3.E: Basic Concepts of Probability (Exercises)

These are homework exercises to accompany the Textmap created for "Introductory Statistics" by Shafer and Zhang.

3.1: Sample Spaces, Events, and Their Probabilities

Basic

Q3.1.1

A box contains 10 white and 10 black marbles. Construct a sample space for the experiment of randomly drawing out, with replacement, two marbles in succession and noting the color each time. (To draw "with replacement" means that the first marble is put back before the second marble is drawn.)

Q3.1.2

A box contains 16 white and 16 black marbles. Construct a sample space for the experiment of randomly drawing out, with replacement, three marbles in succession and noting the color each time. (To draw "with replacement" means that each marble is put back before the next marble is drawn.)

Q3.1.3

A box contains 8 red, 8 yellow, and 8 green marbles. Construct a sample space for the experiment of randomly drawing out, with replacement, two marbles in succession and noting the color each time.

Q3.1.4

A box contains 6 red, 6 yellow, and 6 green marbles. Construct a sample space for the experiment of randomly drawing out, with replacement, three marbles in succession and noting the color each time.

Q3.1.5

In the situation of **Exercise 1**, list the outcomes that comprise each of the following events.

a. At least one marble of each color is drawn.
b. No white marble is drawn.

Q3.1.6

In the situation of **Exercise 2**, list the outcomes that comprise each of the following events.

a. At least one marble of each color is drawn.
b. No white marble is drawn.
c. More black than white marbles are drawn.

Q3.1.7

In the situation of **Exercise 3**, list the outcomes that comprise each of the following events.

a. No yellow marble is drawn.
b. The two marbles drawn have the same color.
c. At least one marble of each color is drawn.

Q3.1.8

In the situation of **Exercise 4**, list the outcomes that comprise each of the following events.

a. No yellow marble is drawn.
b. The three marbles drawn have the same color.
c. At least one marble of each color is drawn.

Q3.1.9

Assuming that each outcome is equally likely, find the probability of each event in **Exercise 5**.

Q3.1.10

Assuming that each outcome is equally likely, find the probability of each event in **Exercise 6**.

Q3.1.11

Assuming that each outcome is equally likely, find the probability of each event in **Exercise 7**.

Q3.1.12

Assuming that each outcome is equally likely, find the probability of each event in **Exercise 8**.

Q3.1.13

A sample space is $S=\{a,b,c,d,e\}$. Identify two events as $U=\{a,b,d\}$ and $V=\{b,c,d\}$. Suppose $P(a)$ and $P(b)$ are each 0.2 and $P(c)$ and $P(d)$ are each 0.1.

a. Determine what $P(e)$ must be.
b. Find $P(U)$.
c. Find $P(V)$

Q3.1.14

A sample space is $S=\{u,v,w,x\}$. Identify two events as $A=\{v,w\}$ and $B=\{u,w,x\}$. Suppose $P(u)=0.22$, $P(w)=0.36$, and $P(x)=0.27$.

a. Determine what $P(v)$ must be.
b. Find $P(A)$.
c. Find $P(B)$.

Q3.1.15

A sample space is $S=\{m,n,q,r,s\}$. Identify two events as $U=\{m,q,s\}$ and $V=\{n,q,r\}$. The probabilities of some of the outcomes are given by the following table:

$$\begin{array}{c|ccccc} Outcome & m & n & q & r & s \\ \hline Probability & 0.18 & 0.16 & & 0.24 & 0.21 \end{array} \tag{3.E.1}$$

a. Determine what $P(q)$ must be.
b. Find $P(U)$.
c. Find $P(V)$.

Q3.1.16

A sample space is $S=\{d,e,f,g,h\}$. Identify two events as $M=\{e,f,g,h\}$ and $N=\{d,g\}$. The probabilities of some of the outcomes are given by the following table:

$$\begin{array}{c|ccccc} Outcome & d & e & f & g & h \\ \hline Probability & 0.22 & 0.13 & 0.27 & & 0.19 \end{array} \tag{3.E.2}$$

a. Determine what $P(g)$ must be.
b. Find $P(M)$.
c. Find $P(N)$.

Applications

Q3.1.17

The sample space that describes all three-child families according to the genders of the children with respect to birth order was constructed in "Example 3.1.4". Identify the outcomes that comprise each of the following events in the experiment of selecting a three-child family at random.

a. At least one child is a girl.
b. At most one child is a girl.
c. All of the children are girls.

d. Exactly two of the children are girls.
e. The first born is a girl.

Q3.1.18

The sample space that describes three tosses of a coin is the same as the one constructed in "Example 3.1.4" with “boy” replaced by “heads” and “girl” replaced by “tails.” Identify the outcomes that comprise each of the following events in the experiment of tossing a coin three times.

a. The coin lands heads more often than tails.
b. The coin lands heads the same number of times as it lands tails.
c. The coin lands heads at least twice.
d. The coin lands heads on the last toss.

Q3.1.19

Assuming that the outcomes are equally likely, find the probability of each event in **Exercise 17**.

