11.E: Chi-Square Tests and F-Tests (Exercises)

These are homework exercises to accompany the Textmap created for "Introductory Statistics" by Shafer and Zhang.

11.1: Chi-Square Tests for Independence

Basic

Q11.1.1

Find $\chi^2_{0.01}$ for each of the following number of degrees of freedom.

a. $df = 5$
b. $df = 11$
c. $df = 25$

Q11.1.2

Find $\chi^2_{0.05}$ for each of the following number of degrees of freedom.

a. $df = 6$
b. $df = 12$
c. $df = 30$

Q11.1.3

Find $\chi^2_{0.10}$ for each of the following number of degrees of freedom.

a. $df = 6$
b. $df = 12$
c. $df = 30$

Q11.1.4

Find $\chi^2_{0.01}$ for each of the following number of degrees of freedom.

a. $df = 7$
b. $df = 10$
c. $df = 20$

Q11.1.5

For $df = 7$ and $\alpha = 0.05$

a. χ^2_{α}
b. $\chi^2_{\frac{\alpha}{2}}$

Q11.1.6

For $df = 17$ and $\alpha = 0.01$

a. χ^2_{α}
b. $\chi^2_{\frac{\alpha}{2}}$

Q11.1.7

A data sample is sorted into a 2×2 contingency table based on two factors, each of which has two levels.

		Factor 1		
		Level 1	Level 2	Row Total
Factor 2	Level 1	20	10	R
	Level 2	15	5	R

		Factor 1		
		Level 1 Level 2		Row Total
Column Total		C	C	n

a. Find the column totals, the row totals, and the grand total, n, of the table.
b. Find the expected number E of observations for each cell based on the assumption that the two factors are independent (that is, just use the formula $E = (R \times C)/n$).
c. Find the value of the chi-square test statistic χ^2.
d. Find the number of degrees of freedom of the chi-square test statistic.

Q11.1.8

A data sample is sorted into a 3×2 contingency table based on two factors, one of which has three levels and the other of which has two levels.

		Factor 1		
		Level 1 Level 2		Row Total
Factor 2	Level 1	20	10	R
	Level 2	15	5	R
	Level 3	10	20	R
Column Total		C	C	n

a. Find the column totals, the row totals, and the grand total, n, of the table.
b. Find the expected number E of observations for each cell based on the assumption that the two factors are independent (that is, just use the formula $E = (R \times C)/n$).
c. Find the value of the chi-square test statistic χ^2.
d. Find the number of degrees of freedom of the chi-square test statistic.

Applications

Q11.1.9

A child psychologist believes that children perform better on tests when they are given perceived freedom of choice. To test this belief, the psychologist carried out an experiment in which 200 third graders were randomly assigned to two groups, A and B. Each child was given the same simple logic test. However in group B, each child was given the freedom to choose a text booklet from many with various drawings on the covers. The performance of each child was rated as Very Good, Good, and Fair. The results are summarized in the table provided. Test, at the 5% level of significance, whether there is sufficient evidence in the data to support the psychologist's belief.

		Group	
		A	B
Performance	Very Good	32	29
	Good	55	61
	Fair	10	13

Q11.1.10

In regard to wine tasting competitions, many experts claim that the first glass of wine served sets a reference taste and that a different reference wine may alter the relative ranking of the other wines in competition. To test this claim, three wines, A, B and C, were served at a wine tasting event. Each person was served a single glass of each wine, but in different orders for different

guests. At the close, each person was asked to name the best of the three. One hundred seventy-two people were at the event and their top picks are given in the table provided. Test, at the 1% level of significance, whether there is sufficient evidence in the data to support the claim that wine experts' preference is dependent on the first served wine.

		Top Pick		
		A	*B*	*C*
First Glass	*A*	12	31	27
	B	15	40	21
	C	10	9	7

9. Is being left-handed hereditary? To answer this question, 250 adults are randomly selected and their handedness and their parents' handedness are noted. The results are summarized in the table provided. Test, at the 1% level of significance, whether there is sufficient evidence in the data to conclude that there is a hereditary element in handedness.

		Number of Parents Left-Handed		
		0	1	2
Handedness	Left	8	10	12
	Right	178	21	21

10. Some geneticists claim that the genes that determine left-handedness also govern development of the language centers of the brain. If this claim is true, then it would be reasonable to expect that left-handed people tend to have stronger language abilities. A study designed to text this claim randomly selected 807 students who took the Graduate Record Examination (GRE). Their scores on the language portion of the examination were classified into three categories: *low*, *average*, and *high*, and their handedness was also noted. The results are given in the table provided. Test, at the 5% level of significance, whether there is sufficient evidence in the data to conclude that left-handed people tend to have stronger language abilities.

