4.E: Discrete Random Variables (Exercises)

These are homework exercises to accompany the Textmap created for "Introductory Statistics" by Shafer and Zhang.

4.1: Random Variables

Basic

1. Classify each random variable as either discrete or continuous.
   a. The number of arrivals at an emergency room between midnight and $6:00\ a.m.$
   b. The weight of a box of cereal labeled "18 ounces."
   c. The duration of the next outgoing telephone call from a business office.
   d. The number of kernels of popcorn in a 1-pound container.
   e. The number of applicants for a job.
2. Classify each random variable as either discrete or continuous.
   a. The time between customers entering a checkout lane at a retail store.
   b. The weight of refuse on a truck arriving at a landfill.
   c. The number of passengers in a passenger vehicle on a highway at rush hour.
   d. The number of clerical errors on a medical chart.
   e. The number of accident-free days in one month at a factory.
3. Classify each random variable as either discrete or continuous.
   a. The number of boys in a randomly selected three-child family.
   b. The temperature of a cup of coffee served at a restaurant.
   c. The number of no-shows for every 100 reservations made with a commercial airline.
   d. The number of vehicles owned by a randomly selected household.
   e. The average amount spent on electricity each July by a randomly selected household in a certain state.
4. Classify each random variable as either discrete or continuous.
   a. The number of patrons arriving at a restaurant between $5:00\ p.m.$ and $6:00\ p.m.$
   b. The number of new cases of influenza in a particular county in a coming month.
   c. The air pressure of a tire on an automobile.
   d. The amount of rain recorded at an airport one day.
   e. The number of students who actually register for classes at a university next semester.
5. Identify the set of possible values for each random variable. (Make a reasonable estimate based on experience, where necessary.)
   a. The number of heads in two tosses of a coin.
   b. The average weight of newborn babies born in a particular county one month.
   c. The amount of liquid in a 12-ounce can of soft drink.
   d. The number of games in the next World Series (best of up to seven games).
   e. The number of coins that match when three coins are tossed at once.
6. Identify the set of possible values for each random variable. (Make a reasonable estimate based on experience, where necessary.)
   a. The number of hearts in a five-card hand drawn from a deck of 52 cards that contains 13 hearts in all.
   b. The number of pitches made by a starting pitcher in a major league baseball game.
   c. The number of breakdowns of city buses in a large city in one week.
   d. The distance a rental car rented on a daily rate is driven each day.
   e. The amount of rainfall at an airport next month.

Answers

1. a. discrete
   b. continuous
   c. continuous
   d. discrete

e. discrete

2.

3. a. discrete
   b. continuous
   c. discrete
   d. discrete
   e. continuous

4.

5. a. $\{0.1.2\}$
   b. an interval (a, b) (answers vary)
   c. an interval (a, b) (answers vary)
   d. $\{4, 5, 6, 7\}$
   e. $\{2, 3\}$

4.2: Probability Distributioins for Discrete Random Variables

Basic

1. Determine whether or not the table is a valid probability distribution of a discrete random variable. Explain fully.

a.

$$\begin{array}{c|cccc} x & -2 & 0 & 2 & 4 \\ \hline P(x) & 0.3 & 0.5 & 0.2 & 0.1 \end{array} \tag{4.E.1}$$

b.

$$\begin{array}{c|ccc} x & 0.5 & 0.25 & 0.25 \\ \hline P(x) & -0.4 & 0.6 & 0.8 \end{array} \tag{4.E.2}$$

c.

$$\begin{array}{c|ccccc} x & 1.1 & 2.5 & 4.1 & 4.6 & 5.3 \\ \hline P(x) & 0.16 & 0.14 & 0.11 & 0.27 & 0.22 \end{array} \tag{4.E.3}$$

2. Determine whether or not the table is a valid probability distribution of a discrete random variable. Explain fully.

a.

$$\begin{array}{c|ccccc} x & 0 & 1 & 2 & 3 & 4 \\ \hline P(x) & -0.25 & 0.50 & 0.35 & 0.10 & 0.30 \end{array} \tag{4.E.4}$$

b.