Q3.1.20

Assuming that the outcomes are equally likely, find the probability of each event in **Exercise 18**.

Additional Exercises

Q3.1.21

The following two-way contingency table gives the breakdown of the population in a particular locale according to age and tobacco usage:

Age	Tobacco Use	
	Smoker	Non-smoker
Under 30	0.05	0.20
Over 30	0.20	0.55

A person is selected at random. Find the probability of each of the following events.

a. The person is a smoker.
b. The person is under 30.
c. The person is a smoker who is under 30.

Q3.1.22

The following two-way contingency table gives the breakdown of the population in a particular locale according to party affiliation ($A, B, C,$ or None) and opinion on a bond issue:

Affiliation	Opinion		
	Favors	Opposes	Undecided
A	0.12	0.09	0.07
B	0.16	0.12	0.14
C	0.04	0.03	0.06
None	0.08	0.06	0.03

A person is selected at random. Find the probability of each of the following events.

a. The person is affiliated with party B.
b. The person is affiliated with some party.
c. The person is in favor of the bond issue.

d. The person has no party affiliation and is undecided about the bond issue.

Q3.1.23

The following two-way contingency table gives the breakdown of the population of married or previously married women beyond child-bearing age in a particular locale according to age at first marriage and number of children:

Age	Number of Children		
	0	$1\ or\ 2$	3 or More
$Under$ 20	0.02	0.14	0.08
$20-29$	0.07	0.37	0.11
30 and above	0.10	0.10	0.01

A woman is selected at random. Find the probability of each of the following events.

a. The woman was in her twenties at her first marriage.
b. The woman was 20 or older at her first marriage.
c. The woman had no children.
d. The woman was in her twenties at her first marriage and had at least three children.

Q3.1.24

The following two-way contingency table gives the breakdown of the population of adults in a particular locale according to highest level of education and whether or not the individual regularly takes dietary supplements:

Education	Use of Supplements	
	Takes	Does Not Take
No High School Diploma	0.04	0.06
High School Diploma	0.06	0.44
Undergraduate Degree	0.09	0.28
Graduate Degree	0.01	0.02

An adult is selected at random. Find the probability of each of the following events.

a. The person has a high school diploma and takes dietary supplements regularly.
b. The person has an undergraduate degree and takes dietary supplements regularly.
c. The person takes dietary supplements regularly.
d. The person does not take dietary supplements regularly.

Large Data Set Exercises

Q3.1.25

Large Data Set 4 and Data Set 4A record the results of 500 tosses of a coin. Find the relative frequency of each outcome $1, 2, 3, 4, 5,\ and\ 6$. Does the coin appear to be “balanced” or “fair”?

Q3.1.26

Large Data Set 6, Data Set 6A, and Data Set 6B record results of a random survey of 200 voters in each of two regions, in which they were asked to express whether they prefer Candidate A for a U.S. Senate seat or prefer some other candidate.

a. Find the probability that a randomly selected voter among these 400 prefers Candidate A.
b. Find the probability that a randomly selected voter among the 200 who live in Region 1 prefers Candidate A (separately recorded in Large Data Set 6A).

c. Find the probability that a randomly selected voter among the 200 who live in Region 2 prefers Candidate A (separately recorded in Large Data Set 6B).

Answers

S3.1.1

$S=\{bb,bw,wb,ww\}$

S3.1.3

$S=\{rr,ry,rg,yr,yy,yg,gr,gy,gg\}$

S3.1.5

a. $\{bw,wb\}$
b. $\{bb\}$

S3.1.7

a. $\{rr,rg,gr,gg\}$
b. $\{rr,yy,gg\}$
c. $\varnothing$

S3.1.9

a. $1/4$
b. $2/4$

S3.1.11

a. $4/9$
b. $3/9$
c. 0

S3.1.13

a. 0.4
b. 0.5
c. 0.4

S3.1.15

a. 0.61
b. 0.6
c. 0.21

S3.1.17

a. $\{gbb,gbg,ggb,ggg\}$
b. $\{bgg,gbg,ggb\}$
c. $\{ggg\}$
d. $\{bbb,bbg,bgb,gbb\}$
e. $\{bbg,bgb,bgg,gbb,gbg,ggb,ggg\}$

S3.1.19

a. $4/8$
b. $3/8$
c. $1/8$
d. $4/8$
e. $7/8$

S3.1.21

a. 0.05
b. 0.25
c. 0.25

S3.1.23

a. 0.11
b. 0.19
c. 0.76
d. 0.55

S3.1.25

The relative frequencies for 1 through 6 are $0.16, 0.194, 0.162, 0.164, 0.154$ *and* 0.166 It would appear that the die is not balanced.

