11.5: Test for Homogeneity

The goodness–of–fit test can be used to decide whether a population fits a given distribution, but it will not suffice to decide whether two populations follow the same unknown distribution. A different test, called the test for homogeneity, can be used to draw a conclusion about whether two populations have the same distribution. To calculate the test statistic for a test for homogeneity, follow the same procedure as with the test of independence.

> The expected value for each cell needs to be at least five in order for you to use this test.

Hypotheses

- H_0: The distributions of the two populations are the same.
- H_a: The distributions of the two populations are not the same.

Test Statistic

- Use a χ^2 test statistic. It is computed in the same way as the test for independence.

Degrees of Freedom (df)

- $df = \text{number of columns} - 1$

Requirements

- All values in the table must be greater than or equal to five.

Common Uses

Comparing two populations. For example: men vs. women, before vs. after, east vs. west. The variable is categorical with more than two possible response values.

Example 11.5.1

Do male and female college students have the same distribution of living arrangements? Use a level of significance of 0.05. Suppose that 250 randomly selected male college students and 300 randomly selected female college students were asked about their living arrangements: dormitory, apartment, with parents, other. The results are shown in Table $11.5.1$. Do male and female college students have the same distribution of living arrangements?

Table 11.5.1: Distribution of Living Arragements for College Males and College Females

	Dormitory	Apartment	With Parents	Other
Males	72	84	49	45
Females	91	86	88	35

Answer

- H_0: The distribution of living arrangements for male college students is the same as the distribution of living arrangements for female college students.
- H_a: The distribution of living arrangements for male college students is not the same as the distribution of living arrangements for female college students.

Degrees of Freedom (df):

$df = \text{number of columns} - 1 = 4 - 1 = 3$

Distribution for the test: χ^2_3

Calculate the test statistic: $\chi^2 = 10.1287$ (calculator or computer)

Probability statement: $p\text{-value} = P(\chi^2 > 10.1287) = 0.0175$

Press the

```
MATRX
```

key and arrow over to

```
EDIT
```

. Press

```
1:[A]
```

. Press

```
2 ENTER 4 ENTER
```

. Enter the table values by row. Press

```
ENTER
```

after each. Press

```
2nd QUIT
```

. Press

```
STAT
```

and arrow over to

```
TESTS
```

. Arrow down to

```
C:χ2-TEST
```

. Press

```
ENTER
```

. You should see

```
Observed:[A] and Expected:[B]
```

. Arrow down to

```
Calculate
```

. Press

```
ENTER
```

. The test statistic is 10.1287 and the $p\text{-value} = 0.0175$. Do the procedure a second time but arrow down to

```
Draw
```

instead of

```
calculate
```

.

Compare α and the p-value: Since no α is given, assume $\alpha = 0.05$. $p\text{-value} = 0.0175$. $\alpha > p\text{-value}$.

Make a decision: Since $\alpha > p\text{-value}$, reject H_0. This means that the distributions are not the same.

Conclusion: At a 5% level of significance, from the data, there is sufficient evidence to conclude that the distributions of living arrangements for male and female college students are not the same.

Notice that the conclusion is only that the distributions are not the same. We cannot use the test for homogeneity to draw any conclusions about how they differ.

Exercise 11.5.1

Do families and singles have the same distribution of cars? Use a level of significance of 0.05. Suppose that 100 randomly selected families and 200 randomly selected singles were asked what type of car they drove: sport, sedan, hatchback, truck, van/SUV. The results are shown in Table $11.5.2$ Do families and singles have the same distribution of cars? Test at a level of significance of 0.05.

Table 11.5.1

	Sport	Sedan	Hatchback	Truck	Van/SUV
Family	5	15	35	17	28
Single	45	65	37	46	7

Answer

With a p-value of almost zero, we reject the null hypothesis. The data show that the distribution of cars is not the same for families and singles.

Example 11.5.2

Both before and after a recent earthquake, surveys were conducted asking voters which of the three candidates they planned on voting for in the upcoming city council election. Has there been a change since the earthquake? Use a level of significance of 0.05. Table shows the results of the survey. Has there been a change in the distribution of voter preferences since the earthquake?

	Perez	Chung	Stevens
Before	167	128	135
After	214	197	225

Answer

H_0: The distribution of voter preferences was the same before and after the earthquake.

H_a: The distribution of voter preferences was not the same before and after the earthquake.

Degrees of Freedom (df):

$df =$ number of columns $- 1 = 3 - 1 = 2$

Distribution for the test: χ^2_2

Calculate the test statistic: $\chi^2 = 3.2603$ (calculator or computer)

Probability statement: $p\text{-value} = P(\chi^2 > 3.2603) = 0.1959$

Press the `MATRX` key and arrow over to `EDIT` . Press `1:[A]` . Press `2 ENTER 3 ENTER` . Enter the table values by row. Press `ENTER` after each. Press `2nd QUIT` . Press `STAT` and arrow over to `TESTS` . Arrow down to `C:χ2-TEST` . Press `ENTER` . You should see `Observed:[A] and Expected:[B]` . Arrow down to `Calculate` . Press `ENTER` . The test statistic is 3.2603 and the *p*-value = 0.1959. Do the procedure a second time but arrow down to `Draw` instead of `calculate` .

