

6.3: Finding Probabilities for the Normal Distribution

The Empirical Rule is just an approximation and only works for certain values. What if you want to find the probability for x values that are not integer multiples of the standard deviation? The probability is the area under the curve. To find areas under the curve, you need calculus. Before technology, you needed to convert every x value to a standardized number, called the z -score or z -value or simply just z . The z -score is a measure of how many standard deviations an x value is from the mean. To convert from a normally distributed x value to a z -score, you use the following formula.

Definition 6.3.1: z -score

$$z = \frac{x - \mu}{\sigma} \quad (6.3.1)$$

where μ = mean of the population of the x value and σ = standard deviation for the population of the x value

The z -score is normally distributed, with a mean of 0 and a standard deviation of 1. It is known as the standard normal curve. Once you have the z -score, you can look up the z -score in the standard normal distribution table.

Definition 6.3.2: standard normal distribution

The **standard normal distribution**, z , has a mean of $\mu = 0$ and a standard deviation of $\sigma = 1$.

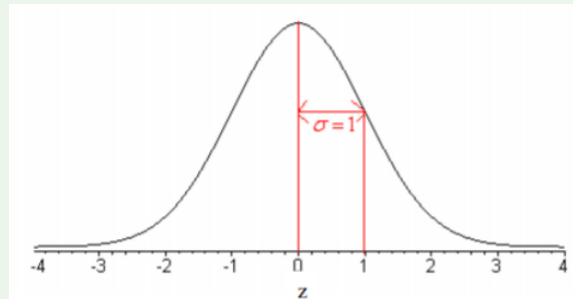


Figure 6.3.1: Standard Normal Curve

Luckily, these days technology can find probabilities for you without converting to the z -score and looking the probabilities up in a table. There are many programs available that will calculate the probability for a normal curve including Excel and the TI-83/84. There are also online sites available. The following examples show how to do the calculation on the TI-83/84 and with R. The command on the TI-83/84 is in the DISTR menu and is `normalcdf`(. You then type in the lower limit, upper limit, mean, standard deviation in that order and including the commas. The command on R to find the area to the left is `pnorm`(z -value or x -value, mean, standard deviation).

Example 6.3.1 general normal distribution

The length of a human pregnancy is normally distributed with a mean of 272 days with a standard deviation of 9 days (Bhat & Kushtagi, 2006).

- State the random variable.
- Find the probability of a pregnancy lasting more than 280 days.
- Find the probability of a pregnancy lasting less than 250 days.
- Find the probability that a pregnancy lasts between 265 and 280 days.
- Find the length of pregnancy that 10% of all pregnancies last less than.
- Suppose you meet a woman who says that she was pregnant for less than 250 days. Would this be unusual and what might you think?

Solution

- x = length of a human pregnancy
- First translate the statement into a mathematical statement.

$$P(x > 280)$$

Now, draw a picture. Remember the center of this normal curve is 272.

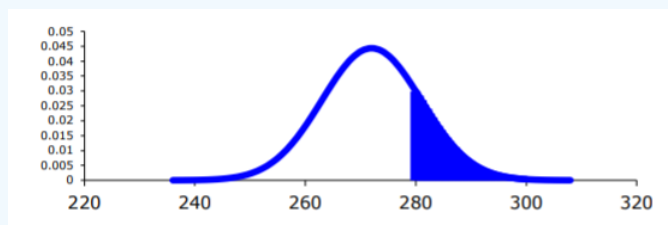


Figure for Example 6.3.1b

To find the probability on the TI-83/84, looking at the picture you realize the lower limit is 280. The upper limit is infinity. The calculator doesn't have infinity on it, so you need to put in a really big number. Some people like to put in 1000, but if you are working with numbers that are bigger than 1000, then you would have to remember to change the upper limit. The safest number to use is 1×10^{99} , which you put in the calculator as 1E99 (where E is the EE button on the calculator). The command looks like:

