3.3: Ranking

Along with the center and the variability, another useful numerical measure is the ranking of a number. A **percentile** is a measure of ranking. It represents a location measurement of a data value to the rest of the values. Many standardized tests give the results as a percentile. Doctors also use percentiles to track a child's growth.

The **kth percentile** is the data value that has k% of the data at or below that value.

Example 3.3.1 interpreting percentile

a. What does a score of the 90th percentile mean?
b. What does a score of the 70th percentile mean?

Solution

a. This means that 90% of the scores were at or below this score. (A person did the same as or better than 90% of the test takers.)
b. This means that 70% of the scores were at or below this score.

Example 3.3.2 percentile versus score

If the test was out of 100 points and you scored at the 80th percentile, what was your score on the test?

Solution

You don't know! All you know is that you scored the same as or better than 80% of the people who took the test. If all the scores were really low, you could have still failed the test. On the other hand, if many of the scores were high you could have gotten a 95% or so.

There are special percentiles called quartiles. Quartiles are numbers that divide the data into fourths. One fourth (or a quarter) of the data falls between consecutive quartiles.

Definition 3.3.1

To find the quartiles:

1. Sort the data in increasing order.
2. Find the median, this divides the data list into 2 halves.
3. Find the median of the data below the median. This value is *Q1*.
4. Find the median of the data above the median. This value is Q3.
   Ignore the median in both calculations for *Q1* and *Q3*

If you record the quartiles together with the maximum and minimum you have five numbers. This is known as the five-number summary. The five-number summary consists of the minimum, the first quartile (*Q1*), the median, the third quartile (*Q3*), and the maximum (in that order).

The interquartile range, *IQR*, is the difference between the first and third quartiles, *Q1* and *Q3*. Half of the data (50%) falls in the interquartile range. If the *IQR* is "large" the data is spread out and if the *IQR* is "small" the data is closer together.

Definition 3.3.2

Interquartile Range (*IQR*)

$IQR = Q3 - Q1$

Determining probable outliers from IQR: *fences*

A value that is less than $Q1 - 1.5 * IQR$ (this value is often referred to as a ***low fence***) is considered an outlier.

Similarly, a value that is more than $Q3 + 1.5 * IQR$ (the ***high fence***) is considered an outlier.

A box plot (or box-and-whisker plot) is a graphical display of the five-number summary. It can be drawn vertically or horizontally. The basic format is a box from *Q1* to *Q3*, a vertical line across the box for the median and horizontal lines as whiskers extending out each end to the minimum and maximum. The minimum and maximum can be represented with dots. Don't forget to label the tick marks on the number line and give the graph a title.

An alternate form of a Box-and-Whiskers Plot, known as a modified box plot, only extends the left line to the smallest value greater than the *low fence*, and extends the left line to the largest value less than the *high fence*, and displays markers (dots, circles or asterisks) for each outlier.

If the data are *symmetrical*, then the box plot will be visibly symmetrical. If the data distribution has a left skew or a right skew, the line on that side of the box plot will be visibly long. If the plot is symmetrical, and the four quartiles are all about the same length, then the data are likely a near *uniform* distribution. If a box plot is symmetrical, and both outside lines are noticeably longer than the Q1 to median and median to Q3 distance, the distribution is then probably *bell-shaped*.

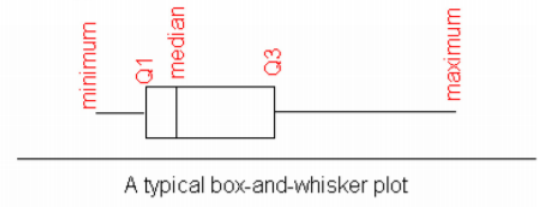


Figure 3.3.1: Typical Box Plot

Example 3.3.3 five-number summary for an even number of data points

The total assets in billions of Australian dollars (AUD) of Australian banks for the year 2012 are given in Example 3.3.1 ("Reserve bank of," 2013). Find the five-number summary and the interquartile range (IQR), and draw a box-and-whiskers plot.

Table 3.3.1: Total Assets (in billions of AUD) of Australian Banks

2855	2862	2861	2884	3014	2965
2971	3002	3032	2950	2967	2964

Solution

Variable: $x =$ total assets of Australian banks

First sort the data.

Table 3.3.2: Sorted Data for Total Assets

2855	2861	2862	2884	2950	2964	2965	2967	2971	3002	3014	3032

The minimum is 2855 billion AUD and the maximum is 3032 billion AUD.

