# 4.5: Variance

Another important measure of variability is the variance. **Variance** is a descriptive statistic that summarizes how far raw scores tended to fall from the mean in squared units. It can also be thought of as the squared estimate of the error that occurs when the mean is used to estimate the value of a raw score. Thus, the variance is just the standard deviation left in squared units.

However, we generally best understand and, thus, report results in standard units. For this reason, variances are rarely reported as final results. Instead, variances are often used as important parts of other formulas (such as in an independent samples *t*-test which we will cover in Chapter 8).

## Decoding the Symbols

The symbols within the variance formulas are the same as those within the respective standard deviation formulas: $x$ refers to an individual raw score, $\mu$ refers to the population mean, $\bar{x}$ refers to the sample mean, $N$ refers to the population size, and $n$ refers to the sample size. The variance is just the standard deviation left in squared units. For the sake of simplicity and to keep this clear, the symbols for variance are just the standard deviation symbols adjusted to reflect this. Specifically, $\sigma^2$ is used to refer to variances of populations and $SD^2$ or $s^2$ are used to refer to the variances of samples.

## Calculating the Variance

Just as was true for standard deviation, there are two formulas for variance: one for populations and one for samples. The formulas use the same symbols and many of the same steps as the standard deviation.

### Population Variance

Here is the formula and required steps for finding the variance of a population:

Variance for a Population

| Formula | Calculation Steps |
|---|---|
| $\sigma^2 = \dfrac{\sum(x-\mu)^2}{N}$ | 1. Find the mean.<br>2. Subtract the mean from each raw score to find each deviation.<br>3. Square each deviation.<br>4. Sum the squared deviations to find SS.<br>5. Divide SS by $N$. |

Notice that these are the same as the first five steps to the population standard deviation formula. This leaves the result for variance in squared units, as is indicated by the symbol for variance which is $\sigma^2$.

### Sample Variance Formula

You can see the sample variance formula below. This formula is very similar to the population formula with two exceptions:

1. It uses sample symbols such as $\bar{x}$ in place of $\mu$ and
2. It uses an adjusted sample size in its denominator.

These are the same exceptions we saw when we moved from the formula for population standard deviations to the formula for sample standard deviations and they serve the same purpose here.

Variance for a Sample

| Formula | Calculation Steps |
|---|---|
| $s^2 = \dfrac{\sum(\mathrm{x}-\bar{\mathrm{x}})^2}{n-1}$ | 1. Find the mean.<br>2. Subtract the mean from each raw score to find each deviation.<br>3. Square each deviation.<br>4. Sum the squared deviations to find *SS*.<br>5. Find the adjusted sample size by subtracting 1 from $n$.<br>6. Divide *SS* by the adjusted sample size. |

Note that because the variance is calculated using the first six steps to finding a standard deviation, we have already calculated the variance for Data Set 4.1 when we found its standard deviation. Take a look at Table 2 which shows calculations for variation. Everything is the same as in Table 1 which showed the calculations for standard deviation for Data Set 4.1 until the last step. Specifically, instead of using the symbol *s* for standard deviation the symbol $s^2$ for variance appears and the square root sign is missing. Therefore, the result, when rounded to the hundredths place, is simply 88.10. This is the same value we had under the square root sign in the penultimate step for calculating standard deviation in Table 4.1. By comparing the two tables we can see how the variance was found in Table 4.1 in the process of ultimately calculating the standard deviation.

Table 2 Standard Deviation Calculations for Data Set 4.1 (n = 22)

| Raw Score | Mean | Deviation | Squared Deviation |
|---|---|---|---|
| 36 | 29 | 7 | 49 |
| 34 | 29 | 5 | 25 |
| 33 | 29 | 4 | 16 |
| 33 | 29 | 4 | 16 |
| 32 | 29 | 3 | 9 |
| 29 | 29 | 0 | 0 |
| 29 | 29 | 0 | 0 |
| 29 | 29 | 0 | 0 |
| 28 | 29 | -1 | 1 |
| 27 | 29 | -2 | 4 |
| 25 | 29 | -4 | 16 |
| 23 | 29 | -6 | 36 |
| 20 | 29 | -9 | 81 |
| 19 | 29 | -10 | 100 |
| 19 | 29 | -10 | 100 |
| 18 | 29 | -11 | 121 |
| 16 | 29 | -13 | 169 |
| 14 | 29 | -15 | 225 |
| | | Sum of Squared | |
| | | Sum of Deviations = 0 | Deviations (SS) = 1,850 |

$$s^2 = \frac{1,850}{22-1} = \frac{1,850}{21} = 88.0952381\ldots \approx 88.10$$