		GRE English Scores		
		Low	Average	High
Handedness	Left	18	40	22
	Right	201	360	166

11. It is generally believed that children brought up in stable families tend to do well in school. To verify such a belief, a social scientist examined 290 randomly selected students' records in a public high school and noted each student's family structure and academic status four years after entering high school. The data were then sorted into a 2×3 contingency table with two factors. $\mathrm{Factor\ 1}$ has two levels: *graduated* and *did not graduate*. $\mathrm{Factor\ 2}$ has three levels: *no parent*, *one parent*, and *two parents*. The results are given in the table provided. Test, at the 1% level of significance, whether there is sufficient evidence in the data to conclude that family structure matters in school performance of the students.

		Academic Status	
		Graduated	Did Not Graduate
Family	No parent	18	31
	One parent	101	44
	Two parents	70	26

12. A large middle school administrator wishes to use celebrity influence to encourage students to make healthier choices in the school cafeteria. The cafeteria is situated at the center of an open space. Everyday at lunch time students get their lunch and a drink in three separate lines leading to three separate serving stations. As an experiment, the school administrator displayed a

poster of a popular teen pop star drinking milk at each of the three areas where drinks are provided, except the milk in the poster is different at each location: one shows white milk, one shows strawberry-flavored pink milk, and one shows chocolate milk. After the first day of the experiment the administrator noted the students' milk choices separately for the three lines. The data are given in the table provided. Test, at the 1% level of significance, whether there is sufficient evidence in the data to conclude that the posters had some impact on the students' drink choices.

	Student Choice		
	Regular	Strawberry	Chocolate
Poster Choice			
Regular	38	28	40
Strawberry	18	51	24
Chocolate	32	32	53

Large Data Set Exercise

Large Data Sets not available

15. Large Data Set 8 records the result of a survey of 300 randomly selected adults who go to movie theaters regularly. For each person the gender and preferred type of movie were recorded. Test, at the 5% level of significance, whether there is sufficient evidence in the data to conclude that the factors "gender" and "preferred type of movie" are dependent.

Answers

1. a. 15.09
   b. 24.72
   c. 44.31
2.
3. a. 10.64
   b. 18.55
   c. 40.26
4.
5. a. 14.07
   b. 16.01
6.
7. a. $C_1 = 35,\ C_2 = 15,\ R_1 = 30,\ R_2 = 20,\ n = 50$
   b. $E_{11} = 21,\ E_{12} = 9,\ E_{21} = 14,\ E_{22} = 6$
   c. $\chi^2 = 0.3968$
   d. $df = 1$
8.
9. $\chi^2 = 0.6698,\ \chi^2_{0.05} = 5.99$, do not reject H_0
10.
11. $\chi^2 = 72.35,\ \chi^2_{0.01} = 9.21$, reject H_0
12.
13. $\chi^2 = 21.2784,\ \chi^2_{0.01} = 9.21$, reject H_0
14.
15. $\chi^2 = 28.4539,\ df = 3$, Rejection Region: $[7.815, \infty)$, Decision: reject H_0 of independence

11.2: Chi-Square One-Sample Goodness-of-Fit Tests

Basic

1. A data sample is sorted into five categories with an assumed probability distribution.

Factor Levels	Assumed Distribution	Observed Frequency

Factor Levels	Assumed Distribution	Observed Frequency
1	$p_1 = 0.1$	10
2	$p_2 = 0.4$	35
3	$p_3 = 0.4$	45
4	$p_4 = 0.1$	10

a. Find the size n of the sample.
b. Find the expected number E of observations for each level, if the sampled population has a probability distribution as assumed (that is, just use the formula $E_i = n \times p_i$).
c. Find the chi-square test statistic χ^2.
d. Find the number of degrees of freedom of the chi-square test statistic.

2. A data sample is sorted into five categories with an assumed probability distribution.

Factor Levels	Assumed Distribution	Observed Frequency
1	$p_1 = 0.3$	23
2	$p_2 = 0.3$	30
3	$p_3 = 0.2$	19
4	$p_4 = 0.1$	8
5	$p_5 = 0.1$	10

a. Find the size n of the sample.
b. Find the expected number E of observations for each level, if the sampled population has a probability distribution as assumed (that is, just use the formula $E_i = n \times p_i$).
c. Find the chi-square test statistic χ^2.
d. Find the number of degrees of freedom of the chi-square test statistic.

