.846	1.291	1.662	1.986	2.368	2.631	2.877	3.182	3.401
92	0.846	1.291	1.662	1.986	2.368	2.630	2.876	3.181	3.399
93	0.846	1.291	1.661	1.986	2.367	2.630	2.876	3.180	3.398
94	0.846	1.291	1.661	1.986	2.367	2.629	2.875	3.179	3.397
95	0.845	1.291	1.661	1.985	2.366	2.629	2.874	3.178	3.396
96	0.845	1.290	1.661	1.985	2.366	2.628	2.873	3.177	3.395
97	0.845	1.290	1.661	1.985	2.365	2.627	2.873	3.176	3.394
98	0.845	1.290	1.661	1.984	2.365	2.627	2.872	3.175	3.393
99	0.845	1.290	1.660	1.984	2.365	2.626	2.871	3.175	3.392
100	0.845	1.290	1.660	1.984	2.364	2.626	2.871	3.174	3.390
∞ [z]	0.842	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Figure $7.2.1.6$: Critical Values of t

Solution:

1. In the next section we will learn about a continuous random variable that has a probability distribution called the Student t-distribution. Figure $7.2.1.6$gives the value t_c that cuts off a right tail of area c for different values of c. The last line of that table, the one whose heading is the symbol ∞ for infinity and $[z]$, gives the corresponding z-value z_c that cuts off a right tail of the same area c. In particular, $z_{0.05}$ is the number in that row and in the column with the heading $t_{0.05}$. We read off directly that $z_{0.05} = 1.645$.
2. In Figure $7.2.1.6$ $z_{0.005}$ is the number in the last row and in the column headed $t_{0.005}$, namely 2.576.

Figure $7.2.1.6$can be used to find z_c only for those values of c for which there is a column with the heading t_c appearing in the table; otherwise we must use Figure $7.2.1.5$in reverse. But when it can be done it is both faster and more accurate to use the last line of Figure $7.2.1.6$to find z_c than it is to do so using Figure $7.2.1.5$in reverse.

✔ Example $7.2.1.3$

A sample of size 49 has sample mean 35 and sample standard deviation 14. Construct a 98% confidence interval for the population mean using this information. Interpret its meaning.

Solution:

For confidence level 98%, $\alpha = 1 - 0.98 = 0.02$, so $z_{\alpha/2} = z_{0.01}$. From Figure $7.2.1.6$ we read directly that $z_{0.01} = 2.326$.Thus

$$\bar{x} \pm z_{\alpha/2} \frac{s}{\sqrt{n}} = 35 \pm 2.326 \left(\frac{14}{\sqrt{49}} \right) = 35 \pm 4.652 \approx 35 \pm 4.7$$

We are 98% confident that the population mean μ lies in the interval $[30.3, 39.7]$ in the sense that in repeated sampling 98% of all intervals constructed from the sample data in this manner will contain μ.

Example 7.2.1.4

A random sample of 120 students from a large university yields mean GPA 2.71 with sample standard deviation 0.51. Construct a 90% confidence interval for the mean GPA of all students at the university.

Solution:

For confidence level 90%, $\alpha = 1 - 0.90 = 0.10$, so $z_{\alpha/2} = z_{0.05}$. From Figure $7.2.1.6$ we read directly that $z_{0.05} = 1.645$. Since $n = 120$, $\bar{x} = 2.71$, and $s = 0.51$,

$$\bar{x} \pm z_{\alpha/2} \frac{s}{\sqrt{n}} = 2.71 \pm 1.645 \left(\frac{0.51}{\sqrt{120}} \right) = 2.71 \pm 0.0766$$

One may be 90% confident that the true average GPA of all students at the university is contained in the interval $(2.71 - 0.08, 2.71 + 0.08) = (2.63, 2.79)$

- A confidence interval for a population mean is an estimate of the population mean together with an indication of reliability.
- There are different formulas for a confidence interval based on the sample size and whether or not the population standard deviation is known.
- The confidence intervals are constructed entirely from the sample data (or sample data and the population standard deviation, when it is known).