7.7: Movie Popcorn

Let's see how hypothesis testing works in action by working through an example. Say that a movie theater owner likes to keep a very close eye on how much popcorn goes into each bag sold, so he knows that the average bag has 8 cups of popcorn and that this varies a little bit, about half a cup. That is, the known population mean is μ = 8.00 and the known population standard deviation is σ = 0.50. The owner wants to make sure that the newest employee is filling bags correctly, so over the course of a week he randomly assesses 25 bags filled by the employee to test for a difference (N = 25). He doesn't want bags overfilled or under filled, so he looks for differences in both directions. This scenario has all of the information we need to begin our hypothesis testing procedure.

Step 1: State the Hypotheses Our manager is looking for a difference in the mean weight of popcorn bags compared to the population mean of 8. We will need both a null and an alternative hypothesis written both mathematically and in words. We'll always start with the null hypothesis:

H_0: There is no difference in the weight of popcorn bags from this employee

H_0: $\mu = 8.00$

Notice that we phrase the hypothesis in terms of the population parameter μ, which in this case would be the true average weight of bags filled by the new employee. Our assumption of no difference, the null hypothesis, is that this mean is exactly the same as the known population mean value we want it to match, 8.00. Now let's do the alternative:

H_A: There is a difference in the weight of popcorn bags from this employee

H_A: $\mu \neq 8.00$

In this case, we don't know if the bags will be too full or not full enough, so we do a two-tailed alternative hypothesis that there is a difference.

Step 2: Find the Critical Values Our critical values are based on two things: the directionality of the test and the level of significance. We decided in step 1 that a two-tailed test is the appropriate directionality. We were given no information about the level of significance, so we assume that α = 0.05 is what we will use. As stated earlier in the chapter, the critical values for a two-tailed z-test at α = 0.05 are $z*$ = ±1.96. This will be the criteria we use to test our hypothesis. We can now draw out our distribution so we can visualize the rejection region and make sure it makes sense.

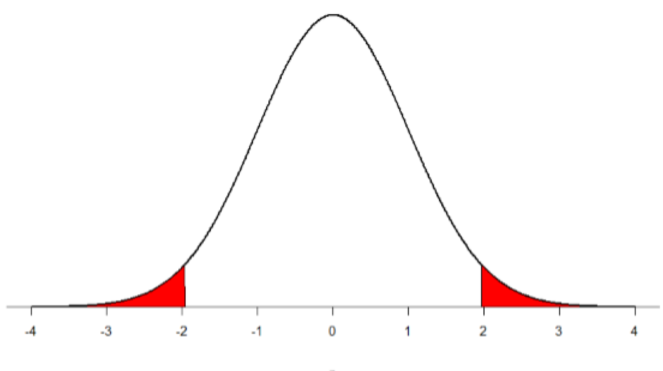

Figure 7.7.1: Rejection region for $z*$ = ±1.96

Step 3: Calculate the Test Statistic Now we come to our formal calculations. Let's say that the manager collects data and finds that the average weight of this employee's popcorn bags is $\overline{\mathrm{X}}$= 7.75 cups. We can now plug this value, along with the values presented in the original problem, into our equation for z:

$$z = \frac{7.75 - 8.00}{0.50/\sqrt{25}} = \frac{-0.25}{0.10} = -2.50$$

So our test statistic is z = -2.50, which we can draw onto our rejection region distribution:

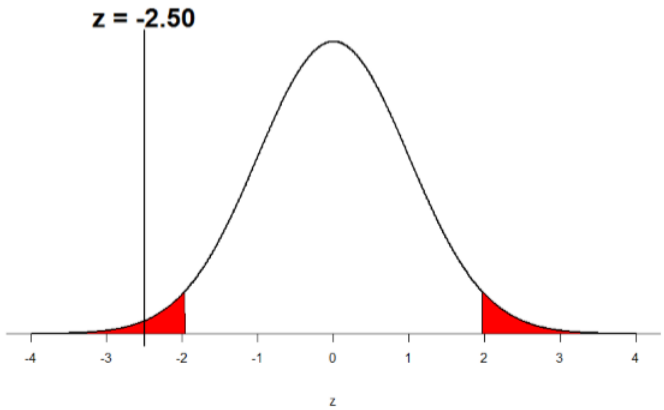

Figure $7.7.2$: Test statistic location

Step 4: Make the Decision Looking at Figure $7.7.2$, we can see that our obtained z-statistic falls in the rejection region. We can also directly compare it to our critical value: in terms of absolute value, -2.50 > -1.96, so we reject the null hypothesis. We can now write our conclusion:

> Reject H_0. Based on the sample of 25 bags, we can conclude that the average popcorn bag from this employee is smaller ($\overline{\mathrm{X}}$= 7.75 cups) than the average weight of popcorn bags at this movie theater, z = 2.50, p < 0.05.

When we write our conclusion, we write out the words to communicate what it actually means, but we also include the average sample size we calculated (the exact location doesn't matter, just somewhere that flows naturally and makes sense) and the z-statistic and p-value. We don't know the exact p-value, but we do know that because we rejected the null, it must be less than α.