Applications

3. Retailers of collectible postage stamps often buy their stamps in large quantities by weight at auctions. The prices the retailers are willing to pay depend on how old the postage stamps are. Many collectible postage stamps at auctions are described by the proportions of stamps issued at various periods in the past. Generally the older the stamps the higher the value. At one particular auction, a lot of collectible stamps is advertised to have the age distribution given in the table provided. A retail buyer took a sample of 73 stamps from the lot and sorted them by age. The results are given in the table provided. Test, at the 5% level of significance, whether there is sufficient evidence in the data to conclude that the age distribution of the lot is different from what was claimed by the seller.

Year	Claimed Distribution	Observed Frequency
Before 1940	0.10	6
1940 to 1959	0.25	15
1960 to 1979	0.45	30
After 1979	0.20	22

4. The litter size of Bengal tigers is typically two or three cubs, but it can vary between one and four. Based on long-term observations, the litter size of Bengal tigers in the wild has the distribution given in the table provided. A zoologist believes that Bengal tigers in captivity tend to have different (possibly smaller) litter sizes from those in the wild. To verify this belief, the zoologist searched all data sources and found 316 litter size records of Bengal tigers in captivity. The results are given in the table provided. Test, at the 5% level of significance, whether there is sufficient evidence in the data to conclude that the distribution of litter sizes in captivity differs from that in the wild.

Litter Size	Wild Litter Distribution	Observed Frequency
1	0.11	41
2	0.69	243
3	0.18	27
4	0.02	5

5. An online shoe retailer sells men's shoes in sizes 8 to 13. In the past orders for the different shoe sizes have followed the distribution given in the table provided. The management believes that recent marketing efforts may have expanded their customer base and, as a result, there may be a shift in the size distribution for future orders. To have a better understanding of its future sales, the shoe seller examined $1,040$ sales records of recent orders and noted the sizes of the shoes ordered. The results are given in the table provided. Test, at the 1% level of significance, whether there is sufficient evidence in the data to conclude that the shoe size distribution of future sales will differ from the historic one.

Shoe Size	Past Size Distribution	Recent Size Frequency
8.0	0.03	25
8.5	0.06	43
9.0	0.09	88
9.5	0.19	221
10.0	0.23	272
10.5	0.14	150
11.0	0.10	107
11.5	0.06	51
12.0	0.05	37
12.5	0.03	35
13.0	0.02	11

6. An online shoe retailer sells women's shoes in sizes 5 to 10. In the past orders for the different shoe sizes have followed the distribution given in the table provided. The management believes that recent marketing efforts may have expanded their customer base and, as a result, there may be a shift in the size distribution for future orders. To have a better understanding of its future sales, the shoe seller examined $1,174$ sales records of recent orders and noted the sizes of the shoes ordered. The results are given in the table provided. Test, at the 1% level of significance, whether there is sufficient evidence in the data to conclude that the shoe size distribution of future sales will differ from the historic one.

Shoe Size	Past Size Distribution	Recent Size Frequency
5.0	0.02	20
5.5	0.03	23
6.0	0.07	88
6.5	0.08	90
7.0	0.20	222
7.5	0.20	258
8.0	0.15	177
8.5	0.11	121

Shoe Size	Past Size Distribution	Recent Size Frequency
9.0	0.08	91
9.5	0.04	53
10.0	0.02	31

7. A chess opening is a sequence of moves at the beginning of a chess game. There are many well-studied named openings in chess literature. French Defense is one of the most popular openings for black, although it is considered a relatively weak opening since it gives black probability 0.344 of winning, probability 0.405 of losing, and probability 0.251 of drawing. A chess master believes that he has discovered a new variation of French Defense that may alter the probability distribution of the outcome of the game. In his many Internet chess games in the last two years, he was able to apply the new variation in 77 games. The wins, losses, and draws in the 77 games are given in the table provided. Test, at the 5% level of significance, whether there is sufficient evidence in the data to conclude that the newly discovered variation of French Defense alters the probability distribution of the result of the game.

Result for Black	Probability Distribution	New Variation Wins
Win	0.344	31
Loss	0.405	25
Draw	0.251	21

8. The Department of Parks and Wildlife stocks a large lake with fish every six years. It is determined that a healthy diversity of fish in the lake should consist of 10% largemouth bass, 15% smallmouth bass, 10% striped bass, 10% trout, and 20% catfish. Therefore each time the lake is stocked, the fish population in the lake is restored to maintain that particular distribution. Every three years, the department conducts a study to see whether the distribution of the fish in the lake has shifted away from the target proportions. In one particular year, a research group from the department observed a sample of 292 fish from the lake with the results given in the table provided. Test, at the 5% level of significance, whether there is sufficient evidence in the data to conclude that the fish population distribution has shifted since the last stocking.