There are 12 data points so the median is the average of the 6th and 7th numbers.

2855	2861	2862	2884	2950	2964	2965	2967	2971	3002	3014	3032

$$\frac{2964+2965}{2} = 2964.5 \text{ billion AUD}$$

Table 3.3.3: *Sorted Data for Total Assets with Median*

To find *QI*, find the median of the first half of the list.

2855	2861	2862	2884	2950	2964

Q1

$$Q1 = \frac{2862 + 2884}{2} = 2873 \text{ billion AUD}$$

Table 3.3.4: *Finding QI*

To find *Q3*, find the median of the second half of the list.

2965	2967	2971	3002	3014	3032

Q3

$$Q3 = \frac{2971 + 3002}{2} = 2986.5 \text{ billion AUD}$$

Table 3.3.5: *Finding Q3*

The five-number summary is (all numbers in billion AUD)

Minimum: 2855

Q1: 2873

Median: 2964.5

Q3: 2986.5

Maximum: 3032

To find the interquartile range, *IQR*, find *Q3-Q1*

IQR = 2986.5 - 2873 = 113.5 billion AUD

This tells you the middle 50% of assets were within 113.5 billion AUD of each other.

You can use the five-number summary to draw the box-and-whiskers plot.

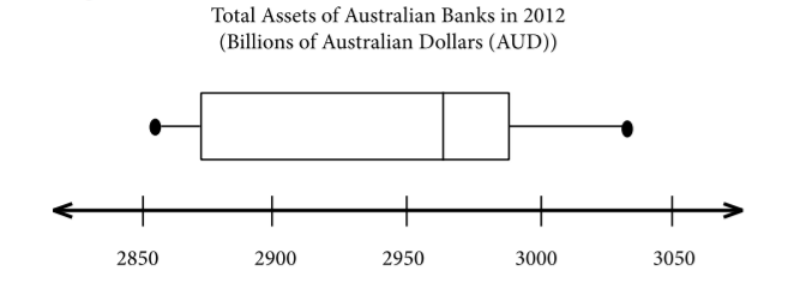


Figure 3.3.1: Box Plot of Total Assets of Australian Banks

The distribution is skewed right because the right tail is longer.

Example 3.3.4 five-number summary for an odd number of data points

The life expectancy for a person living in one of 11 countries in the region of South East Asia in 2012 is given below ("Life expectancy in," 2013). Find the five-number summary for the data and the *IQR*, then draw a box-and-whiskers plot.

Table 3.3.6: Life Expectancy of a Person Living in South-East Asia

70	67	69	65	69	77
65	68	75	74	64	

Solution

Variable: x = life expectancy of a person.

Sort the data first.

Table 3.3.7: Sorted Life Expectancies

64	65	65	67	68	69	69	70	74	75	77

The minimum is 64 years and the maximum is 77 years.

There are 11 data points so the median is the 6th number in the list.

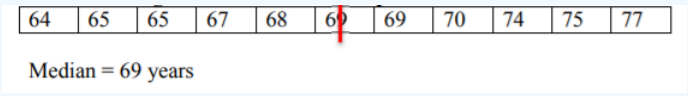


Table 3.3.8: *Finding the Median of Life Expectancies*

Finding the *Q1* and *Q3* you need to find the median of the numbers below the median and above the median. The median is not included in either calculation.

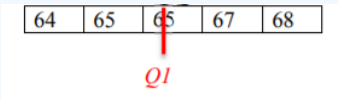


Table 3.3.9: *Finding Q1*

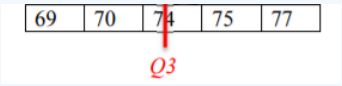


Table 3.3.10: *Finding Q3*

Q1=65 years and *Q3*=74 years

The five-number summary is (in years)

Minimum: 64

Q1: 65

Median: 69

Q3: 74

Maximum: 77

To find the interquartile range (*IQR*)

$$IQR = Q3 - Q1 = 74 - 65 = 9 \text{ years}$$

The middle 50% of life expectancies are within 9 years.

