CHAPTER OVERVIEW

7: Estimation

If we wish to estimate the mean μ of a population for which a census is impractical, say the average height of all 18-year-old men in the country, a reasonable strategy is to take a sample, compute its mean $\bar{x}$, and estimate the unknown number μ by the known number $\bar{x}$. For example, if the average height of 100 randomly selected men aged 18 is 70.6 inches, then we would say that the average height of all 18-year-old men is (at least approximately) 70.6 inches.

Estimating a population parameter by a single number like this is called point estimation; in the case at hand the statistic $\bar{x}$ is a point estimate of the parameter μ. The terminology arises because a single number corresponds to a single point on the number line.

A problem with a point estimate is that it gives no indication of how reliable the estimate is. In contrast, in this chapter we learn about interval estimation. In brief, in the case of estimating a population mean μ we use a formula to compute from the data a number E, called the margin of error of the estimate, and form the interval $[\bar{x}-E,\bar{x}+E]$. We do this in such a way that a certain proportion, say 95%, of all the intervals constructed from sample data by means of this formula contain the unknown parameter μ. Such an interval is called a 95% confidence interval for μ.

Continuing with the example of the average height of 18-year-old men, suppose that the sample of 100 men mentioned above for which $\bar{x}=70.6$ inches also had sample standard deviation $s = 1.7$ inches. It then turns out that $E = 0.33$ and we would state that we are 95% confident that the average height of all 18-year-old men is in the interval formed by 70.6 ± 0.33 inches, that is, the average is between 70.27 and 70.93 inches. If the sample statistics had come from a smaller sample, say a sample of 50 men, the lower reliability would show up in the 95% confidence interval being longer, hence less precise in its estimate. In this example the 95% confidence interval for the same sample statistics but with $n = 50$ is 70.6 ± 0.47 inches, or from 70.13 to 71.07 inches.