Fish	Target Distribution	Fish in Sample
Largemouth Bass	0.10	14
Smallmouth Bass	0.15	49
Striped Bass	0.10	21
Trout	0.10	22
Catfish	0.20	75
Other	0.35	111

Large Data Set Exercise

Large Data Sets not available

9. Large Data Set 4 records the result of 500 tosses of six-sided die. Test, at the 10% level of significance, whether there is sufficient evidence in the data to conclude that the die is not "fair" (or "balanced"), that is, that the probability distribution differs from probability $1/6$ for each of the six faces on the die.

Answers

S11.2.1

a. $n = 100$
b. $E = 10, E = 40, E = 40, E = 10$
c. $\chi^2 = 1.25$

d. $df = 3$

S11.2.3

$\chi^2 = 4.8082,\ \chi^2_{0.05} = 7.81,$ do not reject H_0

S11.2.5

$\chi^2 = 26.5765,\ \chi^2_{0.01} = 23.21,$ reject H_0

S11.2.7

$\chi^2 = 2.1401,\ \chi^2_{0.05} = 5.99,$ do not reject H_0

S11.2.9

$\chi^2 = 2.944,\ df = 5,$ Rejection Region: $[9.236, \infty),$ Decision: Fail to reject H_0 of balance

11.3 F-tests for Equality of Two Variances

Basic

1. Find $F_{0.01}$ for each of the following degrees of freedom.
   a. $df_1 = 5$ and $df_2 = 5$
   b. $df_1 = 5$ and $df_2 = 12$
   c. $df_1 = 12$ and $df_2 = 20$
2. Find $F_{0.05}$ for each of the following degrees of freedom.
   a. $df_1 = 6$ and $df_2 = 6$
   b. $df_1 = 6$ and $df_2 = 12$
   c. $df_1 = 12$ and $df_2 = 30$
3. Find $F_{0.95}$ for each of the following degrees of freedom.
   a. $df_1 = 6$ and $df_2 = 6$
   b. $df_1 = 6$ and $df_2 = 12$
   c. $df_1 = 12$ and $df_2 = 30$
4. Find $F_{0.90}$ for each of the following degrees of freedom.
   a. $df_1 = 5$ and $df_2 = 5$
   b. $df_1 = 5$ and $df_2 = 12$
   c. $df_1 = 12$ and $df_2 = 20$
5. For $df_1 = 7$, $df_2 = 10$ and $\alpha = 0.05$, find
   a. F_{α}
   b. $F_{1-\alpha}$
   c. $F_{\alpha/2}$
   d. $F_{1-\alpha/2}$
6. For $df_1 = 15$, $df_2 = 8$ and $\alpha = 0.01$, find
   a. F_{α}
   b. $F_{1-\alpha}$
   c. $F_{\alpha/2}$
   d. $F_{1-\alpha/2}$
7. For each of the two samples

$$\begin{aligned} &\text{Sample 1} : \{8, 2, 11, 0, -2\} \\ &\text{Sample 2} : \{-2, 0, 0, 0, 2, 4, -1\} \end{aligned} \tag{11.E.1}$$

   find
   a. the sample size
   b. the sample mean
   c. the sample variance

8. For each of the two samples

$$\text{Sample } 1 : \{0.8, 1.2, 1.1, 0.8, -2.0\} \qquad (11.E.2)$$
$$\text{Sample } 2 : \{-2.0, 0.0, 0.7, 0.8, 2.2, 4.1, -1.9\}$$

find

   a. the sample size
   b. the sample mean
   c. the sample variance

9. Two random samples taken from two normal populations yielded the following information:

Sample	Sample Size	Sample Variance
1	$n_1 = 16$	$s_1^2 = 53$
2	$n_2 = 21$	$s_2^2 = 32$

   a. Find the statistic $F = s_1^2 / s_2^2$
   b. Find the degrees of freedom df_1 and df_2.
   c. Find $F_{0.05}$ using df_1 and df_2 computed above.
   d. Perform the test the hypotheses $H_0 : \alpha_1^2 = \alpha_2^2$ *vs* $H_a : \alpha_1^2 > \alpha_2^2$ at the 5% level of significance.

10. Two random samples taken from two normal populations yielded the following information:

Sample	Sample Size	Sample Variance
1	$n_1 = 11$	$s_1^2 = 61$
2	$n_2 = 8$	$s_2^2 = 44$

   a. Find the statistic $F = s_1^2 / s_2^2$.
   b. Find the degrees of freedom df_1 and df_2.
   c. Find $F_{0.05}$ using df_1 and df_2computed above.
   d. Perform the test the hypotheses $H_0 : \alpha_1^2 = \alpha_2^2$ *vs* $H_a : \alpha_1^2 > \alpha_2^2$ at the 5% level of significance.