3.2: Complements, Intersections and Unions

Basic

1. For the sample space $S = \{a, b, c, d, e\}$ identify the complement of each event given.
   a. $A = \{a, d, e\}$
   b. $B = \{b, c, d, e\}$
   c. S
2. For the sample space $S = \{r, s, t, u, v\}$ identify the complement of each event given.
   a. $R = \{t, u\}$
   b. $T = \{r\}$
   c. $\varnothing$ (the "empty" set that has no elements)
3. The sample space for three tosses of a coin is $S = \{hhh, hht, hth, htt, thh, tht, tth, ttt\}$ Define events

$$\begin{gathered}\text{H:at least one head is observed}\\ \text{M:more heads than tails are observed}\end{gathered} \tag{3.E.3}$$

   a. List the outcomes that comprise H and M.
   b. List the outcomes that comprise $H \cap M$, $H \cup M$, and H^c.
   c. Assuming all outcomes are equally likely, find $P(H \cap M)$, $P(H \cup M)$, and $P(H^c)$.
   d. Determine whether or not H^c and M are mutually exclusive. Explain why or why not.
4. For the experiment of rolling a single six-sided die once, define events

$$\begin{gathered}\text{T:the number rolled is three}\\ \text{G:the number rolled is four or greater}\end{gathered} \tag{3.E.4}$$

   a. List the outcomes that comprise T and G.
   b. List the outcomes that comprise $T \cap G$, $T \cup G$, T^c, and $(T \cup G)^c$.
   c. Assuming all outcomes are equally likely, find $P(T \cap G)$, $P(T \cup G)$, and $P(T^c)$.
   d. Determine whether or not T and G are mutually exclusive. Explain why or why not.
5. A special deck of 16 cards has 4 that are blue, 4 yellow, 4 green, and 4 red. The four cards of each color are numbered from one to four. A single card is drawn at random. Define events

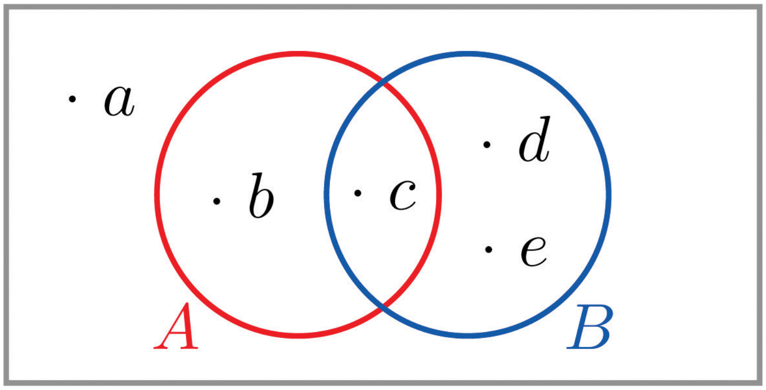
$$\begin{gathered}\text{B:the card is blue}\\ \text{R:the card is red}\\ \text{N:the number on the card is at most two}\end{gathered} \tag{3.E.5}$$

   a. List the outcomes that comprise B, R, and N.
   b. List the outcomes that comprise $B \cap R$, $B \cup R$, $B \cap N$, $R \cup N$, B^c, and $(B \cup R)^c$.
   c. Assuming all outcomes are equally likely, find the probabilities of the events in the previous part.
   d. Determine whether or not B and N are mutually exclusive. Explain why or why not.
6. In the context of the previous problem, define events

$$\begin{aligned}&\text{Y:the card is yellow}\\&\text{I:the number on the card is not a one}\\&\text{J:the number on the card is a two or a four}\end{aligned} \tag{3.E.6}$$

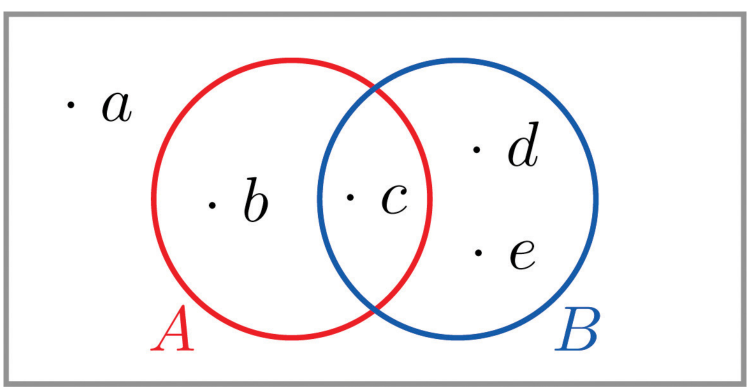
a. List the outcomes that comprise Y, I, and J.
b. List the outcomes that comprise $Y \cap I$, $Y \cup J$, $I \cap J$, I^c, and $(Y \cup J)^c$.
c. Assuming all outcomes are equally likely, find the probabilities of the events in the previous part.
d. Determine whether or not I^c and J are mutually exclusive. Explain why or why not.

7. The Venn diagram provided shows a sample space and two events A and B. Suppose $P(a) = 0.13, P(b) = 0.09, P(c) = 0.27, P(d) = 0.20,\ \text{and } P(e) = 0.31$. Confirm that the probabilities of the outcomes add up to 1, then compute the following probabilities.



a. $P(A)$.
b. $P(B)$.
c. $P(A^c)$. Two ways: (i) by finding the outcomes in A^c and adding their probabilities, and (ii) using the Probability Rule for Complements.
d. $P(A \cap B)$.
e. $P(A \cup B)$ Two ways: (i) by finding the outcomes in $A \cup B$ and adding their probabilities, and (ii) using the Additive Rule of Probability.

