

9.3: Independent Samples for Two Means

This section will look at how to analyze when two samples are collected that are independent. As with all other hypothesis tests and confidence intervals, the process is the same though the formulas and assumptions are different. The only difference with the independent t-test, as opposed to the other tests that have been done, is that there are actually two different formulas to use depending on if a particular assumption is met or not.

Hypothesis Test for Independent t-Test (2-Sample t-Test)

1. State the random variables and the parameters in words.

x_1 = random variable 1

x_2 = random variable 2

μ_1 = mean of random variable 1

μ_2 = mean of random variable 2

2. State the null and alternative hypotheses and the level of significance The normal hypotheses would be

$$H_o : \mu_1 = \mu_2 \quad \text{or} \quad H_o : \mu_1 - \mu_2 = 0$$

$$H_A : \mu_1 < \mu_2 \quad H_A : \mu_1 - \mu_2 < 0$$

$$H_A : \mu_1 > \mu_2 \quad H_A : \mu_1 - \mu_2 > 0$$

$$H_A : \mu_1 \neq \mu_2 \quad H_A : \mu_1 - \mu_2 \neq 0$$

Also, state your α level here.

3. State and check the assumptions for the hypothesis test

- a. A random sample of size n_1 is taken from population 1. A random sample of size n_2 is taken from population 2.

Note

The samples do not need to be the same size, but the test is more robust if they are.

- b. The two samples are independent.

- c. Population 1 is normally distributed. Population 2 is normally distributed. Just as before, the t-test is fairly robust to the assumption if the sample size is large. This means that if this assumption isn't met, but your sample sizes are quite large (over 30), then the results of the t-test are valid.

- d. The population variances are unknown and not assumed to be equal. The old assumption is that the variances are equal. However, this assumption is no longer an assumption that most statisticians use. This is because it isn't really realistic to assume that the variances are equal. So we will just assume the assumption of the variances being unknown and not assumed to be equal is true, and it will not be checked.

4. Find the sample statistic, test statistic, and p-value

Sample Statistic:

Calculate $\bar{x}_1, \bar{x}_2, s_1, s_2, n_1, n_2$

Test Statistic:

Since the assumption that $\sigma_1^2 = \sigma_2^2$ isn't being satisfied, then

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Usually $\mu_1 - \mu_2 = 0$, since $H_o : \mu_1 - \mu_2 = 0$

Degrees of freedom: (the Welch-Satterthwaite equation)

$$df = \frac{(A + B)^2}{\frac{A^2}{n_1 - 1} + \frac{B^2}{n_2 - 1}}$$

where $A = \frac{s_1^2}{n_1}$ and $B = \frac{s_2^2}{n_2}$

p-value:

Using the TI-83/84: tcdf(lower limit, upper limit, df)

Note

If $H_A : \mu_1 - \mu_2 < 0$, then lower limit is $-1E99$ and upper limit is your test statistic. If $H_A : \mu_1 - \mu_2 > 0$, then lower limit is your test statistic and the upper limit is $1E99$. If $H_A : \mu_1 - \mu_2 \neq 0$, then find the p-value for $H_A : \mu_1 - \mu_2 < 0$, and multiply by 2.

Using R: `pt(t, df)`

Note

If $H_A : \mu_1 - \mu_2 < 0$, then use `pt(t, df)`. If $H_A : \mu_1 - \mu_2 > 0$, then use `1 - pt(t, df)`. If $H_A : \mu_1 - \mu_2 \neq 0$, then find the p-value for $H_A : \mu_1 - \mu_2 < 0$, and multiply by 2.

5. Conclusion

This is where you write reject H_o or fail to reject H_o . The rule is: if the p-value $< \alpha$, then reject H_o . If the p-value $\geq \alpha$, then fail to reject H_o .

6. Interpretation

This is where you interpret in real world terms the conclusion to the test. The conclusion for a hypothesis test is that you either have enough evidence to show H_A is true, or you do not have enough evidence to show H_A is true.

Confidence Interval for the Difference in Means from Two Independent Samples (2 Samp T-Int)

The confidence interval for the difference in means has the same random variables and means and the same assumptions as the hypothesis test for independent samples. If you have already completed the hypothesis test, then you do not need to state them again. If you haven't completed the hypothesis test, then state the random variables and means and state and check the assumptions before completing the confidence interval step.

1. Find the sample statistic and confidence interval

Sample Statistic:

Calculate Confidence Interval: $\bar{x}_1, \bar{x}_2, s_1, s_2, n_1, n_2$

The confidence interval estimate of the difference is $\mu_1 - \mu_2$

Since the assumption that $\sigma_1^2 = \sigma_2^2$ isn't being satisfied, then

$$(\bar{x}_1 - \bar{x}_2) - E < \mu_1 - \mu_2 < (\bar{x}_1 - \bar{x}_2) + E$$

$$\text{where } E = t_c \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

where t_c is the critical value with degrees of freedom:

Degrees of freedom: (the Welch-Satterthwaite equation)

$$df = \frac{(A + B)^2}{\frac{A^2}{n_1 - 1} + \frac{B^2}{n_2 - 1}}$$

$$\text{where } A = \frac{s_1^2}{n_1} \text{ and } B = \frac{s_2^2}{n_2}$$

2. Statistical Interpretation:

In general this looks like, "there is a C% chance that $(\bar{x}_1 - \bar{x}_2) - E < \mu_1 - \mu_2 < (\bar{x}_1 - \bar{x}_2) + E$ contains the true mean difference."

3. Real World Interpretation:

This is where you state what interval contains the true difference in means, though often you state how much more (or less) the first mean is from the second mean.

The critical value is a value from the Student's t-distribution. Since a confidence interval is found by adding and subtracting a margin of error amount from the difference in sample means, and the interval has a probability of containing the true difference in means, then you can think of this as the statement $P((\bar{x}_1 - \bar{x}_2) - E < \mu_1 - \mu_2 < (\bar{x}_1 - \bar{x}_2) + E) = C$. To find the critical value you use table A.2 in the Appendix.

How to check the assumptions of two sample t-test and confidence interval:

In order for the t-test or confidence interval to be valid, the assumptions of the test must be true. So whenever you run a t-test or confidence interval, you must make sure the assumptions are true. So you need to check them. Here is how you do this:

1. For the random sample assumption, describe how you took the two samples. Make sure your sampling technique is random for both samples.
2. For the independent assumption, describe how they are independent samples.
3. For the assumption about each population being normally distributed, remember the process of assessing normality from chapter 6. Make sure you assess each sample separately.
4. You do not need to check the equal variance assumption since it is not being assumed.

Example 9.3.1 hypothesis test for two means

The cholesterol level of patients who had heart attacks was measured two days after the heart attack. The researchers want to see if patients who have heart attacks have higher cholesterol levels over healthy people, so they also measured the cholesterol level of healthy adults who show no signs of heart disease. The data is in *Table 9.3.1* ("Cholesterol levels after," 2013). Do the data show that people who have had heart attacks have higher cholesterol levels over patients that have not had heart attacks? Test at the 1% level.