11. Two random samples taken from two normal populations yielded the following information:
   a. Find the statistic $F = s_1^2 / s_2^2$.
   b. Find the degrees of freedom df_1 and df_2.
   c. For $\alpha = 0.05$ find $F_{1-\alpha}$ using df_1 and df_2 computed above.
   d. Perform the test the hypotheses $H_0 : \alpha_1^2 = \alpha_2^2$ *vs* $H_a : \alpha_1^2 < \alpha_2^2$ at the 5% level of significance.

12.

Sample	Sample Size	Sample Variance
1	$n_1 = 10$	$s_1^2 = 12$
2	$n_2 = 13$	$s_2^2 = 23$

13. Two random samples taken from two normal populations yielded the following information:

Sample	Sample Size	Sample Variance
1	$n_1 = 8$	$s_1^2 = 102$
2	$n_2 = 8$	$s_2^2 = 603$

   a. Find the statistic $F = s_1^2 / s_2^2$
   b. Find the degrees of freedom df_1 and df_2.
   c. For $\alpha = 0.05$ find $F_{1-\alpha}$using df_1 and df_2computed above.
   d. Perform the test the hypotheses $H_0 : \alpha_1^2 = \alpha_2^2$ *vs* $H_a : \alpha_1^2 < \alpha_2^2$ at the 5% level of significance.

14. Two random samples taken from two normal populations yielded the following information:

Sample	Sample Size	Sample Variance
1	$n_1 = 9$	$s_1^2 = 123$
2	$n_2 = 31$	$s_2^2 = 543$

a. Find the statistic $F = s_1^2 / s_2^2$
b. Find the degrees of freedom df_1 and df_2.
c. For $\alpha = 0.05$ find $F_{1-\alpha/2}$ and $F_{\alpha/2}$ using df_1 and df_2computed above.
d. Perform the test the hypotheses $H_0 : \alpha_1^2 = \alpha_2^2$ *vs* $H_a : \alpha_1^2 \neq \alpha_2^2$ at the 5% level of significance.

15. Two random samples taken from two normal populations yielded the following information:

Sample	Sample Size	Sample Variance
1	$n_1 = 21$	$s_1^2 = 199$
2	$n_2 = 21$	$s_2^2 = 66$

a. Find the statistic $F = s_1^2 / s_2^2$
b. Find the degrees of freedom df_1 and df_2.
c. For $\alpha = 0.05$ find df_1 and df_2computed above.
d. Perform the test the hypotheses $H_0 : \alpha_1^2 = \alpha_2^2$ *vs* $H_a : \alpha_1^2 \neq \alpha_2^2$ at the 5% level of significance.

Applications

15. Japanese sturgeon is a subspecies of the sturgeon family indigenous to Japan and the Northwest Pacific. In a particular fish hatchery newly hatched baby Japanese sturgeon are kept in tanks for several weeks before being transferred to larger ponds. Dissolved oxygen in tank water is very tightly monitored by an electronic system and rigorously maintained at a target level of 6.5 milligrams per liter (mg/l). The fish hatchery looks to upgrade their water monitoring systems for tighter control of dissolved oxygen. A new system is evaluated against the old one currently being used in terms of the variance in measured dissolved oxygen. Thirty-one water samples from a tank operated with the new system were collected and 16 water samples from a tank operated with the old system were collected, all during the course of a day. The samples yield the following information:

$$\begin{aligned} &\text{New Sample 1: } n_1 = 31 \; s_1^2 = 0.0121 \\ &\text{Old Sample 2: } n_1 = 16 \; s_2^2 = 0.0319 \end{aligned} \tag{11.E.3}$$

Test, at the 10% level of significance, whether the data provide sufficient evidence to conclude that the new system will provide a tighter control of dissolved oxygen in the tanks.

16. The risk of investing in a stock is measured by the volatility, or the variance, in changes in the price of that stock. Mutual funds are baskets of stocks and offer generally lower risk to investors. Different mutual funds have different focuses and offer different levels of risk. Hippolyta is deciding between two mutual funds, A and B, with similar expected returns. To make a final decision, she examined the annual returns of the two funds during the last ten years and obtained the following information:

$$\begin{aligned} &\text{Mutual Fund A Sample 1: } n_1 = 10 \; s_1^2 = 0.012 \\ &\text{Mutual Fund B Sample 2: } n_1 = 10 \; s_2^2 = 0.005 \end{aligned} \tag{11.E.4}$$

Test, at the 5% level of significance, whether the data provide sufficient evidence to conclude that the two mutual funds offer different levels of risk.

