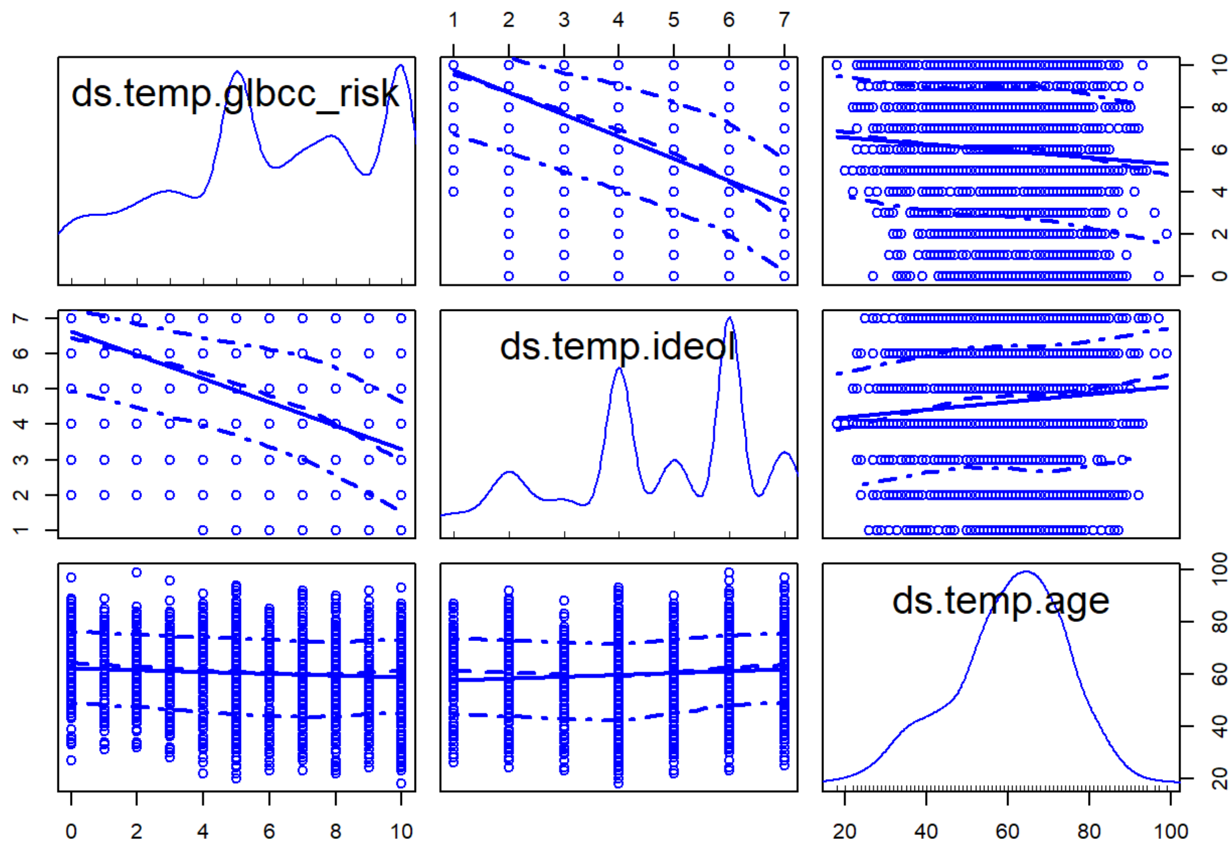
12.3: Multiple Regression Example

```
library(psych)
describe(data.frame(ds.temp$glbcc_risk,ds.temp$ideol,
                    ds.temp$age))
```

```
##                     vars    n  mean    sd median trimmed   mad min max
## ds.temp.glbcc_risk     1 2513  5.95  3.07      6    6.14  2.97   0  10
## ds.temp.ideol          2 2513  4.66  1.73      5    4.76  1.48   1   7
## ds.temp.age            3 2513 60.38 14.19     62   61.01 13.34  18  99
##                     range  skew kurtosis   se
## ds.temp.glbcc_risk     10 -0.32    -0.94 0.06
## ds.temp.ideol           6 -0.45    -0.79 0.03
## ds.temp.age            81 -0.38    -0.23 0.28
```

```
library(car)
scatterplotMatrix(data.frame(ds.temp$glbcc_risk,
                             ds.temp$ideol,ds.temp$age),
                  diagonal="density")
```



