

## 5.6: Continuous Random Variables (Exercises)

These are homework exercises to accompany the Textmap created for "Introductory Statistics" by OpenStax.

### Continuous Probability Functions

For each probability and percentile problem, draw the picture.

#### Q 5.2.1

Consider the following experiment. You are one of 100 people enlisted to take part in a study to determine the percent of nurses in America with an R.N. (registered nurse) degree. You ask nurses if they have an R.N. degree. The nurses answer "yes" or "no." You then calculate the percentage of nurses with an R.N. degree. You give that percentage to your supervisor.

- What part of the experiment will yield discrete data?
- What part of the experiment will yield continuous data?

#### Q 5.2.2

When age is rounded to the nearest year, do the data stay continuous, or do they become discrete? Why?

#### S 5.2.2

Age is a measurement, regardless of the accuracy used.

#### Exercise 5.2.2

Which type of distribution does the graph illustrate?

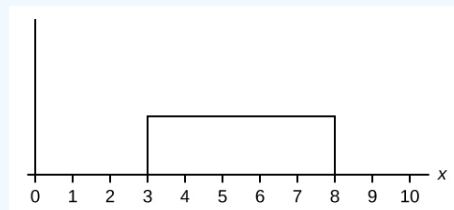


Figure 5.2.9.

**Answer**

Uniform Distribution

#### Exercise 5.2.3

Which type of distribution does the graph illustrate?

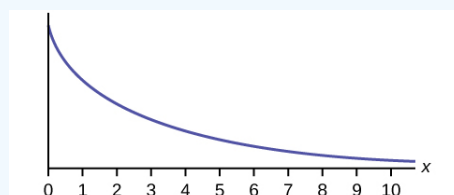


Figure 5.2.10.

#### Exercise 5.2.4

Which type of distribution does the graph illustrate?

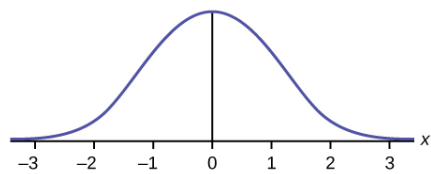


Figure 5.2.11.

**Answer**

Normal Distribution

#### Exercise 5.2.5

What does the shaded area represent?  $P(\_\_\_\_\_\_ < x < \_\_\_\_\_\_)$

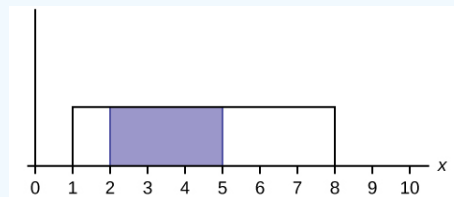


Figure 5.2.12.

#### Exercise 5.2.6

What does the shaded area represent?  $P(\_\_\_\_\_\_ < x < \_\_\_\_\_\_)$

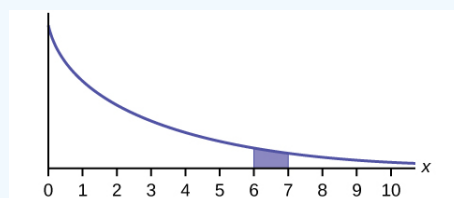


Figure 5.2.13.

**Answer**

$P(6 < x < 7)$

#### Exercise 5.2.7

For a continuous probability distribution,  $0 \leq x \leq 15$ . What is  $P(x > 15)$ ?

#### Exercise 5.2.8

What is the area under  $f(x)$  if the function is a continuous probability density function?

**Answer**

one

#### Exercise 5.2.9

For a continuous probability distribution,  $0 \leq x \leq 10$ . What is  $P(x = 7)$ ?

#### Exercise 5.2.11

A **continuous** probability function is restricted to the portion between  $x = 0$  and 7. What is  $P(x = 10)$ ?

**Answer**

zero

**Exercise 5.2.12**

$f(x)$  for a continuous probability function is  $\frac{1}{5}$ , and the function is restricted to  $0 \leq x \leq 5$ . What is  $P(x < 0)$ ?

**Exercise 5.2.13**

$f(x)$ , a continuous probability function, is equal to  $\frac{1}{12}$ , and the function is restricted to  $0 \leq x \leq 12$ . What is  $P(0 < x) < 12)$ ?

**Answer**

one

**Exercise 5.2.14**

Find the probability that  $x$  falls in the shaded area.