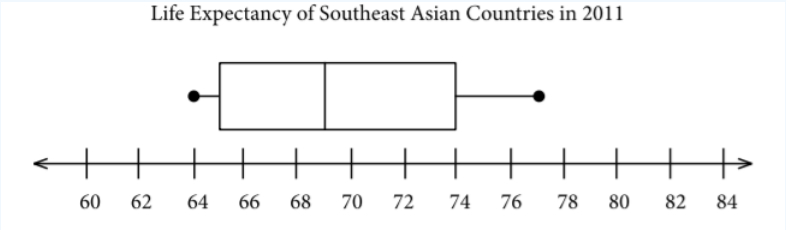


Figure 3.3.2: Box Plot of Life Expectancy

This distribution looks somewhat skewed right, since the whisker is longer on the right. However, it could be considered almost symmetric too since the box looks somewhat symmetric.

You can draw 2 box plots side by side (or one above the other) to compare 2 samples. Since you want to compare the two data sets, make sure the box plots are on the same axes. As an example, suppose you look at the box-and-whiskers plot for life expectancy

for European countries and Southeast Asian countries.

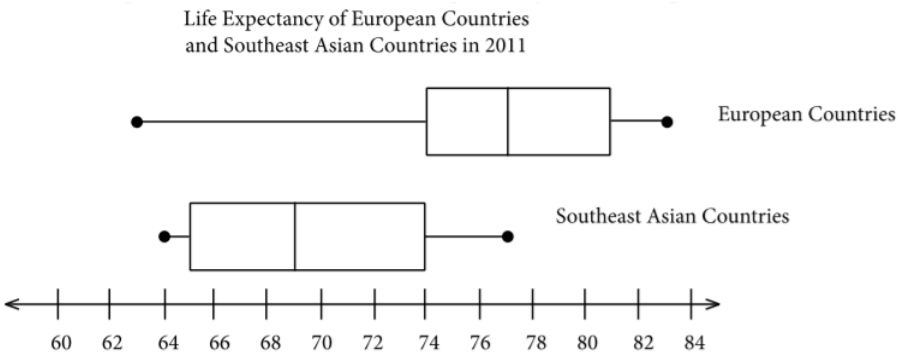


Figure 3.3.3: Box Plot of Life Expectancy of Two Regions

Looking at the box-and-whiskers plot, you will notice that the three quartiles for life expectancy are all higher for the European countries, yet the minimum life expectancy for the European countries is less than that for the Southeast Asian countries. The life expectancy for the European countries appears to be skewed left, while the life expectancies for the Southeast Asian countries appear to be more symmetric. There are of course more qualities that can be compared between the two graphs.

To find the five-number summary using R, the command is:

variable<-c(type in data with commas)
summary(variable)

This command will give you the five number summary and the mean.

For Example 3.3.4, the commands would be

expectancy<-c(70, 67, 69, 65, 69, 77, 65, 68, 75, 74, 64)
summary(expectancy)

The output would be:

Min.	Ist Qu.	Median	Mean	3rd Qu.	Max.
64.00	66.00	69.00	69.36	72.00	77.00

To draw the box plot the command is boxplot(variable, main="title you want", xlab="label you want", horizontal = TRUE). The horizontal = TRUE orients the box plot to be horizontal. If you leave that part off, the box plot will be vertical by default.

For Example 3.3.4, the command is
boxplot(expectancy, main="Life Expectancy of Southeast Asian Countries in 2011",horizontal=TRUE, xlab="Life Expectancy")

You should get the box plot in *Graph 3.3.4*.

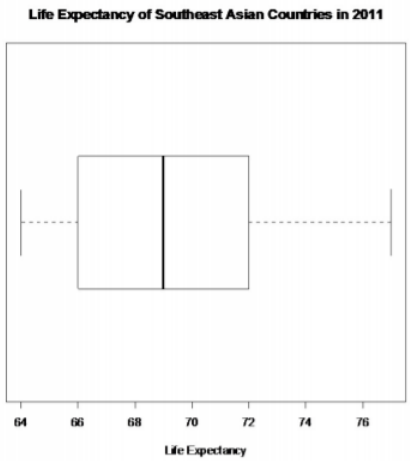


Figure 3.3.4: Box plot for Life Expectance in Southeast Asian Countries

This is known as a modified box plot. Instead of plotting the maximum and minimum, the box plot has as a lower line *Q1*-1.5**IQR* , and as an upper line, *Q3*+1.5**IQR*. Any values below the lower line or above the upper line are considered outliers. Outliers are plotted as dots on the modified box plot. This data set does not have any outliers.