In this section, we walk through another example of multiple regression. First, we start with our two IV model.

```
ols1 <- lm(glbcc_risk ~ age+ideol, data=ds.temp)
summary(ols1)
```

```
##
## Call:
## lm(formula = glbcc_risk ~ age + ideol, data = ds.temp)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -8.7913 -1.6252  0.2785  1.4674  6.6075
##
## Coefficients:
##              Estimate Std. Error t value            Pr(>|t|)
## (Intercept) 11.096064   0.244640  45.357 <0.0000000000000002 ***
## age         -0.004872   0.003500  -1.392               0.164
## ideol       -1.042748   0.028674 -36.366 <0.0000000000000002 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.479 on 2510 degrees of freedom
## Multiple R-squared:  0.3488, Adjusted R-squared:  0.3483
## F-statistic: 672.2 on 2 and 2510 DF,  p-value: < 0.00000000000000022
```

The results show that the relationship between age and perceived risk (glbccrsk) is negative and insignificant. The relationship between ideology and perceived risk is negative and significant. The coefficients of the XX's are interpreted in the same way as with simple regression, except that we are now controlling for the effect of the other XX's by removing their influence on the estimated coefficient. Therefore, we say that as ideology increases one unit, perceptions of the risk of climate change (glbccrsk) decrease by -1.0427478, controlling for the effect of age.

As was the case with simple regression, multiple regression finds the intercept and slopes that minimize the sum of the squared residuals. With only one IV the relationship can be represented in a two-dimensional plane (a graph) as a line, but each IV adds another dimension. Two IVs create a regression plane within a cube, as shown in Figure $12.3.3$. The Figure shows a scatterplot of perceived climate change risk, age, and ideology coupled with the regression plane. Note that this is a sample of 200 observations from the larger data set. Were we to add more IVs, we would generate a hypercube… and we haven't found a clever way to draw that yet.

```
ds200 <- ds.temp[sample(1:nrow(ds.temp), 200, replace=FALSE),]
library(scatterplot3d)
s3d <-scatterplot3d(ds200$age,
                    ds200$ideol,
                    ds200$glbcc_risk
                    ,pch=16, highlight.3d=TRUE,
type="h", main="3D Scatterplot")
s3d$plane3d(ols1)
```