$$\begin{array}{c|ccc} x & 1 & 2 & 3 \\ \hline P(x) & 0.325 & 0.406 & 0.164 \end{array} \tag{4.E.5}$$

c.

$$\begin{array}{c|ccccc} x & 25 & 26 & 27 & 28 & 29 \\ \hline P(x) & 0.13 & 0.27 & 0.28 & 0.18 & 0.14 \end{array} \tag{4.E.6}$$

3. A discrete random variable X has the following probability distribution:

$$\begin{array}{c|ccccc} x & 77 & 78 & 79 & 80 & 81 \\ \hline P(x) & 0.15 & 0.15 & 0.20 & 0.40 & 0.10 \end{array} \tag{4.E.7}$$

Compute each of the following quantities.

a. $P(80)$.
b. $P(X > 80)$.
c. $P(X \leq 80)$.
d. The mean μ of X.
e. The variance σ^2 of X.
f. The standard deviation σ of X.

4. A discrete random variable X has the following probability distribution:

x	13	18	20	24	27
$P(x)$	0.22	0.25	0.20	0.17	0.16

(4.E.8)

Compute each of the following quantities.

a. $P(18)$.
b. $P(X > 18)$.
c. $P(X \leq 18)$.
d. The mean μ of X.
e. The variance σ^2 of X.
f. The standard deviation σ of X.

5. If each die in a pair is "loaded" so that one comes up half as often as it should, six comes up half again as often as it should, and the probabilities of the other faces are unaltered, then the probability distribution for the sum X of the number of dots on the top faces when the two are rolled is

x	2	3	4	5	6	7
$P(x)$	$\frac{1}{144}$	$\frac{4}{144}$	$\frac{8}{144}$	$\frac{12}{144}$	$\frac{16}{144}$	$\frac{22}{144}$

(4.E.9)

x	8	9	10	11	12
$P(x)$	$\frac{24}{144}$	$\frac{20}{144}$	$\frac{16}{144}$	$\frac{12}{144}$	$\frac{9}{144}$

(4.E.10)

Compute each of the following.

a. $P(5 \leq X \leq 9)$.
b. $P(X \geq 7)$.
c. The mean μ of X. (For fair dice this number is 7).
d. The standard deviation σ of X. (For fair dice this number is about 2.415).

Applications

6. Borachio works in an automotive tire factory. The number X of sound but blemished tires that he produces on a random day has the probability distribution

x	2	3	4	5
$P(x)$	0.48	0.36	0.12	0.04

(4.E.11)

a. Find the probability that Borachio will produce more than three blemished tires tomorrow.
b. Find the probability that Borachio will produce at most two blemished tires tomorrow.
c. Compute the mean and standard deviation of X. Interpret the mean in the context of the problem.

7. In a hamster breeder's experience the number X of live pups in a litter of a female not over twelve months in age who has not borne a litter in the past six weeks has the probability distribution

x	3	4	5	6	7	8	9
$P(x)$	0.04	0.10	0.26	0.31	0.22	0.05	0.02

(4.E.12)

a. Find the probability that the next litter will produce five to seven live pups.
b. Find the probability that the next litter will produce at least six live pups.
c. Compute the mean and standard deviation of X. Interpret the mean in the context of the problem.

8. The number X of days in the summer months that a construction crew cannot work because of the weather has the probability distribution

x	6	7	8	9	10
$P(x)$	0.03	0.08	0.15	0.20	0.19

(4.E.13)

x	11	12	13	14
$P(x)$	0.16	0.10	0.07	0.02

(4.E.14)

a. Find the probability that no more than ten days will be lost next summer.
b. Find the probability that from 8 to 12 days will be lost next summer.
c. Find the probability that no days at all will be lost next summer.
d. Compute the mean and standard deviation of X. Interpret the mean in the context of the problem.