8. The Venn diagram provided shows a sample space and two events A and B. Suppose $P(a) = 0.32, P(b) = 0.17, P(c) = 0.28,\ \text{and } P(d) = 0.23$. Confirm that the probabilities of the outcomes add up to 1, then compute the following probabilities.



a. $P(A)$.
b. $P(B)$.
c. $P(A^c)$. Two ways: (i) by finding the outcomes in A^c and adding their probabilities, and (ii) using the Probability Rule for Complements.
d. $P(A \cap B)$.
e. $P(A \cup B)$ Two ways: (i) by finding the outcomes in $A \cup B$ and adding their probabilities, and (ii) using the Additive Rule of Probability.

9. Confirm that the probabilities in the two-way contingency table add up to 1, then use it to find the probabilities of the events indicated.

	U	V	W
A	0.15	0.00	0.23
B	0.22	0.30	0.10

 a. $P(A), P(B), P(A \cap B)$.
 b. $P(U), P(W), P(U \cap W)$.
 c. $P(U \cup W)$.
 d. $P(V^c)$.
 e. Determine whether or not the events A and U are mutually exclusive; the events A and V.

10. Confirm that the probabilities in the two-way contingency table add up to 1, then use it to find the probabilities of the events indicated.

	R	S	T
M	0.09	0.25	0.19
N	0.31	0.16	0.00

 a. $P(R), P(S), P(R \cap S)$.
 b. $P(M), P(N), P(M \cap N)$.
 c. $P(R \cup S)$.
 d. $P(R^c)$.
 e. Determine whether or not the events N and S are mutually exclusive; the events N and T.

Applications

11. Make a statement in ordinary English that describes the complement of each event (do not simply insert the word “not”).
 a. In the roll of a die: “five or more.”
 b. In a roll of a die: “an even number.”
 c. In two tosses of a coin: “at least one heads.”
 d. In the random selection of a college student: “Not a freshman.”
12. Make a statement in ordinary English that describes the complement of each event (do not simply insert the word “not”).
 a. In the roll of a die: “two or less.”
 b. In the roll of a die: “one, three, or four.”
 c. In two tosses of a coin: “at most one heads.”
 d. In the random selection of a college student: “Neither a freshman nor a senior.”
13. The sample space that describes all three-child families according to the genders of the children with respect to birth order is $S = \{bbb, bbg, bgb, bgg, gbb, gbg, ggb, ggg\}$. For each of the following events in the experiment of selecting a three-child family at random, state the complement of the event in the simplest possible terms, then find the outcomes that comprise the event and its complement.
 a. At least one child is a girl.
 b. At most one child is a girl.
 c. All of the children are girls.
 d. Exactly two of the children are girls.
 e. The first born is a girl.
14. The sample space that describes the two-way classification of citizens according to gender and opinion on a political issue is $S = \{mf, ma, mn, ff, fa, fn\}$, where the first letter denotes gender (m: male, f: female) and the second opinion (f: for, a: against, n: neutral). For each of the following events in the experiment of selecting a citizen at random, state the complement of the event in the simplest possible terms, then find the outcomes that comprise the event and its complement.

a. The person is male.
b. The person is not in favor.
c. The person is either male or in favor.
d. The person is female and neutral.

15. A tourist who speaks English and German but no other language visits a region of Slovenia. If 35% of the residents speak English, 15% speak German, and 3% speak both English and German, what is the probability that the tourist will be able to talk with a randomly encountered resident of the region?
16. In a certain country 43% of all automobiles have airbags, 27% have anti-lock brakes, and 13% have both. What is the probability that a randomly selected vehicle will have both airbags and anti-lock brakes?
17. A manufacturer examines its records over the last year on a component part received from outside suppliers. The breakdown on source (supplier A, supplier B) and quality ($\text{H: high, U: usable, D: defective}$) is shown in the two-way contingency table.

	H	U	D
A	0.6937	0.0049	0.0014
B	0.2982	0.0009	0.0009

The record of a part is selected at random. Find the probability of each of the following events.

a. The part was defective.
b. The part was either of high quality or was at least usable, in two ways: (i) by adding numbers in the table, and (ii) using the answer to (a) and the Probability Rule for Complements.
c. The part was defective and came from supplier B.
d. The part was defective or came from supplier B, in two ways: by finding the cells in the table that correspond to this event and adding their probabilities, and (ii) using the Additive Rule of Probability.

18. Individuals with a particular medical condition were classified according to the presence (T) or absence (N) of a potential toxin in their blood and the onset of the condition ($\text{E: early, M: midrange, L: late}$). The breakdown according to this classification is shown in the two-way contingency table.

