3.7: Factorials and Combination Notation

Learning Outcomes

1. Evaluate a factorial.
2. Use combination notation for statistics applications.

When we need to compute probabilities, we often need to multiple descending numbers. For example, if there is a deck of 52 cards and we want to pick five of them without replacement, then there are 52 choices for the first pick, 51 choices for the second pick since one card has already been picked, 50 choices for the third, 49 choices for the fourth, and 48 for the fifth. If we want to find out how many different outcomes there are, we can use what we call the multiplication principle and multiple them: $52 \times 51 \times 50 \times 49 \times 48$. If we wanted to pick all 52 of the cards one at a time, then this list would be excessively long. Instead there is a notation that describes multiplying all the way down to 1, called the factorial. It must be exciting, since we use the symbol "!" for the factorial.

Example 3.7.1

Calculate $4!$

Solution

We use the definition which says start at 4 and multiply until we get to 1:

$$4! \ = \ 4 \times 3 \times 2 \times 1 \ = \ 24$$

Example 3.7.2

If we pick 5 cards from a 52 card deck without replacement and the same two sets of 5 cards, but in different orders, are considered different, how many sets of 5 cards are there?

Solution

From the introduction, the number of sets is just:

$$52 \times 51 \times 50 \times 49 \times 48$$

This is not quite a factorial since it stops at 48; however, we can think of this as $52!$ with $47!$ removed from it. In other words we need to find

$$\frac{52!}{47!}$$

We could just multiply the numbers from the original list, but it is a good idea to practice with your calculator or computer to find this using the ! symbol. When you do use technology, you should get:

$$\frac{52!}{47!} = 311,875,200$$

Combinations

One of the most important applications of factorials is combinations which count the number of ways of selecting a smaller collection from a larger collection when order is not important. For example if there are 12 people in a room and you want to select a team of 4 of them, then the number of possibilities uses combinations. Here is the definition:

Definition: Combinations

The number of ways of selecting k items without replacement from a collection of n items when order does not matter is:

$$\binom{n}{r} \ = {}_nC_r \ = \ \frac{n!}{r!\,(n-r)!} \tag{3.7.1}$$

Notice that there are a few notations. The first is more of a mathematical notation while the second is the notation that a calculator uses. For example, in the TI 84+ calculator, the notation for the number of combinations when selecting 4 from a collection of 12 is:

$$12\ {}_nC_r\ 4$$

There are many internet sites that will perform combinations. For example the math is fun site asks you to put in n and r and also state whether order is important and repetition is allowed. If you click to make both "no" then you will get the combinations.

Example 3.7.3

Calculate

$$\binom{15}{11} = {}_{15}C_{11}$$

Solution

Whether you use a hand calculator or a computer you should get the number: 1365

Example 3.7.4

The probability of winning the Powerball lottery if you buy one ticket is:

$$P(win) = \frac{1}{{}_{69}C_5 \times 26}$$

Calculate this probability.

Solution

First, let's calculate ${}_{69}C_5$. Using a calculator or computer, you should get 11,238,513. Next, multiply by 26 to get

$$11,238,513 \times 26 = 292,201,338$$

Thus, there is a one in 292,201,338 chance of winning the Powerball lottery if you buy a ticket. We can also write this as a decimal by dividing:

$$P\left(win\right) = \frac{1}{292,201,338} = 0.000000003422$$

As you can see, your chances of winning the Powerball are very small.

Exercise

A classroom is full of 28 students and there will be one president of the class and a "Congress" of 4 others selected. The number of different leadership group possibilities is:

$$28 \times {}_{27}C_4$$

Calculate this number to find out how many different leadership group possibilities there are.

Ex 1: Simplify Expressions with Factorials

Combinations

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