8.4: Example of how to Test a Hypothesis by Computing t.

Let us assume that a researcher believed that high school students would have a higher mean for hours of sleep than college students. Suppose that sleep was measured in hours for a group of 5 high school students and a separate group of 5 college students. Assume that Data Set 8.1 includes data from these two independent samples. Let's use this information to follow the steps in hypothesis testing.

Data Set 8.1. Hours of Sleep Reported by High School and College Students (n = 10)

High Schoolers	College Students
5	3
7	5
6	4
8	6
9	7

Steps in Hypothesis Testing

In order to test a hypothesis, follow these steps:

1. **State the hypothesis.**

A directional hypothesis is the best fit because the goal is to see if the high school students have a *higher* mean score that the college students. A summary of the research hypothesis and corresponding null hypothesis in sentence and symbol format are shown below. However, researchers often only state the research hypothesis using a format like this: *It is hypothesized that the mean hours of sleep will be higher among high schooler students than college students.*

Directional Hypothesis for an Independent Samples t-Test

Research hypothesis	The mean hours of sleep for high school students will be greater than the mean for college students.	$H_A : \mu_1 > \mu_2$
Null hypothesis	The mean hours of sleep for high school students will not be greater (i.e. will be less than or equal to) the mean for college students.	$H_0 : \mu_1 \leq \mu_2$

2. **Choose the inferential test (formula) that best fits the hypothesis.**

The means of two independent samples are being compared so the appropriate test is an independent samples *t*-test.

3. **Determine the critical value.**

In order to determine the critical value, three things must be identified:

i. the alpha level,
ii. whether the hypothesis requires a one-tailed test or a two-tailed test, and,
iii. the degrees of freedom (df).

The alpha level is often set at .05 unless there is reason to adjust it such as when multiple hypotheses are being tested in one study or when a Type I Error could be particularly problematic. The default alpha level can be used for this example because only one hypothesis is being tested and there is no clear indication that a Type I Error would be especially problematic. Thus, alpha can be set to 5%, which can be summarized as α = .05.

The hypothesis is directional so a one-tailed test should be used.

The df must also be calculated. Each inferential test has a unique formula for calculating df. In short, the formula for df for the independent samples t-test is as follows: $n_1 + n_2 - 2$. It appears in the lower left side of the independent samples t-test formula. The sample size for Group 1 (the high school students) was $5 (n_1 = 5)$. The sample size for Group 2 (the college students) was also $5 (n_2 = 5)$. Thus, df = 5 + 5 – 2 so the df for this scenario is 8.

These three pieces of information are used to locate the critical value from the test. The full tables of the critical values for *t*-tests are located in Appendix D. Below is an excerpt of the section of the *t*-tables that fits the current hypothesis and data. Under the conditions of and alpha level of .05, a one-tailed test, and 8 degrees of freedom, the critical value is 1.860.

Critical Values Table

	one-tailed test	
alpha level:	α = 0.05	α = 0.01
Degrees of Freedom: 8	1.860	2.896

The critical value represents the absolute value which must be exceeded in order to declare a result significant. It represents the threshold of evidence needed to be confident a hypothesis is true. The obtained value (which is called *t* in a *t*-test) is the amount of evidence present. When using a two-tailed test, only the absolute value of the critical value must be considered. However, the current hypothesis requires a one tailed test because the hypothesis is directional. Therefore, both the magnitude and direction of the obtained *t*-value must be considered. Because the hypothesis states that Group 1 will have the higher mean than Group 2, the *t*-value must be positive in addition to exceeding the magnitude of the critical value. Thus, in order for the result to significantly support the hypothesis is needs to be positive and exceed the critical value of 1.860.