Table 9.3.1: Cholesterol Levels in mg/dL

Cholesterol Level of Heart Attack Patients	Cholesterol Level of Healthy Individual
270	196
236	232
210	200
142	242
280	206
272	178
160	184
220	198
226	160
242	182
186	182
266	198
206	182
318	238
294	198
282	188
234	166
224	204
276	182
282	178
360	212
310	164
280	230
278	186

Cholesterol Level of Heart Attack Patients	Cholesterol Level of Healthy Individual
288	162
288	182
244	218
236	170
	200
	176

1. State the random variables and the parameters in words.
2. State the null and alternative hypotheses and the level of significance.
3. State and check the assumptions for the hypothesis test.
4. Find the sample statistic, test statistic, and p-value.
5. Conclusion
6. Interpretation

Solution

1. x_1 = Cholesterol level of patients who had a heart attack

x_2 = Cholesterol level of healthy individuals

μ_1 = mean cholesterol level of patients who had a heart attack

μ_2 = mean cholesterol level of healthy individuals

2. The normal hypotheses would be

$$H_o : \mu_1 = \mu_2 \quad \text{or} \quad H_o : \mu_1 - \mu_2 = 0$$

$$H_A : \mu_1 > \mu_2 \quad H_A : \mu_1 - \mu_2 > 0$$

$$\alpha = 0.01$$

3.

- a. A random sample of 28 cholesterol levels of patients who had a heart attack is taken. A random sample of 30 cholesterol levels of healthy individuals is taken. The problem does not state if either sample was randomly selected. So this assumption may not be valid.
- b. The two samples are independent. This is because either they were dealing with patients who had heart attacks or healthy individuals.
- c. Population of all cholesterol levels of patients who had a heart attack is normally distributed. Population of all cholesterol levels of healthy individuals is normally distributed.

Patients who had heart attacks:

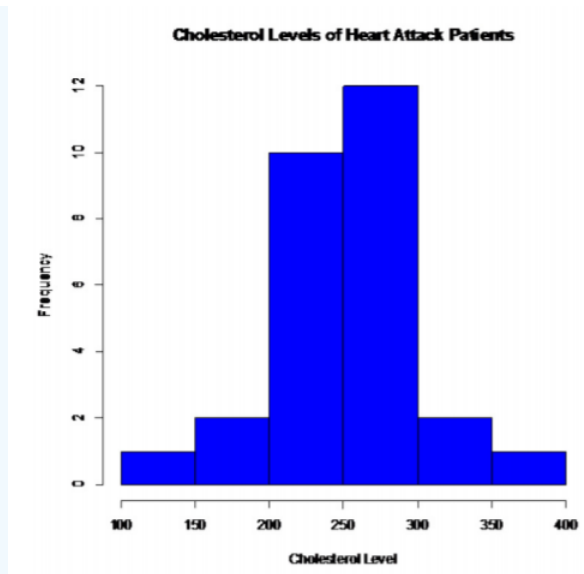


Figure 9.3.1: Histogram of Cholesterol Levels of Patients who had Heart Attacks

This looks somewhat bell shaped.

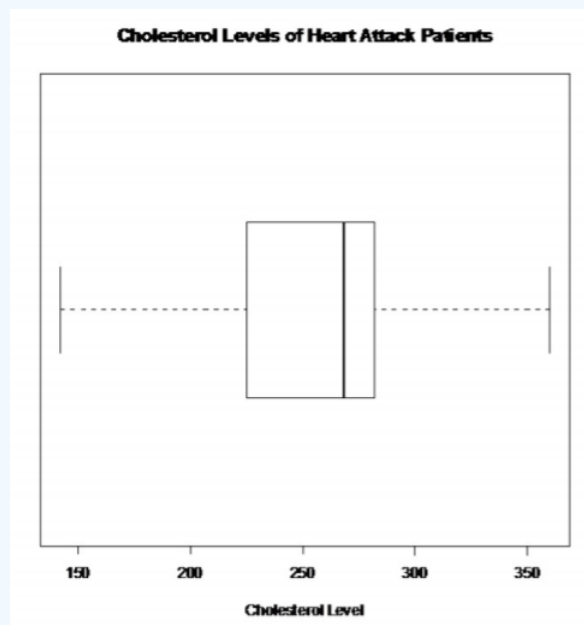


Figure 9.3.2: Modified Box Plot of Cholesterol Levels of Patients who had Heart Attacks

There are no outliers

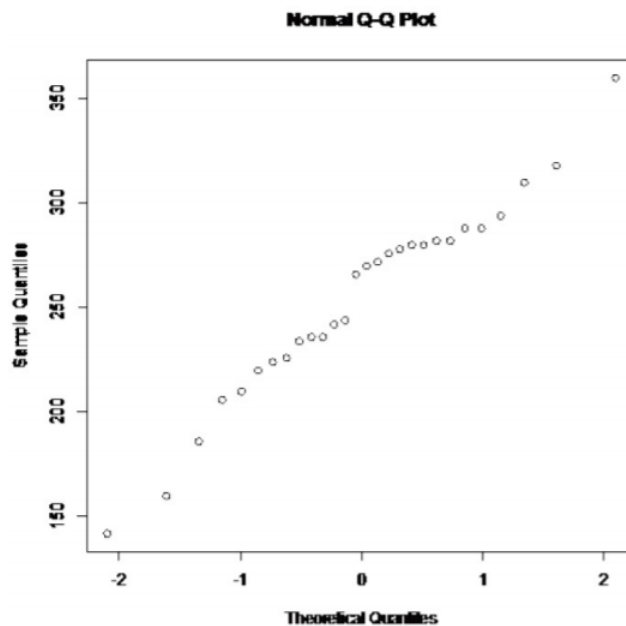


Figure 9.3.3: Normal Quantile Plot of Cholesterol Levels of Patients who had Heart Attacks

This looks somewhat linear.

So, the population of all cholesterol levels of patients who had heart attacks is probably somewhat normally distributed.

Healthy individuals:

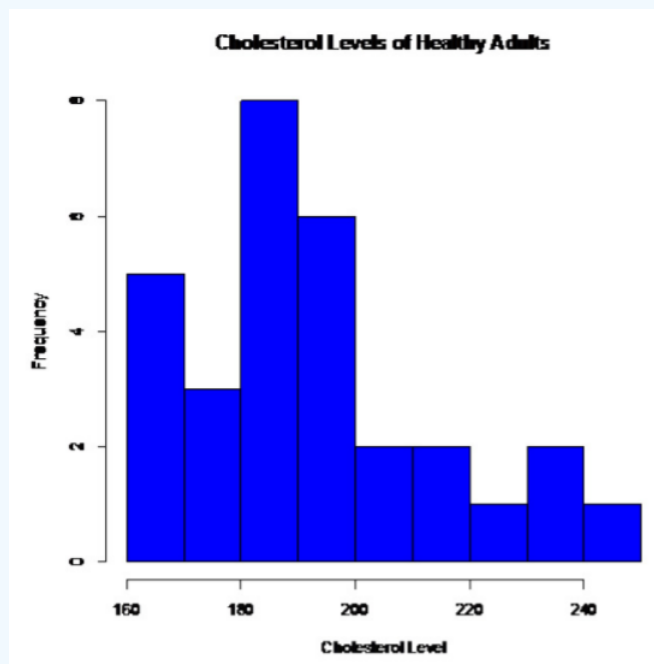


Figure 9.3.4: Histogram of Cholesterol Levels of Healthy Individuals

This does not look bell shaped.