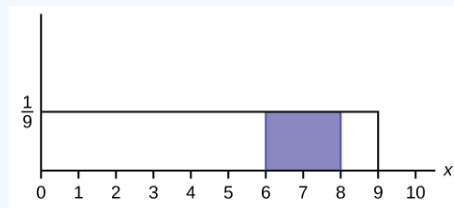


Figure 5.2.14.

**Exercise 5.2.15**

Find the probability that  $x$  falls in the shaded area.

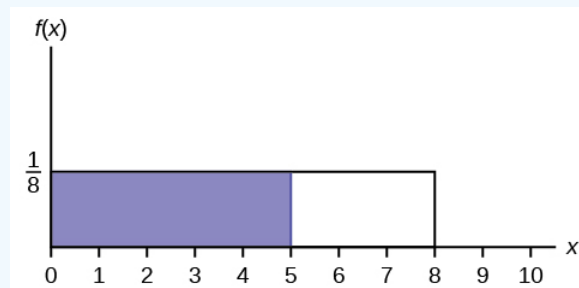


Figure 5.2.15.

**Answer**

0.625

**Exercise 5.2.16**

Find the probability that  $x$  falls in the shaded area.

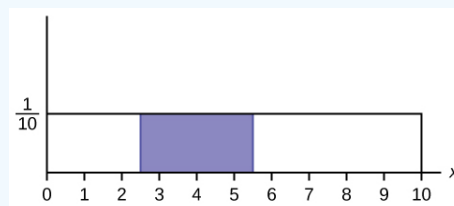


Figure 5.2.16.

**Exercise 5.2.17**

$f(x)$ , a continuous probability function, is equal to  $\frac{1}{3}$  and the function is restricted to  $1 \leq x \leq 4$ . Describe  $P(x > \frac{3}{2})$ .

**Answer**

The probability is equal to the area from  $x = \frac{3}{2}$  to  $x = 4$  above the x-axis and up to  $f(x) = \frac{1}{3}$ .

## The Uniform Distribution

For each probability and percentile problem, draw the picture.

### Q 5.3.1

Births are approximately uniformly distributed between the 52 weeks of the year. They can be said to follow a uniform distribution from one to 53 (spread of 52 weeks).

- $X \sim$  \_\_\_\_\_
- Graph the probability distribution.
- $f(x) =$  \_\_\_\_\_
- $\mu =$  \_\_\_\_\_
- $\sigma =$  \_\_\_\_\_
- Find the probability that a person is born at the exact moment week 19 starts. That is, find  $P(x = 19) =$  \_\_\_\_\_
- $P(2 < x < 31) =$  \_\_\_\_\_
- Find the probability that a person is born after week 40.
- $P(12 < x < 28) =$  \_\_\_\_\_
- Find the 70<sup>th</sup> percentile.
- Find the minimum for the upper quarter.

### Q 5.3.2

A random number generator picks a number from one to nine in a uniform manner.

- $X \sim$  \_\_\_\_\_
- Graph the probability distribution.
- $f(x) =$  \_\_\_\_\_
- $\mu =$  \_\_\_\_\_
- $\sigma =$  \_\_\_\_\_
- $P(3.5 < x < 7.25) =$  \_\_\_\_\_
- $P(x > 5.67) =$  \_\_\_\_\_
- $P(x > 5 | x > 3) =$  \_\_\_\_\_
- Find the 90<sup>th</sup> percentile.

### S 5.3.2

- $X \sim U(1, 9)$
- Check student's solution.
- $f(x) = 18$  where  $1 \leq x \leq 9$
- five
- 2.3
- $\frac{15}{32}$
- $\frac{333}{800}$
- $\frac{2}{3}$
- 8.2

### Q 5.3.3

According to a study by Dr. John McDougall of his live-in weight loss program at St. Helena Hospital, the people who follow his program lose between six and 15 pounds a month until they approach trim body weight. Let's suppose that the weight loss is uniformly distributed. We are interested in the weight loss of a randomly selected individual following the program for one month.

- Define the random variable.  $X =$  \_\_\_\_\_
- $X \sim$  \_\_\_\_\_
- Graph the probability distribution.
- $f(x) =$  \_\_\_\_\_

- e.  $\mu =$  \_\_\_\_\_
- f.  $\sigma =$  \_\_\_\_\_
- g. Find the probability that the individual lost more than ten pounds in a month.
- h. Suppose it is known that the individual lost more than ten pounds in a month. Find the probability that he lost less than 12 pounds in the month.
- i.  $P(7 < x < 13 | x > 9) =$  \_\_\_\_\_. State this in a probability question, similarly to parts g and h, draw the picture, and find the probability.