Degrees of Freedom for an Independent Samples t-Test

Degrees of Freedom (df) indicate how much information you have that is free to vary. The degrees of freedom is equal to the total of each sample size minus 1 per group. It appears in the independent samples *t*-test like this:

$$df = n_1 + n_2 - 2$$

Thus, in an independent samples *t*-test, the degrees of freedom are calculated as the total number of cases minus the number of groups (because we subtract one from each group). Total number of cases can be summarized with the symbol N. N is calculated as follows:

$$N = n_1 + n_2$$

The number of independent groups is summarized with the symbol *k*. Degrees of freedom for each *t*-test is calculated as the total sample size minus the number of groups. This can be summarizes as follows:

$$df = N - k$$

4. **Calculate the test statistic.**

A test statistic can also be referred to as an obtained value. The formula needed to find the test statistic known as *t* for this scenario is as follows:

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\left[\frac{(n_1 - 1)\, s_1^2 + (n_2 - 1)\, s_2^2}{n_1 + n_2 - 2}\right]\left[\frac{n_1 + n_2}{n_1 \times n_2}\right]}}$$

<u>Section A: Preparation</u>

Start each inferential formula by identifying and solving for the pieces that must go into the formula. For the independent samples *t*-test, this preparatory work is as follows:

1. Find n for Group 1.
   - This value is found using Data Set 8.1 and is summarized as $n_1 = 5$
2. Find $\bar{x}$ for Group 1.
   - This value is found using Data Set 8.1 and is summarized as $\bar{X}_1 = 7.00$
3. Find s^2 for Group 1.
   - This value is found using Data Set 8.1 and is summarized as $s_1^2 = 2.50$

4. Find n for Group 2.
   - This value is found using Data Set 8.1 and is summarized as $n_2 = 5$
5. Find $\bar{x}$ for Group 2.
   - This value is found using Data Set 8.1 and is summarized as $\bar{X}_2 = 7.00$
6. Find s^2 for Group 2.
   - This value is found using Data Set 8.1 and is summarized as $s_2^2 = 2.50$

> **Note**
>
> The summarized calculations are shown below for reference. For a detailed review of how to calculate a sample mean, see Chapter 3. For a detailed review of how to calculate a sample variance, see Chapter 4.

Data Set 8.1. Descriptive Statistics for Hours of Sleep Reported by High School and College Students (n = 10)

High Schoolers	$X-\bar{X}_1$	$(X-\bar{X}_1)^2$	College Students	$(X-\bar{X}_2)^2$	$(X-\bar{X}_2)^2$
5	(-2)	4	3	(-2)	4
7	0	0	5	0	0
6	(-1)	1	4	(-1)	1
8	1	1	6	1	1
9	2	4	7	2	4
$n_1 = 5$		$SS_1 = 10$	$n_2 = 5$		$SS_1 = 10$
$\boldsymbol{\bar{X}_1 = 7.00}$		$s_1^2 = 2.50$	$\boldsymbol{\bar{X}_2 = 5.00}$		$s_2^2 = 2.50$

Now that the pieces needed for the formula have been found, we can move to Section B.

Section B: Solving

The inferential formula is used to compute the obtained value. For the independent samples *t*-test, this work is as follows:

1. Write the formula with the values found in section A plugged into their respective locations.

Writing the formula in symbol format before filling it in with the values can help you recognize and memorize it. Here is the formula with the symbols:

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\left[\frac{(n_1-1)\,s_1^2 + (n_2-1)\,s_2^2}{n_1+n_2-2}\right]\left[\frac{n_1+n_2}{n_1 \times n_2}\right]}}$$

Here is the formula with values filled into their appropriate locations in place of their symbols:

$$t = \frac{7.00 - 5.00}{\sqrt{\left[\frac{(5-1)2.50 + (5-1)2.50}{5+5-2}\right]\left[\frac{5+5}{5 \times 5}\right]}}$$

2. Solve the numerator by subtracting the mean of Group 2 from the mean of Group 1. Note: Steps will appear in bold to show when they have been computed.

$$t = \frac{\mathbf{2.00}}{\sqrt{\left[\frac{(5-1)2.50 + (5-1)2.50}{5+5-2}\right]\left[\frac{5+5}{5 \times 5}\right]}}$$

3. Solve for the left side of the denominator as follows:

a. Multiply variance for Group 1 by $n-1$ for Group 1 to get the SS for Group 1.

$$t=\frac{2.00}{\sqrt{\left[\frac{\mathbf{10.00}+(5-1)2.50}{5+5-2}\right]\left[\frac{5+5}{5\times5}\right]}}$$

b. Multiply variance for Group 2 by $n-1$ for Group 2 to get the SS for Group 2.

$$t=\frac{2.00}{\sqrt{\left[\frac{10.00+\mathbf{10.00}}{5+5-2}\right]\left[\frac{5+5}{5\times5}\right]}}$$

c. Add the SS for Group 1 (which is the result of step 3a) to the SS for Group 2 (which is the result of step 3b). This gives you total SS.