`normalcdf(280, 1E99, 272, 9)`

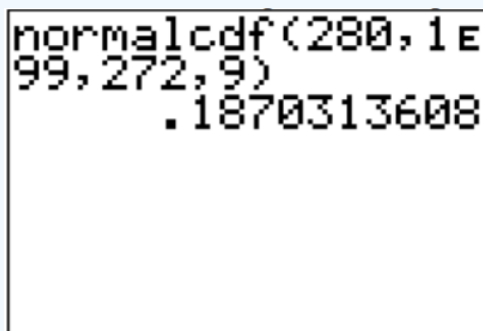


Figure 6.3.3: TI-83/84 Output for Example 6.3.1b

To find the probability on R, R always gives the probability to the left of the value. The total area under the curve is 1, so if you want the area to the right, then you find the area to the left and subtract from 1. The command looks like:

`1 - pnorm(280, 272, 9)`

Thus, $P(x > 280) \approx 0.187$

Thus 18.7% of all pregnancies last more than 280 days. This is not unusual since the probability is greater than 5%.

c. First translate the statement into a mathematical statement.

$$P(x < 250)$$

Now, draw a picture. Remember the center of this normal curve is 272.

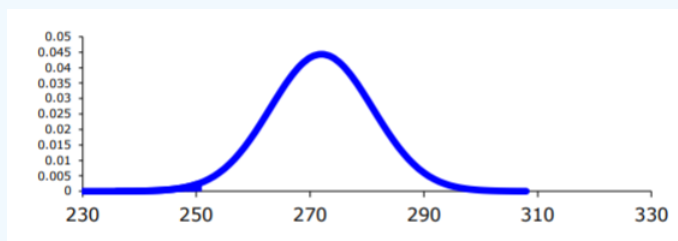


Figure for Example 6.3.1c

To find the probability on the TI-83/84, looking at the picture, though it is hard to see in this case, the lower limit is negative infinity. Again, the calculator doesn't have this on it, put in a really small number, such as $-1 \times 10^{99} = -1E99$ on the calculator.

```
normalcdf(-1E99,
250,272,9)
.0072537738
```

Figure 6.3.5: TI-83/84 Output for Example 6.3.1c

$$P(x < 250) = \text{normalcdf}(-1E99, 250, 272, 9) = 0.0073$$

To find the probability on R, R always gives the probability to the left of the value. Looking at the figure, you can see the area you want is to the left. The command looks like:

$$P(x < 250) = \text{pnorm}(250, 272, 9) = 0.0073$$

Thus 0.73% of all pregnancies last less than 250 days. This is unusual since the probability is less than 5%.

d. First translate the statement into a mathematical statement.

$$P(265 < x < 280)$$

Now, draw a picture. Remember the center of this normal curve is 272.

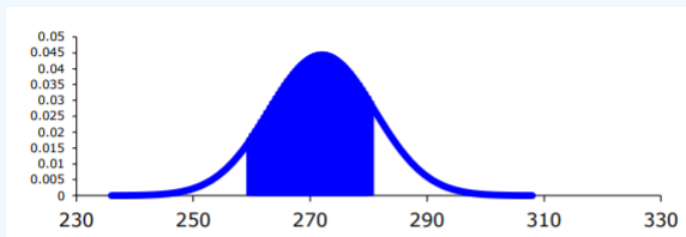


Figure for Example 6.3.1d

In this case, the lower limit is 265 and the upper limit is 280.

Using the calculator

```
normalcdf(265,280,272,9)
.5946186931
```

Figure 6.3.7: TI-83/84 Output for Example 6.3.1d

$$P(265 < x < 280) = \text{normalcdf}(265, 280, 272, 9) = 0.595$$

To use R, you have to remember that R gives you the area to the left. So $P(x < 280) = \text{pnorm}(280, 272, 9)$ is the area to the left of 280 and $P(x < 265) = \text{pnorm}(265, 272, 9)$ is the area to the left of 265. So the area is between the two would be the bigger one minus the smaller one. So, $P(265 < x < 280) = \text{pnorm}(280, 272, 9) - \text{pnorm}(265, 272, 9) = 0.595$. Thus 59.5% of all pregnancies last between 265 and 280 days.

e. This problem is asking you to find an x value from a probability. You want to find the x value that has 10% of the length of pregnancies to the left of it. On the TI-83/84, the command is in the DISTR menu and is called `invNorm`. The `invNorm` command needs the area to the left. In this case, that is the area you are given. For the command on the calculator, once you

have invNorm(on the main screen you type in the probability to the left, mean, standard deviation, in that order with the commas.