#### Q 5.3.4

A subway train on the Red Line arrives every eight minutes during rush hour. We are interested in the length of time a commuter must wait for a train to arrive. The time follows a uniform distribution.

- a. Define the random variable.  $X =$  \_\_\_\_\_
- b.  $X \sim$  \_\_\_\_\_
- c. Graph the probability distribution.
- d.  $f(x) =$  \_\_\_\_\_
- e.  $\mu =$  \_\_\_\_\_
- f.  $\sigma =$  \_\_\_\_\_
- g. Find the probability that the commuter waits less than one minute.
- h. Find the probability that the commuter waits between three and four minutes.
- i. Sixty percent of commuters wait more than how long for the train? State this in a probability question, similarly to parts g and h, draw the picture, and find the probability.

#### S 5.3.5

- a.  $X$  represents the length of time a commuter must wait for a train to arrive on the Red Line.
- b.  $X \sim U(0, 8)$
- c.  $f(x) = \frac{1}{8}$  where  $0 \leq x \leq 8$
- d. four
- e. 2.31
- f.  $\frac{1}{8}$
- g.  $\frac{1}{8}$
- h. 3.2

#### Q 5.3.6

The age of a first grader on September 1 at Garden Elementary School is uniformly distributed from 5.8 to 6.8 years. We randomly select one first grader from the class.

- a. Define the random variable.  $X =$  \_\_\_\_\_
- b.  $X \sim$  \_\_\_\_\_
- c. Graph the probability distribution.
- d.  $f(x) =$  \_\_\_\_\_
- e.  $\mu =$  \_\_\_\_\_
- f.  $\sigma =$  \_\_\_\_\_
- g. Find the probability that she is over 6.5 years old.
- h. Find the probability that she is between four and six years old.
- i. Find the 70<sup>th</sup> percentile for the age of first graders on September 1 at Garden Elementary School.

Use the following information to answer the next three exercises. The Sky Train from the terminal to the rental-car and long-term parking center is supposed to arrive every eight minutes. The waiting times for the train are known to follow a uniform distribution.

#### Q 5.3.7

What is the average waiting time (in minutes)?

- a. zero
- b. two

- c. three
- d. four

### S 5.3.7

d

### Q 5.3.8

Find the 30<sup>th</sup> percentile for the waiting times (in minutes).

- a. two
- b. 2.4
- c. 2.75
- d. three

### Q 5.3.9

The probability of waiting more than seven minutes given a person has waited more than four minutes is?

- a. 0.125
- b. 0.25
- c. 0.5
- d. 0.75

### S 5.3.9

b

### Q 5.3.10

The time (in minutes) until the next bus departs a major bus depot follows a distribution with  $f(x) = \frac{1}{20}$  where  $x$  goes from 25 to 45 minutes.

- a. Define the random variable.  $X =$  \_\_\_\_\_
- b.  $X \sim$  \_\_\_\_\_
- c. Graph the probability distribution.
- d. The distribution is \_\_\_\_\_ (name of distribution). It is \_\_\_\_\_ (discrete or continuous).
- e.  $\mu =$  \_\_\_\_\_
- f.  $\sigma =$  \_\_\_\_\_
- g. Find the probability that the time is at most 30 minutes. Sketch and label a graph of the distribution. Shade the area of interest. Write the answer in a probability statement.
- h. Find the probability that the time is between 30 and 40 minutes. Sketch and label a graph of the distribution. Shade the area of interest. Write the answer in a probability statement.
- i.  $P(25 < x < 55) =$  \_\_\_\_\_. State this in a probability statement, similarly to parts g and h, draw the picture, and find the probability.
- j. Find the 90<sup>th</sup> percentile. This means that 90% of the time, the time is less than \_\_\_\_\_ minutes.
- k. Find the 75<sup>th</sup> percentile. In a complete sentence, state what this means. (See part j.)
- l. Find the probability that the time is more than 40 minutes given (or knowing that) it is at least 30 minutes.

### Q 5.3.11

Suppose that the value of a stock varies each day from \$16 to \$25 with a uniform distribution.

- a. Find the probability that the value of the stock is more than \$19.
- b. Find the probability that the value of the stock is between \$19 and \$22.
- c. Find the upper quartile - 25% of all days the stock is above what value? Draw the graph.
- d. Given that the stock is greater than \$18, find the probability that the stock is more than \$21.