9. Let X denote the number of boys in a randomly selected three-child family. Assuming that boys and girls are equally likely, construct the probability distribution of X.
10. Let X denote the number of times a fair coin lands heads in three tosses. Construct the probability distribution of X.
11. Five thousand lottery tickets are sold for $\$1$ each. One ticket will win $\$1,000$, two tickets will win $\$500$ each, and ten tickets will win $\$100$ each. Let X denote the net gain from the purchase of a randomly selected ticket.
    a. Construct the probability distribution of X.
    b. Compute the expected value $E(X)$ of X. Interpret its meaning.
    c. Compute the standard deviation σ of X.
12. Seven thousand lottery tickets are sold for $\$5$ each. One ticket will win $\$2,000$, two tickets will win $\$750$ each, and five tickets will win $\$100$ each. Let X denote the net gain from the purchase of a randomly selected ticket.
    a. Construct the probability distribution of X.
    b. Compute the expected value $E(X)$ of X. Interpret its meaning.
    c. Compute the standard deviation σ of X.
13. An insurance company will sell a $\$90,000$ one-year term life insurance policy to an individual in a particular risk group for a premium of $\$478$. Find the expected value to the company of a single policy if a person in this risk group has a 99.62% chance of surviving one year.
14. An insurance company will sell a $\$10,000$ one-year term life insurance policy to an individual in a particular risk group for a premium of $\$368$. Find the expected value to the company of a single policy if a person in this risk group has a 97.25% chance of surviving one year.
15. An insurance company estimates that the probability that an individual in a particular risk group will survive one year is 0.9825. Such a person wishes to buy a $\$150,000$ one-year term life insurance policy. Let C denote how much the insurance company charges such a person for such a policy.
    a. Construct the probability distribution of X. (Two entries in the table will contain C).
    b. Compute the expected value $E(X)$ of X.
    c. Determine the value C must have in order for the company to break even on all such policies (that is, to average a net gain of zero per policy on such policies).
    d. Determine the value C must have in order for the company to average a net gain of $\$250$ per policy on all such policies.
16. An insurance company estimates that the probability that an individual in a particular risk group will survive one year is 0.99. Such a person wishes to buy a $\$75,000$ one-year term life insurance policy. Let C denote how much the insurance company charges such a person for such a policy.
    a. Construct the probability distribution of X. (Two entries in the table will contain C).
    b. Compute the expected value $E(X)$ of X.
    c. Determine the value C must have in order for the company to break even on all such policies (that is, to average a net gain of zero per policy on such policies).
    d. Determine the value C must have in order for the company to average a net gain of $\$150$ per policy on all such policies.
17. A roulette wheel has 38 slots. Thirty-six slots are numbered from 1 to 36; half of them are red and half are black. The remaining two slots are numbered 0 and 00 and are green. In a $\$1$ bet on red, the bettor pays $\$1$ to play. If the ball lands in a red slot, he receives back the dollar he bet plus an additional dollar. If the ball does not land on red he loses his dollar. Let X denote the net gain to the bettor on one play of the game.
    a. Construct the probability distribution of X.
    b. Compute the expected value $E(X)$ of X, and interpret its meaning in the context of the problem.
    c. Compute the standard deviation of X.
18. A roulette wheel has 38 slots. Thirty-six slots are numbered from 1 to 36; the remaining two slots are numbered 0 and 00. Suppose the “number” 00 is considered not to be even, but the number 0 is still even. In a $\$1$ bet on even, the bettor pays $\$1$ to play. If the ball lands in an even numbered slot, he receives back the dollar he bet plus an additional dollar. If the ball does not land on an even numbered slot, he loses his dollar. Let X denote the net gain to the bettor on one play of the game.

a. Construct the probability distribution of X.
b. Compute the expected value $E(X)$ of X, and explain why this game is not offered in a casino (where 0 is not considered even).
c. Compute the standard deviation of X.

19. The time, to the nearest whole minute, that a city bus takes to go from one end of its route to the other has the probability distribution shown. As sometimes happens with probabilities computed as empirical relative frequencies, probabilities in the table add up only to a value other than 1.00 because of round-off error.

x	42	43	44	45	46	47
$P(x)$	0.10	0.23	0.34	0.25	0.05	0.02

(4.E.15)

a. Find the average time the bus takes to drive the length of its route.
b. Find the standard deviation of the length of time the bus takes to drive the length of its route.

20. Tybalt receives in the mail an offer to enter a national sweepstakes. The prizes and chances of winning are listed in the offer as: $\$5$ million, one chance in 65 million; $\$150,000$, one chance in 6.5 million; $\$5,000$, one chance in $650,000$; and $\$1,000$, one chance in $65,000$. If it costs Tybalt 44 cents to mail his entry, what is the expected value of the sweepstakes to him?