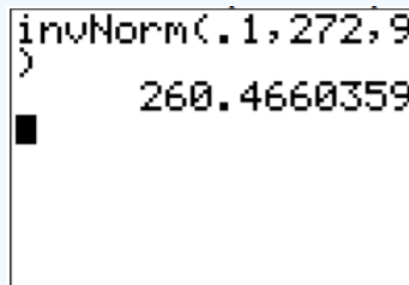


Figure 6.3.8: TI-83/84 Output for Example 6.3.1e

On R, the command is `qnorm(area to the left, mean, standard deviation)`. For this example that would be `qnorm(0.1, 272, 9)`

Thus 10% of all pregnancies last less than approximately 260 days.

f. From part (c) you found the probability that a pregnancy lasts less than 250 days is 0.73%. Since this is less than 5%, it is very unusual. You would think that either the woman had a premature baby, or that she may be wrong about when she actually became pregnant.

Example 6.3.2 general normal distribution

The mean mathematics SAT score in 2012 was 514 with a standard deviation of 117 ("Total group profile," 2012). Assume the mathematics SAT score is normally distributed.

- State the random variable.
- Find the probability that a person has a mathematics SAT score over 700.
- Find the probability that a person has a mathematics SAT score of less than 400.
- Find the probability that a person has a mathematics SAT score between a 500 and a 650.
- Find the mathematics SAT score that represents the top 1% of all scores.

Solution

a. x = mathematics SAT score

b. First translate the statement into a mathematical statement.

$$P(x > 700)$$

Now, draw a picture. Remember the center of this normal curve is 514.

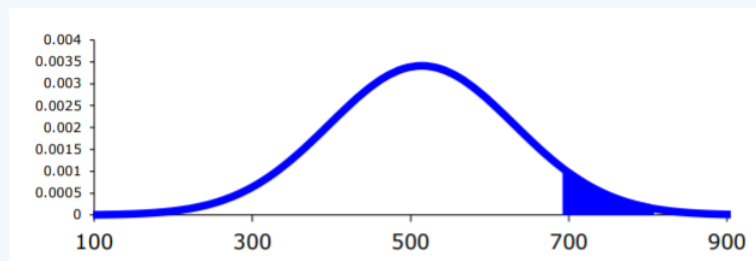


Figure for Example 6.3.2b

On TI-83/84: $P(x > 700) = \text{normalcdf}(700, 1E99, 514, 117) \approx 0.056$

On R: $P(x > 700) = 1 - \text{pnorm}(700, 514, 117) \approx 0.056$

There is a 5.6% chance that a person scored above a 700 on the mathematics SAT test. This is not unusual.

c. First translate the statement into a mathematical statement.

$$P(x < 400)$$

Now, draw a picture. Remember the center of this normal curve is 514.

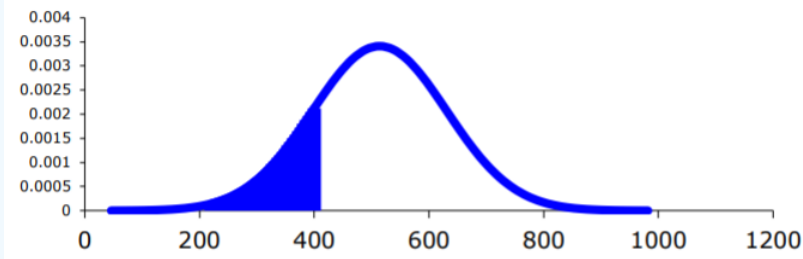


Figure for Example 6.3.2c

On TI-83/84: $P(x < 400) = \text{normalcdf}(-1E99, 400, 514, 117) \approx 0.165$

On R: $P(x < 400) = \text{pnorm}(400, 514, 117) \approx 0.165$

So, there is a 16.5% chance that a person scores less than a 400 on the mathematics part of the SAT.

d. First translate the statement into a mathematical statement.

$$P(500 < x < 650)$$

Now, draw a picture. Remember the center of this normal curve is 514.

