

## 11.2: Facts About the Chi-Square Distribution

The notation for the chi-square distribution is:

$$\chi \sim \chi_{df}^2 \quad (11.2.1)$$

where  $df$  = degrees of freedom which depends on how chi-square is being used. (If you want to practice calculating chi-square probabilities then use  $df = n - 1$ . The degrees of freedom for the three major uses are each calculated differently.)

For the  $\chi^2$  distribution, the population mean is  $\mu = df$  and the population standard deviation is

$$\sigma = \sqrt{2(df)}. \quad (11.2.2)$$

The random variable is shown as  $\chi^2$ , but may be any upper case letter. The random variable for a chi-square distribution with  $k$  degrees of freedom is the sum of  $k$  independent, squared standard normal variables.

$$\chi^2 = (Z_1)^2 + \dots + (Z_k)^2 \quad (11.2.3)$$

- The curve is nonsymmetrical and skewed to the right.
- There is a different chi-square curve for each  $df$ .

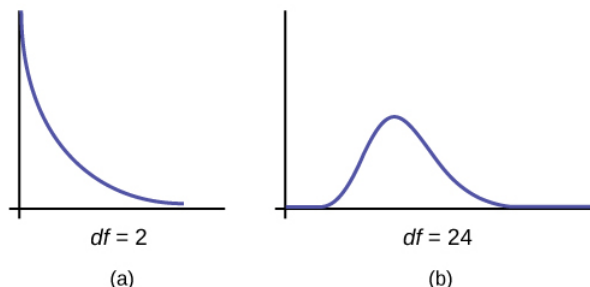


Figure 11.2.1

- The test statistic for any test is always greater than or equal to zero.
- When  $df > 90$ , the chi-square curve approximates the normal distribution. For  $\chi \sim \chi_{1,000}^2$  the mean,  $\mu = df = 1,000$  and the standard deviation,  $\mu = \sqrt{2(1,000)}$ . Therefore,  $X \sim N(1,000, 44.7)$  approximately.
- The mean,  $\mu$ , is located just to the right of the peak.

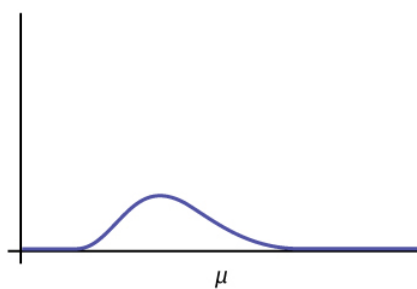
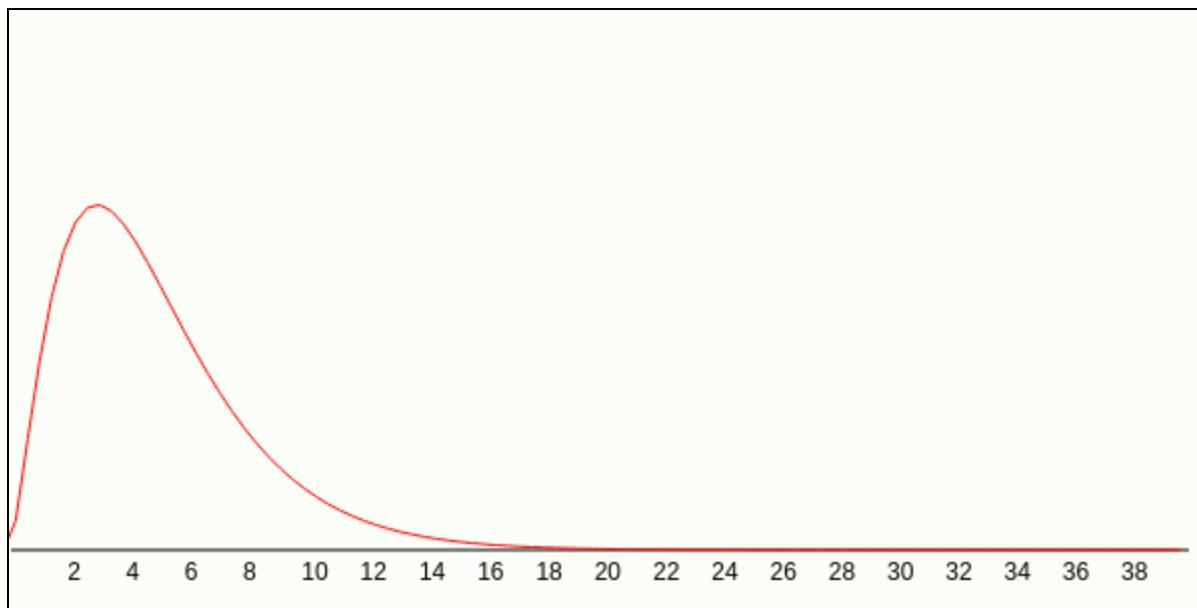


Figure 11.2.2

### Visualize the $\chi^2$ distribution

Move the slider to see how changing the degrees of freedom, changes the  $\chi^2$  distribution.



df = 5

## References

1. Data from *Parade Magazine*.
2. "HIV/AIDS Epidemiology Santa Clara County." Santa Clara County Public Health Department, May 2011.

## Chapter Review

The chi-square distribution is a useful tool for assessment in a series of problem categories. These problem categories include primarily (i) whether a data set fits a particular distribution, (ii) whether the distributions of two populations are the same, (iii) whether two events might be independent, and (iv) whether there is a different variability than expected within a population.

An important parameter in a chi-square distribution is the degrees of freedom  $df$  in a given problem. The random variable in the chi-square distribution is the sum of squares of  $df$  standard normal variables, which must be independent. The key characteristics of the chi-square distribution also depend directly on the degrees of freedom.

The chi-square distribution curve is skewed to the right, and its shape depends on the degrees of freedom  $df$ . For  $df > 90$ , the curve approximates the normal distribution. Test statistics based on the chi-square distribution are always greater than or equal to zero. Such application tests are almost always right-tailed tests.

## Formula Review

$$\chi^2 = (Z_1)^2 + (Z_2)^2 + \dots + (Z_{df})^2 \quad (11.2.4)$$

chi-square distribution random variable

$\mu_{\chi^2} = df$  chi-square distribution population mean

$\sigma_{\chi^2} = \sqrt{2(df)}$  Chi-Square distribution population standard deviation

### Exercise 11.2.1

If the number of degrees of freedom for a chi-square distribution is 25, what is the population mean and standard deviation?

**Answer**

mean = 25 and standard deviation = 7.0711

### Exercise 11.2.2

If  $df > 90$ , the distribution is \_\_\_\_\_. If  $df = 15$ , the distribution is \_\_\_\_\_.

### Exercise 11.2.3

When does the chi-square curve approximate a normal distribution?

**Answer**

when the number of degrees of freedom is greater than 90

### Exercise 11.2.4

Where is  $\mu$  located on a chi-square curve?

### Exercise

Is it more likely the  $df$  is 90, 20, or two in the graph?

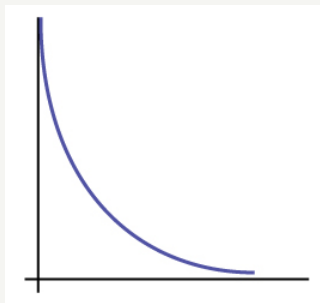


Figure 11.2.3.

**Answer**

$df = 2$

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