1.1.3: Order of Operations and Introduction to Expressions

Learning Objectives

By the end of this section, you will be able to:

- Decide which operations to do first
- Simplify numerical expressions with multiple operations

Be Prepared

Before we get started, take this readiness quiz.

1. Evaluate $-5 \cdot 2$.

2. Evaluate $-5+3$.

3. Evaluate $-6 \div (-2)$.

What is the meaning of the expression '3 times 4 plus 5'. Some will answer 17, while others may answer 27. Why? To take the ambiguity out, we can write

$$(3 \cdot 4)+5 = 17$$

and

$$3 \cdot (4+5) = 27,$$

where we must first evaluate the quantity in parentheses. Since it can be somewhat cumbersome to write a lot of parentheses, there is an important convention or agreement that if we just write $3 \cdot 4+5$ we mean $(3 \cdot 4)+5$. That is, in the absence of parentheses, we should multiply before we add. This is part of what is called **the order of operations**. This must be remembered.

The Order of Operations

When evaluating an expression, which only involves addition, subtraction, multiplication and division (no parentheses), we first perform, from left to right, all of the multiplications and divisions. Then, from left to right, the additions and subtractions. If there are parts of the expression enclosed by parentheses, what is within the parentheses must be evaluated first.

Remark 1.1.3.1

Subtraction can be turned into addition and then addition can be done in any order, not necessarily from left to right. This explains why addition and subtraction come together in the order of operations. For example, $5-2$ is also $5+(-2)$.

There is a similar statement for multiplication and division. For example, $8 \div 2$ is also $8 \cdot \frac{1}{2}$.

PE(MD)(AS) is a mnemonic device to remember the order of operations. This means that the order is: Parentheses, Exponents (this will be incorporated later), Multiplication and Division (taken together from left to right), and finally, Addition and Subtraction (taken together from left to right).

Let us try a few problems.

Example 1.1.3.2

Simplify:

a. $3+2(3+5)$

b. $3-2(-4+7)$

Solution

a. $3+2(3+5) = 3+2(8) = 3+16 = 19$

b. $3-2(-4+7)=3-2(3)=3-6=-3$

Try It 1.1.3.3

Simplify:

a. $-3-4-2(-2\cdot 6-5)$

b. $-(3-(-6))-(1-4\cdot(-5)+4)$

c. $-2(-14\div 7+7)$

Answer

a. $-3-4-2(-2\cdot 6-5)=-3-4-2(-12-5)=-3-4-2(-17)=-3-4-(-34)=-3-4+34=27$

b.
$-(3-(-6))-(1-4\cdot(-5)+4)=-(3+6)-(1-(-20)+4)=-9-(1+20+4)=-9-25=-9+(-25)=-34$

c. $-2(-14\div 7+7)=-2(-2+7)=-2(5)=-10$

Try It 1.1.3.4

Simplify:

a. $-3(-2\cdot 7-(-5)(4)\div 2)$

b. $6\div 2\times 3$

c. $-2(3-1)2-(8-24)\div 4$

Answer

a. $-3(-2\cdot 7-(-5)(4)\div 2)=-3(-14-(-20)\div 2)=-3(-14-(-10))=-3(-4)=12$

b. $6\div 2\times 3=3\times 3=9$

c. $-2(3-1)2-(8-24)\div 4=-2(2)2-(-16)\div 4=-4\cdot 2-(-4)=-8+4=-4$

An expression is a combination of numbers, variables (letters that represent numbers), operations and parentheses that can be evaluated when 'appropriate' numbers are substituted in place of the variables following the order of operations which will be discussed. Which numbers are appropriate will depend on the particular example at hand. For example, $3\cdot(x+2)$ is an expression where 3 and 2 are numbers, x is a variable, $+$ and $\cdot$ are operations and the parentheses enclose a sub-expression, that is, an expression within an expression.

We concern ourselves with expressions in order to express a quantity that depends on another quantity which can vary. Keep the general idea in mind as we work through different ways of combining letters and numbers to form expressions in this unit. We will start with the simplest expressions that just involve multiplication of a number by a variable and addition of the same with numbers.

Expression An expression is a combination of numbers, variables (letters that represent numbers), operations and parentheses that can be evaluated when 'appropriate' numbers are substituted in place of the variables following the order of operations which will be discussed.

Variable letters that represent numbers

Given an expression, we can **evaluate** it by replacing every instance of a variable with a single number. Which numbers are being substituted for which variables should be made clear in the wording. The following examples involve only one variable so there is little chance of ambiguity.

Example 1.1.3.5

Evaluate the expression $-3x+2-2(x-2)$ at $x=-3$.

Solution

We replace every instance of x by -3 and evaluate the result using the order of operations.

We see that

$$-3(-3)+2-2((-3)-2)=9+2-2(-5)=9+2-(-10)=9+2+10=21.$$

The expression evaluated at -3 is 21.

? Try It 1.1.3.6

Evaluate the expression $5x+1-3(-x+2)$ at $x=2$.

Answer

The expression evaluated at 2 is 11

? Try It 1.1.3.7

Evaluate the expression $-3(2t-2)+5+2(7-t)$ at $t=-2$.

Answer

The expression evaluated at -2 is 41.

? Writing Exercises 1.1.3.8

1. Explain the purpose of parentheses and give an example.
2. Explain why multiplication and division should be treated on the same level in the order of operations.
3. Give an example of an expression with one variable.
4. What reasons my we be interested in using variables?

Exit Problem

Evaluate $(2\cdot 3+5)\div 4-4(7-2)$.

Key Concepts

- The order of operations
- Expression
- Variable