Additional Exercises

21. The number X of nails in a randomly selected 1-pound box has the probability distribution shown. Find the average number of nails per pound.

x	100	101	102
$P(x)$	0.01	0.96	0.03

(4.E.16)

22. Three fair dice are rolled at once. Let X denote the number of dice that land with the same number of dots on top as at least one other die. The probability distribution for X is

x	0	u	3
$P(x)$	p	$\frac{15}{36}$	$\frac{1}{36}$

(4.E.17)

a. Find the missing value u of X.
b. Find the missing probability p.
c. Compute the mean of X.
d. Compute the standard deviation of X.

23. Two fair dice are rolled at once. Let X denote the difference in the number of dots that appear on the top faces of the two dice. Thus for example if a one and a five are rolled, $X=4$, and if two sixes are rolled, $X=0$.
a. Construct the probability distribution for X.
b. Compute the mean μ of X.
c. Compute the standard deviation σ of X.

24. A fair coin is tossed repeatedly until either it lands heads or a total of five tosses have been made, whichever comes first. Let X denote the number of tosses made.
a. Construct the probability distribution for X.
b. Compute the mean μ of X.
c. Compute the standard deviation σ of X.

25. A manufacturer receives a certain component from a supplier in shipments of 100 units. Two units in each shipment are selected at random and tested. If either one of the units is defective the shipment is rejected. Suppose a shipment has 5 defective units.
a. Construct the probability distribution for the number X of defective units in such a sample. (A tree diagram is helpful).
b. Find the probability that such a shipment will be accepted.

26. Shylock enters a local branch bank at $4:30\ p.m.$ every payday, at which time there are always two tellers on duty. The number X of customers in the bank who are either at a teller window or are waiting in a single line for the next available teller has the following probability distribution.

$$\begin{array}{c|cccc} x & 0 & 1 & 2 & 3 \\ \hline P(x) & 0.135 & 0.192 & 0.284 & 0.230 \end{array} \tag{4.E.18}$$

$$\begin{array}{c|ccc} x & 4 & 5 & 6 \\ \hline P(x) & 0.103 & 0.051 & 0.005 \end{array} \tag{4.E.19}$$

a. What number of customers does Shylock most often see in the bank the moment he enters?
b. What number of customers waiting in line does Shylock most often see the moment he enters?
c. What is the average number of customers who are waiting in line the moment Shylock enters?

27. The owner of a proposed outdoor theater must decide whether to include a cover that will allow shows to be performed in all weather conditions. Based on projected audience sizes and weather conditions, the probability distribution for the revenue X per night if the cover is not installed is

$$\begin{array}{c|c|c} Weather & x & P(x) \\ \hline Clear & \$3,000 & 0.61 \\ Threatening & \$2,800 & 0.17 \\ Light Rain & \$1,975 & 0.11 \\ Show-cancelling\ rain & \$0 & 0.11 \end{array} \tag{4.E.20}$$

The additional cost of the cover is \$410,000. The owner will have it built if this cost can be recovered from the increased revenue the cover affords in the first ten 90-night seasons.

a. Compute the mean revenue per night if the cover is not installed.
b. Use the answer to (a) to compute the projected total revenue per 90-night season if the cover is not installed.
c. Compute the projected total revenue per season when the cover is in place. To do so assume that if the cover were in place the revenue each night of the season would be the same as the revenue on a clear night.
d. Using the answers to (b) and (c), decide whether or not the additional cost of the installation of the cover will be recovered from the increased revenue over the first ten years. Will the owner have the cover installed?

