4.2: Range

We can summarize quantitative data using one of several versions of a measure of variability called the **range**. The way the range is computed depends on the nature of the variable and the intended use of the range.

The Exclusive Range

When the quantitative distance between the midpoints of the highest and lowest scores is desired, the exclusive range is used. This is a common version of the range which is often the one which is presumed used when a range is reported. The **exclusive range** refers to the difference between the highest value and the lowest value in a set of scores measured on an interval or ratio scale. These can be referred to as the maximum (or “max”) and minimum (or “min”), respectively.

The formula has two parts:

1. X_{max} which represents the highest value raw score for the variable and
2. X_{min} which represents the lowest value raw score for the variable.

There is only one step to the formula which is to subtract because the result of subtraction is a *difference*.

Look over the ages in Data Set 4.1 and try to deduce the exclusive range. Note that, unless otherwise specified, you should assume that all data sets were generated from samples rather than populations. For the age data in Data Set 4.1, the exclusive range is 33. This is found by subtracting 14 (the X_{min}) from 47 (the X_{max}).

Data Set 4.1

Age
47
46
42
39
36
34
33
33
32
29
29
29
28
27
25
23
20
19
19
18
16
14

Exclusive Range

Formula	Calculation
$\mathrm{ER} = X_{\max} - X_{\min}$	$ER = 47 - 14$ $ER = 33$

The Inclusive Range

The **inclusive rage** refers to the difference between the highest value and one less than the lowest value in a set of scores measured on a quantitative scale of measurement. This version is generally useful when working with discrete values rather than continuous and can be applied for many forms of ordinal data as well as with data measured on the ratio or interval scales. This is because the inclusive range emphasized how many numbers are in the range, *including* the lowest value observed among the raw scores. Compare this to the exclusive range which *excludes* the lowest raw score and everything below it by subtracting it. In contrast, the inclusive range is only trying to remove everything below the lowest raw score while retaining the lowest score. Therefore, inclusive range is always one unit higher than the exclusive range.

The inclusive range is essentially showing how many numbers exist in the range. It could be applied to the data for ages for which the inclusive range would be 34. However, it is rarely used for a variable like this. Instead, you may see it used to identify how many score categories were in the range observed. Consider a quiz with 10 questions, each worth 1 point if correct and 0 if incorrect. In this example, there are 11 possible total scores an individual can get on the quiz: 10, 9, 8, 7, 6, 5, 4, 3, 2, 1 and 0. If someone in the sample gets all 10 items correct (earning then 10 points) and another person get 0 items correct (earning them 0 points), the inclusive range will be 11 to represent all 11 possible scores in the range of observations, including the lowest score (which in this example is a 0).

Being specific about the exact statistic or formula used can reduce misconceptions and reduce the potential for unintentional misinformation. When the inclusive range is used it can be particularly important to state its full name because it is rarely used and, thus, readers may assume the more commonly used exclusive range is being reported when a statistician simple refers to their result as “the range.” Different statistics are more or less common in different fields and specialties. Therefore, when a vague name is provided, a reader may incorrectly assume the term refers to the one they are most familiar with because it most easily comes to mind. In fact, we will see many terms, concepts, and statistics (both descriptive and inferential) with similar names. Thus, providing the full names for things helps ensure that information is accurately conveyed by minimizing opportunities for assumptions and misunderstanding.

Inclusive Range

Formula	Calculation
$\mathrm{IR} = X_{\max} - (X_{\min} - 1)$	$IR = 47 - (14 - 1)$ $IR = 47 - 13$ $IR = 34$

Note

Here $X_{\max}$ refers to the highest raw score in the data set and $X_{\min}$ refers to the lowest.

Alternatively, some prefer to use this version of the inclusive range formula:

$$\mathrm{IR} = X_{\max} - X_{\min} + 1$$

This is just another way to write the same formula which was shown before and both versions will always yield the same result. Some may prefer this version (which finds the exclusive range and then adds the lowest value back in via the + 1) because the order of operations is simpler. These two are both acceptable ways to compute IR. These versions of the inclusive range is appropriate for use with variables that are not quantitative and discontinuous (discrete) in nature.

The Real Limit Range

Another option for computing the range focuses on the real score limits. This is similar to the inclusive range but is a conceptually better fit for variables that are quantitative and continuous in nature. **Real limits** refer to the boundaries used to differentiate scores which are important for variables that are continuous in nature. When something continuous is measured, the precision of the quantity yielded is limited by the precision of the measurement instrument and operationalization of the variable. What essentially must happen when something continuous is measured is that it is cut off or rounded to some level of specificity. When something is truly continuous, the midpoints between possible scores yielded by the measurement tool used are the real score limits. For example, IQ scores are presented in whole numbers (rounded to the ones place) but the theoretical nature of intellectual functioning, if quantifiable, is presumed to be continuous. Therefore, an IQ test can yield scores of 99, 100, or 101, for example, each of which is actually functioning as a little interval rather than a single point on the number line. Theoretical IQ scores of 99.5 to $100.4\overline{9}$ (or 100.50 when rounded to the hundredths place) would all be represented as 100. Thus, the real lower limit of 100 is 99.5 and the real upper limit of 100 is $100.4\overline{9}$.