### S 5.3.11

- a. The probability density function of  $X$  is  $\frac{1}{25-16} = \frac{1}{9}$ .  
 $P(X > 19) = (25 - 19) \left(\frac{1}{9}\right) = \frac{6}{9} = \frac{2}{3}$ .



Figure 5.3.2.

- b. The area must be 0.25, and  $0.25 = (\text{width}) \left(\frac{1}{9}\right)$ , so  $\text{width} = (0.25)(9) = 2.25$ . Thus, the value is  $25 - 2.25 = 22.75$
- c. This is a conditional probability question.  $P(x > 21 | x > 18)$ . You can do this two ways:
- Draw the graph where a is now 18 and b is still 25. The height is  $\frac{1}{25-18} = \frac{1}{7}$ .  
So,  $P(x > 21 | x > 18) = (25-21) \left(\frac{1}{7}\right) = \frac{4}{7}$ .
  - Use the formula:  $P(x > 21 | x > 18) = \frac{P(x > 21 \text{ AND } x > 18)}{P(x > 18)} = \frac{P(x > 21)}{P(x > 18)} = \frac{(25-21)}{25-18} = \frac{4}{7}$ .

### Q 5.3.12

A fireworks show is designed so that the time between fireworks is between one and five seconds, and follows a uniform distribution.

- Find the average time between fireworks.
- Find probability that the time between fireworks is greater than four seconds.

### Q 5.3.13

The number of miles driven by a truck driver falls between 300 and 700, and follows a uniform distribution.

- Find the probability that the truck driver goes more than 650 miles in a day.
- Find the probability that the truck drivers goes between 400 and 650 miles in a day.
- At least how many miles does the truck driver travel on the furthest 10% of days?

### S 5.3.13

- $P(X > 650) = \frac{700-650}{700-300} = \frac{500}{400} = \frac{1}{8} = 0.125$ .
- $P(400 < X < 650) = \frac{700-650}{700-300} = \frac{250}{400} = 0.625$
- $0.10 = \frac{\text{width}}{700-300}$ , so  $\text{width} = 400(0.10) = 40$ . Since  $700 - 40 = 660$ , the drivers travel at least 660 miles on the furthest 10% of days.

## The Exponential Distribution

Use the following information to answer the next ten exercises. A customer service representative must spend different amounts of time with each customer to resolve various concerns. The amount of time spent with each customer can be modeled by the following distribution:  $X \sim \text{Exp}(0.2)$

### Exercise 5.4.8

What type of distribution is this?

### Exercise 5.4.9

Are outcomes equally likely in this distribution? Why or why not?

#### Answer

No, outcomes are not equally likely. In this distribution, more people require a little bit of time, and fewer people require a lot of time, so it is more likely that someone will require less time.

### Exercise 5.4.10

What is  $m$ ? What does it represent?

### Exercise 5.4.11

What is the mean?

#### Answer

five

**Exercise 5.4.12**

What is the standard deviation?

**Exercise 5.4.13**

State the probability density function.

**Answer**

$$f(x) = 0.2e^{-0.2x}$$

**Exercise 5.4.14**

Graph the distribution.

**Exercise 5.4.15**

Find  $P(2 < x < 10)$ .

**Answer**

0.5350

**Exercise 5.4.16**

Find  $P(x > 6)$ .

**Exercise 5.4.17**

Find the 70<sup>th</sup> percentile.

**Answer**

6.02

Use the following information to answer the next seven exercises. A distribution is given as  $X \sim Exp(0.75)$ .

**Exercise 5.4.18**

What is  $m$ ?

**Exercise 5.4.19**

What is the probability density function?

**Answer**

$$f(x) = 0.75e^{-0.75x}$$

**Exercise 5.4.20**

What is the cumulative distribution function?

**Exercise 5.4.21**

Draw the distribution.

**Answer**



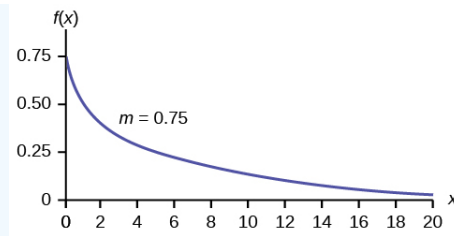


Figure 5.4.4

**Exercise 5.4.22**

Find  $P(x < 4)$ .

**Exercise 5.4.23**

Find the 30<sup>th</sup> percentile.

**Answer**

0.4756

**Exercise 5.4.24**

Find the median.

**Exercise 5.4.25**

Which is larger, the mean or the median?