Example 3.3.5 putting it all together

A random sample was collected on the health expenditures (as a % of GDP) of countries around the world. The data is in Example 3.3.11. Using graphical and numerical descriptive statistics, analyze the data and use it to predict the health expenditures of all countries in the world.

Table 3.3.11: Health Expenditures as a Percentage of GDP

3.35	5.94	10.64	5.24	3.79	5.65	7.66	7.38	5.87	11.15
5.96	4.78	7.75	2.72	9.50	7.69	10.05	11.96	8.18	6.74
5.89	6.20	5.98	8.83	6.78	6.66	9.45	5.41	5.16	8.55

Solution

First, it might be useful to look at a visualization of the data, so create a histogram.

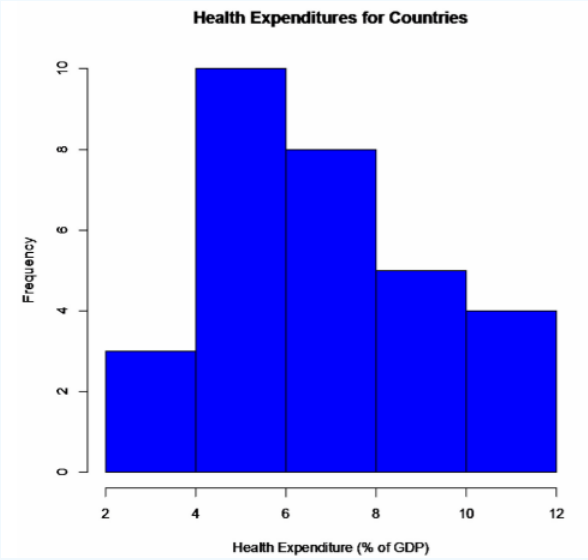


Figure 3.3.5: Histogram of Health Expenditure

From the graph, the data appears to be somewhat skewed right. So there are some countries that spend more on health based on a percentage of GDP than other countries, but the majority of countries appear to spend around 4 to 8% of their GDP on health.

Numerical descriptions might also be useful. Using technology, the mean is 7.03%, the standard deviation is 2.27%, and the five-number summary is minimum = 2.72%, Q1 = 5.71%, median = 6.70%, Q3 = 8.46%, and maximum = 11.96%. To visualize the five-number summary, create a box plot.

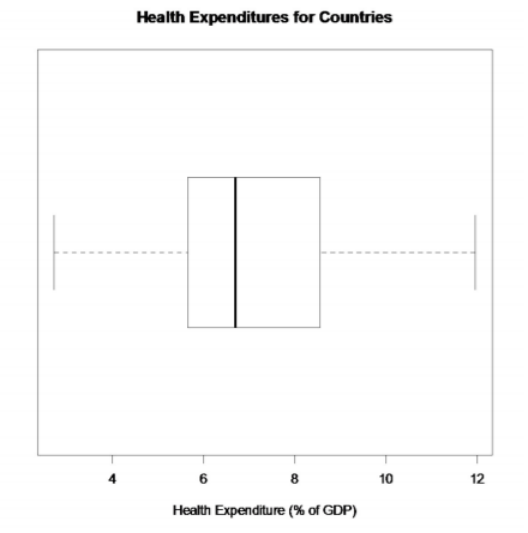


Figure 3.3.6: Box Plot of Health Expenditure

So it appears that countries spend on average about 7% of their GPD on health. The spread is somewhat low, since the standard deviation is fairly small, which means that the data is fairly consistent. The five-number summary confirms that the data is slightly skewed right. The box plot shows that there are no outliers. So from all of this information, one could say that countries spend a small percentage of their GDP on health and that most countries spend around the same amount. There doesn't appear to be any country that spends much more than other countries or much less than other countries.

Homework

Exercise 3.3.1

1. Suppose you take a standardized test and you are in the 10th percentile. What does this percentile mean? Can you say that you failed the test? Explain.
2. Suppose your child takes a standardized test in mathematics and scores in the 96th percentile. What does this percentile mean? Can you say your child passed the test? Explain.
3. Suppose your child is in the 83rd percentile in height and 24th percentile in weight. Describe what this tells you about your child's stature.
4. Suppose your work evaluates the employees and places them on a percentile ranking. If your evaluation is in the 65th percentile, do you think you are working hard enough? Explain.
5. Cholesterol levels were collected from patients two days after they had a heart attack (Ryan, Joiner & Ryan, Jr, 1985) and are in Example 3.3.12 Find the five-number summary and interquartile range (IQR), and draw a box-and-whiskers plot.