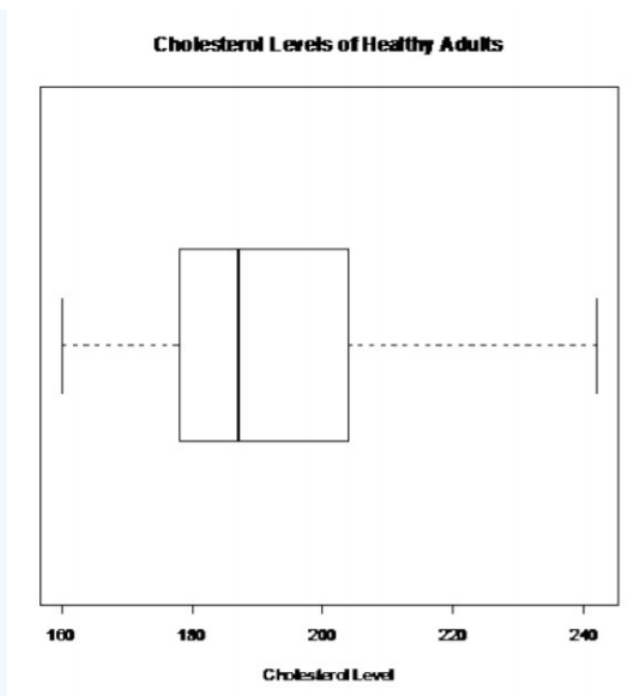


Figure 9.3.5: Modified Box Plot of Cholesterol Levels of Healthy Individuals

There are no outliers.

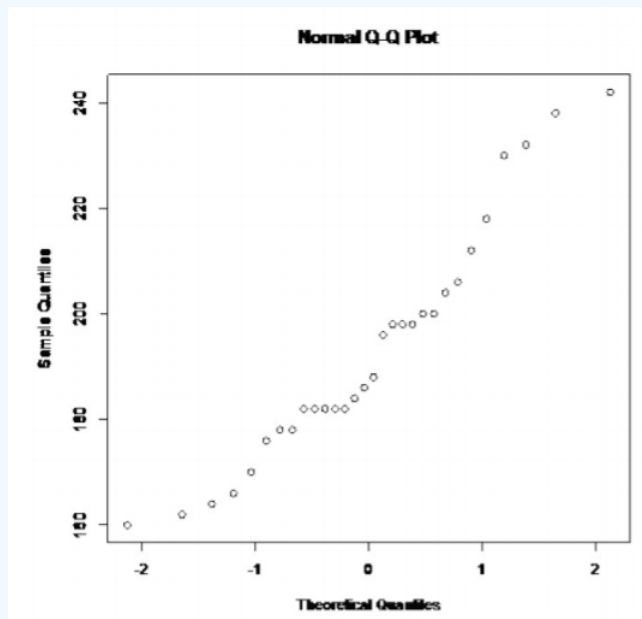


Figure 9.3.6: Normal Quantile Plot of Cholesterol Levels of Healthy Individuals

This doesn't look linear.

So, the population of all cholesterol levels of healthy individuals is probably not normally distributed.

This assumption is not valid for the second sample. Since the sample is fairly large, and the t-test is robust, it may not be an issue. However, just realize that the conclusions of the test may not be valid.

4. Sample Statistic:

$$\bar{x}_1 \approx 252.32, \bar{x}_2 \approx 193.13, s_1 \approx 47.0642, s_2 \approx 22.3000, n_1 = 28, n_2 = 30$$

Test Statistic:

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$= \frac{(252.32 - 193.13) - 0}{\sqrt{\frac{47.0642^2}{28} + \frac{22.3000^2}{30}}}$$

$$\approx 6.051$$

Degrees of freedom: (the Welch-Satterthwaite equation)

$$A = \frac{s_1^2}{n_1} = \frac{47.0642^2}{28} \approx 79.1085$$

$$B = \frac{s_2^2}{n_2} = \frac{22.3000^2}{30} \approx 16.5763$$

$$df = \frac{(A + B)^2}{\frac{A^2}{n_1 - 1} + \frac{B^2}{n_2 - 1}} = \frac{(79.1085 + 16.5763)^2}{\frac{79.1085^2}{28 - 1} + \frac{16.5763^2}{30 - 1}} \approx 37.9493$$

p-value:

Using TI-83/84: $\text{tcdf}(6.051, 1E99, 37.9493) \approx 2.44 \times 10^{-7}$

Using R: $1 - \text{pt}(6.051, 37.9493) \approx 2.44 \times 10^{-7}$

Using Technology: Using the TI-83/84:

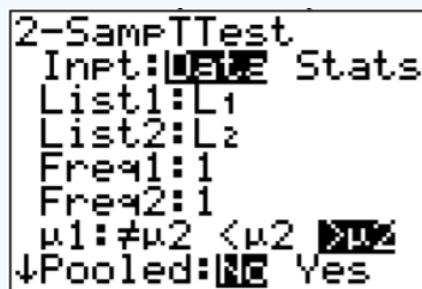


Figure 9.3.7: Setup for 2-SampTTest on TI-83/84 Calculator

Note

The Pooled question on the calculator is for whether you are assuming the variances are equal. Since this assumption is not being made, then the answer to this question is no. Pooled means that you assume the variances are equal and can pool the sample variances together.

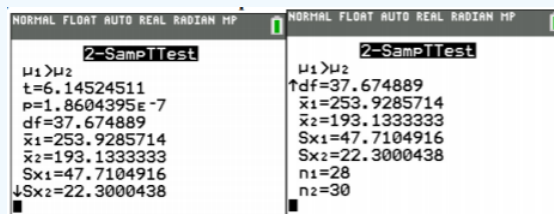


Figure 9.3.8: Results for 2-SampTTest on TI-83/84 Calculator

Using R: command in general: `t.test(variable1, variable2, alternative = "less" or "greater")`

For this example, the R command is:

`t.test(heartattack, healthy, alternative="greater")`

Welch Two Sample t-test

data: heartattack and healthy

$t = 6.1452$, $df = 37.675$, $p\text{-value} = 1.86e-07$

alternative hypothesis: true difference in means is greater than 0

95 percent confidence interval:

44.1124 Inf

sample estimates:

mean of x mean of y

253.9286 193.1333

The test statistic is $t = 6.1452$. The $p\text{-value}$ is 1.86×10^{-7}

5. Reject H_0 since the $p\text{-value} < \alpha$.

6. This is enough evidence to show that patients who have had heart attacks have higher cholesterol level on average from healthy individuals. (Though do realize that some of assumptions are not valid, so this interpretation may be invalid.)

Example 9.3.2 confidence interval for $\mu_1 - \mu_2$

The cholesterol level of patients who had heart attacks was measured two days after the heart attack. The researchers want to see if patients who have heart attacks have higher cholesterol levels over healthy people, so they also measured the cholesterol level of healthy adults who show no signs of heart disease. The data is in Example 9.3.1 ("Cholesterol levels after," 2013). Find a 99% confidence interval for the mean difference in cholesterol levels between heart attack patients and healthy individuals.

