16.1: Generalized Linear Models

Generalized Linear Models (GLMs) provide a modeling structure that can relate a linear model to response variables that do not have normal distributions. The distribution of YY is assumed to belong to one of an exponential family of distributions, including the Gaussian, Binomial, and Poisson distributions. GLMs are fit to the data by the method of maximum likelihood.

Like OLS, GLMs contain a stochastic component and a systematic component. The systematic component is expressed as:

$$\eta = \alpha + \beta 1Xi1 + \beta 2Xi2 + \ldots + \beta kXik(16.1)(16.1)\eta = \alpha + \beta 1Xi1 + \beta 2Xi2 + \ldots + \beta kXik \tag{16.1.1}$$

However, GLMs also contain a link function" that relates the response variable, YiYi, to the systematic linear component, ηη. Table 16.1 shows the major exponential “families”" of GLM models, and indicates the kinds of link functions involved in each. Note that OLS models would fall within the Gaussian family. In the next section we focus on the binomial family, and on logit estimation in particular.

Family	***Default “Link”***	***Range of y_i***
gaussian	identity	$(-\infty, +\infty)$
binomial	logit	$\frac{0,1,\ldots,n_i}{n_i}$
poisson	log	$0, 1, 2, \ldots$
Gamma	inverse	$(0, \infty)$
inverse gaussian	$1/\mu^2$	$(0, \infty)$

Figure 16.1.1: Exponential ‘Families’ of GLM Models