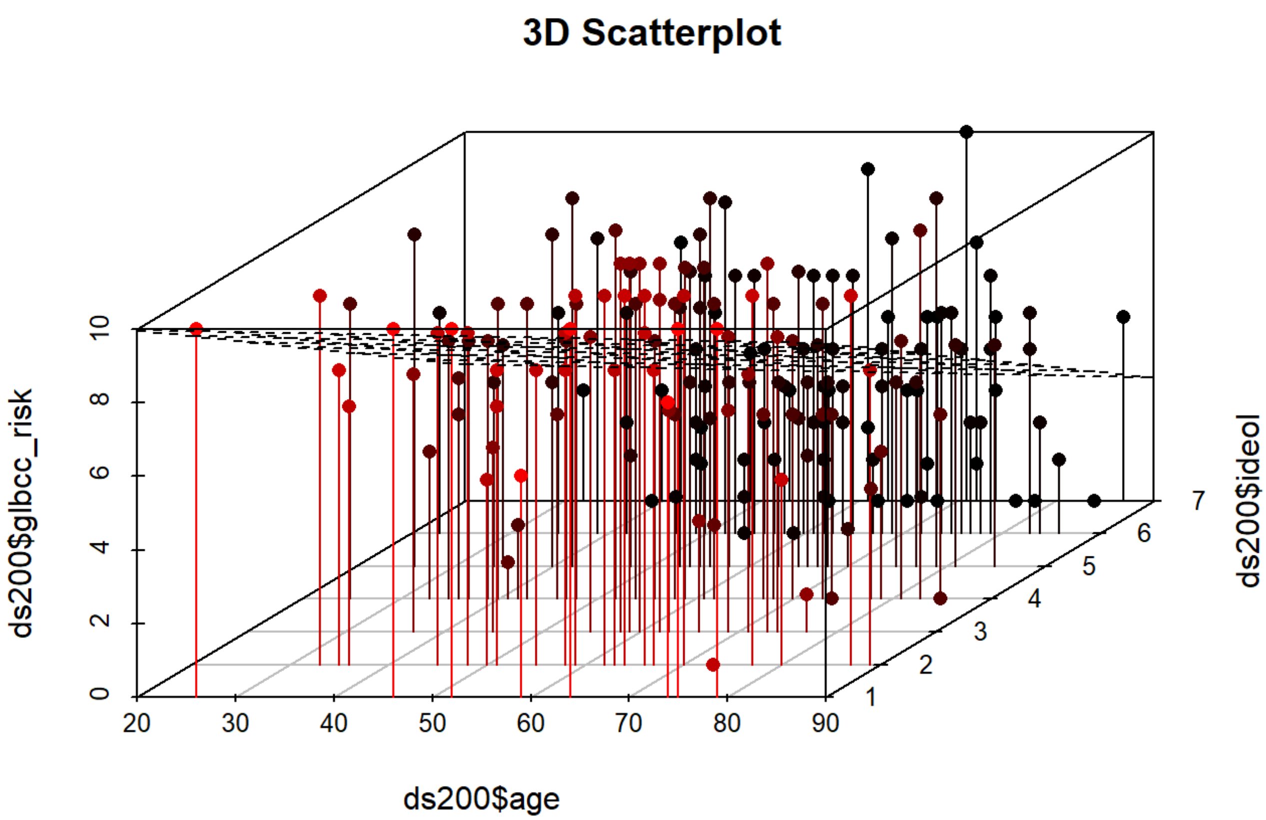


Figure 12.3.3: Scatterplot and Regression Plane of gcc risk, age, and ideology

In the next example education is added to the model.

```
ds.temp <- filter(ds) %>%
  dplyr::select(glbcc_risk, age, education, income, ideol) %>%
  na.omit()

ols2 <- lm(glbcc_risk ~ age + education + ideol, data = ds.temp)
summary(ols2)
```

```
##
## Call:
## lm(formula = glbcc_risk ~ age + education + ideol, data = ds.temp)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -8.8092 -1.6355  0.2388  1.4279  6.6334
##
## Coefficients:
##              Estimate Std. Error t value            Pr(>|t|)
## (Intercept) 10.841669   0.308416  35.153 <0.0000000000000002 ***
## age         -0.003246   0.003652  -0.889               0.374
## education    0.036775   0.028547   1.288               0.198
## ideol       -1.044827   0.029829 -35.027 <0.0000000000000002 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.437 on 2268 degrees of freedom
## Multiple R-squared:  0.3607, Adjusted R-squared:  0.3598
## F-statistic: 426.5 on 3 and 2268 DF,  p-value: < 0.00000000000000022
```

