10.5: Confidence Intervals for the Difference of Two Means

The form of the confidence interval is

$$(\bar{x}_1 - \bar{x}_2) - E < (\mu_1 - \mu_2) < (\bar{x}_1 - \bar{x}_2) + E \tag{10.5.1}$$

but, as with hypothesis testing, we have two cases to choose from to get the formula for E :

Case 1 : Variances of the 2 populations unequal}

$$E = t_{\mathcal{C}} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \tag{10.5.2}$$

where the degrees of freedom to use when looking up $t_{\mathcal{C}}$ in the ***t* Distribution Table** is

$$\nu = \min[(n_1 - 1), (n_2 - 1)] \tag{10.5.3}$$

Case 2 : Variances of the 2 populations equal

$$E = t_{\mathcal{C}} \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}} \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \tag{10.5.4}$$

where we use

$$\nu = n_1 + n_2 - 2 \tag{10.5.5}$$

when looking up $t_{\mathcal{C}}$.

To select the appropriate formula for E we need to do a preliminary hypothesis test on $H_0 : \sigma_1^2 = \sigma_2^2$. An odd combination of hypothesis test followed by confidence interval calculation.

Insight! By now you should have noticed that the formulae for E are just t times standard error of the mean. This whole z-transformation thing should be becoming somewhat transparent.

Example 10.6 : Find the 95% confidence interval for $\mu_1 - \mu_2$ for the data of Example 10.4 :

$s_1 = 38$	$\bar{x}_1 = 191$	$n_1 = 8$
$s_2 = 12$	$\bar{x}_2 = 199$	$n_2 = 100$

Solution :

First use F-test to see which formula to use. We did this already in Example 10.4 (the data come from that question) and found that we believed $\sigma_1^2 \neq \sigma_2^2$ with $\alpha = 0.05$.

Next, look up $t_{\mathcal{C}}$ in the ***t* Distribution Table** for 95% confidence interval for $\nu = 7$:

$$t_{95\%} = 2.365 \tag{10.5.6}$$

Compute

$$E = t_{95\%} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$
$$E = 2.365 \sqrt{\frac{38^2}{8} + \frac{12^2}{10}} = 33.01$$

So

$$\begin{aligned} (\bar{x}_1 - \bar{x}_2) - E &< \mu_1 - \mu_2 &< (\bar{x}_1 - \bar{x}_2) + E \\ (191 - 199) - 33.02 &< \mu_1 - \mu_2 &< (191 - 199) + 33.02 \\ -8 - 33.02 &< \mu_1 - \mu_2 &< -8 + 33.02 \\ -41.02 &< \mu_1 - \mu_2 &< 25.02 \end{aligned}$$

be careful of the order!

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