17. It is commonly acknowledged that grading of the writing part of a college entrance examination is subject to inconsistency. Every year a large number of potential graders are put through a rigorous training program before being given grading assignments. In order to gauge whether such a training program really enhances consistency in grading, a statistician conducted an experiment in which a reference essay was given to 61 trained graders and 31 untrained graders. Information on the scores given by these graders is summarized below:

$$\text{Trained Sample 1: } n_1 = 61 \; s_1^2 = 2.15 \qquad (11.E.5)$$
$$\text{Untrained Sample 2: } n_1 = 31 \; s_2^2 = 3.91$$

Test, at the 5% level of significance, whether the data provide sufficient evidence to conclude that the training program enhances the consistency in essay grading.

18. A common problem encountered by many classical music radio stations is that their listeners belong to an increasingly narrow band of ages in the population. The new general manager of a classical music radio station believed that a new playlist offered by a professional programming agency would attract listeners from a wider range of ages. The new list was used for a year. Two random samples were taken before and after the new playlist was adopted. Information on the ages of the listeners in the sample are summarized below:

$$\text{Before Sample 1: } n_1 = 21 \; s_1^2 = 56.25 \qquad (11.E.6)$$
$$\text{After Sample 2: } n_1 = 16 \; s_2^2 = 76.56$$

Test, at the 10% level of significance, whether the data provide sufficient evidence to conclude that the new playlist has expanded the range of listener ages.

19. A laptop computer maker uses battery packs supplied by two companies, A and B. While both brands have the same average battery life between charges (LBC), the computer maker seems to receive more complaints about shorter LBC than expected for battery packs supplied by company B. The computer maker suspects that this could be caused by higher variance in LBC for Brand B. To check that, ten new battery packs from each brand are selected, installed on the same models of laptops, and the laptops are allowed to run until the battery packs are completely discharged. The following are the observed LBCs in hours.

Brand A	Brand B
3.2	3.0
3.4	3.5
2.8	2.9
3.0	3.1
3.0	2.3
3.0	2.0
2.8	3.0
2.9	2.9
3.0	3.0
3.0	4.1

Test, at the 5% level of significance, whether the data provide sufficient evidence to conclude that the LBCs of Brand B have a larger variance that those of Brand A.

20. A manufacturer of a blood-pressure measuring device for home use claims that its device is more consistent than that produced by a leading competitor. During a visit to a medical store a potential buyer tried both devices on himself repeatedly during a short period of time. The following are readings of systolic pressure.

Manufacturer	Competitor
132	129
134	132
129	129
129	138
130	
132	

a. Test, at the 5% level of significance, whether the data provide sufficient evidence to conclude that the manufacturer's claim is true.
b. Repeat the test at the 10% level of significance. Quote as many computations from part (a) as possible.

Large Data Set Exercises

Large Data Sets not available

21. Large Data Sets 1A and 1B record SAT scores for 419 male and 581 female students. Test, at the 1% level of significance, whether the data provide sufficient evidence to conclude that the variances of scores of male and female students differ.
22. Large Data Sets 7, 7A, and 7B record the survival times of 140 laboratory mice with thymic leukemia. Test, at the 10% level of significance, whether the data provide sufficient evidence to conclude that the variances of survival times of male mice and female mice differ.

Answers

1. a. 11.0
   b. 5.06
   c. 3.23
2.
3. a. 0.23
   b. 0.25
   c. 0.40
4.
5. a. 3.14
   b. 0.27
   c. 3.95
   d. 0.21
6.
7. I. Sample 1
      a. $n_1 = 5$
      b. $\bar{x_1} = 3.8$
      c. $s_1^2 = 30.2$
   II. Sample 1
      a. $n_2 = 7$
      b. $\bar{x_1} = 0.4286$
      c. $s_2^2 = 3.95$
8.
9. a. 1.6563
   b. $df_1 = 15,\ df_2 = 20$
   c. $F_{0.05} = 2.2$
   d. do not reject H_0
10.
11. a. 0.5217
    b. $df_1 = 9,\ df_2 = 12$
    c. $F_{0.95} = 0.3254$
    d. do not reject H_0
12.
13. a. 0.1692
    b. $df_1 = 8,\ df_2 = 30$
    c. $F_{0.975} = 0.26,\ F_{0.025} = 2.65$
    d. reject H_0

14.
15. $F = 0.3793,\ F_{0.90} = 0.58$, reject H_0
16.
17. $F = 0.5499,\ F_{0.95} = 0.61$, reject H_0
18.
19. $F = 0.0971,\ F_{0.95} = 0.31$, reject H_0
20.
21. $F = 0.893131, df_1 = 418,\ df_2 = 580$. Rejection Region: $(0, 0.7897] \cup [1.2614, \infty)$ Decision: Fail to reject H_0 of equal variances.