**Answer**

The mean is larger. The mean is  $\frac{1}{m} = \frac{1}{0.75} \approx 1.33$ , which is greater than 0.9242

Use the following information to answer the next 16 exercises. Carbon-14 is a radioactive element with a half-life of about 5,730 years. Carbon-14 is said to decay exponentially. The decay rate is 0.000121. We start with one gram of carbon-14. We are interested in the time (years) it takes to decay carbon-14.

**Exercise 5.4.26**

What is being measured here?

**Exercise 5.4.27**

Are the data discrete or continuous?

**Answer**

continuous

**Exercise 5.4.28**

In words, define the random variable  $X$ .

**Exercise 5.4.29**

What is the decay rate ( $m$ )?

**Answer**

$m = 0.000121$

### Exercise 5.4.30

The distribution for  $X$  is \_\_\_\_\_.

### Exercise 5.4.31

Find the amount (percent of one gram) of carbon-14 lasting less than 5,730 years. This means, find  $P(x < 5,730)$ .

- a. Sketch the graph, and shade the area of interest.

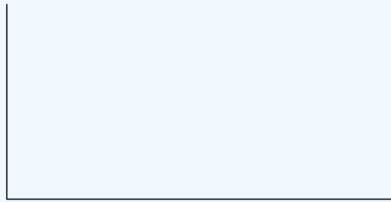


Figure 5.4.5

- b. Find the probability.  $P(x < 5,730) =$  \_\_\_\_\_

#### Answer

- a. Check student's solution  
b.  $P(x < 5,730) = 0.5001$

### Exercise 5.4.32

Find the percentage of carbon-14 lasting longer than 10,000 years.

- a. Sketch the graph, and shade the area of interest.



Figure 5.4.6

- b. Find the probability.  $P(x > 10,000) =$  \_\_\_\_\_

### Exercise 5.2.33

Thirty percent (30%) of carbon-14 will decay within how many years?

- a. Sketch the graph, and shade the area of interest.



Figure 5.4.7

- b. Find the value  $k$  such that  $P(x < k) = 0.30$ .

#### Answer

- a. Check student's solution.  
b.  $k = 2947.73$

## Q 5.4.1

Suppose that the length of long distance phone calls, measured in minutes, is known to have an exponential distribution with the average length of a call equal to eight minutes.

- Define the random variable.  $X =$  \_\_\_\_\_.
- Is  $X$  continuous or discrete?
- $X \sim$  \_\_\_\_\_
- $\mu =$  \_\_\_\_\_
- $\sigma =$  \_\_\_\_\_
- Draw a graph of the probability distribution. Label the axes.
- Find the probability that a phone call lasts less than nine minutes.
- Find the probability that a phone call lasts more than nine minutes.
- Find the probability that a phone call lasts between seven and nine minutes.
- If 25 phone calls are made one after another, on average, what would you expect the total to be? Why?

## Q 5.4.2

Suppose that the useful life of a particular car battery, measured in months, decays with parameter 0.025. We are interested in the life of the battery.

- Define the random variable.  $X =$  \_\_\_\_\_.
- Is  $X$  continuous or discrete?
- $X \sim$  \_\_\_\_\_
- On average, how long would you expect one car battery to last?
- On average, how long would you expect nine car batteries to last, if they are used one after another?
- Find the probability that a car battery lasts more than 36 months.
- Seventy percent of the batteries last at least how long?

## S 5.4.2

- $X =$  the useful life of a particular car battery, measured in months.
- $X$  is continuous.
- $X \sim \text{Exp}(0.025)$
- 40 months
- 360 months
- 0.4066
- 14.27

## Q 5.4.3

The percent of persons (ages five and older) in each state who speak a language at home other than English is approximately exponentially distributed with a mean of 9.848. Suppose we randomly pick a state.

- Define the random variable.  $X =$  \_\_\_\_\_.
- Is  $X$  continuous or discrete?
- $X \sim$  \_\_\_\_\_
- $\mu =$  \_\_\_\_\_
- $\sigma =$  \_\_\_\_\_
- Draw a graph of the probability distribution. Label the axes.
- Find the probability that the percent is less than 12.
- Find the probability that the percent is between eight and 14.
- The percent of all individuals living in the United States who speak a language at home other than English is 13.8.
  - Why is this number different from 9.848%?
  - What would make this number higher than 9.848%?

## Q 5.4.4

The time (in years) **after** reaching age 60 that it takes an individual to retire is approximately exponentially distributed with a mean of about five years. Suppose we randomly pick one retired individual. We are interested in the time after age 60 to retirement.