Table 3.3.12: Cholesterol Levels

270	236	210	142	280	272	160
220	226	242	186	266	206	318
294	282	234	224	276	282	360
310	280	278	288	288	244	236

6. The lengths (in kilometers) of rivers on the South Island of New Zealand that flow to the Pacific Ocean are listed in Example 3.3.13(Lee, 1994). Find the five-number summary and interquartile range (IQR), and draw a box-and-whiskers plot.

Table 3.3.13: Lengths of Rivers (km) Flowing to Pacific Ocean

River	Length (km)	River	Length (km)
Clarence	209	Clutha	322

River	Length (km)	River	Length (km)
Conway	48	Taieri	288
Waiau	169	Shag	72
Hurunui	169	Kakanui	64
Waipara	64	Waitaki	209
Ashley	97	Waihao	64
Waimakariri	161	Pareora	56
Selwyn	95	Rangitata	121
Rakaia	145	Ophi	80
Ashburton	90		

7. The lengths (in kilometers) of rivers on the South Island of New Zealand that flow to the Tasman Sea are listed in Example 3.3.14(Lee, 1994). Find the five-number summary and interquartile range (IQR), and draw a box-and-whiskers plot.

Table 3.3.14: Lengths of Rivers (km) Flowing to Tasman Sea

River	Length (km)	River	Length (km)
Hollyford	76	Waimea	48
Cascade	64	Motueka	108
Arawhata	68	Takaka	72
Haast	64	Aorere	72
Karangarua	37	Heaphy	35
Cook	32	Karamea	80
Waiho	32	Mokihinui	56
Whataroa	51	Buller	177
Wanganui	56	Grey	121
Waitaha	40	Taramakau	80
Hokitika	64	Arahura	56

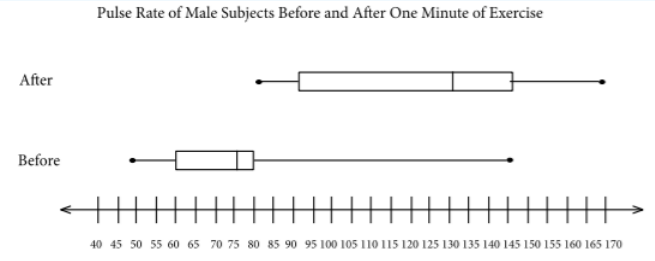
8. Eyeglassmatic manufactures eyeglasses for their retailers. They test to see how many defective lenses they made the time period of January 1 to March 31. Example 3.3.15gives the defect and the number of defects. Find the five-number summary and interquartile range (IQR), and draw a box-and-whiskers plot.

Table 3.3.15: Number of Defective Lenses

Defect type	Number of defects
Scratch	5865
Right shaped - small	4613
Flaked	1992
Wrong axis	1838
Chamfer wrong	1596
Crazing, cracks	1546

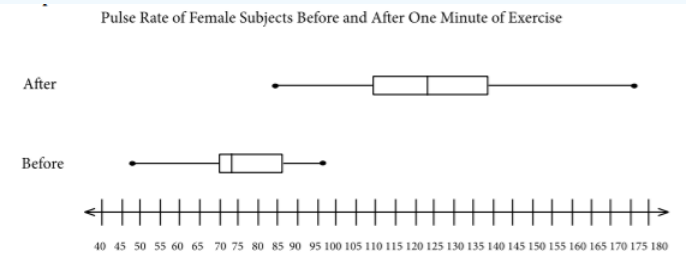
Defect type	Number of defects
Wrong shape	1485
Wrong PD	1398
Spots and bubbles	1371
Wrong height	1130
Right shape - big	1105
Lost in lab	976
Spots/bubble - intern	976

9. A study was conducted to see the effect of exercise on pulse rate. Male subjects were taken who do not smoke, but do drink. Their pulse rates were measured ("Pulse rates before," 2013). Then they ran in place for one minute and then measured their pulse rate again. *Graph 3.3.7* is of box-and-whiskers plots that were created of the before and after pulse rates. Discuss any conclusions you can make from the graphs.