1. State the random variables and the parameters in words.
2. State and check the assumptions for the hypothesis test.
3. Find the sample statistic and confidence interval.
4. Statistical Interpretation
5. Real World Interpretation

Solution

1. These were stated in Example 9.3.1, but are reproduced here for reference.

x_1 = Cholesterol level of patients who had a heart attack

x_2 = Cholesterol level of healthy individuals

μ_1 = mean cholesterol level of patients who had a heart attack

μ_2 = mean cholesterol level of healthy individuals

2. The assumptions were stated and checked in Example 9.3.1.

3. Sample Statistic:

$\bar{x}_1 \approx 252.32$, $\bar{x}_2 \approx 193.13$, $s_1 \approx 47.0642$, $s_2 \approx 22.3000$, $n_1 = 28$, $n_2 = 30$

Test Statistic:

Degrees of freedom: (the Welch–Satterthwaite equation)

$$A = \frac{s_1^2}{n_1} = \frac{47.0642^2}{28} \approx 79.1085$$

$$B = \frac{s_2^2}{n_2} = \frac{22.3000^2}{30} \approx 16.5763$$

$$df = \frac{(A+B)^2}{\frac{A^2}{n_1-1} + \frac{B^2}{n_2-1}} = \frac{(79.1085 + 16.5763)^2}{\frac{79.1085^2}{28-1} + \frac{16.5763^2}{30-1}} \approx 37.9493$$

Since this df is not in the table, round to the nearest whole number.

$$t_c = 2.712$$

$$E = t_c \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = 2.712 \sqrt{\frac{47.0642^2}{28} + \frac{22.3000^2}{30}} \approx 26.53$$

$$(\bar{x}_1 - \bar{x}_2) - E < \mu_1 - \mu_2 < (\bar{x}_1 - \bar{x}_2) + E$$

$$(252.32 - 193.13) - 26.53 < \mu_1 - \mu_2 < (252.32 - 193.13) + 26.53$$

$$32.66\text{mg/dL} < \mu_1 - \mu_2 < 85.72\text{mg/dL}$$

Using Technology:

Using TI-83/84:

```
2-SampTInt
Inpt: DATA Stats
List1:L1
List2:L2
Freq1:1
Freq2:1
C-Level:.99
↓Pooled: NO Yes
```

Figure 9.3.9: Setup for 2-SampTInt on TI-83/84 Calculator

Note

The Pooled question on the calculator is for whether you are assuming the variances are equal. Since this assumption is not being made, then the answer to this question is no. Pooled means that you assume the variances are equal and can pool the sample variances together.

```
2-SampTInt
(32.662,85.714)
df=37.94930949
x1=252.3214286
x2=193.1333333
sx1=47.0642221
sx2=22.3000438
↓
2-SampTInt
(32.662,85.714)
↑x2=193.1333333
sx1=47.0642221
sx2=22.3000438
n1=28
n2=30
```

Figure 9.3.10: Results for 2-SampTInt on TI-83/84 Calculator

Using R: the commands is `t.test(variable1, variable2, conf.level=C)`, where C is in decimal form.

For this example, the command is

```
t.test(heartattack, healthy, conf.level=.99)
```

Output:

Welch Two Sample t-test

data: heartattack and healthy

t = 6.1452, df = 37.675, p-value = 3.721e-07

alternative hypothesis: true difference in means is not equal to 0

99 percent confidence interval:

33.95750 87.63298

sample estimates:

mean of x mean of y

253.9286 193.1333

The confidence interval is $33.96 < \mu_1 - \mu_2 < 87.63$

4. There is a 99% chance that $33.96 < \mu_1 - \mu_2 < 87.63$ contains the true difference in means.

5. The mean cholesterol level for patients who had heart attacks is anywhere from 32.66 mg/dL to 85.72 mg/dL more than the mean cholesterol level for healthy patients. (Though do realize that many of assumptions are not valid, so this interpretation may be invalid.)

If you do assume that the variances are equal, that is $\sigma_1^2 = \sigma_2^2$, then the test statistic is:

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$\text{where } s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 - 1) + (n_2 - 1)}}$$

s_p = pooled standard deviation

The Degrees of Freedom is: $df = n_1 + n_2 - 2$

The confidence interval if you do assume that $\sigma_1^2 = \sigma_2^2$ has been met, is

$$(\bar{x}_1 - \bar{x}_2) - E < \mu_1 - \mu_2 < (\bar{x}_1 - \bar{x}_2) + E$$

$$\text{where } E = t_c s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$\text{and } s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 - 1) + (n_2 - 1)}}$$

Degrees of Freedom: $df = n_1 + n_2 - 2$

t_c is the critical value where $C = 1 - \alpha$

To show that the variances are equal, just show that the ratio of your sample variances is not unusual (probability is greater than 0.05). In other words, make sure the following is true.

$P(F > s_1^2/s_2^2) \geq 0.05$ (or $P(F > s_2^2/s_1^2) \geq 0.05$ so that the larger variance is in the numerator). This probability is from an F-distribution. To find the probability on the TI-83/84 calculator use $Fcdf(s_1^2/s_2^2, 1E99, n_1 - 1, n_2 - 1)$. To find the probability on R, use $1 - pf(s_1^2/s_2^2, n_1 - 1, n_2 - 1)$.

Note

The F-distribution is very sensitive to the normal distribution. A better test for equal variances is Levene's test, though it is more complicated. It is best to do Levene's test when using statistical software (such as SPSS or Minitab) to perform the two-sample independent t-test.

Example 9.3.3 hypothesis test for two means

The amount of sodium in beef hotdogs was measured. In addition, the amount of sodium in poultry hotdogs was also measured ("SOCR 012708 id," 2013). The data is in Example 9.3.2. Is there enough evidence to show that beef has less sodium on average than poultry hotdogs? Use a 5% level of significance.

Table 9.3.2: Hotdog Data

Sodium in Beef Hotdogs	Sodium in Poultry Hotdogs
------------------------	---------------------------

Sodium in Beef Hotdogs	Sodium in Poultry Hotdogs
495	430
477	375
425	396
322	383
482	387
587	542
370	359
322	357
479	528
375	513
330	426
300	513
386	358
401	581
645	588
440	522
317	545
319	430
298	375
253	396

1. State the random variables and the parameters in words.
2. State the null and alternative hypotheses and the level of significance.
3. State and check the assumptions for the hypothesis test.
4. Find the sample statistic, test statistic, and p-value.
5. Conclusion
6. Interpretation

Solution

1. x_1 = sodium level in beef hotdogs
 x_2 = sodium level in poultry hotdogs
 μ_1 = mean sodium level in beef hotdogs
 μ_2 = mean sodium level in poultry hotdogs
2. The normal hypotheses would be
 $H_o : \mu_1 = \mu_2$ or $H_o : \mu_1 - \mu_2 = 0$
 $H_A : \mu_1 < \mu_2$ $H_A : \mu_1 - \mu_2 < 0$
 $\alpha = 0.05$
- 3.

