2.7: Rolling your own descriptive statistics

We spent many paragraphs talking about variation in numbers, and how to use calculate the **variance** and **standard deviation** to summarize the average differences between numbers in a data set. The basic process was to 1) calculate some measure of the differences, then 2) average the differences to create a summary. We found that we couldn't average the raw difference scores, because we would always get a zero. So, we squared the differences from the mean, then averaged the squared differences differences. Finally, we square rooted our measure to bring the summary back down to the scale of the original numbers.

Perhaps you haven't heard, but there is more than one way to skin a cat, but we prefer to think of this in terms of petting cats, because some of us love cats. Jokes aside, perhaps you were also thinking that the problem of summing differences scores (so that they don't equal zero), can be solved in more than one way. Can you think of a different way, besides squaring?

Absolute deviations

How about just taking the absolute value of the difference scores. Remember, the absolute value converts any number to a positive value. Check out the following table:

scores	values	mean	Difference_from_Mean	Absolute_Deviations
1	1	4.5	-3.5	3.5
2	6	4.5	1.5	1.5
3	4	4.5	-0.5	0.5
4	2	4.5	-2.5	2.5
5	6	4.5	1.5	1.5
6	8	4.5	3.5	3.5
Sums	27	27	0	13
Means	4.5	4.5	0	2.16666666666667

This works pretty well too. By converting the difference scores from the mean to positive values, we can now add them up and get a non-zero value (if there are differences). Then, we can find the mean of the sum of the absolute deviations. If we were to map the terms sum of squares (SS), variance and standard deviation onto these new measures based off of the absolute deviation, how would the mapping go? For example, what value in the table corresponds to the SS? That would be the sum of absolute deviations in the last column. How about the variance and standard deviation, what do those correspond to? Remember that the variance is mean (SS/N), and the standard deviation is a square-rooted mean ($\sqrt{SS/N}$). In the table above we only have one corresponding mean, the mean of the sum of the absolute deviations. So, we have a **variance** measure that does not need to be square rooted. We might say the mean absolute deviation, is doing double-duty as a variance and a standard-deviation. Neat.

Other sign-inverting operations

In principle, we could create lots of different summary statistics for variance that solve the summing to zero problem. For example, we could raise every difference score to any even numbered power beyond 2 (which is the square). We could use, 4, 6, 8, 10, etc. There is an infinity of even numbers, so there is an infinity of possible variance statistics. We could also use odd numbers as powers, and then take their absolute value. Many things are possible. The important aspect to any of this is to have a reason for what you are doing, and to choose a method that works for the data-analysis problem you are trying to solve. Note also, we bring up this general issue because we want you to understand that statistics is a creative exercise. We invent things when we need them, and we use things that have already been invented when they work for the problem at hand.