- Define the random variable.  $X =$  \_\_\_\_\_.
- Is  $X$  continuous or discrete?
- $X \sim$  \_\_\_\_\_
- $\mu =$  \_\_\_\_\_
- $\sigma =$  \_\_\_\_\_
- Draw a graph of the probability distribution. Label the axes.
- Find the probability that the person retired after age 70.
- Do more people retire before age 65 or after age 65?
- In a room of 1,000 people over age 80, how many do you expect will NOT have retired yet?

## S 5.4.4

- $X =$  the time (in years) after reaching age 60 that it takes an individual to retire
- $X$  is continuous.
- $X \sim \text{Exp}\left(\frac{1}{5}\right)$
- five
- five
- Check student's solution.
- 0.1353
- before
- 18.3

## Q 5.4.5

The cost of all maintenance for a car during its first year is approximately exponentially distributed with a mean of \$150.

- Define the random variable.  $X =$  \_\_\_\_\_.
- $X \sim$  \_\_\_\_\_
- $\mu =$  \_\_\_\_\_
- $\sigma =$  \_\_\_\_\_
- Draw a graph of the probability distribution. Label the axes.
- Find the probability that a car required over \$300 for maintenance during its first year.

Use the following information to answer the next three exercises. The average lifetime of a certain new cell phone is three years. The manufacturer will replace any cell phone failing within two years of the date of purchase. The lifetime of these cell phones is known to follow an exponential distribution.

## Q 5.4.6

The decay rate is:

- 0.3333
- 0.5000
- 2
- 3

## S 5.4.6

a

## Q 5.4.7

What is the probability that a phone will fail within two years of the date of purchase?

- 0.8647
- 0.4866
- 0.2212

d. 0.9997

#### Q 5.4.8

What is the median lifetime of these phones (in years)?

- a. 0.1941
- b. 1.3863
- c. 2.0794
- d. 5.5452

#### S 5.4.8

c

#### Q 5.4.9

Let  $X \sim \text{Exp}(0.1)$ .

- a. decay rate = \_\_\_\_\_
- b.  $\mu =$  \_\_\_\_\_
- c. Graph the probability distribution function.
- d. On the graph, shade the area corresponding to  $P(x < 6)$  and find the probability.
- e. Sketch a new graph, shade the area corresponding to  $P(3 < x < 6)$  and find the probability.
- f. Sketch a new graph, shade the area corresponding to  $P(x < 7)$  and find the probability.
- g. Sketch a new graph, shade the area corresponding to the 40<sup>th</sup> percentile and find the value.
- h. Find the average value of  $x$ .

#### Q 5.4.10

Suppose that the longevity of a light bulb is exponential with a mean lifetime of eight years.

- a. Find the probability that a light bulb lasts less than one year.
- b. Find the probability that a light bulb lasts between six and ten years.
- c. Seventy percent of all light bulbs last at least how long?
- d. A company decides to offer a warranty to give refunds to light bulbs whose lifetime is among the lowest two percent of all bulbs. To the nearest month, what should be the cutoff lifetime for the warranty to take place?
- e. If a light bulb has lasted seven years, what is the probability that it fails within the 8<sup>th</sup> year.

#### S 5.4.10

Let  $T =$  the life time of a light bulb.

The decay parameter is  $m = \frac{1}{8}$ , and  $T \sim \text{Exp}(\frac{1}{8})$ . The cumulative distribution function is  $P(T < t) = 1 - e^{-\frac{t}{8}}$

- a. Therefore,  $P(T < t) = 1 - e^{-\frac{t}{8}} \approx 0.1175$ .
- b. We want to find  $P(6 < t < 10)$ .

To do this,  $P(6 < t < 10) = P(t < 10) - P(t < 6) = (1 - e^{-\frac{1}{8} \cdot 10}) - (1 - e^{-\frac{1}{8} \cdot 6}) \approx 0.7135 - 0.5276 = 0.1859$

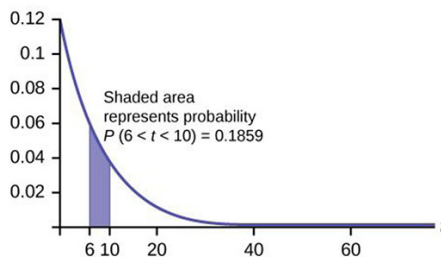


Figure 5.4.1.