- A random sample of 20 sodium levels in beef hotdogs is taken. A random sample of 20 sodium levels in poultry hotdogs. The problem does not state if either sample was randomly selected. So this assumption may not be valid.
- The two samples are independent since these are different types of hotdogs.
- Population of all sodium levels in beef hotdogs is normally distributed. Population of all sodium levels in poultry hotdogs is normally distributed. Beef Hotdogs:

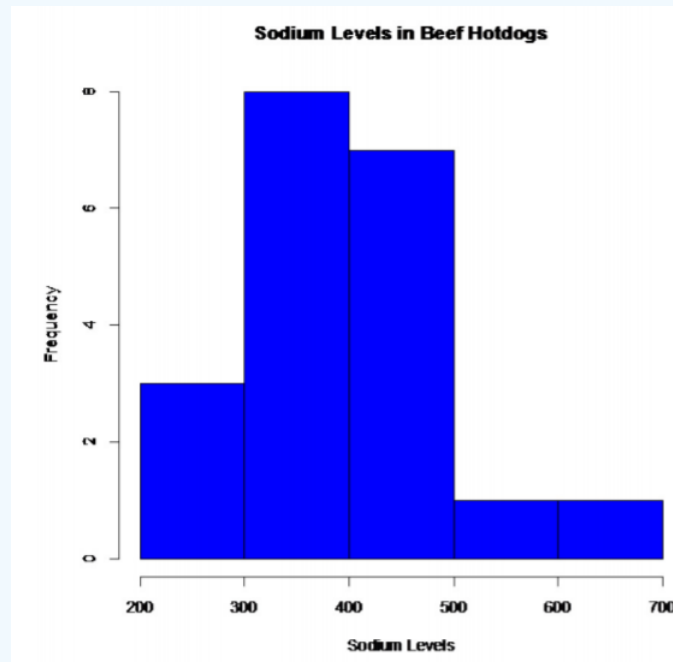


Figure 9.3.11: Histogram of Sodium Levels in Beef Hotdogs

This looks somewhat bell shaped.

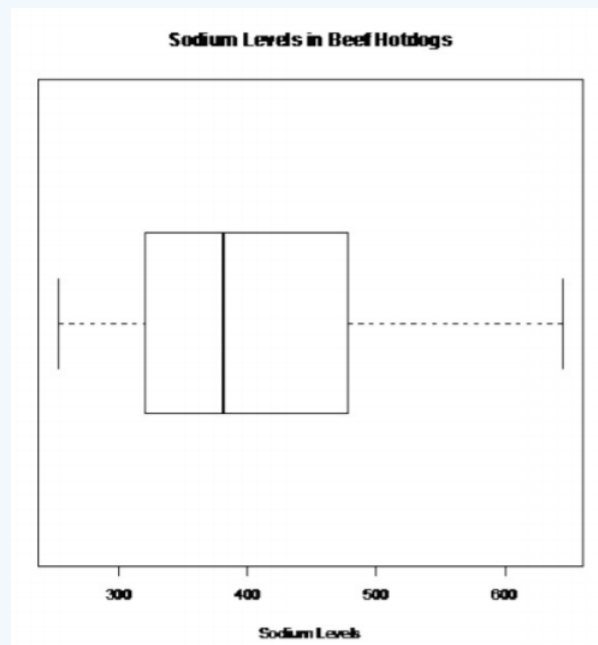


Figure 9.3.12: Modified Box Plot of Sodium Levels in Beef Hotdogs

There are no outliers.

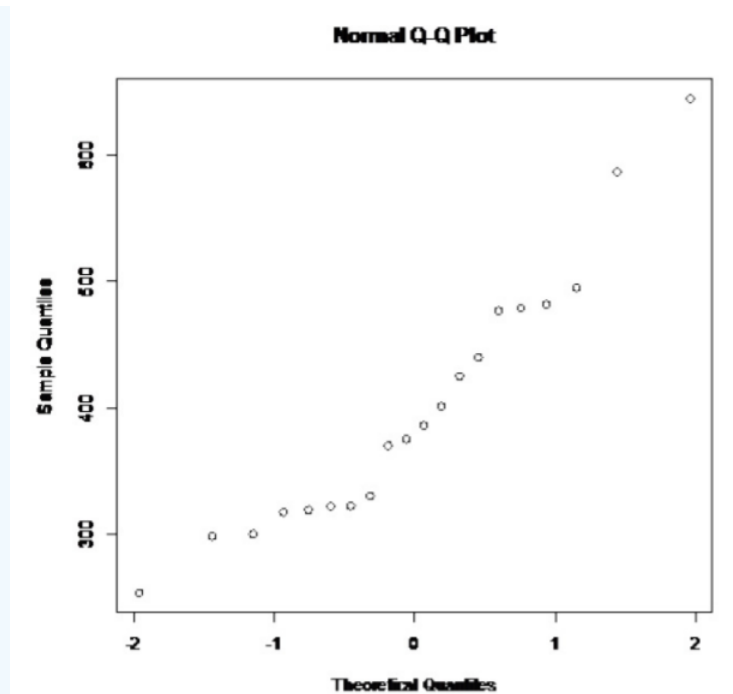


Figure 9.3.13: Normal Quantile Plot of Sodium Levels in Beef Hotdogs

This looks somewhat linear.

So, the population of all sodium levels in beef hotdogs may be normally distributed.

Poultry Hotdogs:

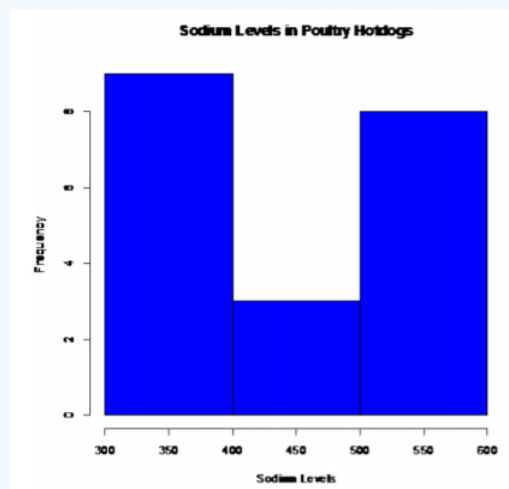


Figure 9.3.14: Histogram of Sodium Levels in Poultry Hotdogs

This does not look bell shaped.

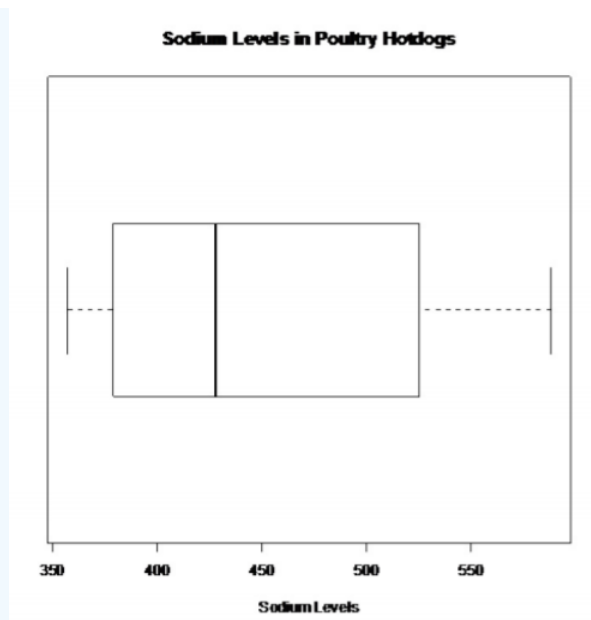


Figure 9.3.15: Modified Box Plot of Sodium Levels in Poultry Hotdogs

There are no outliers.