Compare α and the *p*-value: $\alpha = 0.05$ and the $p\text{-value} = 0.1959$. $\alpha < p\text{-value}$.

Make a decision: Since $\alpha < p\text{-value}$, do not reject H_0.

Conclusion: At a 5% level of significance, from the data, there is insufficient evidence to conclude that the distribution of voter preferences was not the same before and after the earthquake.

? Exercise 11.5.2

Ivy League schools receive many applications, but only some can be accepted. At the schools listed in Table, two types of applications are accepted: regular and early decision.

Application Type Accepted	Brown	Columbia	Cornell	Dartmouth	Penn	Yale
Regular	2,115	1,792	5,306	1,734	2,685	1,245
Early Decision	577	627	1,228	444	1,195	761

We want to know if the number of regular applications accepted follows the same distribution as the number of early applications accepted. State the null and alternative hypotheses, the degrees of freedom and the test statistic, sketch the graph of the *p*-value, and draw a conclusion about the test of homogeneity.

Answer

H_0: The distribution of regular applications accepted is the same as the distribution of early applications accepted.

H_a: The distribution of regular applications accepted is not the same as the distribution of early applications accepted.

$df = 5$

$\chi^2 \text{test statistic} = 430.06$

This is a nonsymmetric chi-square curve with df = 5. The values 0, 5, and 430.06 are labeled on the horizontal axis. The value 5 coincides with the peak of the curve. A vertical upward line extends from 430.06 to the curve, and the region to the right of this line is shaded. The shaded area is equal to the p-value.

Figure 11.5.1.

Press the `MATRX` key and arrow over to `EDIT` . Press `1:[A]` . Press `3 ENTER 3 ENTER` . Enter the table values by row. Press `ENTER` after each. Press `2nd QUIT` . Press `STAT` and arrow over to `TESTS` . Arrow down to `C:χ2-TEST` . Press `ENTER` . You should see `Observed:[A] and Expected:[B]` . Arrow down to `Calculate` . Press `ENTER` . The test statistic is 430.06 and the $p\text{-value} = 9.80E-91$. Do the procedure a second time but arrow down to `Draw` instead of `calculate` .

References

1. Data from the Insurance Institute for Highway Safety, 2013. Available online at www.iihs.org/iihs/ratings (accessed May 24, 2013).

2. "Energy use (kg of oil equivalent per capita)." The World Bank, 2013. Available online at http://data.worldbank.org/indicator/...G.OE/countries (accessed May 24, 2013).
3. "Parent and Family Involvement Survey of 2007 National Household Education Survey Program (NHES)," U.S. Department of Education, National Center for Education Statistics. Available online at http://nces.ed.gov/pubsearch/pubsinf...?pubid=2009030 (accessed May 24, 2013).
4. "Parent and Family Involvement Survey of 2007 National Household Education Survey Program (NHES)," U.S. Department of Education, National Center for Education Statistics. Available online at http://nces.ed.gov/pubs2009/2009030_sup.pdf (accessed May 24, 2013).

Review

To assess whether two data sets are derived from the same distribution—which need not be known, you can apply the test for homogeneity that uses the chi-square distribution. The null hypothesis for this test states that the populations of the two data sets come from the same distribution. The test compares the observed values against the expected values if the two populations followed the same distribution. The test is right-tailed. Each observation or cell category must have an expected value of at least five.

Formula Review

$\sum_{i \cdot j} \frac{(O-E)^2}{E}$ Homogeneity test statistic where: $O =$ observed values

$E =$ expected values

$i =$ number of rows in data contingency table

$j =$ number of columns in data contingency table

$df = (i-1)(j-1)$ Degrees of freedom

Exercise 11.5.3

A math teacher wants to see if two of her classes have the same distribution of test scores. What test should she use?

Answer

test for homogeneity

Exercise 11.5.4

What are the null and alternative hypotheses for Exercise?

Exercise 11.5.5

A market researcher wants to see if two different stores have the same distribution of sales throughout the year. What type of test should he use?

Answer

test for homogeneity

Exercise 11.5.6

A meteorologist wants to know if East and West Australia have the same distribution of storms. What type of test should she use?

Exercise 11.5.7

What condition must be met to use the test for homogeneity?

Answer

All values in the table must be greater than or equal to five.

Use the following information to answer the next five exercises: Do private practice doctors and hospital doctors have the same distribution of working hours? Suppose that a sample of 100 private practice doctors and 150 hospital doctors are selected at random and asked about the number of hours a week they work. The results are shown in Table.

	20–30	**30–40**	**40–50**	**50–60**
Private Practice	16	40	38	6
Hospital	8	44	59	39

Exercise 11.5.8

State the null and alternative hypotheses.

Exercise 11.5.9

$df =$ _______

Answer

3

Exercise 11.5.10

What is the test statistic?

Exercise 11.5.11

What is the p-value?

Answer

0.00005

Exercise 11.5.12

What can you conclude at the 5% significance level?