11.4 F-Tests in One-Way ANOVA

Basic

1. The following three random samples are taken from three normal populations with respective means μ_1, μ_2 and μ_3, and the same variance σ^2.

Sample 1	Sample 2	Sample 3
2	3	0
2	5	1
3	7	2
5		1
3		

   a. Find the combined sample size n.
   b. Find the combined sample mean $\bar{x}$.
   c. Find the sample mean for each of the three samples.
   d. Find the sample variance for each of the three samples.
   e. Find MST.
   f. Find MSE.
   g. Find $F = MST/MSE$.

2. The following three random samples are taken from three normal populations with respective means μ_1, μ_2 and μ_3, and the same variance σ^2.

Sample 1	Sample 2	Sample 3
0.0	1.3	0.2
0.1	1.5	0.2
0.2	1.7	0.3
0.1		0.5
		0.0

   a. Find the combined sample size n.
   b. Find the combined sample mean $\bar{x}$.
   c. Find the sample mean for each of the three samples.
   d. Find the sample variance for each of the three samples.
   e. Find MST.
   f. Find MSE.
   g. Find $F = MST/MSE$.

3. Refer to Exercise 1.
   a. Find the number of populations under consideration K.

b. Find the degrees of freedom $df_1 = K - 1$ and $df_2 = n - K$
c. For $\alpha = 0.05$, find F_α with the degrees of freedom computed above.
d. At $\alpha = 0.05$, test hypotheses

$$H_0 : \mu_1 = \mu_2 = \mu_3 \quad (11.E.7)$$
$$vs\ H_a : \text{at least one pair of the population means are not equal}$$

4. Refer to Exercise 2.
   a. Find the number of populations under consideration K.
   b. Find the degrees of freedom $df_1 = K - 1$ and $df_2 = n - K$
   c. For $\alpha = 0.01$, find F_α with the degrees of freedom computed above.
   d. At $\alpha = 0.01$, test hypotheses

$$H_0 : \mu_1 = \mu_2 = \mu_3 \quad (11.E.8)$$
$$vs\ H_a : \text{at least one pair of the population means are not equal}$$

Applications

5. The Mozart effect refers to a boost of average performance on tests for elementary school students if the students listen to Mozart's chamber music for a period of time immediately before the test. In order to attempt to test whether the Mozart effect actually exists, an elementary school teacher conducted an experiment by dividing her third-grade class of 15 students into three groups of 5. The first group was given an end-of-grade test without music; the second group listened to Mozart's chamber music for 10 minutes; and the third groups listened to Mozart's chamber music for 20 minutes before the test. The scores of the 15 students are given below:

Group 1	Group 2	Group 3
80	79	73
63	73	82
74	74	79
71	77	82
70	81	84

Using the ANOVA F-test at $\alpha = 0.10$, is there sufficient evidence in the data to suggest that the Mozart effect exists?

6. The Mozart effect refers to a boost of average performance on tests for elementary school students if the students listen to Mozart's chamber music for a period of time immediately before the test. Many educators believe that such an effect is not necessarily due to Mozart's music per se but rather a relaxation period before the test. To support this belief, an elementary school teacher conducted an experiment by dividing her third-grade class of 15 students into three groups of 5. Students in the first group were asked to give themselves a self-administered facial massage; students in the second group listened to Mozart's chamber music for 15 minutes; students in the third group listened to Schubert's chamber music for 15 minutes before the test. The scores of the 15 students are given below:

Group 1	Group 2	Group 3
79	82	80
81	84	81
80	86	71
89	91	90
86	82	86

Test, using the ANOVA F-test at the 10% level of significance, whether the data provide sufficient evidence to conclude that any of the three relaxation method does better than the others.

7. Precision weighing devices are sensitive to environmental conditions. Temperature and humidity in a laboratory room where such a device is installed are tightly controlled to ensure high precision in weighing. A newly designed weighing device is claimed to be more robust against small variations of temperature and humidity. To verify such a claim, a laboratory tests the new device under four settings of temperature-humidity conditions. First, two levels of *high* and *low* temperature and two levels of *high* and *low* humidity are identified. Let T stand for temperature and H for humidity. The four experimental settings are defined and noted as (T, H): (high, high), (high, low), (low, high), and (low, low). A pre-calibrated standard weight of 1 kg was weighed by the new device four times in each setting. The results in terms of error (in micrograms mcg) are given below:

(high, high)	(high, low)	(low, high)	(low, low)
−1.50	11.47	−14.29	5.54
−6.73	9.28	−18.11	10.34
11.69	5.58	−11.16	15.23
−5.72	10.80	−10.41	−5.69

Test, using the ANOVA F-test at the 1% level of significance, whether the data provide sufficient evidence to conclude that the mean weight readings by the newly designed device vary among the four settings.