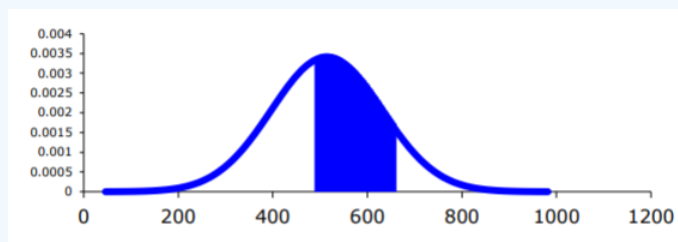


Figure for Example 6.3.2d

On TI-83/84: $P(500 < x < 650) = \text{normalcdf}(500, 650, 514, 117) \approx 0.425$

On R: $P(500 < x < 650) = \text{pnorm}(650, 514, 117) - \text{pnorm}(500, 514, 117) \approx 0.425$

So, there is a 42.5% chance that a person has a mathematical SAT score between 500 and 650.

e. This problem is asking you to find an x value from a probability. You want to find the x value that has 1% of the mathematics SAT scores to the right of it. Remember, the calculator and R always need the area to the left, you need to find the area to the left by $1 - 0.01 = 0.99$.

On TI-83/84: $\text{invNorm}(0.99, 514, 117) \approx 786$

On R: $\text{qnorm}(0.99, 514, 117) \approx 786$

So, 1% of all people who took the SAT scored over about 786 points on the mathematics SAT.

Homework

Exercise 6.3.1

- Find each of the probabilities, where z is a z -score from the standard normal distribution with mean of $\mu = 0$ and standard deviation $\sigma = 1$. Make sure you draw a picture for each problem.
 - $P(z < 2.36)$
 - $P(z > 0.67)$
 - $P(0 < z < 2.11)$
 - $P(-2.78 < z < 1.97)$
- Find the z -score corresponding to the given area. Remember, z is distributed as the standard normal distribution with mean of $\mu = 0$ and standard deviation $\sigma = 1$.
 - The area to the left of z is 15%.

- b. The area to the right of z is 65%.
 - c. The area to the left of z is 10%.
 - d. The area to the right of z is 5%.
 - e. The area between $-z$ and z is 95%. (Hint draw a picture and figure out the area to the left of the $-z$.)
 - f. The area between $-z$ and z is 99%.
3. If a random variable that is normally distributed has a mean of 25 and a standard deviation of 3, convert the given value to a z -score.
 - a. $x = 23$
 - b. $x = 33$
 - c. $x = 19$
 - d. $x = 45$
4. According to the WHO MONICA Project the mean blood pressure for people in China is 128 mmHg with a standard deviation of 23 mmHg (Kuulasmaa, Hense & Tolonen, 1998). Assume that blood pressure is normally distributed.
 - a. State the random variable.
 - b. Find the probability that a person in China has blood pressure of 135 mmHg or more.
 - c. Find the probability that a person in China has blood pressure of 141 mmHg or less.
 - d. Find the probability that a person in China has blood pressure between 120 and 125 mmHg.
 - e. Is it unusual for a person in China to have a blood pressure of 135 mmHg? Why or why not?
 - f. What blood pressure do 90% of all people in China have less than?
5. The size of fish is very important to commercial fishing. A study conducted in 2012 found the length of Atlantic cod caught in nets in Karlskrona to have a mean of 49.9 cm and a standard deviation of 3.74 cm (Ovegard, Berndt & Lunneryd, 2012). Assume the length of fish is normally distributed.
 - a. State the random variable.
 - b. Find the probability that an Atlantic cod has a length less than 52 cm.
 - c. Find the probability that an Atlantic cod has a length of more than 74 cm.
 - d. Find the probability that an Atlantic cod has a length between 40.5 and 57.5 cm.
 - e. If you found an Atlantic cod to have a length of more than 74 cm, what could you conclude?
 - f. What length are 15% of all Atlantic cod longer than?
6. The mean cholesterol levels of women age 45-59 in Ghana, Nigeria, and Seychelles is 5.1 mmol/l and the standard deviation is 1.0 mmol/l (Lawes, Hoorn, Law & Rodgers, 2004). Assume that cholesterol levels are normally distributed.
 - a. State the random variable.
 - b. Find the probability that a woman age 45-59 in Ghana, Nigeria, or Seychelles has a cholesterol level above 6.2 mmol/l (considered a high level).
 - c. Find the probability that a woman age 45-59 in Ghana, Nigeria, or Seychelles has a cholesterol level below 5.2 mmol/l (considered a normal level).
 - d. Find the probability that a woman age 45-59 in Ghana, Nigeria, or Seychelles has a cholesterol level between 5.2 and 6.2 mmol/l (considered borderline high).
 - e. If you found a woman age 45-59 in Ghana, Nigeria, or Seychelles having a cholesterol level above 6.2 mmol/l, what could you conclude?
 - f. What value do 5% of all woman ages 45-59 in Ghana, Nigeria, or Seychelles have a cholesterol level less than?
7. In the United States, males between the ages of 40 and 49 eat on average 103.1 g of fat every day with a standard deviation of 4.32 g ("What we eat," 2012). Assume that the amount of fat a person eats is normally distributed.
 - a. State the random variable.
 - b. Find the probability that a man age 40-49 in the U.S. eats more than 110 g of fat every day.
 - c. Find the probability that a man age 40-49 in the U.S. eats less than 93 g of fat every day.
 - d. Find the probability that a man age 40-49 in the U.S. eats less than 65 g of fat every day.
 - e. If you found a man age 40-49 in the U.S. who says he eats less than 65 g of fat every day, would you believe him? Why or why not?
 - f. What daily fat level do 5% of all men age 40-49 in the U.S. eat more than?