- c. We want to find  $0.70 = P(T < t) = 1 - (1 - e^{-\frac{t}{8}}) = e^{-\frac{t}{8}}$ .

Solving for  $t$ ,  $e^{-\frac{t}{8}} = 0.70$ , so  $-\frac{t}{8} = \ln(0.70)$  and  $t = -8 \cdot \ln(0.70) \approx 2.85$  years.

Or use  $t = \frac{\ln(\text{area\_to\_the\_right})}{(-m)} = \frac{\ln(0.70)}{-\frac{1}{8}} \approx 2.85$  years

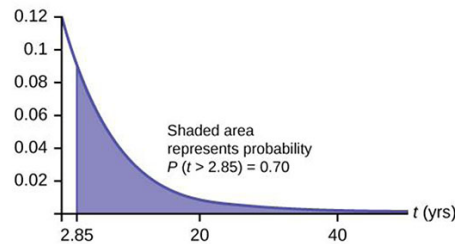


Figure 5.4.2.

d. We want to find  $0.02 = P(T < t) = 1 - e^{-\frac{t}{8}}$

Solving for  $t$ ,  $e^{-\frac{t}{8}} = 0.98$ , so  $\frac{t}{8} = \ln(0.98)$ , and  $t = -8 \cdot \ln(0.98) \approx 0.1616$  years, or roughly two months.

The warranty should cover light bulbs that last less than 2 months.

Or use  $\frac{\ln(\text{area\_to\_the\_right})}{(-m)} = \frac{\ln(1-0.2)}{-\frac{1}{8}} = 0.1616$ .

e. We must find  $P(T < 8 | T > 7)$ .

Notice that by the rule of complement events,  $P(T < 8 | T > 7) = 1 - P(T > 8 | T > 7)$ .

By the memoryless property ( $P(X > r + t | X > r) = P(X > t)$ ).

So  $P(T > 8 | T > 7) = P(T > 1) = 1 - \left(1 - e^{-\frac{1}{8}}\right) = e^{-\frac{1}{8}} \approx 0.8825$

Therefore,  $P(T < 8 | T > 7) = 1 - 0.8825 = 0.1175$ .

#### Q 5.4.11

At a 911 call center, calls come in at an average rate of one call every two minutes. Assume that the time that elapses from one call to the next has the exponential distribution.

- On average, how much time occurs between five consecutive calls?
- Find the probability that after a call is received, it takes more than three minutes for the next call to occur.
- Ninety-percent of all calls occur within how many minutes of the previous call?
- Suppose that two minutes have elapsed since the last call. Find the probability that the next call will occur within the next minute.
- Find the probability that less than 20 calls occur within an hour.

#### Q 5.4.12

In major league baseball, a no-hitter is a game in which a pitcher, or pitchers, doesn't give up any hits throughout the game. No-hitters occur at a rate of about three per season. Assume that the duration of time between no-hitters is exponential.

- What is the probability that an entire season elapses with a single no-hitter?
- If an entire season elapses without any no-hitters, what is the probability that there are no no-hitters in the following season?
- What is the probability that there are more than 3 no-hitters in a single season?

#### S 5.4.12

Let  $X$  = the number of no-hitters throughout a season. Since the duration of time between no-hitters is exponential, the number of no-hitters per season is Poisson with mean  $\lambda = 3$ .

Therefore,  $(X = 0) = \frac{3^0 e^{-3}}{0!} = e^{-3} \approx 0.0498$

NOTE

You could let  $T$  = duration of time between no-hitters. Since the time is exponential and there are 3 no-hitters per season, then the time between no-hitters is  $\frac{1}{3}$  season. For the exponential,  $\mu = \frac{1}{3}$ .

Therefore,  $m = \frac{1}{\mu} = 3$  and  $T \sim \text{Exp}(3)$ .

- The desired probability is  $P(T > 1) = 1 - P(T < 1) = 1 - (1 - e^{-3}) = e^{-3} \approx 0.0498$ .
- Let  $T$  = duration of time between no-hitters. We find  $P(T > 2 | T > 1)$ , and by the **memoryless property** this is simply  $P(T > 1)$ , which we found to be 0.0498 in part a.

- c. Let  $X$  = the number of no-hitters in a season. Assume that  $X$  is Poisson with mean  $\lambda = 3$ . Then  $P(X > 3) = 1 - P(X \leq 3) = 0.3528$ .

#### Q 5.4.13

During the years 1998–2012, a total of 29 earthquakes of magnitude greater than 6.5 have occurred in Papua New Guinea. Assume that the time spent waiting between earthquakes is exponential.