Answers

1. a. no: the sum of the probabilities exceeds 1
   b. no: a negative probability
   c. no: the sum of the probabilities is less than 1
2.
3. a. 0.4
   b. 0.1
   c. 0.9
   d. 79.15
   e. $\sigma^2 = 1.5275$
   f. $\sigma = 1.2359$
4.
5. a. 0.6528
   b. 0.7153
   c. $\mu = 7.8333$
   d. $\sigma^2 = 5.4866$
   e. $\sigma = 2.3424$
6.
7. a. 0.79
   b. 0.60
   c. $\mu = 5.8$, $\sigma = 1.2570$
8.
9.

$$\begin{array}{c|cccc} x & 0 & 1 & 2 & 3 \\ \hline P(x) & 1/8 & 3/8 & 3/8 & 1/8 \end{array} \tag{4.E.21}$$

10.
11. a.

$$\begin{array}{c|cccc} x & -1 & 999 & 499 & 99 \\ \hline P(x) & \frac{4987}{5000} & \frac{1}{5000} & \frac{2}{5000} & \frac{10}{5000} \end{array} \tag{4.E.22}$$

b. -0.4
c. 17.8785

12.
13. 136
14.
15. a.

$$\begin{array}{c|ccc} x & C & C & -150,000 \\ \hline P(x) & 0.9825 & & 0.0175 \end{array} \tag{4.E.23}$$

b. $C - 2625$
c. $C \geq 2625$
d. $C \geq 2875$

16.
17. a.

$$\begin{array}{c|cc} x & -1 & 1 \\ \hline P(x) & \frac{20}{38} & \frac{18}{38} \end{array} \tag{4.E.24}$$

b. $E(X) = -0.0526$. In many bets the bettor sustains an average loss of about 5.25 cents per bet.
c. 0.9986

18.
19. a. 43.54
b. 1.2046

20.
21. 101.02
22.
23. a.

$$\begin{array}{c|cccccc} x & 0 & 1 & 2 & 3 & 4 & 5 \\ \hline P(x) & \frac{6}{36} & \frac{10}{36} & \frac{8}{36} & \frac{6}{36} & \frac{4}{36} & \frac{2}{36} \end{array} \tag{4.E.25}$$

b. 1.9444
c. 1.4326

24.
25. a.

$$\begin{array}{c|ccc} x & 0 & 1 & 2 \\ \hline P(x) & 0.902 & 0.096 & 0.002 \end{array} \tag{4.E.26}$$

b. 0.902

26.
27. a. 2523.25
b. $227,092.5$
c. $270,000$
d. The owner will install the cover.

4.3: The Binomial Distribution

Basic

1. Determine whether or not the random variable X is a binomial random variable. If so, give the values of n and$p$. If not, explain why not.
   a. X is the number of dots on the top face of fair die that is rolled.
   b. X is the number of hearts in a five-card hand drawn (without replacement) from a well-shuffled ordinary deck.
   c. X is the number of defective parts in a sample of ten randomly selected parts coming from a manufacturing process in which 0.02% of all parts are defective.
   d. X is the number of times the number of dots on the top face of a fair die is even in six rolls of the die.
   e. X is the number of dice that show an even number of dots on the top face when six dice are rolled at once.
2. Determine whether or not the random variable X is a binomial random variable. If so, give the values of n and p. If not, explain why not.
   a. X is the number of black marbles in a sample of 5 marbles drawn randomly and without replacement from a box that contains 25 white marbles and 15 black marbles.
   b. X is the number of black marbles in a sample of 5 marbles drawn randomly and with replacement from a box that contains 25 white marbles and 15 black marbles.
   c. X is the number of voters in favor of proposed law in a sample $1,200$ randomly selected voters drawn from the entire electorate of a country in which 35% of the voters favor the law.
   d. X is the number of fish of a particular species, among the next ten landed by a commercial fishing boat, that are more than 13 inches in length, when 17% of all such fish exceed 13 inches in length.
   e. X is the number of coins that match at least one other coin when four coins are tossed at once.
3. X is a binomial random variable with parameters $n = 12$ and $p = 0.82$. Compute the probability indicated.
   a. $P(11)$
   b. $P(9)$
   c. $P(0)$
   d. $P(13)$
4. X is a binomial random variable with parameters $n = 16$ and $p = 0.74$. Compute the probability indicated.
   a. $P(14)$
   b. $P(4)$
   c. $P(0)$
   d. $P(20)$
5. X is a binomial random variable with parameters $n = 5$, $p = 0.5$. Use the tables in 7.1: Large Sample Estimation of a Population Mean to compute the probability indicated.
   a. $P(X \leq 3)$
   b. $P(X \geq 3)$
   c. $P(3)$
   d. $P(0)$
   e. $P(5)$
6. X is a binomial random variable with parameters $n = 5$, $p = 0.\bar{3}$. Use the tables in 7.1: Large Sample Estimation of a Population Mean to compute the probability indicated.
   a. $P(X \leq 2)$
   b. $P(X \geq 2)$
   c. $P(2)$
   d. $P(0)$
   e. $P(5)$
7. X is a binomial random variable with the parameters shown. Use the tables in 7.1: Large Sample Estimation of a Population Mean to compute the probability indicated.
   a. $n = 10, p = 0.25, P(X \leq 6)$
   b. $n = 10, p = 0.75, P(X \leq 6)$

