

10.7: Paired t-Test

Here two measurements x_1 and x_2 are taken from every subject. We could say that we measure a vector $\vec{x} = [x_1 \ x_2]^T$ as the independent variable for every subject instead of just a number as the independent variable^[1]. This is a *within subject* design. Within subject designs tend to be more statistically powerful than independent or between subjects designs that have two completely different bunches of people for each variable. The extra power comes because we take the difference $D = x_1 - x_2$ for every subject. So any overall differences, or variances, in x_1 or x_2 due to individuals has been removed from the data.

The paired t -test is a *univariate test*. The difference between univariate and multivariate statistics is the the independent variables are numbers for univariate statistics and vectors for multivariate statistics. For the paired t -test, the vector is converted to a number by taking a difference. To convert vector data to difference data, make a table :

x_1	x_2	$D = x_1 - x_2$
1	2	-1
2	3	-1
3	5	-2
1	-2	3

Note here that the differences in individuals are gone after we take differences D .

The data from the D column are what you will work with. Compute \bar{D} and s_D the mean and sample standard deviation of these data. With D the procedure becomes a single sample t -test of D against zero. Specifically we can test :

Two-tailed	Left-tailed	Right-tailed
$H_0 : \mu_D = 0$	$H_0 : \mu_D \geq 0$	$H_0 : \mu_D \leq 0$
$H_1 : \mu_D \neq 0$	$H_1 : \mu_D < 0$	$H_1 : \mu_D > 0$

The test statistic is

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with $\nu = n - 1$ (for finding t_{crit}).

Example 10.7 : A Physical Education director claims that a vitamin will increase a weight lifter's strength. Eight athletes are selected and tested on how much they can bench press. They are each tested once before taking the vitamin and again after taking the vitamin for two weeks. We want to test the director's claim at $\alpha = 0.05$

The data are :

Athlete	Before(x_1)	After(x_2)	$D = x_1 - x_2$
1	210	219	-9
2	230	236	-6
3	182	179	3
4	205	204	1
5	262	270	-8
6	253	250	3
7	219	222	-3
8	216	216	0

Here we have listed the differences which is actually part of step 0 of the solution. The x_1 and x_2 columns are what you enter into SPSS as your independent variables. With SPSS you never see the differences.

Solution :

0. Data reduction.

Compute $\bar{D} = -2.375$, $s_D = 4.84$ by entering the difference data into your calculator.

1. Hypothesis.

$$H_0 : \mu_D \geq 0 \quad (10.7.1)$$

$$H_1 : \mu_D < 0 \quad (10.7.2)$$

Note that a negative difference, based on $x_1 - x_2$ (always consistently give population 1 priority if you want to stay out of trouble without thinking), indicates an increase in strength. It is important to interpret positive or negative differences correctly by thinking about what they mean.

2. Critical statistic.

Using the **t Distribution Table** with the column for one-tailed tests, $\alpha = 0.05$, and row $\nu = n - 1 = 8 - 1 = 7$, find

$$t_{\text{crit}} = -1.895 \quad (10.7.3)$$

(We added the negative sign because this is a left-tailed test.)

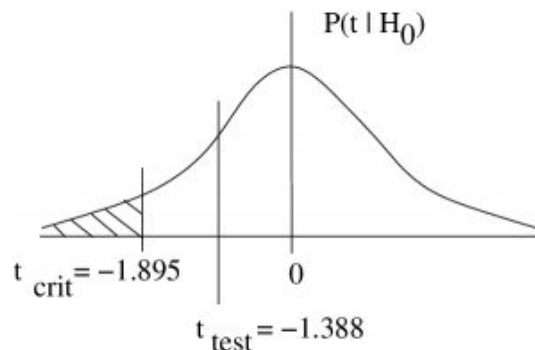
3. Test statistic.

$$t_{\text{test}} = \frac{\bar{D} - \mu_D}{\left(\frac{s_D}{\sqrt{n}}\right)} = \frac{-2.375 - 0}{\left(\frac{4.84}{\sqrt{8}}\right)} \quad (10.7.4)$$

$$t_{\text{test}} = -1.388 \quad (10.7.5)$$

To estimate the p -value, from the **t Distribution Table**, $\nu = 7$ line, find $0.10 < p < 0.25$.

4. Decision.



Do not reject H_0 . ($0.10 < p < 0.25$) $>$ ($\alpha = 0.05$).

5. Interpretation.

Under a paired t -test, at $\alpha = 0.05$, there is not enough evidence to conclude that the vitamin increases strength.

□

1. An introduction to vectors will be given in Chapter 17. [↩](#)

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