$$t=\frac{2.00}{\sqrt{\left[\frac{\mathbf{20.00}}{5+5-2}\right]\left[\frac{5+5}{5\times5}\right]}}$$

d. Find the degrees of freedom (df) by adding the sample size for Group 1 to the sample size for Group 2 and then subtracting 2 from that total.

$$t=\frac{2.00}{\sqrt{\left[\frac{20.00}{\mathbf{8}}\right]\left[\frac{5+5}{5\times5}\right]}}$$

e. Divide the total SS (which is result of step 3c) by the df (which is the result of step 3d).

$$t=\frac{2.00}{\sqrt{[\mathbf{2.50}]\left[\frac{5+5}{5\times5}\right]}}$$

4. Solve right side of the denominator as follows:
   a. Add the sample size of Group 1 to the sample size of Group 2 to get the total sample size.

$$t=\frac{2.00}{\sqrt{[2.50]\left[\frac{\mathbf{10}}{5\times5}\right]}}$$

   b. Multiply the sample size of Group 1 by the sample size of Group 2 to get the product of the sample sizes.

$$t=\frac{2.00}{\sqrt{[2.50]\left[\frac{10}{\mathbf{25}}\right]}}$$

   c. Divide the total sample size (which is the result of step 4a) by the product of sample sizes (which is the result of step 4b).

$$t=\frac{2.00}{\sqrt{[2.50][\mathbf{0.40}]}}$$

5. Multiply the left side of the denominator (which is the result of step 3e) by the right side of the denominator (which is the result of step 4c).

$$t=\frac{2.00}{\sqrt{[\mathbf{1.00}]}}$$

6. Square root the denominator (which means square root the result of step 5) to get the pooled standard error for the formula.

$$t = \frac{2.00}{\mathbf{1.00}}$$

7. Finally, divide the numerator (which is the result of step 2) by the pooled standard error (which is the result of step 6) to get the obtained *t*-value.

$$t = 2.00$$

This result, known as a test statistic or *t*-value, can also be referred to by the general term "obtained value." This result is positive meaning Group 1 (the high schoolers) had a higher mean than Group 2 (the college students). The means of these two groups were two standard errors apart as indicated by the magnitude of the result.

5. **Apply a decision rule and determine whether the result is significant.**

   Assess whether the obtained value for t exceeds the critical value as follows:

   The critical value is 1.860.

   The obtained *t* value is 2.00

   The obtained *t* value does exceed (i.e. is greater than) the critical value. However, because this is a one-tailed test (due to the directional hypothesis), we must also check the direction of the result during this step. The hypothesis stated that Group 1 (high schoolers) would have the higher mean so the result must be positive to support this hypothesis. Thankfully, it is. Therefore, both criteria are met to declare the result significant:

i. The magnitude of the obtained value is greater than the critical value and
ii. The result is in the hypothesized direction.

   Thus, the result is significant and supports the hypothesis

> **Note**
>
> If the hypothesis had been non-directional (and, thus, a two-tailed test was being performed), we would only need to check that the magnitude exceeded that of the critical value to declare the result significant.

6. **Calculate the effect size and/or other relevant secondary analyses.**

   When it is determined that the result is significant, effect sizes should be computed. Because the result was determined to be significant in step 5, the effect size is needed before proceeding to step 7 to complete the process.

   The effect size that is appropriate for *t*-tests under standard conditions is known as Cohen's d (Cohen, 1988). The formula for Cohen's d is as follows when working with an independent samples *t*-test:

$$d = \frac{\bar{x}_1 - \bar{x}_2}{S_p}$$

   Cohen's d, when used for an independent samples *t*-test, calculates how many pooled standard deviations the sample mean of Group 1 is from the mean of Group 2. Thus, the numerator finds the difference in the two means and the denominator is used to divide that by the pooled standard deviation (S_p). Notice that this is very similar to the format of the *t*-test formula only that standard deviation is being used in place of standard error.