c. $n=15, p=0.75, P(X \leq 6)$
d. $n=15, p=0.75, P(12)$
e. $n=15, p=0.\bar{6}, P(10 \leq X \leq 12)$

8. X is a binomial random variable with the parameters shown. Use the tables in 7.1: Large Sample Estimation of a Population Mean to compute the probability indicated.
   a. $n=5, p=0.05, P(X \leq 1)$
   b. $n=5, p=0.5, P(X \leq 1)$
   c. $n=10, p=0.75, P(X \leq 5)$
   d. $n=10, p=0.75, P(12)$
   e. $n=10, p=0.\bar{6}, P(5 \leq X \leq 8)$
9. X is a binomial random variable with the parameters shown. Use the special formulas to compute its mean μ and standard deviation σ.
   a. $n=8, p=0.43$
   b. $n=47, p=0.82$
   c. $n=1200, p=0.44$
   d. $n=2100, p=0.62$
10. X is a binomial random variable with the parameters shown. Use the special formulas to compute its mean μ and standard deviation σ.
    a. $n=14, p=0.55$
    b. $n=83, p=0.05$
    c. $n=957, p=0.35$
    d. $n=1750, p=0.79$
11. X is a binomial random variable with the parameters shown. Compute its mean μ and standard deviation σ in two ways, first using the tables in 7.1: Large Sample Estimation of a Population Mean in conjunction with the general formulas $\mu=\sum x P(x)$ and $\sigma=\sqrt{\left[\sum x^2 P(x)\right]-\mu^2}$, then using the special formulas $\mu=n p$ and $\sigma=\sqrt{n p q}$.
    a. $n=5, p=0.\bar{3}$
    b. $n=10, p=0.75$
12. X is a binomial random variable with the parameters shown. Compute its mean μ and standard deviation σ in two ways, first using the tables in 7.1: Large Sample Estimation of a Population Mean in conjunction with the general formulas $\mu=\sum x P(x)$ and $\sigma=\sqrt{\left[\sum x^2 P(x)\right]-\mu^2}$, then using the special formulas $\mu=n p$ and $\sigma=\sqrt{n p q}$.
    a. $n=10, p=0.25$
    b. $n=15, p=0.1$
13. X is a binomial random variable with parameters $n=10$ and $p=1/3$. Use the cumulative probability distribution for X that is given in 7.1: Large Sample Estimation of a Population Mean to construct the probability distribution of X.
14. X is a binomial random variable with parameters $n=15$ and $p=1/2$. Use the cumulative probability distribution for X that is given in 7.1: Large Sample Estimation of a Population Mean to construct the probability distribution of X.
15. In a certain board game a player's turn begins with three rolls of a pair of dice. If the player rolls doubles all three times there is a penalty. The probability of rolling doubles in a single roll of a pair of fair dice is $1/6$. Find the probability of rolling doubles all three times.
16. A coin is bent so that the probability that it lands heads up is $2/3$. The coin is tossed ten times.
    a. Find the probability that it lands heads up at most five times.
    b. Find the probability that it lands heads up more times than it lands tails up.

Applications

17. An English-speaking tourist visits a country in which 30% of the population speaks English. He needs to ask someone directions.
    a. Find the probability that the first person he encounters will be able to speak English.
    b. The tourist sees four local people standing at a bus stop. Find the probability that at least one of them will be able to speak English.
18. The probability that an egg in a retail package is cracked or broken is 0.025.

a. Find the probability that a carton of one dozen eggs contains no eggs that are either cracked or broken.
b. Find the probability that a carton of one dozen eggs has (i) at least one that is either cracked or broken; (ii) at least two that are cracked or broken.
c. Find the average number of cracked or broken eggs in one dozen cartons.