	E	M	L
T	0.012	0.124	0.013
N	0.170	0.638	0.043

One of these individuals is selected at random. Find the probability of each of the following events.

a. The person experienced early onset of the condition.
b. The onset of the condition was either midrange or late, in two ways: (i) by adding numbers in the table, and (ii) using the answer to (a) and the Probability Rule for Complements.
c. The toxin is present in the person's blood.
d. The person experienced early onset of the condition and the toxin is present in the person's blood.
e. The person experienced early onset of the condition or the toxin is present in the person's blood, in two ways: (i) by finding the cells in the table that correspond to this event and adding their probabilities, and (ii) using the Additive Rule of Probability.

19. The breakdown of the students enrolled in a university course by class ($\text{F: freshman, So: sophomore, J: junior, Se: senior}$) and academic major ($\text{S: science, mathematics, or engineering, L: liberal arts, O: other}$) is shown in the two-way classification table.

Major	Class			
	F	So	J	Se
S	92	42	20	13
L	368	167	80	53

Major	Class			
	F	So	J	Se
O	460	209	100	67

A student enrolled in the course is selected at random. Adjoin the row and column totals to the table and use the expanded table to find the probability of each of the following events.

a. The student is a freshman.
b. The student is a liberal arts major.
c. The student is a freshman liberal arts major.
d. The student is either a freshman or a liberal arts major.
e. The student is not a liberal arts major.

20. The table relates the response to a fund-raising appeal by a college to its alumni to the number of years since graduation.

Response	Years Since Graduation			
	$0-5$	$6-20$	$21-35$	Over 35
Positive	120	440	210	90
None	1380	3560	3290	910

An alumnus is selected at random. Adjoin the row and column totals to the table and use the expanded table to find the probability of each of the following events.

a. The alumnus responded.
b. The alumnus did not respond.
c. The alumnus graduated at least 21 years ago.
d. The alumnus graduated at least 21 years ago and responded.

Additional Exercises

21. The sample space for tossing three coins is $S=\{hhh, hht, hth, htt, thh, tht, tth, ttt\}$
    a. List the outcomes that correspond to the statement "All the coins are heads."
    b. List the outcomes that correspond to the statement "Not all the coins are heads."
    c. List the outcomes that correspond to the statement "All the coins are not heads."

Answers

1. a. $\{b, c\}$
   b. $\{a\}$
   c. $\varnothing$
2.
3. a. $H=\{hhh, hht, hth, htt, thh, tht, tth\},\ M=\{hhh, hht, hth, thh\}$
   b. $H\cap M=\{hhh, hht, hth, thh\}, H\cup M=H, H^c=\{ttt\}$
   c. $P(H\cap M)=4/8, P(H\cup M)=7/8, P(H^c)=1/8$
   d. Mutually exclusive because they have no elements in common.
4.
5. a. $B=\{b1, b2, b3, b4\},\ R=\{r1, r2, r3, r4\},\ N=\{b1, b2, y1, y2, g1, g2, r1, r2\}$
   b. $B\cap R=\varnothing, B\cup R=\{b1, b2, b3, b4, r1, r2, r3, r4\},\ B\cap N=\{b1, b2\},$
   $R\cup N=\{b1, b2, y1, y2, g1, g2, r1, r2, r3, r4\},$
   $B^c=\{y1, y2, y3, y4, g1, g2, g3, g4, r1, r2, r3, r4\},\ (B\cup R)^c=\{y1, y2, y3, y4, g1, g2, g3, g4\}$
   c. $P(B\cap R)=0,\ P(B\cup R)=8/16,\ P(B\cap N)=2/16,\ P(R\cup N)=10/16,\ P(B^c)=12/16,\ P((B\cup R)^c)=8/16$
   d. Not mutually exclusive because they have an element in common.

6.
7. a. 0.36
   b. 0.78
   c. 0.64
   d. 0.27
   e. 0.87
8.
9. a. $P(A)=0.38,\ P(B)=0.62,\ P(A\cap B)=0$
   b. $P(U)=0.37,\ P(W)=0.33,\ P(U\cap W)=0$
   c. 0.7
   d. 0.7
   e. A and U are not mutually exclusive because $P(A\cap U)$ is the nonzero number 0.15. A and V are mutually exclusive because $P(A\cap V)=0$.
10.
11. a. “four or less”
    b. “an odd number”
    c. “no heads” or “all tails”
    d. “a freshman”
12.
13. a. “All the children are boys.” Event: $\{bbg, bgb, bgg, gbb, gbg, ggb, ggg\}$, Complement: $\{bbb\}$
    b. “At least two of the children are girls” or “There are two or three girls.” Event: $\{bbb, bbg, bgb, gbb\}$, Complement: $\{bgg, gbg, ggb, ggg\}$
    c. “At least one child is a boy.” Event: $\{ggg\}$, Complement: $\{bbb, bbg, bgb, bgg, gbb, gbg, ggb\}$
    d. “There are either no girls, exactly one girl, or three girls.” Event: $\{bgg, gbg, ggb\}$, Complement: $\{bbb, bbg, bgb, gbb, ggg\}$
    e. “The first born is a boy.” Event: $\{gbb, gbg, ggb, ggg\}$, Complement: $\{bbb, bbg, bgb, bgg\}$
14.
15. 0.47
16.
17. a. 0.0023
    b. 0.9977
    c. 0.0009
    d. 0.3014
18.
19. a. $920/1671$
    b. $668/1671$
    c. $368/1671$
    d. $1220/1671$
    e. $1003/1671$
20.
21. a. $\{hhh\}$
    b. $\{hht, hth, htt, thh, tht, tth, ttt\}$
    c. $\{ttt\}$