   Recall, that the S_p is in the boom of the denominator of the *t*-formula and can be isolated and used to compute S_p for Data Set 8.1 as follows:

$$S_p = \sqrt{\left[\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}\right]}$$
$$S_p = \sqrt{\left[\frac{[(5-1)2.50+(5-1)2.50}{5+5-2}\right]}$$
$$S_p = \sqrt{\left[\frac{10.00+10.00}{8}\right]}$$
$$S_p = \sqrt{\left[\frac{20.00}{8}\right]}$$
$$S_p = \sqrt{2.50}$$
$$S_p = 1.5811\ldots$$

The calculations for Cohen's d for Data Set 8.1 are as follows:

$$d = \frac{7.00-5.00}{1.5811\ldots}$$
$$d = \frac{2.00}{1.5811\ldots}$$
$$d \approx 1.2649$$

Effect sizes, like most values, are rounded and reported to the hundredths place. Thus, this effect size is reported as $d = 1.26$. Cohen's *d* can be interpreted using the following rules of thumb (Cohen, 1988; Navarro, 2014):

Interpreting Cohen's d Effect Sizes

~0.80	Large effect
~0.50	Moderate effect
~0.20	Small effect

Note that the magnitude of the *d* value is what is used to interpret effect, not the direction; thus, the absolute value of *d* is compared to the rules of thumb to estimate the effect size of the result.

The rules of thumb are general guidance and do not dictate precise or required interpretations. Instead, they provide some generally agreed upon approximations to aid in interpretations. As is true of all analyses, it is best to consider their situated (or practical) relevance. Nevertheless, the rules of thumb are useful in providing an initial guideline for interpreting effect sizes. Following these rules of thumb, the current finding of *d* = 1.26 would be considered a large effect.

7. **Report the results in American Psychological Associate (APA) format.**

Results for inferential tests are often best summarized using a paragraph that states the following:

a. the hypothesis and specific inferential test used,
b. the main results of the test and whether they were significant,
c. any additional results that clarify or add details about the results,
d. whether the results support or refute the hypothesis.

Keep in mind that results are reported in past tense because they report on what has already been found. In addition, the research hypothesis must be stated but the null hypothesis is usually not needed for summary paragraphs because it can be deduced

from the research hypothesis. Finally, APA format requires a specific format be used for reporting the results of a test. This includes a specific format for reporting relevant symbols and details for the formula and data used.

Note: Standard deviations are reported with means, not variances. Thus, the standard deviation must also be computed before completing the write-up. Standard deviation for each group can be computed by square rooting their respective variances like so:

High School Group: $s_1^2 = 2.50$ Thus, $\sqrt{s_1^2} = \sqrt{2.50}$ so s = 1.5811… which rounds to 1.58

College Group: $s_2^2 = 2.50$ Thus, $\sqrt{s_2^2} = \sqrt{2.50}$ so s = 1.5811… which rounds to 1.58

Following this, the results for our hypothesis with Data Set 8.1 can be written as shown in the summary example.

APA Formatted Summary Example

An independent samples *t*-test was used to test the hypothesis that the mean hours of sleep would be higher among high schooler students than college students. Consistent with the hypothesis, the mean hours of sleep was significantly higher for the high schooler students (M = 7.00; SD = 1.58) than for the college students (M = 5.00; SD = 1.58), t(8) = 2.00, $p < .05$. The Cohen's d effect size of 1.26 was large.

This succinct summary in APA format provides a lot of detail and uses specific symbols in a particular order. To understand how to read and create a summary like this, see the brief review of the structure for APA format below.

Summary of APA-Formatted Results for the Independent Samples t-Test

In your APA-formatted write up for an independent samples *t*-test you should state:

1. Which test was used and the hypothesis which warranted its use.
2. Whether the aforementioned hypothesis was supported or not. To do so properly, three components must be reported:
   a. The mean and standard deviation for each group
   b. The test results in an evidence string as follows: $t(df)$ = obtained value
   c. The significance part of the evidence string as $p < .05$ if significant or $p > .05$, *ns* if not significant
   d. The effect size, if the result was significant

The following breaks down what each part represents in the evidence string for Data Set 8.1:

Anatomy of an Evidence String

Symbol for the test	Degrees of Freedom	Obtained Value	*p*-Value
t	(8)	= 2.00,	$p < .05$

Reading Review 8.3

1. Which two things should be stated first in a results summary paragraph?
2. What are the four parts of the evidence string and what does each one report?
3. Which set of symbols should be used at the end of a sentence to indicate that a result was significant?
4. What does reporting Cohen's *d* add to a results summary?