19. An appliance store sells 20 refrigerators each week. Ten percent of all purchasers of a refrigerator buy an extended warranty. Let X denote the number of the next 20 purchasers who do so.
    a. Verify that X satisfies the conditions for a binomial random variable, and find n and p.
    b. Find the probability that X is zero.
    c. Find the probability that X is two, three, or four.
    d. Find the probability that X is at least five.
20. Adverse growing conditions have caused 5% of grapefruit grown in a certain region to be of inferior quality. Grapefruit are sold by the dozen.
    a. Find the average number of inferior quality grapefruit per box of a dozen.
    b. A box that contains two or more grapefruit of inferior quality will cause a strong adverse customer reaction. Find the probability that a box of one dozen grapefruit will contain two or more grapefruit of inferior quality.
21. The probability that a 7-ounce skein of a discount worsted weight knitting yarn contains a knot is 0.25. Goneril buys ten skeins to crochet an afghan.
    a. Find the probability that (i) none of the ten skeins will contain a knot; (ii) at most one will.
    b. Find the expected number of skeins that contain knots.
    c. Find the most likely number of skeins that contain knots.
22. One-third of all patients who undergo a non-invasive but unpleasant medical test require a sedative. A laboratory performs 20 such tests daily. Let X denote the number of patients on any given day who require a sedative.
    a. Verify that X satisfies the conditions for a binomial random variable, and find n and p.
    b. Find the probability that on any given day between five and nine patients will require a sedative (include five and nine).
    c. Find the average number of patients each day who require a sedative.
    d. Using the cumulative probability distribution for X in 7.1: Large Sample Estimation of a Population Mean find the minimum number x_{min} of doses of the sedative that should be on hand at the start of the day so that there is a 99% chance that the laboratory will not run out.
23. About 2% of alumni give money upon receiving a solicitation from the college or university from which they graduated. Find the average number monetary gifts a college can expect from every $2,000$ solicitations it sends.
24. Of all college students who are eligible to give blood, about 18% do so on a regular basis. Each month a local blood bank sends an appeal to give blood to 250 randomly selected students. Find the average number of appeals in such mailings that are made to students who already give blood.
25. About 12% of all individuals write with their left hands. A class of 130 students meets in a classroom with 130 individual desks, exactly 14 of which are constructed for people who write with their left hands. Find the probability that exactly 14 of the students enrolled in the class write with their left hands.
26. A traveling salesman makes a sale on 65% of his calls on regular customers. He makes four sales calls each day.
    a. Construct the probability distribution of X, the number of sales made each day.
    b. Find the probability that, on a randomly selected day, the salesman will make a sale.
    c. Assuming that the salesman makes 20 sales calls per week, find the mean and standard deviation of the number of sales made *per week*.
27. A corporation has advertised heavily to try to insure that over half the adult population recognizes the brand name of its products. In a random sample of 20 adults, 14 recognized its brand name. What is the probability that 14 or more people in such a sample would recognize its brand name if the actual proportion p of all adults who recognize the brand name were only 0.50?

Additional Exercises

28. When dropped on a hard surface a thumbtack lands with its sharp point touching the surface with probability $2/3$; it lands with its sharp point directed up into the air with probability $1/3$. The tack is dropped and its landing position observed 15 times.
    a. Find the probability that it lands with its point in the air at least 7 times.

b. If the experiment of dropping the tack 15 times is done repeatedly, what is the average number of times it lands with its point in the air?