We see that as a respondent's education increases one unit on the education scale, perceived risk appears to increase by 0.0367752, keeping age and ideology constant. However, this result is not significant. In the final example, income is added to the model. Note that the size and significance of education actually increases once income is included, indicating that education only has bearing on the perceived risks of climate change once the independent effect of income is considered.

```
options(scipen = 999) #to turn off scientific notation
ols3 <- lm(glbcc_risk ~ age + education + income + ideol, data = ds.temp)
summary(ols3)
```

```
##
## Call:
## lm(formula = glbcc_risk ~ age + education + income + ideol, data = ds.temp)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -8.7991 -1.6654  0.2246  1.4437  6.5968
##
## Coefficients:
##                  Estimate    Std. Error t value             Pr(>|t|)
## (Intercept) 10.9232861851  0.3092149750  35.326 < 0.0000000000000002 ***
## age         -0.0044231931  0.0036688855  -1.206              0.22810
## education    0.0632823391  0.0299443094   2.113              0.03468 *
## income      -0.0000026033  0.0000009021  -2.886              0.00394 **
## ideol       -1.0366154295  0.0299166747 -34.650 < 0.0000000000000002 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.433 on 2267 degrees of freedom
## Multiple R-squared:  0.363,  Adjusted R-squared:  0.3619
## F-statistic:   323 on 4 and 2267 DF,  p-value: < 0.00000000000000022
```

12.3.1 Hypothesis Testing and t-tests

The logic of hypothesis testing with multiple regression is a straightforward extension from simple regression as described in Chapter 7. Below we will demonstrate how to use the standard error of the ideology variable to test whether ideology influences perceptions of the perceived risk of global climate change. Specifically, we posit:

> *H_1: As respondents become more conservative, they will perceive climate change to be less risky, all else equal.*

Therefore, $\beta_{ideology} < 0$. The null hypothesis is that $\beta_{ideology} = 0$.

To test H_1 we first need to find the standard error of the B for ideology, (B_j).

$$SE(B_j) = \frac{S_E}{\sqrt{RSS_j}} \tag{12.1}$$

where $RSS_j =$ the residual sum of squares from the regression of X_j (ideology) on the other Xs (age, education, income) in the model. RSS_j captures all of the **independent** variation in X_j. Note that the bigger RSS_j, the smaller $SE(B_j)$, and the smaller $SE(B_j)$, the more precise the estimate of B_j.

S_E (the standard error of the model) is:

$$S_E = \sqrt{\frac{RSS}{n-k-1}}$$

We can use `R` to find the RSS for ideology in our model. First we find the S_E of the model:

```
Se <- sqrt((sum(ols3$residuals^2))/(length(ds.temp$ideol)-5-1))
Se
```

```
## [1] 2.43312
```

Then we find the RSS, for ideology:

```
ols4 <- lm(ideol ~ age + education + income, data = ds.temp)
summary(ols4)
```

```
##
## Call:
## lm(formula = ideol ~ age + education + income, data = ds.temp)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -4.2764 -1.1441  0.2154  1.4077  3.1288
##
## Coefficients:
##                 Estimate    Std. Error t value             Pr(>|t|)
## (Intercept)  4.5945481422  0.1944108986  23.633 < 0.0000000000000002 ***
## age          0.0107541759  0.0025652107   4.192   0.0000286716948757 ***
## education   -0.1562812154  0.0207596525  -7.528   0.0000000000000738 ***
## income       0.0000028680  0.0000006303   4.550   0.0000056434561990 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.707 on 2268 degrees of freedom
## Multiple R-squared:  0.034,  Adjusted R-squared:  0.03272
## F-statistic:  26.6 on 3 and 2268 DF,  p-value: < 0.00000000000000022
```

```
RSSideol <- sum(ols4$residuals^2)
RSSideol
```

```
## [1] 6611.636
```

Finally, we calculate the SE for ideology:

```
SEideol <- Se/sqrt(RSSideol)
SEideol
```

```
## [1] 0.02992328
```

Once the $SE(B_j)$ is known, the t-test for the ideology coefficient can be calculated. The t value is the ratio of the estimated coefficient to its standard error.

$$t=\frac{B_j}{SE(B_j)} \tag{12.2}$$

This can be calculated using `R`.

```
ols3$coef[5]/SEideol
```

```
##     ideol
## -34.64245
```

As we see, the result is statistically significant, and therefore we reject the null hypothesis. Also note that the results match those from the `R` output for the full model, as was shown earlier.