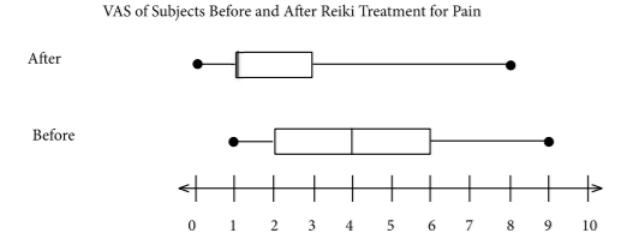
***Graph 3.3.7**: Box-and-Whiskers Plot of Pulse Rates for Males*

10. A study was conducted to see the effect of exercise on pulse rate. Female subjects were taken who do not smoke, but do drink. Their pulse rates were measured ("Pulse rates before," 2013). Then they ran in place for one minute, and after measured their pulse rate again. *Graph 3.3.8* is of box-and-whiskers plots that were created of the before and after pulse rates. Discuss any conclusions you can make from the graphs.



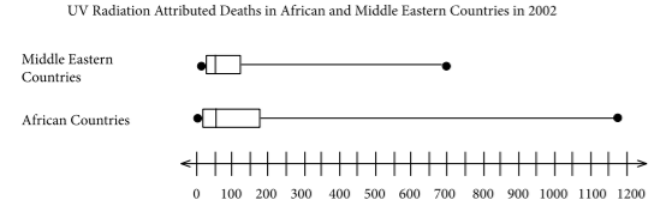
***Graph 3.3.8**: Box-and-Whiskers Plot of Pulse Rates for Females*

11. To determine if Reiki is an effective method for treating pain, a pilot study was carried out where a certified second-degree Reiki therapist provided treatment on volunteers. Pain was measured using a visual analogue scale (VAS) immediately before and after the Reiki treatment (Olson & Hanson, 1997). *Graph 3.3.9* is of box-and-whiskers plots that were created of the before and after VAS ratings. Discuss any conclusions you can make from the graphs.



***Graph 3.3.9**: Box-and-Whiskers Plot of Pain Using Reiki*

12. The number of deaths attributed to UV radiation in African countries and Middle Eastern countries in the year 2002 were collected by the World Health Organization ("UV radiation: Burden," 2013). *Graph 3.3.10* is of box-and-whiskers plots that were created of the deaths in African countries and deaths in Middle Eastern countries. Discuss any conclusions you can make from the graphs.



***Graph 3.3.10**: Box-and-Whiskers Plot of UV Radiation Deaths in Different Regions*

Answer

Note: Q1, Q3, and IQR may differ slightly due to how technology finds them.

1. See solutions

3. See solutions

5. min = 142, Q1 = 225, med = 268, Q3 = 282, max = 360, IQR = 57, see solutions

7. min = 32 km, Q1 = 46 km, med = 64 km, Q3 = 77 km, max = 177 km, IQR = 31 km, see solutions

9. See solutions

11. See solutions

Data Sources:

Annual maximums of daily rainfall in Sydney. (2013, September 25). Retrieved from http://www.statsci.org/data/oz/sydrain.html

Lee, A. (1994). *Data analysis: An introduction based on r. Auckland.* Retrieved from http://www.statsci.org/data/oz/nzrivers.html

Life expectancy in southeast Asia. (2013, September 23). Retrieved from http://apps.who.int/gho/data/node.main.688

Olson, K., & Hanson, J. (1997). Using reiki to manage pain: a preliminary report. *Cancer Prev Control*, 1(2), 108-13. Retrieved from http://www.ncbi.nlm.nih.gov/pubmed/9765732

Pulse rates before and after exercise. (2013, September 25). Retrieved from http://www.statsci.org/data/oz/ms212.html

Reserve bank of Australia. (2013, September 23). Retrieved from http://data.gov.au/dataset/banks-assets

Ryan, B. F., Joiner, B. L., & Ryan, Jr, T. A. (1985). *Cholesterol levels after heart attack*. Retrieved from http://www.statsci.org/data/general/cholest.html

Time between nerve pulses. (2013, September 25). Retrieved from http://www.statsci.org/data/general/nerve.html

Time of passages of play in rugby. (2013, September 25). Retrieved from http://www.statsci.org/data/oz/rugby.html

U.S. tornado climatology. (17, May 2013). Retrieved from www.ncdc.noaa.gov/oa/climate/...tornadoes.html

UV radiation: Burden of disease by country. (2013, September 4). Retrieved from http://apps.who.int/gho/data/node.main.165?lang=en