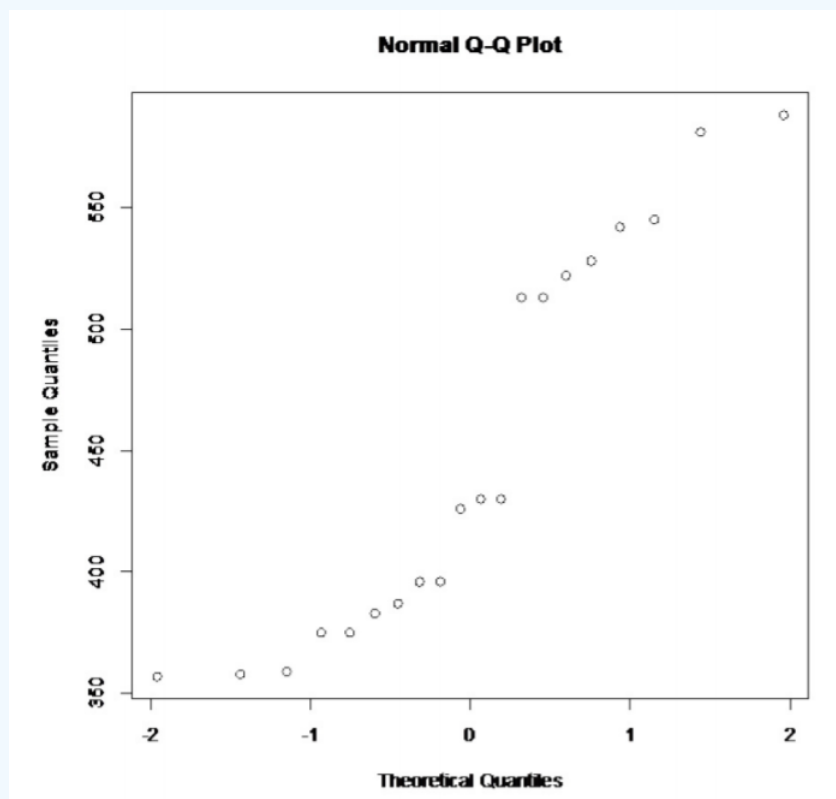


Figure 9.3.16: Normal Quantile Plot of Sodium Levels in Poultry Hotdogs

This does not look linear.

So, the population of all sodium levels in poultry hotdogs is probably not normally distributed.

This assumption is not valid. Since the samples are fairly large, and the t-test is robust, it may not be a large issue. However, just realize that the conclusions of the test may not be valid.

d. The population variances are equal, i.e. $\sigma_1^2 = \sigma_2^2$.

$$s_1 \approx 102.4347$$

$$s_2 \approx 81.1786$$

$$\frac{s_1^2}{s_2^2} = \frac{102.4347^2}{81.1786^2} \approx 1.592$$

Using TI-83/84: $\text{Fcdf}(1.592, 1E99, 19, 19) \approx 0.1597 \geq 0.05$

Using R: $1 - \text{pf}(1.592, 19, 19) \approx 0.1597 \geq 0.05$

So you can say that these variances are equal.

4. Find the sample statistic, test statistic, and p-value

Sample Statistic:

$$\bar{x}_1 = 401.15, \bar{x}_2 = 450.2, s_1 \approx 102.4347, s_2 \approx 81.1786, n_1 = 20, n_2 = 20$$

Test Statistic:

The assumption $\sigma_1^2 = \sigma_2^2$ has been met, so

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 - 1) + (n_2 - 1)}}$$

$$= \sqrt{\frac{102.4347^2 * 19 + 81.1786^2 * 19}{(20 - 1) + (20 - 1)}}$$

$$\approx 92.4198$$

Though you should try to do the calculations in the problem so you don't create round off error.

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$= \frac{(401.15 - 450.2) - 0}{92.4198 \sqrt{\frac{1}{20} + \frac{1}{20}}}$$

$$\approx -1.678$$

$$df = 20 + 20 - 2 = 38$$

p-value:

Using TI-83/84: $\text{tcdf}(-1E99, -1.678, 38) \approx 0.0508$

Using R: $\text{pt}(-1.678, 38) \approx 0.0508$

Using technology to find the t and p-value:

Using TI-83/84:

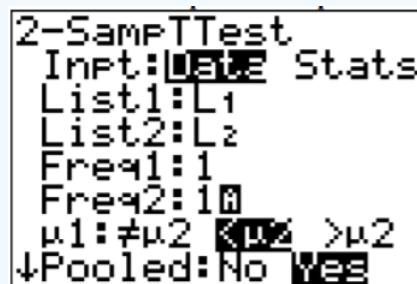


Figure 9.3.17: Setup for 2-SampTTest on TI-83/84 Calculator

Note

The Pooled question on the calculator is for whether you are using the pooled standard deviation or not. In this example, the pooled standard deviation was used since you are assuming the variances are equal. That is why the answer to the question is Yes.

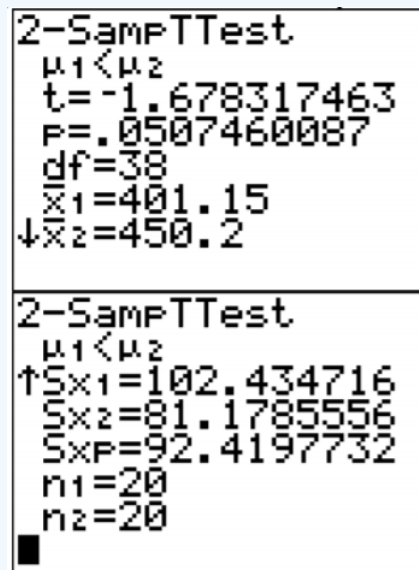


Figure 9.3.18: Results for 2-SampTTest on TI-83/84 Calculator

Using R: the command is `t.test(variable1, variable2, alternative="less" or "greater")`

For this example, the command is

```
t.test(beef, poultry, alternative="less", equalvar=TRUE)
```

Welch Two Sample t-test

data: beef and poultry

$t = -1.6783$, $df = 36.115$, $p\text{-value} = 0.05096$

alternative hypothesis: true difference in means is less than 0

95 percent confidence interval:

$-\text{Inf}$ 0.2875363

sample estimates:

mean of x mean of y

401.15 450.20

The $t = -1.6783$ and the $p\text{-value} = 0.05096$.

5. Fail to reject H_0 since the $p\text{-value} > \alpha$.

6. This is not enough evidence to show that beef hotdogs have less sodium than poultry hotdogs. (Though do realize that many of assumptions are not valid, so this interpretation may be invalid.)