3.3: Conditional Probability and Independent Events

Basic

1. Q3.3.1For two events A and B, $P(A)=0.73,\ P(B)=0.48$ and $P(A\cap B)=0.29$.
   a. Find $P(A\mid B)$.
   b. Find $P(B\mid A)$.
   c. Determine whether or not A and B are independent.

2. Q3.3.1For two events A and B, $P(A)=0.26,\ P(B)=0.37$ and $P(A\cap B)=0.11$.
   a. Find $P(A\mid B)$.
   b. Find $P(B\mid A)$.
   c. Determine whether or not A and B are independent.
3. Q3.3.1For independent events A and B, $P(A)=0.81$ and $P(B)=0.27$.
   a. $P(A\cap B)$.
   b. Find $P(A\mid B)$.
   c. Find $P(B\mid A)$.
4. Q3.3.1For independent events A and B, $P(A)=0.68$ and $P(B)=0.37$.
   a. $P(A\cap B)$.
   b. Find $P(A\mid B)$.
   c. Find $P(B\mid A)$.
5. Q3.3.1For mutually exclusive events A and B, $P(A)=0.17$ and $P(B)=0.32$.
   a. Find $P(A\mid B)$.
   b. Find $P(B\mid A)$.
6. Q3.3.1For mutually exclusive events A and B, $P(A)=0.45$ and $P(B)=0.09$.
   a. Find $P(A\mid B)$.
   b. Find $P(B\mid A)$.
7. Q3.3.1Compute the following probabilities in connection with the roll of a single fair die.
   a. The probability that the roll is even.
   b. The probability that the roll is even, given that it is not a two.
   c. The probability that the roll is even, given that it is not a one.
8. Q3.3.1Compute the following probabilities in connection with two tosses of a fair coin.
   a. The probability that the second toss is heads.
   b. The probability that the second toss is heads, given that the first toss is heads.
   c. The probability that the second toss is heads, given that at least one of the two tosses is heads.
9. Q3.3.1A special deck of 16 cards has 4 that are blue, 4 yellow, 4 green, and 4 red. The four cards of each color are numbered from one to four. A single card is drawn at random. Find the following probabilities.
   a. The probability that the card drawn is red.
   b. The probability that the card is red, given that it is not green.
   c. The probability that the card is red, given that it is neither red nor yellow.
   d. The probability that the card is red, given that it is not a four.
10. Q3.3.1A special deck of 16 cards has 4 that are blue, 4 yellow, 4 green, and 4 red. The four cards of each color are numbered from one to four. A single card is drawn at random. Find the following probabilities.
    a. The probability that the card drawn is a two or a four.
    b. The probability that the card is a two or a four, given that it is not a one.
    c. The probability that the card is a two or a four, given that it is either a two or a three.
    d. The probability that the card is a two or a four, given that it is red or green.
11. Q3.3.1A random experiment gave rise to the two-way contingency table shown. Use it to compute the probabilities indicated.

	R	S
A	0.12	0.18
B	0.28	0.42

    a. $P(A),\ P(R),\ P(A\cap B)$.
    b. Based on the answer to (a), determine whether or not the events A and R are independent.
    c. Based on the answer to (b), determine whether or not $P(A\mid R)$ can be predicted without any computation. If so, make the prediction. In any case, compute $P(A\mid R)$ using the Rule for Conditional Probability.

12. Q3.3.1A random experiment gave rise to the two-way contingency table shown. Use it to compute the probabilities indicated.

	R	S
A	0.13	0.07
B	0.61	0.19

   a. $P(A),\ P(R),\ P(A\cap B)$.
   b. Based on the answer to (a), determine whether or not the events A and R are independent.
   c. Based on the answer to (b), determine whether or not $P(A \mid R)$ can be predicted without any computation. If so, make the prediction. In any case, compute $P(A \mid R)$ using the Rule for Conditional Probability.
13. Q3.3.1Suppose for events A and B in a random experiment $P(A)=0.70$ and $P(B)=0.30$.Compute the indicated probability, or explain why there is not enough information to do so.
   a. $P(A\cap B)$.
   b. $P(A\cap B)$, with the extra information that A and B are independent.
   c. $P(A\cap B)$, with the extra information that A and B are mutually exclusive.
14. Q3.3.1Suppose for events A and B in a random experiment $P(A)=0.50$ and $P(B)=0.50$. Compute the indicated probability, or explain why there is not enough information to do so.
   a. $P(A\cap B)$.
   b. $P(A\cap B)$, with the extra information that A and B are independent.
   c. $P(A\cap B)$, with the extra information that A and B are mutually exclusive.
15. Q3.3.1Suppose for events $A,\ B,\ and\ C$ connected to some random experiment, $A,\ B,\ and\ C$ are independent and $P(A)=0.50$, $P(B)=0.50$ and $P(C)=0.44$. Compute the indicated probability, or explain why there is not enough information to do so.
   a. $P(A\cap B\cap C)$.
   b. $P(A^c\cap B^c\cap C^c)$.
16. Q3.3.1Suppose for events $A,\ B,\ and\ C$ connected to some random experiment, $A,\ B,\ and\ C$ are independent and $P(A)=0.95$, $P(B)=0.73$ and $P(C)=0.62$. Compute the indicated probability, or explain why there is not enough information to do so.
   a. $P(A\cap B\cap C)$.
   b. $P(A^c\cap B^c\cap C^c)$.