8. To investigate the real cost of owning different makes and models of new automobiles, a consumer protection agency followed 16 owners of new vehicles of four popular makes and models, call them TC,HA,NA, and FT, and kept a record of each of the owner's real cost in dollars for the first five years. The five-year costs of the 16 car owners are given below:

TC	HA	NA	FT
8423	7776	8907	10333
7889	7211	9077	9217
8665	6870	8732	10540
	7129	9747	
	7359	8677	

Test, using the ANOVA F-test at the 5% level of significance, whether the data provide sufficient evidence to conclude that there are differences among the mean real costs of ownership for these four models.

9. Helping people to lose weight has become a huge industry in the United States, with annual revenue in the hundreds of billion dollars. Recently each of the three market-leading weight reducing programs claimed to be the most effective. A consumer research company recruited 33 people who wished to lose weight and sent them to the three leading programs. After six months their weight losses were recorded. The results are summarized below:

Statistic	Prog. 1	Prog. 2	Prog. 3
Sample Mean	$\bar{x_1} = 10.65$	$\bar{x_2} = 8.90$	$\bar{x_3} = 9.33$
Sample Variance	$s_1^2 = 27.20$	$s_2^2 = 16.86$	$s_3^2 = 32.40$
Sample Size	$n_1 = 11$	$n_2 = 11$	$n_3 = 11$

The mean weight loss of the combined sample of all 33 people was $\bar{x} = 9.63$. Test, using the ANOVA F-test at the 5% level of significance, whether the data provide sufficient evidence to conclude that some program is more effective than the others.

10. A leading pharmaceutical company in the disposable contact lenses market has always taken for granted that the sales of certain peripheral products such as contact lens solutions would automatically go with the established brands. The long-standing culture in the company has been that lens solutions would not make a significant difference in user experience. Recent market research surveys, however, suggest otherwise. To gain a better understanding of the effects of contact lens solutions on user experience, the company conducted a comparative study in which 63 contact lens users were randomly divided into three groups, each of which received one of three top selling lens solutions on the market, including one of the company's own. After

using the assigned solution for two weeks, each participant was asked to rate the solution on the scale of 1 to 5 for satisfaction, with 5 being the highest level of satisfaction. The results of the study are summarized below:

Statistics	Sol. 1	Sol. 2	Sol. 3
Sample Mean	$\bar{x_1} = 3.28$	$\bar{x_2} = 3.96$	$\bar{x_3} = 4.10$
Sample Variance	$s_1^2 = 0.15$	$s_2^2 = 0.32$	$s_3^2 = 0.36$
Sample Size	$n_1 = 18$	$n_1 = 23$	$n_1 = 22$

The mean satisfaction level of the combined sample of all 63 participants was $\bar{x} = 3.81$. Test, using the ANOVA F-test at the 5% level of significance, whether the data provide sufficient evidence to conclude that not all three average satisfaction levels are the same

Large Data Set Exercise

Large Data Set not available

11. Large Data Set 9 records the costs of materials (textbook, solution manual, laboratory fees, and so on) in each of ten different courses in each of three different subjects, chemistry, computer science, and mathematics. Test, at the 1% level of significance, whether the data provide sufficient evidence to conclude that the mean costs in the three disciplines are not all the same.

Answers

1. a. $n = 12$
   b. $\bar{x} = 2.8333$
   c. $\bar{x_1} = 3,\ \bar{x_2} = 5,\ \bar{x_3} = 1$
   d. $s_1^2 = 1.5,\ s_2^2 = 4,\ s_3^2 = 0.6667$
   e. $MST = 13.83$
   f. $MSE = 1.78$
   g. $F = 7.7812$
2.
3. a. $K = 3$
   b. $df_1 = 2,\ df_2 = 9$
   c. $F_{0.05} = 4.26$
   d. $F = 5.53$, reject H_0
4.
5. $F = 3.9647,\ F_{0.10} = 2.81$, reject H_0
6.
7. $F = 9.6018,\ F_{0.01} = 5.95$, reject H_0
8.
9. $F = 0.3589,\ F_{0.05} = 5.32$, do not reject H_0
10.
11. $F = 1.418, df_1 = 2,\ and\ df_2 = 27$, Rejection Region: $[5.4881, \infty)$, Decision: Fail to reject H_0 of equal means.