

6.1: The Mean and Standard Deviation of the Sample Mean

Learning Objectives

- To become familiar with the concept of the probability distribution of the sample mean.
- To understand the meaning of the formulas for the mean and standard deviation of the sample mean.

Suppose we wish to estimate the mean μ of a population. In actual practice we would typically take just one sample. Imagine however that we take sample after sample, all of the same size n , and compute the sample mean \bar{x} each time. The sample mean \bar{x} is a random variable: it varies from sample to sample in a way that cannot be predicted with certainty. We will write \bar{X} when the sample mean is thought of as a random variable, and write x for the values that it takes. The random variable \bar{X} has a mean, denoted $\mu_{\bar{X}}$, and a standard deviation, denoted $\sigma_{\bar{X}}$. Here is an example with such a small population and small sample size that we can actually write down every single sample.

✓ Example 6.1.1

A rowing team consists of four rowers who weigh 152, 156, 160, and 164 pounds. Find all possible random samples with replacement of size two and compute the sample mean for each one. Use them to find the probability distribution, the mean, and the standard deviation of the sample mean \bar{X} .

Solution

The following table shows all possible samples with replacement of size two, along with the mean of each:

Sample	Mean	Sample	Mean	Sample	Mean	Sample	Mean
152, 152	152	156, 152	154	160, 152	156	164, 152	158
152, 156	154	156, 156	156	160, 156	158	164, 156	160
152, 160	156	156, 160	158	160, 160	160	164, 160	162
152, 164	158	156, 164	160	160, 164	162	164, 164	164

The table shows that there are seven possible values of the sample mean \bar{X} . The value $\bar{x} = 152$ happens only one way (the rower weighing 152 pounds must be selected both times), as does the value $\bar{x} = 164$, but the other values happen more than one way, hence are more likely to be observed than 152 and 164 are. Since the 16 samples are equally likely, we obtain the probability distribution of the sample mean just by counting:

\bar{x}	152	154	156	158	160	162	164
$P(\bar{x})$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{4}{16}$	$\frac{3}{16}$	$\frac{2}{16}$	$\frac{1}{16}$

Now we apply the formulas from Section 4.2 to \bar{X} . For $\mu_{\bar{X}}$, we obtain.

$$\begin{aligned}
 \mu_{\bar{X}} &= \sum \bar{x}P(\bar{x}) \\
 &= 152 \left(\frac{1}{16} \right) + 154 \left(\frac{2}{16} \right) + 156 \left(\frac{3}{16} \right) + 158 \left(\frac{4}{16} \right) + 160 \left(\frac{3}{16} \right) + 162 \left(\frac{2}{16} \right) + 164 \left(\frac{1}{16} \right) \\
 &= 158
 \end{aligned}$$

For $\sigma_{\bar{X}}$, we first compute $\sum \bar{x}^2 P(\bar{x})$:

$$\sum \bar{x}^2 P(\bar{x}) = 152^2 \left(\frac{1}{16} \right) + 154^2 \left(\frac{2}{16} \right) + 156^2 \left(\frac{3}{16} \right) + 158^2 \left(\frac{4}{16} \right) + 160^2 \left(\frac{3}{16} \right) + 162^2 \left(\frac{2}{16} \right) + 164^2 \left(\frac{1}{16} \right)$$

which is 24,974 so that

$$\begin{aligned}\sigma_{\bar{x}} &= \sqrt{\sum \bar{x}^2 P(\bar{x}) - \mu_{\bar{x}}^2} \\ &= \sqrt{24,974 - 158^2} \\ &= \sqrt{10}\end{aligned}$$

The mean and standard deviation of the population $\{152, 156, 160, 164\}$ in the example are $\mu = 158$ and $\sigma = \sqrt{20}$. The mean of the sample mean \bar{X} that we have just computed is exactly the mean of the population. The standard deviation of the sample mean \bar{X} that we have just computed is the standard deviation of the population divided by the square root of the sample size: $\sqrt{10} = \sqrt{20}/\sqrt{2}$. These relationships are not coincidences, but are illustrations of the following formulas.

Definition: Sample mean and sample standard deviation

Suppose random samples of size n are drawn from a population with mean μ and standard deviation σ . The mean $\mu_{\bar{X}}$ and standard deviation $\sigma_{\bar{X}}$ of the sample mean \bar{X} satisfy

$$\mu_{\bar{X}} = \mu \quad (6.1.1)$$

and

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} \quad (6.1.2)$$

Equation 6.1.1 says that if we could take every possible sample from the population and compute the corresponding sample mean, then those numbers would center at the number we wish to estimate, the population mean μ . Equation 6.1.2 says that averages computed from samples vary less than individual measurements on the population do, and quantifies the relationship.

✓ Example 6.1.2

The mean and standard deviation of the tax value of all vehicles registered in a certain state are $\mu = \$13,525$ and $\sigma = \$4,180$. Suppose random samples of size 100 are drawn from the population of vehicles. What are the mean $\mu_{\bar{X}}$ and standard deviation $\sigma_{\bar{X}}$ of the sample mean \bar{X} ?

Solution

Since $n = 100$, the formulas yield

$$\mu_{\bar{X}} = \mu = \$13,525$$

and

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{\$4,180}{\sqrt{100}} = \$418$$

Key Takeaway

- The sample mean is a random variable; as such it is written \bar{X} , and \bar{x} stands for individual values it takes.
- As a random variable the sample mean has a probability distribution, a mean $\mu_{\bar{X}}$, and a standard deviation $\sigma_{\bar{X}}$.
- There are formulas that relate the mean and standard deviation of the sample mean to the mean and standard deviation of the population from which the sample is drawn.

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