Example 9.3.4 confidence interval for $\mu_1 - \mu_2$

The amount of sodium in beef hotdogs was measured. In addition, the amount of sodium in poultry hotdogs was also measured ("SOCR 012708 id," 2013). The data is in Example 9.3.2. Find a 95% confidence interval for the mean difference in sodium levels between beef and poultry hotdogs.

1. State the random variables and the parameters in words.

2. State and check the assumptions for the hypothesis test.
3. Find the sample statistic and confidence interval.
4. Statistical Interpretation
5. Real World Interpretation

Solution

1. These were stated in Example 9.3.1, but are reproduced here for reference.

x_1 = sodium level in beef hotdogs

x_2 = sodium level in poultry hotdogs

μ_1 = mean sodium level in beef hotdogs

μ_2 = mean sodium level in poultry hotdogs

2. The assumptions were stated and checked in Example 9.3.3.

3. Sample Statistic:

$\bar{x}_1 = 401.15, \bar{x}_2 = 450.2, s_1 \approx 102.4347, s_2 \approx 81.1786, n_1 = 20, n_2 = 20$

Confidence Interval:

The confidence interval estimate of the difference $\mu_1 - \mu_2$ is

The assumption $\sigma_1^2 = \sigma_2^2$ has been met, so

$$\begin{aligned}
 s_p &= \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 - 1) + (n_2 - 1)}} \\
 &= \sqrt{\frac{102.4347^2 * 19 + 81.1786^2 * 19}{(20 - 1) + (20 - 1)}} \\
 &\approx 92.4198
 \end{aligned}$$

Though you should try to do the calculations in the formula for E so you don't create round off error.

$$df = n_1 + n_2 - 2 = 20 + 20 - 2 = 38$$

$$t_c = 2.024$$

$$E = t_c s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$= 2.024(92.4198) \sqrt{\frac{1}{20} + \frac{1}{20}}$$

$$\approx 59.15$$

$$(\bar{x}_1 - \bar{x}_2) - E < \mu_1 - \mu_2 < (\bar{x}_1 - \bar{x}_2) + E$$

$$(401.15 - 450.2) - 59.15 < \mu_1 - \mu_2 < (401.15 - 450.2) + 59.15$$

$$-108.20g < \mu_1 - \mu_2 < 10.10g$$

Using technology:

Using the TI-83/84:

```

2-SampTInt
Inpt: ☒ Data Stats
List1: L1
List2: L2
Freq1: 1
Freq2: 1
C-Level: .95
↓Pooled: No ☒ Yes

```

Figure 9.3.19: Setup for 2-SampTInt on TI-83/84 Calculator

Note

The Pooled question on the calculator is for whether you are using the pooled standard deviation or not. In this example, the pooled standard deviation was used since you are assuming the variances are equal. That is why the answer to the question is Yes.

```

2-SampTInt
(-108.2, 10.114)
df=38
x̄₁=401.15
x̄₂=450.2
Sx₁=102.434716
↓Sx₂=81.1785556

2-SampTInt
(-108.2, 10.114)
↑Sx₁=102.434716
Sx₂=81.1785556
SxP=92.4197732
n₁=20
n₂=20

```

Figure 9.3.20: Results for 2-SampTInt on TI-83/84 Calculator

Using R: the command is `t.test(variable1, variable2, equalvar=TRUE, conf.level=C)`, where C is in decimal form.

For this example, the command is

```
t.test(beef, poultry, conf.level=.95, equalvar=TRUE)
```

Welch Two Sample t-test

data: beef and poultry

$t = -1.6783$, $df = 36.115$, $p\text{-value} = 0.1019$

alternative hypothesis: true difference in means is not equal to 0

95 percent confidence interval:

-108.31592 10.21592

sample estimates:

mean of x mean of y

401.15 450.20

The confidence interval is $-108.32 < \mu_1 - \mu_2 < 10.22$.

4. There is a 95% chance that $-108.20 < \mu_1 - \mu_2 < 10.10$ contains the true difference in means.

5. The mean sodium level of beef hotdogs is anywhere from 108.20 g less than the mean sodium level of poultry hotdogs to 10.10 g more. (The negative sign on the lower limit implies that the first mean is less than the second mean. The positive sign on the upper limit implies that the first mean is greater than the second mean.)

Realize that many of assumptions are not valid in this example, so the interpretation may be invalid.

Homework

Exercise 9.3.1

In each problem show all steps of the hypothesis test or confidence interval. If some of the assumptions are not met, note that the results of the test or interval may not be correct and then continue the process of the hypothesis test or confidence interval. Unless directed by your instructor, do not assume the variances are equal (except in problems 11 through 16).

1. The income of males in each state of the United States, including the District of Columbia and Puerto Rico, are given in Example 9.3.3, and the income of females is given in table #9.3.4 ("Median income of," 2013). Is there enough evidence to show that the mean income of males is more than of females? Test at the 1% level.

Table 9.3.3: Data of Income for Males

\$42,951	\$52,379	\$42,544	\$37,488	\$49,281	\$50,987	\$60,705
\$50,411	\$66,760	\$40,951	\$43,902	\$45,494	\$41,528	\$50,746
\$45,183	\$43,624	\$43,993	\$41,612	\$46,313	\$43,944	\$56,708
\$60,264	\$50,053	\$50,580	\$40,202	\$43,146	\$41,635	\$42,182
\$41,803	\$53,033	\$60,568	\$41,037	\$50,388	\$41,950	\$44,660
\$46,176	\$41,420	\$45,976	\$47,956	\$22,529	\$48,842	\$41,464
\$40,285	\$41,309	\$43,160	\$47,573	\$44,057	\$52,805	\$53,046
\$42,125	\$46,214	\$51,630				

Table 9.3.4: Data of Income for Females

\$31,862	\$40,550	\$36,048	\$30,752	\$41,817	\$40,236	\$47,476	\$40,500
\$60,332	\$33,823	\$35,438	\$37,242	\$31,238	\$39,150	\$34,023	\$33,745
\$33,269	\$32,684	\$31,844	\$34,599	\$48,748	\$46,185	\$36,931	\$40,416
\$29,548	\$33,865	\$31,067	\$33,424	\$35,484	\$41,021	\$47,155	\$32,316
\$42,113	\$33,459	\$32,462	\$35,746	\$31,274	\$36,027	\$37,089	\$22,117
\$41,412	\$31,330	\$31,329	\$33,184	\$35,301	\$32,843	\$38,177	\$40,969
\$40,993	\$29,688	\$35,890	\$34,381				

2. The income of males in each state of the United States, including the District of Columbia and Puerto Rico, are given in Example 9.3.3, and the income of females is given in Example 9.3.4 ("Median income of," 2013). Compute a 99% confidence interval for the difference in incomes between males and females in the U.S.
3. A study was conducted that measured the total brain volume (TBV) (in mm^3) of patients that had schizophrenia and patients that are considered normal. Example 9.3.5 contains the TBV of the normal patients and Example 9.3.6 contains the TBV of schizophrenia patients ("SOCR data oct2009," 2013). Is there enough evidence to show that the patients with schizophrenia have less TBV on average than a patient that is considered normal? Test at the 10% level.

