7.1: t-Statistics and the One Sample t-tests

The first group of inferential tests covered in this book are the *t*-tests. There are three kinds of *t*-tests: the one sample *t*-test, the dependent samples *t*-test, and the independent samples *t*-test. Each of these is best suited for comparing the means of two groups of data under different conditions. What the *t*-tests have in common is that each compares the means of two groups of scores for the same variable. They differ in what makes up the two groups of scores. The one sample *t*-test is used to compare the mean of a sample to a known or hypothesized population mean. The dependent samples *t*-test is used to compare the mean of a single sample at one time to itself at another time. The independent samples *t*-test is used to compare the mean of one sample to the mean of a different sample. Each of these uses a different formula but all require the use of sample data. Each of these is the focus of one chapter, starting with the one sample *t*-test for this chapter.

One Sample t-Test

The one sample *t*-test is used to test whether the mean from a sample is significantly different from another hypothesized mean such as a known population mean. When the population parameter is known, we can assess how similar our sample statistic is to the population parameter. This makes the one sample *t*-test a bit usual because one of its common applications can only be used when the population parameter is known yet statisticians typically use sample estimates *because* the population parameter has not been or cannot be directly measured. However, this version of the one sample *t*-test does have utility; it can be used to estimate how representative a sample is for its population. For example, say you wanted to conduct research to see how a sample of students felt about their school but that you had concerns that they may not represent the population of students well if they had a significantly different grade point average (GPA) than the student body at the school, broadly. A one sample *t*-test could be conducted first to assess whether the sample's GPA was similar or significantly different from the overall student GPA at the school. If the sample's mean GPA is not significantly different from the known mean GPA of the full population of students at the school, the researcher may comfortably proceed to collecting and analyzing data about other variables in the sample to understand the feelings of the corresponding population of students.

A second use of a one sample *t*-test is to see if a sample mean is similar or significantly different from a hypothesized value for the mean, whether or not that value is known to be true in a population. In this version, a hypothesized or hoped for value is used in place of a known population mean. For example, suppose that a nail polish manufacturer has a machine that is supposed to distribute 15 ml of product into each bottle and that the owner wants to check whether the machine is depositing amounts appropriately. To do so, the owner randomly samples 100 bottles filled by the machine and measures the amount of polish in each bottle. If the machine is working properly, the mean volume for the sample should *not* be significantly different from 15 ml. This scenario can also be tested using a one sample *t*-test.

Data and Hypotheses that Fit the One Sample t-Test

Data

Each statistical test has some assumptions which must be met in order for the formula to function properly. In keeping, there are a few assumptions about the data which must be met before a one sample *t* test is used. First, the one sample *t*-test is univariate. This means that only one variable is being tested or analyzed at a time using data from a sample. Second, the variable should be measured on the interval or ratio scale. If these conditions are not met, the one sample *t*-test should not be used. Third, and warranting further explanation, the data must be independent. Let's take a moment to get a clear understanding of what that means before moving on to a discussion of hypotheses.

Independent Data

The one sample *t*-test is a univariate analysis that is suited for independent data. The word *independent* is used in several ways in statistics, each time distinguishing a particular attribute of a variable or data. When data for a variable are independent, it means that the scores for each case are not dependent upon, directly connected to, or influential of one another. For example, if data are collected about hours of sleep from a random sample of people, the data are likely independent. The number of hours slept by person 1 do not impact how many hours person 2 or person 3 slept. In this example, the scores for the variable Hours of Sleep are independent of each other. In contrast, the data for Hours of Sleep would *not* be independent for two new parents who have decided to sleep in shifts so that one is awake while the other sleeps, and visa-versa. In this example the hours parent 1 sleeps are dependent upon the hours parent 2 sleeps. This is an important consideration when working with univariate data and statistical tests. Hours of sleep could be analyzed using a one sample *t*-test for the random sample of independent scores for sleep hours but not for data from the couples with dependent sleep hours. When data for a variable are independent, the one sample *t*-test can be used.

