14.6: Confidence Interval for y′ at a Given x

At a fixed x (that is important to remember) the confidence interval for y is

$$y' - E < y < y' + E \tag{14.6.1}$$

where

$$E = t_{\mathcal{C}}\, s_{\text{est}} \sqrt{1 + \frac{1}{n} + \frac{n(x - \overline{x})^2}{n(\sum x^2) - (\sum x)^2}} \tag{14.6.2}$$

where, as usual, $t_{\mathcal{C}}$ comes from the ***t* Distribution Table** with $\nu = n - 2$.

Example 14.5 : Continuing from Example 14.4 (so you can see how an exam will go), say we want to predict the grade (y) in terms of a 95$\%$ confidence interval for the number of absences (x) equal to 10.

First, find the value predicted from the regression line, which we previously found to be :

$$y' = 102.493 - 3.622x \tag{14.6.3}$$

at $x = 10$. The result is

$$y' = 102.493 - 3.622(10) = 66.273 \tag{14.6.4}$$

Furthermore, from the last example, we found

$$s_{\text{est}} = 6.06 \tag{14.6.5}$$

and, from the completed data table (Example 14.3)

$$\sum x = 57 \quad \sum x^2 = 579 \tag{14.6.6}$$

We still need $t\mathcal{C}$ and $\overline{x}$. Using our sums:

$$\overline{x} = \frac{\sum x}{n} = \frac{57}{7} = 8.143 \tag{14.6.7}$$

and from ***t* Distribution Table** for the 95$\%$ confidence interval, $\nu = 7 - 2 = 5$ we get

$$t\mathcal{C} = 2.571 \tag{14.6.8}$$

Now we compute E :

$$\begin{aligned}
E &= t_{\mathcal{C}}\, s_{\text{est}} \sqrt{1 + \frac{1}{n} + \frac{n(x - \overline{x})^2}{n(\sum x^2) - (\sum x)^2}} \\
E &= (2.571)\,(6.06) \sqrt{1 + \frac{1}{7} + \frac{7(10 - 8.143)^2}{7(579) - (52)^2}} \\
E &= 15.58026 \sqrt{1 + 0.1428571 + \frac{24.139}{804}} \\
E &= 16.77
\end{aligned}$$

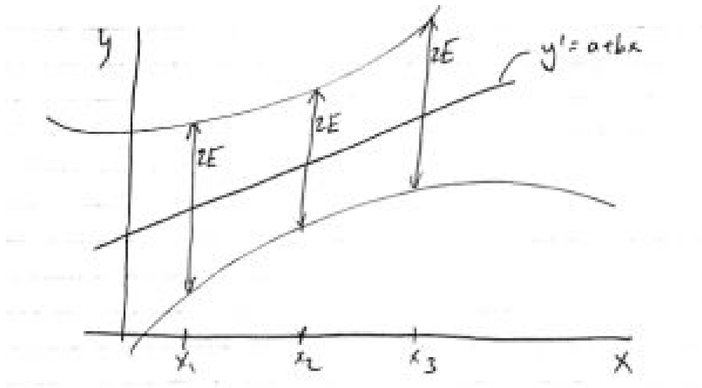
So

$$\begin{aligned}
y' - E &< y < y' + E \\
66.273 - 16.77 &< y < 66.273 + 16.77 \\
49.5 &< y < 83.0
\end{aligned}$$

This is the 95$\%$ confidence interval for predicting the mark of a person who was absent for 10 days.

□

Important: s_{set} is independent of x but E is not. So confidence intervals look like :



The reason for this variance of the width of the confidence interval comes from the uncertainty in the slope b. You can make plots like the one above in SPSS.