Table 9.3.5: Total Brain Volume (in mm^3) of Normal Patients

1663407	1583940	1299470	1535137	1431890	1578698
---------	---------	---------	---------	---------	---------

1453510	1650348	1288971	1366346	1326402	1503005
1474790	1317156	1441045	1463498	1650207	1523045
1441636	1432033	1420416	1480171	1360810	1410213
1574808	1502702	1203344	1319737	1688990	1292641
1512571	1635918				

Table 9.3.6: Total Brain Volume (in mm^3) of Schizophrenia Patients

1331777	1487886	1066075	1297327	1499983	1861991
1368378	1476891	1443775	1337827	1658258	1588132
1690182	1569413	1177002	1387893	1483763	1688950
1563593	1317885	1420249	1363859	1238979	1286638
1325525	1588573	1476254	1648209	1354054	1354649
1636119					

- A study was conducted that measured the total brain volume (TBV) (in mm^3) of patients that had schizophrenia and patients that are considered normal. Example 9.3.5 contains the TBV of the normal patients and Example 9.3.6 contains the TBV of schizophrenia patients ("SOCR data oct2009," 2013). Compute a 90% confidence interval for the difference in TBV of normal patients and patients with Schizophrenia.
- The length of New Zealand (NZ) rivers that travel to the Pacific Ocean are given in Example 9.3.7 and the lengths of NZ rivers that travel to the Tasman Sea are given in Example 9.3.8 ("Length of NZ," 2013). Do the data provide enough evidence to show on average that the rivers that travel to the Pacific Ocean are longer than the rivers that travel to the Tasman Sea? Use a 5% level of significance.

Table 9.3.7: Lengths (in km) of NZ Rivers that Flow into the Pacific Ocean

209	48	169	138	64
97	161	95	145	90
121	80	56	64	209
64	72	288	322	

Table 9.3.8: Lengths (in km) of NZ Rivers that Flow into the Tasman Sea

76	64	68	64	37	32
32	51	56	40	64	56
80	121	177	56	80	35
72	72	108	48		

- The length of New Zealand (NZ) rivers that travel to the Pacific Ocean are given in Example 9.3.7 and the lengths of NZ rivers that travel to the Tasman Sea are given in Example 9.3.8 ("Length of NZ," 2013). Estimate the difference in mean lengths of rivers between rivers in NZ that travel to the Pacific Ocean and ones that travel to the Tasman Sea. Use a 95% confidence level.
- The number of cell phones per 100 residents in countries in Europe is given in Example 9.3.9 for the year 2010. The number of cell phones per 100 residents in countries of the Americas is given in Example 9.3.10 also for the year 2010 ("Population reference bureau," 2013). Is there enough evidence to show that the mean number of cell phones in countries of Europe is more than in countries of the Americas? Test at the 1% level.

Table 9.3.9: Number of Cell Phones per 100 Residents in Europe

100	76	100	130	75	84
112	84	138	133	118	134
126	188	129	93	64	128
124	122	109	121	127	152
96	63	99	95	151	147
123	95	67	67	118	125
110	115	140	115	141	77
98	102	102	112	118	118
54	23	121	126	47	

Table 9.3.10: Number of Cell Phones per 100 Residents in the America

158	117	106	159	53	50
78	66	88	92	42	3
150	72	86	113	50	58
70	109	37	32	85	101
75	69	55	115	95	73
86	157	100	119	81	113
87	105	96			

8. The number of cell phones per 100 residents in countries in Europe is given in Example 9.3.9 for the year 2010. The number of cell phones per 100 residents in countries of the Americas is given in Example 9.3.10 also for the year 2010 ("Population reference bureau," 2013). Find the 98% confidence interval for the difference in mean number of cell phones per 100 residents in Europe and the Americas.
9. A vitamin K shot is given to infants soon after birth. Nurses at Northbay Healthcare were involved in a study to see if how they handle the infants could reduce the pain the infants feel ("SOCR data nips," 2013). One of the measurements taken was how long, in seconds, the infant cried after being given the shot. A random sample was taken from the group that was given the shot using conventional methods (Example 9.3.11), and a random sample was taken from the group that was given the shot where the mother held the infant prior to and during the shot (Example 9.3.12). Is there enough evidence to show that infants cried less on average when they are held by their mothers than if held using conventional methods? Test at the 5% level.

Table 9.3.11: Crying Time of Infants Given Shots Using Conventional Methods

63	0	2	46	33	33
29	23	11	12	48	15
33	14	51	37	24	70
63	0	73	39	54	52
39	34	30	55	58	18

Table 9.3.12: Crying Time of Infants Given Shots Using New Methods

0	32	20	23	14	19
60	59	64	64	72	50

44	14	10	58	19	41
17	5	36	73	19	46
9	43	73	27	25	18

10. A vitamin K shot is given to infants soon after birth. Nurses at Northbay Healthcare were involved in a study to see if how they handle the infants could reduce the pain the infants feel ("SOCR data nips," 2013). One of the measurements taken was how long, in seconds, the infant cried after being given the shot. A random sample was taken from the group that was given the shot using conventional methods (Example 9.3.11), and a random sample was taken from the group that was given the shot where the mother held the infant prior to and during the shot (Example 9.3.12). Calculate a 95% confidence interval for the mean difference in mean crying time after being given a vitamin K shot between infants held using conventional methods and infants held by their mothers.
11. Redo problem 1 testing for the assumption of equal variances and then use the formula that utilizes the assumption of equal variances (follow the procedure in Example 9.3.3).
12. Redo problem 2 testing for the assumption of equal variances and then use the formula that utilizes the assumption of equal variances (follow the procedure in Example 9.3.3).
13. Redo problem 7 testing for the assumption of equal variances and then use the formula that utilizes the assumption of equal variances (follow the procedure in Example 9.3.3).
14. Redo problem 8 testing for the assumption of equal variances and then use the formula that utilizes the assumption of equal variances (follow the procedure in Example 9.3.3).
15. Redo problem 9 testing for the assumption of equal variances and then use the formula that utilizes the assumption of equal variances (follow the procedure in Example 9.3.3).
16. Redo problem 10 testing for the assumption of equal variances and then use the formula that utilizes the assumption of equal variances (follow the procedure in Example 9.3.3).

Answer

For all hypothesis tests, just the conclusion is given. For all confidence intervals, just the interval using technology is given. See solution for the entire answer.

1. Reject H_0
2. $\$65443.80 < \mu_1 - \mu_2 < \13340.80
3. Fail to reject H_0
4. $-51564.6\text{mm}^3 < \mu_1 - \mu_2 < 75656.6\text{mm}^3$
5. Reject H_0
6. $23.2818\text{km} < \mu_1 - \mu_2 < 103.67\text{km}$
7. Reject H_0
8. $4.3641 < \mu_1 - \mu_2 < 37.5276$
9. Fail to reject H_0
10. $-10.9726\text{s} < \mu_1 - \mu_2 < 11.3059\text{s}$
11. Reject H_0
12. $\$6544.98 < \mu_1 - \mu_2 < \13339.60
13. Reject H_0
14. $4.8267 < \mu_1 - \mu_2 < 37.0649$
15. Fail to reject H_0
16. $-10.9713\text{s} < \mu_1 - \mu_2 < 11.3047\text{s}$