Hypotheses

Hypotheses for the one sample *t*-test can be non-directional or directional. For the one sample *t* test, the non-directional research hypothesis is that the sample mean will be different from the population mean (or an otherwise hypothesized mean). The corresponding null hypothesis is that the sample mean will not be different from the population mean (or an otherwise hypothesized mean). Because this research hypothesis is non-directional, it requires a two-tailed test. The non-directional research and corresponding null hypotheses can be summarized as follows:

Non-Directional Hypothesis for a One Sample t-Test

Research hypothesis	The sample mean is not equal to the population mean (or hypothesized mean)	$H_A: \bar{X} \neq \mu$
Null hypothesis	The sample mean is equal to the population mean (or hypothesized mean)	$H_0: \bar{X} = \mu$

There are two directional hypotheses possible for the one sample *t*-test. One possible directional research hypothesis is that the sample mean will be *higher than* the population mean (or an otherwise hypothesized mean). The corresponding null hypothesis is that the sample mean will *not* be higher than the population mean (or an otherwise hypothesized mean). This could mean that the sample mean is less than or that it is equal to the population mean. Because this research hypothesis is directional, it requires a one-tailed test. This version of the research and corresponding null hypotheses can be summarized as follows:

Directional Hypothesis for a One Sample t-Test stating Sample Mean will be Higher

Research hypothesis	The sample mean is higher than the population mean (or hypothesized mean)	$H_A: \bar{X} > \mu$
Null hypothesis	The sample mean is not higher than (i.e. is lower than or equal to) the population mean (or hypothesized mean)	$H_0: \bar{X} \leq \mu$

For the one sample *t*-test, the other possible directional research hypothesis is that the sample mean will be *lower than* the population mean (or an otherwise hypothesized mean). The corresponding null hypothesis is that the sample mean will *not* be lower than the population mean (or an otherwise hypothesized mean). This could mean that the sample mean is greater than or that it is equal to the population mean. Because this research hypothesis is directional, it requires a one-tailed test. This version of the research and corresponding null hypotheses can be summarized as follows:

Directional Hypothesis for a One Sample t-Test stating Sample Mean will be Lower

Research hypothesis	The sample mean is lower than the population mean (or hypothesized mean)	$H_A: \bar{X} < \mu$
Null hypothesis	The sample mean is not lower than (i.e. is higher than or equal to) the population mean (or hypothesized mean)	$H_0: \bar{X} \geq \mu$

These three version of the hypothesis are the broad form and would be refined to include the specific name of the variable that is under investigation when working with specific hypotheses. For example, in the nail polish bottling example, a non-directional hypothesis would be used to test whether the sample bottles were different than the intended 15 ml each if there was no reason to believe that the machine was over-filling or under-filling, specifically. This version of the research and corresponding null hypotheses can be summarized as follows:

Specific, Non-Directional Hypothesis for a One Sample t-Test

Research hypothesis	The sample mean volume of nail polish is not equal to 15 ml.	$H_A: \bar{X} \neq \mu$
Null hypothesis	The sample mean volume of nail polish is equal to 15 ml.	$H_0: \bar{X} = \mu$

In this example, the owner and/or manufacturer is testing the research hypothesis but is likely hoping that, in fact, the null hypothesis is retained. This is because they likely hope that the machine is working as intended (by depositing 15 ml per bottle). You may reasonably wonder why, then, the research hypothesis is not that the sample mean *is* equal to 15 ml instead of the null hypothesis stating this. This is because the default when testing is that there is nothing to find: there is no difference, no change, and/or no pattern. That is, the null statement that there is nothing to find (e.g. there is no difference) is assumed and is only rejected when data indicate otherwise. Formulas and the steps to hypothesis testing require that there is strong enough evidence against this to reject the null and, thus, to be able to conclude that a difference, change, or pattern likely exists. If we believed there was truly no difference to find, no data would need to be collected or tested because the null would already be accepted. However, when we are unsure or believe there is a difference, data need to be collected and tested. The default null assumption that there is nothing to find (no difference, change, or pattern) is foundational to how hypothesis testing is carried out and guarding against the risk of a Type I Error (see Chapter 6 for a review of Type I Error).