

## 20.3: Doing Bayesian Estimation

We ultimately want to use Bayesian statistics to make decisions about hypotheses, but before we do that we need to estimate the parameters that are necessary to make the decision. Here we will walk through the process of Bayesian estimation. Let's use another screening example: Airport security screening. If you fly a lot, it's just a matter of time until one of the random explosive screenings comes back positive; I had the particularly unfortunate experience of this happening soon after September 11, 2001, when airport security staff were especially on edge.

What the security staff want to know is what is the likelihood that a person is carrying an explosive, given that the machine has given a positive test. Let's walk through how to calculate this value using Bayesian analysis.

### 20.3.1 Specifying the prior

To use Bayes' theorem, we first need to specify the prior probability for the hypothesis. In this case, we don't know the real number but we can assume that it's quite small. According to the FAA, there were 971,595,898 air passengers in the U.S. in 2017. Let's say that one out of those travelers was carrying an explosive in their bag — that would give a prior probability of 1 out of 971 million, which is very small! The security personnel may have reasonably held a stronger prior in the months after the 9/11 attack, so let's say that their subjective belief was that one out of every million flyers was carrying an explosive.

### 20.3.2 Collect some data

The data are composed of the results of the explosive screening test. Let's say that the security staff runs the bag through their testing apparatus 3 times, and it gives a positive reading on 3 of the 3 tests.

### 20.3.3 Computing the likelihood

We want to compute the likelihood of the data under the hypothesis that there is an explosive in the bag. Let's say that we know (from the machine's manufacturer) that the sensitivity of the test is 0.99 — that is, when a device is present, it will detect it 99% of the time. To determine the likelihood of our data under the hypothesis that a device is present, we can treat each test as a Bernoulli trial (that is, a trial with an outcome of true or false) with a probability of success of 0.99, which we can model using a binomial distribution.

### 20.3.4 Computing the marginal likelihood

We also need to know the overall likelihood of the data — that is, finding 3 positives out of 3 tests. Computing the marginal likelihood is often one of the most difficult aspects of Bayesian analysis, but for our example it's simple because we can take advantage of the specific form of Bayes' theorem for a binary outcome that we introduced in Section 10.7:

$$P(E|T) = \frac{P(T|E) * P(E)}{P(T|E) * P(E) + P(T|\neg E) * P(\neg E)}$$

where  $E$  refers to the presence of explosives, and  $T$  refers to a positive test result.

The marginal likelihood in this case is a weighted average of the likelihood of the data under either presence or absence of the explosive, multiplied by the probability of the explosive being present (i.e. the prior). In this case, let's say that we know (from the manufacturer) that the specificity of the test is 0.99, such that the likelihood of a positive result when there is no explosive ( $P(T|\neg E)$ ) is 0.01.

### 20.3.5 Computing the posterior

We now have all of the parts that we need to compute the posterior probability of an explosive being present, given the observed 3 positive outcomes out of 3 tests.

This result shows us that the posterior probability of an explosive in the bag given these positive tests (0.492) is just under 50%, again highlighting the fact that testing for rare events is almost always liable to produce high numbers of false positives, even when the specificity and sensitivity are very high.

An important aspect of Bayesian analysis is that it can be sequential. Once we have the posterior from one analysis, it can become the prior for the next analysis!

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