1. What is the probability that the next earthquake occurs within the next three months?
2. Given that six months has passed without an earthquake in Papua New Guinea, what is the probability that the next three months will be free of earthquakes?
3. What is the probability of zero earthquakes occurring in 2014?
4. What is the probability that at least two earthquakes will occur in 2014?

#### Q 5.4.14

According to the American Red Cross, about one out of nine people in the U.S. have Type B blood. Suppose the blood types of people arriving at a blood drive are independent. In this case, the number of Type B blood types that arrive roughly follows the Poisson distribution.

- a. If 100 people arrive, how many on average would be expected to have Type B blood?
- b. What is the probability that over 10 people out of these 100 have type B blood?
- c. What is the probability that more than 20 people arrive before a person with type B blood is found?

#### S 5.4.14

- a.  $\frac{100}{9} = 11.11$
- b.  $P(X > 10) = 1 - P(X \leq 10) = 1 - \text{Poissoncdf}(11.11, 10) \approx 0.5532$
- c. The number of people with Type B blood encountered roughly follows the Poisson distribution, so the number of people  $X$  who arrive between successive Type B arrivals is roughly exponential with mean  $\mu = 9$  and  $m = \frac{1}{9}$ . The cumulative distribution function of  $X$  is  $P(X < x) = 1 - e^{-\frac{x}{9}}$ . Thus,  $P(X > 20) = 1 - P(X \leq 20) = 1 - (1 - e^{-\frac{20}{9}}) \approx 0.1084$ .

#### NOTE

We could also deduce that each person arriving has a  $\frac{8}{9}$  chance of not having Type B blood. So the probability that none of the first 20 people arrive have Type B blood is  $(\frac{8}{9})^{20} \approx 0.0948$ . (The geometric distribution is more appropriate than the exponential because the number of people between Type B people is discrete instead of continuous.)

#### Q 5.4.15

A web site experiences traffic during normal working hours at a rate of 12 visits per hour. Assume that the duration between visits has the exponential distribution.

- a. Find the probability that the duration between two successive visits to the web site is more than ten minutes.
- b. The top 25% of durations between visits are at least how long?
- c. Suppose that 20 minutes have passed since the last visit to the web site. What is the probability that the next visit will occur within the next 5 minutes?
- d. Find the probability that less than 7 visits occur within a one-hour period.

#### Q 5.4.16

At an urgent care facility, patients arrive at an average rate of one patient every seven minutes. Assume that the duration between arrivals is exponentially distributed.

- a. Find the probability that the time between two successive visits to the urgent care facility is less than 2 minutes.
- b. Find the probability that the time between two successive visits to the urgent care facility is more than 15 minutes.
- c. If 10 minutes have passed since the last arrival, what is the probability that the next person will arrive within the next five minutes?
- d. Find the probability that more than eight patients arrive during a half-hour period.

## S 5.4.17

Let  $T$  = duration (in minutes) between successive visits. Since patients arrive at a rate of one patient every seven minutes,  $\mu = 7$  and the decay constant is  $m = \frac{1}{7}$ . The cdf is  $P(T < t) = 1 - e^{-\frac{t}{7}}$

- $P(T < 2) = 1 - 1 - e^{-\frac{2}{7}} \approx 0.2485$  .
- $P(T > 15) = 1 - P(T < 15) = 1 - \left(1 - e^{-\frac{15}{7}}\right) \approx e^{-\frac{15}{7}} \approx 0.1173$  .
- $P(T > 15 | T > 10) = P(T > 5) = 1 - \left(1 - e^{-\frac{5}{7}}\right) = e^{-\frac{5}{7}} \approx 0.4895$  .
- Let  $X$  = # of patients arriving during a half-hour period. Then  $X$  has the Poisson distribution with a mean of  $\frac{30}{7}$ ,  $X \sim \text{Poisson}\left(\frac{30}{7}\right)$ . Find  $P(X > 8) = 1 - P(X \leq 8) \approx 0.0311$  .

## Continuous Distribution

This page titled [5.6: Continuous Random Variables \(Exercises\)](#) is shared under a [CC BY 4.0](#) license and was authored, remixed, and/or curated by [OpenStax](#) via [source content](#) that was edited to the style and standards of the LibreTexts platform.

- 5.E: Continuous Random Variables (Exercises)** by [OpenStax](#) is licensed [CC BY 4.0](#). Original source: <https://openstax.org/details/books/introductory-statistics>.