10.7: Conditional Probability

So far we have limited ourselves to simple probabilities - that is, the probability of a single event or combination of events. However, we often wish to determine the probability of some event given that some other event has occurred, which are known as *conditional probabilities*.

Let's take the 2016 US Presidential election as an example. There are two simple probabilities that we could use to describe the electorate. First, we know the probability that a voter in the US affiliated with the Republican party: $p(Republican) = 0.44$. We also know the probability that a voter cast their vote in favor of Donald Trump: $p(\text{Trump voter})=0.46$. However, let's say that we want to know the following: What is the probability that a person cast their vote for Donald Trump, *given that they are a Republican*?

To compute the conditional probability of A given B (which we write as $P(A|B)$, "probability of A, given B"), we need to know the *joint probability* (that is, the probability of both A and B occurring) as well as the overall probability of B:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

That is, we want to know the probability that both things are true, given that the one being conditioned upon is true.

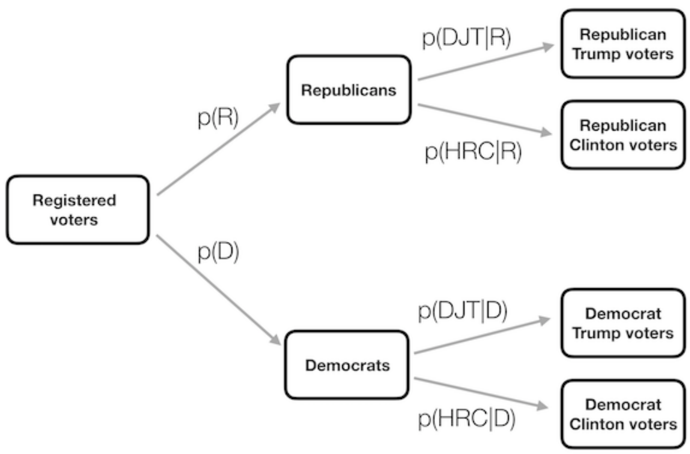

Figure 10.3: A graphical depiction of conditional probability, showing how the conditional probability limits our analysis to a subset of the data.

It can be useful to think of this is graphically. Figure 10.3 shows a flow chart depicting how the full population of voters breaks down into Republicans and Democrats, and how the conditional probability (conditioning on party) further breaks down the members of each party according to their vote.