4.3: Solve Equations with Roots

Learning Outcomes

- Solve equations that include square roots.

Square roots occur frequently in a statistics course, especially when dealing with standard deviations and sample sizes. In this section we will learn how to solve for a variable when that variable lies under the square root sign. The key thing to remember is that the square of a square root is what lies inside. In other words, squaring a square root cancels the square root.

Example 4.3.1

Solve the following equation for x.

$$2+\sqrt{x-3} = 6$$

Solution

What makes this a challenge is the square root. The strategy for solving is to isolate the square root on the left side of the equation and then square both sides. First subtract 2 from both sides:

$$\sqrt{x-3}=4$$

Now that the square root is isolated, we can square both sides of the equation:

$$\left(\sqrt{x-3}\right)^2=4^2$$

Since the square and the square root cancel we get:

$$x-3=16$$

Finally add 3 to both sides to arrive at:

$$x=19$$

It's always a good idea to check your work. We do this by plugging the answer back in and seeing if it works. We plug in $x=19$ to get

$$\begin{aligned} 2+\sqrt{19-3} &= 2+\sqrt{16} \\ &= 2+4 \\ &= 6 \end{aligned}$$

Yes, the solution is correct.

Example 4.3.2

The standard deviation, $\sigma_{\hat{p}}$, of the sampling distribution for a proportion follows the formula:

$$\sigma_{\hat{p}}=\sqrt{\frac{p\left(1-p\right)}{n}}$$

Where p is the population proportion and n is the sample size. If the population proportion is 0.24 and you need the standard deviation of the sampling distribution to be 0.03, how large a sample do you need?

Solution

We are given that $p=0.24$ and $\sigma_{\hat{p}}=0.03$

Plug in to get:

$$0.03=\sqrt{\frac{0.24\left(1-0.24\right)}{n}}$$

We want to solve for n, so we want n on the left hand side of the equation. Just switch to get:

$$\sqrt{\frac{0.24\,(1-0.24)}{n}} \;=\; 0.03$$

Next, we subtract:

$$1-0.24 \;=\; 0.76$$

And them multiply:

$$0.24\,(0.76) = 0.1824$$

This gives us

$$\sqrt{\frac{0.1824}{n}} \;=\; 0.03$$

To get rid of the square root, square both sides:

$$\left(\sqrt{\frac{0.1824}{n}}\right)^2 \;=\; 0.03^2$$

The square cancels the square root, and squaring the right hand side gives:

$$\frac{0.1824}{n} \;=\; 0.0009$$

We can write:

$$\frac{0.1824}{n} \;=\; \frac{0.0009}{1}$$

Cross multiply to get:

$$0.0009\,n \;=\; 0.1824$$

Finally, divide both sides by 0.0009:

$$n \;=\; \frac{0.1824}{0.0009} = 202.66667$$

Round up and we can conclude that we need a sample size of 203 to get a standard error that is 0.03. We can check to see if this is reasonable by plugging $n = 203$ back into the equation. We use a calculator to get:

$$\sqrt{\frac{0.24\,(1-0.24)}{203}} \;=\; 0.029975$$

Since this is very close to 0.03, the answer is reasonable.

Exercise

The standard deviation, $\sigma_{\bar{x}}$, of the sampling distribution for a mean follows the formula:

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

Where σ is the population standard deviation and n is the sample size. If the population standard deviation is 3.8 and you need the standard deviation of the sampling distribution to be 0.5, how large a sample do you need?

- Ex 1: Solve a Basic Radical Equation - Square Roots
- https://youtu.be/u1aGMkJIlMI