29. A professional proofreader has a 98% chance of detecting an error in a piece of written work (other than misspellings, double words, and similar errors that are machine detected). A work contains four errors.
    a. Find the probability that the proofreader will miss at least one of them.
    b. Show that two such proofreaders working independently have a 99.96% chance of detecting an error in a piece of written work.
    c. Find the probability that two such proofreaders working independently will miss at least one error in a work that contains four errors.
30. A multiple choice exam has 20 questions; there are four choices for each question.
    a. A student guesses the answer to every question. Find the chance that he guesses correctly between four and seven times.
    b. Find the minimum score the instructor can set so that the probability that a student will pass just by guessing is 20% or less.
31. In spite of the requirement that all dogs boarded in a kennel be inoculated, the chance that a healthy dog boarded in a clean, well-ventilated kennel will develop kennel cough from a carrier is 0.008.
    a. If a carrier (not known to be such, of course) is boarded with three other dogs, what is the probability that at least one of the three healthy dogs will develop kennel cough?
    b. If a carrier is boarded with four other dogs, what is the probability that at least one of the four healthy dogs will develop kennel cough?
    c. The pattern evident from parts (a) and (b) is that if $K+1$ dogs are boarded together, one a carrier and K healthy dogs, then the probability that at least one of the healthy dogs will develop kennel cough is $P(X \geq 1)=1-(0.992)^K$, where X is the binomial random variable that counts the number of healthy dogs that develop the condition. Experiment with different values of K in this formula to find the maximum number $K+1$ of dogs that a kennel owner can board together so that if one of the dogs has the condition, the chance that another dog will be infected is less than 0.05.
32. Investigators need to determine which of 600 adults have a medical condition that affects 2% of the adult population. A blood sample is taken from each of the individuals.
    a. Show that the expected number of diseased individuals in the group of 600 is 12 individuals.
    b. Instead of testing all 600 blood samples to find the expected 12 diseased individuals, investigators group the samples into 60 groups of 10 each, mix a little of the blood from each of the 10 samples in each group, and test each of the 60 mixtures. Show that the probability that any such mixture will contain the blood of at least one diseased person, hence test positive, is about 0.18.
    c. Based on the result in (b), show that the expected number of mixtures that test positive is about 11. (Supposing that indeed 11 of the 60 mixtures test positive, then we know that none of the 490 persons whose blood was in the remaining 49 samples that tested negative has the disease. We have eliminated 490 persons from our search while performing only 60 tests.)

Answers

1. a. not binomial; not success/failure.
   b. not binomial; trials are not independent.
   c. binomial; $n=10, p=0.0002$
   d. binomial; $n=6, p=0.5$
   e. binomial; $n=6, p=0.5$
2.
3. a. 0.2434
   b. 0.2151
   c. $0.18^{12} \approx 0$
   d. 0
4.
5. a. 0.8125
   b. 0.5000
   c. 0.3125

   d. 0.0313
   e. 0.0312

6.
7. a. 0.9965
   b. 0.2241
   c. 0.0042
   d. 0.2252
   e. 0.5390
8.
9. a. $\mu = 3.44, \sigma = 1.4003$
   b. $\mu = 38.54, \sigma = 2.6339$
   c. $\mu = 528, \sigma = 17.1953$
   d. $\mu = 1302, \sigma = 22.2432$
10.
11. a. $\mu = 1.6667, \sigma = 1.0541$
    b. $\mu = 7.5, \sigma = 1.3693$
12.
13.

$$\begin{array}{c|cccc} x & 0 & 1 & 2 & 3 \\ \hline P(x) & 0.0173 & 0.0867 & 0.1951 & 0.2602 \end{array} \tag{4.E.27}$$

$$\begin{array}{c|cccc} x & 4 & 5 & 6 & 7 \\ \hline P(x) & 0.2276 & 0.1365 & 0.0569 & 0.0163 \end{array} \tag{4.E.28}$$

$$\begin{array}{c|ccc} x & 8 & 9 & 10 \\ \hline P(x) & 0.0030 & 0.0004 & 0.0000 \end{array} \tag{4.E.29}$$

14.
15. 0.0046
16.
17. a. 0.3
    b. 0.7599
18.
19. a. $n = 20, p = 0.1$
    b. 0.1216
    c. 0.5651
    d. 0.0432
20.
21. a. 0.0563 and 0.2440
    b. 2.5
    c. 2
22.
23. 40
24.
25. 0.1019
26.
27. 0.0577
28.
29. a. 0.0776
    b. 0.9996
    c. 0.0016

30.
31. a. 0.0238
 b. 0.0316
 c. 6

Contributor

- Anonymous