Visualizing Real Score Limits

*The real score limit of each value extends to the midpoint between it and each adjacent number (i.e. the values which would round to each whole number). This represents the real score limit range that each whole number represents when used to represent a continuous variable.

Raw Score	97	98	99	100	101	102	103
Real Score Limit of the Raw Score	96.50-$97.4\overline{9}$	97.50-$98.4\overline{9}$	98.50-$99.4\overline{9}$	99.50-$100.4\overline{9}$	100.50-$101.4\overline{9}$	101.50-$102.4\overline{9}$	102.50-$103.4\overline{9}$

We will refer to this version as the Real Limit Range to distinguish it from the exclusive range, though this is not a name that is often used in practice. The **real score limit range** refers to the difference between the upper-score limit of the highest value and the lower-score limit of lowest value in a set of scores measured on an interval or ratio scale when the variable is continuous in nature. These can be referred to as the upper real limit (URL) and lower real limit (or LRL), respectively.

The formula has two parts:

1. maxURL which represents the upper real limit of the highest value raw score for the variable and
2. minLRL which represents the lower real score limit of the lowest value raw score for the variable.

There is only one step to the formula which is to subtract because the result of subtraction is a *difference.*

This version of the range is operationally similar to the exclusive range (because each identifies boundaries of data and subtracts lower from higher) but yields results which are generally the same as the inclusive range. Essentially, the real score limit range is to data for variables which are presumed or known to be continuous what the inclusive range is to data for variables which are presumed or known to be discrete (discontinuous).

Review the dataset for the variable age (Data Set 4.1). The inclusive range is found by subtracting the lowest number in the set (which is 14) from the highest number in the set (which is 47) and then adding 1. For our age data the inclusive range is 34. Now look at the computations for the real score limit range (RSLR) and notice that, though they are conceptualized and computed differently, the RSLR and IR yielded the same result for Data Set 4.1.

Real Score Limit Range

Formula	Calculation
$\text{RSLR} = X_{\max URL} - (X_{\min LRL})$	$\text{RSLR} = 47.50 - 13.50$ $\text{RSLR} = 34.00$

Note

Because decimals were introduced in the steps, it can be appropriate to round and show the answer to the hundredths place.

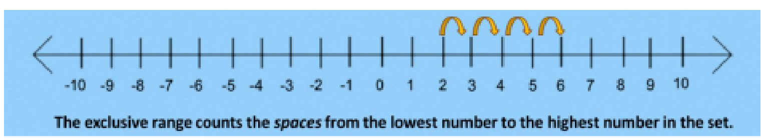
Comparing Ranges

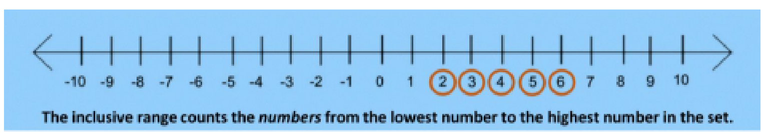
Think about the difference between the inclusive range and exclusive range using the number line. The exclusive range refers to how many spaces we move on the number line to get from the lowest number in the dataset to the highest number. When we move on the number line, the exclusive range is counting how many spaces (also called "units") we have moved. The inclusive range, however, is asking how many numbers are included from the lowest number to the highest number. When we move on the number line, the inclusive range is counting each number we touch, rather than the spaces between them. Therefore, the inclusive range adds one so that it *includes* the lowest number and the highest number that exist in the dataset.

Let’s take another example to help us understand the difference between exclusive and inclusive range. Suppose we gave a group of 10 students a 10-point quiz where the lowest score possible was 0 and the highest score possible was 10 (see the example data to the right). The scores in the dataset reflect the actual scores earned. The exclusive and inclusive ranges are each calculated using the scores earned, NOT the scores that were possible. Therefore, the range of possible scores, the exclusive range, and the inclusive range can all be different.

Quiz Scores	
5	6
3	6
4	2
2	5
3	6

The dataset includes scores of 2s, 3s, 4s, 5s, and 6s. Therefore, not all of the possible scores were earned. The exclusive range is calculated as: 6 – 2 = 4. This means that the highest score was four units from the lowest score. However, this does not reflect how many different scores were in the range because there were five different scores within the range (i.e. scores of 2, 3, 4, 5, and 6 are all in the inclusive range from 2 to 6). If we want to reflect how many numbers were in the range, we use the inclusive range. The inclusive range is calculated as: 6 – (2 –1) = 6 – 1 = 5. This means there were five numbers in the range, including the lowest and highest numbers in the range.





Reading Review 4.1

1. Which ranges can be used with quantitative data?
2. What distinguishes an exclusive range from an inclusive range?
3. How is the upper real score limit of a range defined?