8. A dishwasher has a mean life of 12 years with an estimated standard deviation of 1.25 years ("Appliance life expectancy," 2013). Assume the life of a dishwasher is normally distributed.
 - a. State the random variable.
 - b. Find the probability that a dishwasher will last more than 15 years.
 - c. Find the probability that a dishwasher will last less than 6 years.
 - d. Find the probability that a dishwasher will last between 8 and 10 years.
 - e. If you found a dishwasher that lasted less than 6 years, would you think that you have a problem with the manufacturing process? Why or why not?
 - f. A manufacturer of dishwashers only wants to replace free of charge 5% of all dishwashers. How long should the manufacturer make the warranty period?
9. The mean starting salary for nurses is \$67,694 nationally ("Staff nurse -," 2013). The standard deviation is approximately \$10,333. Assume that the starting salary is normally distributed.
 - a. State the random variable.
 - b. Find the probability that a starting nurse will make more than \$80,000.
 - c. Find the probability that a starting nurse will make less than \$60,000.
 - d. Find the probability that a starting nurse will make between \$55,000 and \$72,000.
 - e. If a nurse made less than \$50,000, would you think the nurse was under paid? Why or why not?
 - f. What salary do 30% of all nurses make more than?
10. The mean yearly rainfall in Sydney, Australia, is about 137 mm and the standard deviation is about 69 mm ("Annual maximums of," 2013). Assume rainfall is normally distributed.
 - a. State the random variable.
 - b. Find the probability that the yearly rainfall is less than 100 mm.
 - c. Find the probability that the yearly rainfall is more than 240 mm.
 - d. Find the probability that the yearly rainfall is between 140 and 250 mm.
 - e. If a year has a rainfall less than 100mm, does that mean it is an unusually dry year? Why or why not?
 - f. What rainfall amount are 90% of all yearly rainfalls more than?

Answer

1. a. $P(z < 2.36) = 0.9909$, b. $P(z > 0.67) = 0.2514$, c. $P(0 < z < 2.11) = 0.4826$, d. $P(-2.78 < z < 1.97) = 0.9729$
3. a. -0.6667, b. -2.6667, c. -2, d. 6.6667
5. a. See solutions, b. $P(x < 52\text{cm}) = 0.7128$, c. $P(x > 74\text{cm}) = 5.852 \times 10^{-11}$, d. $P(40.5\text{cm} < x < 57.5\text{cm}) = 0.9729$ e. See solutions, f. 53.8 cm
7. a. See solutions, b. $P(x > 110\text{g}) = 0.0551$ c. $P(x < 93\text{g}) = 0.0097$, d. $P(x < 65\text{g}) \approx 0$ or 5.57×10^{-19} , e. See solutions, f. 110.2 g
9. a. See solutions, b. $P(x > \$80,000) = 0.1168$, c. $P(x > \$80,000) = 0.2283$, d. $P(\$55,000 < x < \$72,000) = 0.5519$ e. See solutions, f. \$73,112

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