Applications

Q3.3.17

The sample space that describes all three-child families according to the genders of the children with respect to birth order is

$$S=\{bbb, bbg, bgb, bgg, gbb, gbg, ggb, ggg\} \tag{3.E.7}$$

In the experiment of selecting a three-child family at random, compute each of the following probabilities, assuming all outcomes are equally likely.

a. The probability that the family has at least two boys.
b. The probability that the family has at least two boys, given that not all of the children are girls.
c. The probability that at least one child is a boy.
d. The probability that at least one child is a boy, given that the first born is a girl.

Q3.3.18

The following two-way contingency table gives the breakdown of the population in a particular locale according to age and number of vehicular moving violations in the past three years:

Age	Violations		
	0	1	2+

Age	Violations		
	0	1	2+
Under 21	0.04	0.06	0.02
21 – 40	0.25	0.16	0.01
41 – 60	0.23	0.10	0.02
60+	0.08	0.03	0.00

A person is selected at random. Find the following probabilities.

a. The person is under 21.
b. The person has had at least two violations in the past three years.
c. The person has had at least two violations in the past three years, given that he is under 21.
d. The person is under 21, given that he has had at least two violations in the past three years.
e. Determine whether the events “the person is under 21” and “the person has had at least two violations in the past three years” are independent or not.

Q3.3.19

The following two-way contingency table gives the breakdown of the population in a particular locale according to party affiliation (A, B, C, or None) and opinion on a bond issue:

Affiliation	Opinion		
	Favors	Opposes	Undecided
A	0.12	0.09	0.07
B	0.16	0.12	0.14
C	0.04	0.03	0.06
None	0.08	0.06	0.03

A person is selected at random. Find each of the following probabilities.

a. The person is in favor of the bond issue.
b. The person is in favor of the bond issue, given that he is affiliated with party A.
c. The person is in favor of the bond issue, given that he is affiliated with party B.

Q3.3.20

The following two-way contingency table gives the breakdown of the population of patrons at a grocery store according to the number of items purchased and whether or not the patron made an impulse purchase at the checkout counter:

Number of Items	Impulse Purchase	
	Made	Not Made
Few	0.01	0.19
Many	0.04	0.76

A patron is selected at random. Find each of the following probabilities.

a. The patron made an impulse purchase.
b. The patron made an impulse purchase, given that the total number of items purchased was many.
c. Determine whether or not the events “few purchases” and “made an impulse purchase at the checkout counter” are independent.

Q3.3.21

The following two-way contingency table gives the breakdown of the population of adults in a particular locale according to employment type and level of life insurance:

Employment Type	Level of Insurance		
	Low	Medium	High
Unskilled	0.07	0.19	0.00
Semi-skilled	0.04	0.28	0.08
Skilled	0.03	0.18	0.05
Professional	0.01	0.05	0.02

An adult is selected at random. Find each of the following probabilities.

a. The person has a high level of life insurance.
b. The person has a high level of life insurance, given that he does not have a professional position.
c. The person has a high level of life insurance, given that he has a professional position.
d. Determine whether or not the events "has a high level of life insurance" and "has a professional position" are independent.

Q3.3.22

The sample space of equally likely outcomes for the experiment of rolling two fair dice is

$$\begin{array}{cccccc} 11 & 12 & 13 & 14 & 15 & 16 \\ 21 & 22 & 23 & 24 & 25 & 26 \\ 31 & 32 & 33 & 34 & 35 & 36 \\ 41 & 42 & 43 & 44 & 45 & 46 \\ 51 & 52 & 53 & 54 & 55 & 56 \\ 61 & 62 & 63 & 64 & 65 & 66 \end{array} \tag{3.E.8}$$

Identify the events $\text{N: the sum is at least nine, T: at least one of the dice is a two, and F: at least one of the dice is a five}$.

a. Find $P(N)$.
b. Find $P(N \mid F)$.
c. Find $P(N \mid T)$.
d. Determine from the previous answers whether or not the events N and F are independent; whether or not N and T are.

Q3.3.23

The *sensitivity* of a drug test is the probability that the test will be positive when administered to a person who has actually taken the drug. Suppose that there are two independent tests to detect the presence of a certain type of banned drugs in athletes. One has sensitivity 0.75; the other has sensitivity 0.85. If both are applied to an athlete who has taken this type of drug, what is the chance that his usage will go undetected?

