11.2: Confidence Interval for the Difference between Two Proportions

The form of the confidence interval is

$$(\hat{p}_1 - \hat{p}_2) - E < (p_1 - p_2) < (\hat{p}_1 - \hat{p}_2) + E \tag{11.2.1}$$

with

$$E = z_{\mathcal{C}} \sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}} \tag{11.2.2}$$

where, as usual you can get $z_{\mathcal{C}}$ from the last line of the **t Distribution Table**.

Example 11.2 : Using the data from Example 11.1, find the 95% confidence interval for $p_1 - p_2$.

Solution : The relevant numbers from Example 11.1 are: $n_1 = 34$, $\hat{p}_1 = 0.35$, $\hat{q}_1 = 1 - 0.35 = 0.65$ and $n_2 = 24$, $\hat{p}_2 = 0.71$, $\hat{q}_1 = 1 - 0.71 = 0.29$.

Compute (after finding $z_{95\%} = 1.96$ from the **t Distribution Table**)

$$\begin{aligned} E &= z_{95\%} \sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}} \\ E &= 1.96 \sqrt{\frac{(0.35)(0.65)}{34} + \frac{(0.71)(0.29)}{24}} \\ E &= 0.242 \end{aligned}$$

and

$$\hat{p}_1 - \hat{p}_2 = 0.35 - 0.71 = -0.36 \tag{11.2.3}$$

So

$$\begin{aligned} (\hat{p}_1 - \hat{p}_2) - E &< (p_1 - p_2) < & (\hat{p}_1 - \hat{p}_2) + E \\ -0.36 - 0.242 &< (p_1 - p_2) < & -0.36 + 0.242 \\ -0.602 &< (p_1 - p_2) < & -0.118 \end{aligned}$$

with 95% confidence. (Note that this corresponds with the rejection of H_0 in Example 11.1 since 0 is not in the confidence interval.)

$\square$