Q3.3.24

A man has two lights in his well house to keep the pipes from freezing in winter. He checks the lights daily. Each light has probability 0.002 of burning out before it is checked the next day (independently of the other light).

a. If the lights are wired in parallel one will continue to shine even if the other burns out. In this situation, compute the probability that at least one light will continue to shine for the full 24 hours. Note the greatly increased reliability of the system of two bulbs over that of a single bulb.
b. If the lights are wired in series neither one will continue to shine even if only one of them burns out. In this situation, compute the probability that at least one light will continue to shine for the full 24 hours. Note the slightly decreased reliability of the system of two bulbs over that of a single bulb.

Q3.3.25

An accountant has observed that 5% of all copies of a particular two-part form have an error in Part I, and 2% have an error in Part II. If the errors occur independently, find the probability that a randomly selected form will be error-free.

Q3.3.26

A box contains 20 screws which are identical in size, but 12 of which are zinc coated and 8 of which are not. Two screws are selected at random, without replacement.

a. Find the probability that both are zinc coated.
b. Find the probability that at least one is zinc coated.

Additional Exercises

Q3.3.27

Events A and B are mutually exclusive. Find $P(A \mid B)$.

Q3.3.28

The city council of a particular city is composed of five members of party A, four members of party B, and three independents. Two council members are randomly selected to form an investigative committee.

a. Find the probability that both are from party A.
b. Find the probability that at least one is an independent.
c. Find the probability that the two have different party affiliations (that is, not both A, not both B, and not both independent).

Q3.3.29

A basketball player makes 60% of the free throws that he attempts, except that if he has just tried and missed a free throw then his chances of making a second one go down to only 30%. Suppose he has just been awarded two free throws.

a. Find the probability that he makes both.
b. Find the probability that he makes at least one. (A tree diagram could help.)

Q3.3.30

An economist wishes to ascertain the proportion p of the population of individual taxpayers who have purposely submitted fraudulent information on an income tax return. To truly guarantee anonymity of the taxpayers in a random survey, taxpayers questioned are given the following instructions.

a. Flip a coin.
b. If the coin lands heads, answer “Yes” to the question “Have you ever submitted fraudulent information on a tax return?” even if you have not.
c. If the coin lands tails, give a truthful “Yes” or “No” answer to the question “Have you ever submitted fraudulent information on a tax return?”

The questioner is not told how the coin landed, so he does not know if a “Yes” answer is the truth or is given only because of the coin toss.

a. Using the Probability Rule for Complements and the independence of the coin toss and the taxpayers’ status fill in the empty cells in the two-way contingency table shown. Assume that the coin is fair. Each cell except the two in the bottom row will contain the unknown proportion (or probability) p.

Status	Coin		Probability
	H	T	
Fraud			p
No fraud			
Probability			1

b. The only information that the economist sees are the entries in the following table:

$$\begin{array}{c|c|c} Response & "\,Yes\," & "\,No\," \\ \hline Proportion & r & s \end{array} \tag{3.E.9}$$

Equate the entry in the one cell in the table in (a) that corresponds to the answer “No” to the number s to obtain the formula that expresses the unknown number p in terms of the known number s.

c. Equate the sum of the entries in the three cells in the table in (a) that together correspond to the answer “Yes” to the number r to obtain the formula that expresses the unknown number p in terms of the known number r.
d. Use the fact that $r+s=1$ (since they are the probabilities of complementary events) to verify that the formulas in (b) and (c) give the same value for p. (For example, insert $s=1-r$ into the formula in (b) to obtain the formula in (c)).
e. Suppose a survey of $1,200$ taxpayers is conducted and 690 respond “Yes” (truthfully or not) to the question “Have you ever submitted fraudulent information on a tax return?” Use the answer to either (b) or (c) to estimate the true proportion p of all individual taxpayers who have purposely submitted fraudulent information on an income tax return.

Answers

1. a. 0.6
   b. 0.4
   c. not independent
2.
3. a. 0.22
   b. 0.81
   c. 0.27
4.
5. a. 0
   b. 0
6.
7. a. 0.5
   b. 0.4
   c. 0.6
8.
9. a. 0.25
   b. 0.33
   c. 0
   d. 0.25
10.
11. a. $P(A)=0.3,\ P(R)=0.4,\ P(A\cap R)=0.12$
    b. independent
    c. without computation 0.3
12.
13. a. Insufficient information. The events A and B are not known to be either independent or mutually exclusive.
    b. 0.21
    c. 0
14.
15. a. 0.25
    b. 0.02
16.
17. a. 0.5
    b. 0.57
    c. 0.875
    d. 0.75
18.

19. a. 0.4
    b. 0.43
    c. 0.38
20.
21. a. 0.15
    b. 0.14
    c. 0.25
    d. not independent
22.
23. 0.0375
24.
25. 0.931
26.
27. 0
28.
29. a. 0.36
    b. 0.72

Contributor

- Anonymous